Sound and Structural Vibration

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Module No # 02
Lecture No # 10
Physical meaning of terms

Good morning to this next lecture on sound and structural vibration so if you recall we had fully formulated the velocity potential and velocity the inverse.

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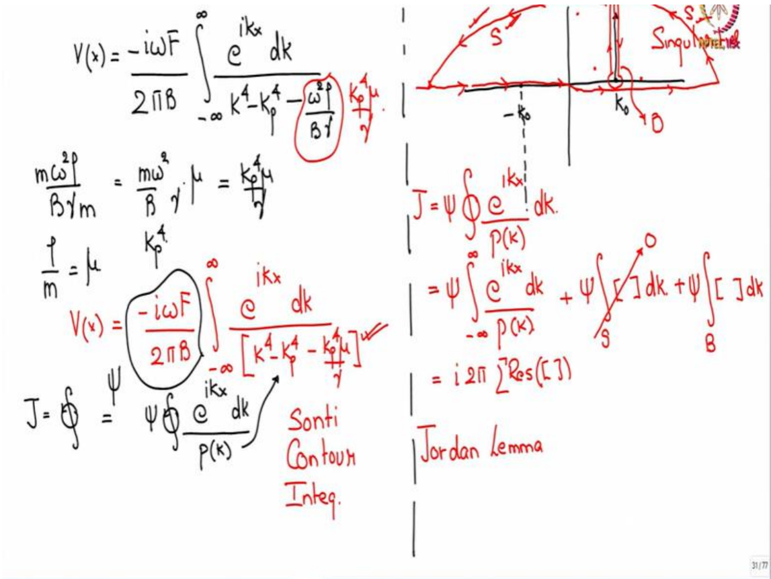

So, velocity of the plate was given

$$v(x) = \frac{-i\omega F}{2\pi B} \int_{-\infty}^{\infty} \frac{e^{ikx}}{\left[k^4 - {k_p}^4 - \frac{\omega^2 \rho}{B\gamma}\right]} dk.$$

So just a word, about $\frac{\omega^2 \rho}{B\gamma}$ if I multiply by m and divide by m I have $\frac{m\omega^2}{B}$ which is ${k_p}^4$. Then I have γ then I have ρ by m which I will call a μ so this becomes a μ.

So, I get $\frac{{k_p}^4 \mu}{\gamma}$ so $\frac{\omega^2 \rho}{B\gamma}$ I will write as $\frac{{k_p}^4 \mu}{\gamma}$. So, if I write it just once

$$v(x) = \frac{-i\omega F}{2\pi B} \int_{-\infty}^{\infty} \frac{e^{ikx}}{\left[k^4 - {k_p}^4 - \frac{{k_p}^4 \mu}{\gamma}\right]} dk.$$

So, it is worth remembering this form. Now this is going to be computed using a contour integral as we just did last class it is going to be computed using a contour integral.

So, what happens here? We will have this whole thing represented I will call $\frac{-i\omega F}{2\pi B}$ part ψ. So, the contour integral is

$$J = \oint = \psi \oint \frac{e^{ikx}}{p(k)} dk,$$

$p(k) = k^4 - {k_p}^4 - \frac{{k_p}^4\mu}{\gamma}$. Now I said last time that is a particular cut we are going to follow which is what? At k_0 I go vertical to $k_0 + i\infty$ at $-k_0$ I have another downward cut.

So how does this integral look like? I come from minus infinity I go down here cross I am below the cut over here I go off to plus infinity and then I come at an arc with infinite radius. And this line is a cut so I cannot cross so I come down here go around take a vertical line back up. And again, with an arc of infinite radius I join up at minus infinity. So that is my closed contour.

And there will be some singularities that are another big topic so we will look at it slowly whatever singularities are there. There will be some singularities so we will look at them separately but that is the contour first. So, if I call these infinite parts arcs as S I called that as S then this part going down and going up. So, this is the direction of the integral I mean along the contour the line integral.

So, this part is called the B portion the branch portion this part is S so now what do I have? I have the J the contour integral given by several branches. So, I have the portion I want which I do not touch again this is the portion I want along the real line. Then I have the arcs at infinity plus I have ψ and the integral over the arcs at infinity then I have the integral over B. And that is equal to 2πi the sum of residues of this integral.

Now by Jordan Lemma these are all theorems in complex variables, and I said that I have a series of 24 lectures on contour integration in the complex variable. So, if you type my name contour integration in you tube you will get this set of lectures so I am not repeating, or you would have seen this in some complex variable class there is a Jordan Lemma. Based on the Jordan Lemma the S branch goes to 0. These integrals on infinite radius arc goes to 0 so what do I have now?

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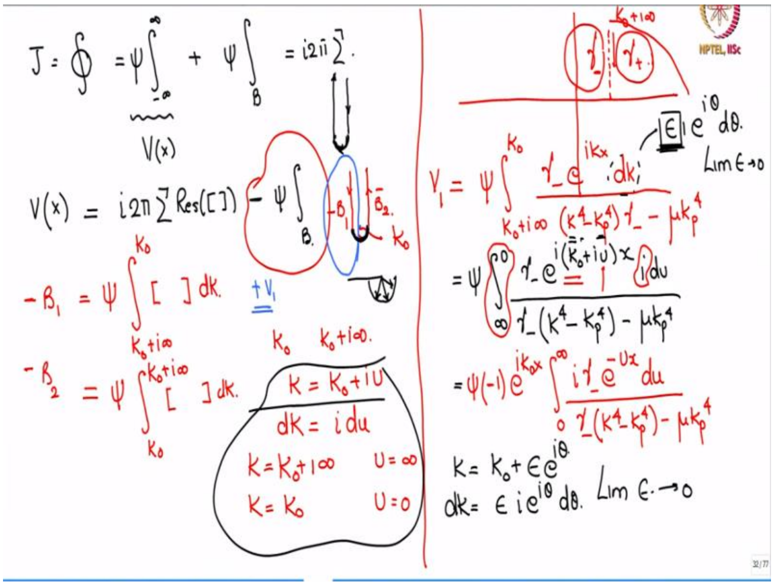

What I have is

$$J = \oint = \psi \int_{-\infty}^{\infty} + \psi \int_B = i2\pi \sum Res.$$

So, my velocity in the space is given by

$$v(x) = i2\pi \sum Res - \psi \int_B [\ ].$$

So, what was B? B was B comes down this way go around and go up that is what B is about.

Now what is –B? Along with the –B is you come down go around and go up this is called B_1 we will call this B_2. So, what is $-B_1$ so this is $-B_1$ and $-B_2$ what is $-B_1$?

$$-B_1 = \psi \int_{k_0+i\infty}^{k_0} [\ ]\, dk.$$

$$-B_2 = \psi \oint_{k_0}^{k_0+i\infty} [\ ]\, dk.$$

So, one end is k_0 the other end is $k_0 + i\infty$ so what I do is I substitute now k is as $k_0 + iU$ so that when what does that mean. So now dk first of all is equal to idU when $k = k_0 + i\infty$ then U is infinity and then $k = k_0$ then $U = 0$. And one more thing so we have k_0 and we are going up, so we come to $k_0 + i\infty$ and we come down. So, this is the cut, so this is called the γ_- portion this called the γ_+ portion.

The cut is because of γ so we will use γ_- definition here and we will use the γ_+ definition here that is the idea. So now related to B_1 I have

$$v_1 = \psi \int_{k_0+i\infty}^{k_0} \frac{\gamma_- e^{ikx}\, dk}{(k^4 - {k_p}^4)\gamma_- - \mu {k_p}^4}.$$

So, if now I implement this transform here if I implement this transform in there what do I get?

I get

$$v_1 = \psi \int_{\infty}^{0} \frac{\gamma_- e^{i(k_0+iU)x}\, idU}{\gamma_-(k^4 - {k_p}^4) - \mu {k_p}^4}.$$

$$= \psi(-1)e^{ik_0x} \int_{0}^{\infty} \frac{i\gamma_- e^{-Ux} dU}{\gamma_-(k^4 - {k_p}^4) - \mu {k_p}^4}.$$

So, that is this part here along with the minus so that means this will get added as $+v_1$ straight away because it carries the minus in it.

So now what about this small lower portions here? The small lower portion around k_0 here let me just briefly show it without elaboration here. My $k = k_0 + \epsilon e^{i\theta}$ because that k if I exaggerate it at k_0 I move on a semicircle of infinitesimally small radius. So, what is d times k? $dk = \epsilon i e^{i\theta}$, ϵ is constant radius is constant.

So, if I implement dk then in the integral the dk will turn out to be $\epsilon i e^{i\theta} d\theta$ and θ value will have its limit. Let us say π to 0 or something like this $d\theta$ will have its limits. But we are going to take limits of ϵ tending to 0 and there will be ϵ always sitting on top. Here also there will be ϵ because k is $k_0 + \epsilon$ but they will be overwhelmed by other terms ϵ is small quantity.

But in the numerator, all by itself there will be one ϵ sitting so as limit ϵ tends to 0 this small half circle integral goes to 0. So, I am not elaborating that it can be rigorously but not elaborating on that.

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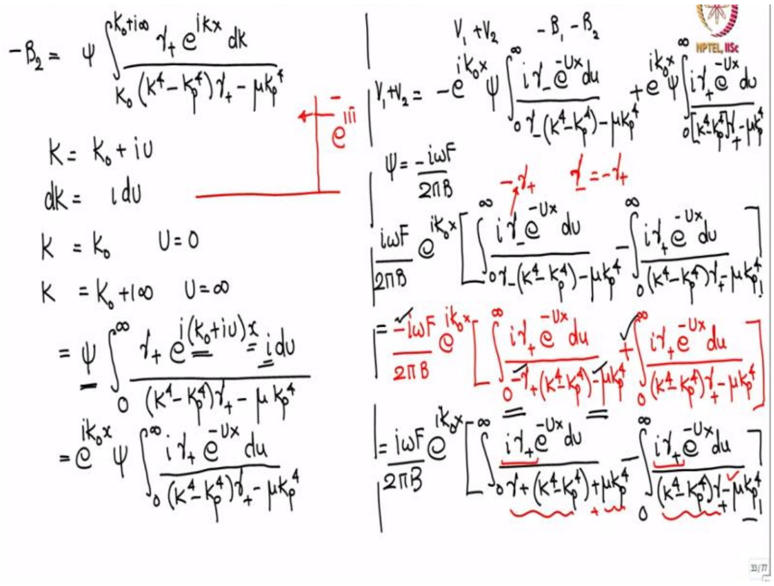

So next my $-B_2$ integral what is that?

$$-B_2 = \psi \int_{k_0}^{k_0+i\infty} \frac{\gamma_+ e^{ikx}\, dk}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4}.$$

So here also we will write

$$k = k_0 + iU,$$

$$dk = idU,$$

when $k = k_0$, $U = 0$ when k = $k_0 + i\infty$, $U = \infty$. So now my

$$-B_2 = \psi \int_0^\infty \frac{\gamma_+ e^{i(k_0+iU)x}\, idU}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4},$$

$$= e^{ik_0x}\, \psi \int_0^\infty \frac{i\,\gamma_+ e^{-Ux}\, dU}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4}.$$

So now we will combine both of them we will combine $v_1 + v_2$ or $-B_1$ $-B_2$ will combine. If we do that what happens I get

$$v_1 + v_2 = -e^{ik_0x}\psi \int_0^\infty \frac{i\gamma_- e^{-Ux} dU}{\gamma_-(k^4 - {k_p}^4) - \mu {k_p}^4} + e^{ik_0x}\, \psi \int_0^\infty \frac{i\,\gamma_+ e^{-Ux}\, dU}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4}.$$

Now ψ itself is $\frac{-i\omega F}{2\pi B}$. So, what do we get?

$$= \frac{-i\omega F}{2\pi B} e^{ik_0x} \left[\int_0^\infty \frac{i\gamma_- e^{-Ux} dU}{\gamma_-(k^4 - {k_p}^4) - \mu {k_p}^4} - \int_0^\infty \frac{i\,\gamma_+ e^{-Ux}\, dU}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4} \right].$$

I will write γ_- is equal to $-\gamma_+$why because at the cut. If you go, there is a minus one jump you will get a jump of $e^{i\pi}$.

$$= \frac{-i\omega F}{2\pi B} e^{ik_0x} \left[\int_0^\infty \frac{i\gamma_+ e^{-Ux} dU}{-\gamma_+ (k^4 - {k_p}^4) - \mu {k_p}^4} + \int_0^\infty \frac{i\,\gamma_+ e^{-Ux}\,dU}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4} \right],$$

$$= \frac{i\omega F}{2\pi B} e^{ik_0x} \left[\int_0^\infty \frac{i\gamma_+ e^{-Ux} dU}{\gamma_+ (k^4 - {k_p}^4) + \mu {k_p}^4} - \int_0^\infty \frac{i\,\gamma_+ e^{-Ux}\,dU}{(k^4 - {k_p}^4)\gamma_+ - \mu {k_p}^4} \right].$$

So, we have to combine this now so how do that, so I have something like $a + b$ I have this, $a + b$ in the same $a - b$. So, $a^2 - b^2$ and then I will take an LCM so that this $i\gamma_+$, $i\gamma_+$ will add up and the other parts will cancel. So, I am going to change the page so if you take the LCM and add it up.

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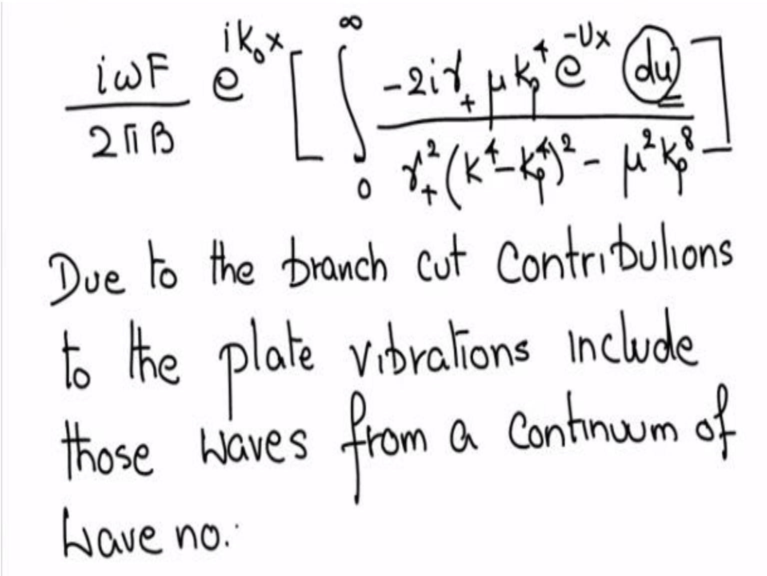

So, I get

$$\frac{i\omega F}{2\pi B} e^{ik_0x} \left[\int_0^\infty \frac{-2i\gamma_+ \mu {k_p}^4 e^{-Ux} dU}{{\gamma_+}^2 (k^4 - {k_p}^4)^2 - \mu^2 {k_p}^8} \right].$$

So now you can see that U is the wave number variable and we have a continuous range of wave number variables contributing to the velocity.

So let us write it due to the branch cut contributions to the plate vibrations include those waves from a continuum of wave numbers. So, this is interesting we are running out of time I will close the lecture here and we will continue from this portion next lecture.