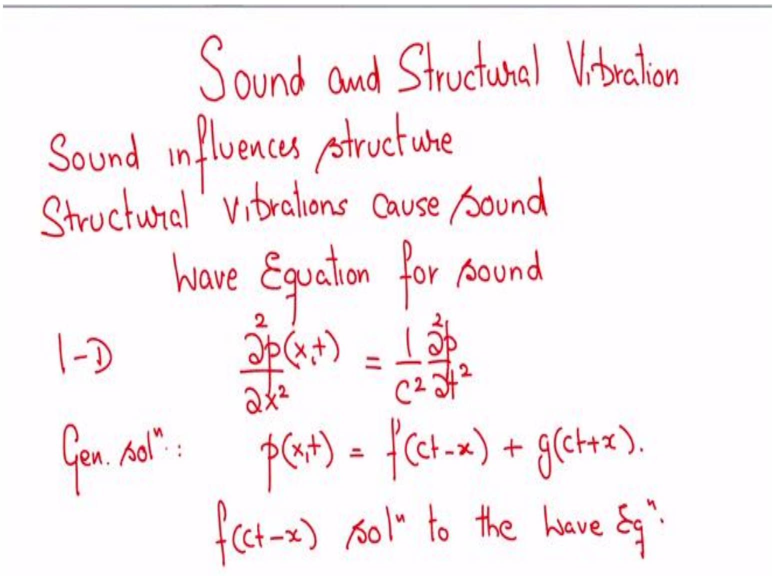
Sound and Structural Vibration

Prof. Dr. Venkata Sonti
Department of Mechanical Engineering
Indian Institute of Science - Bengaluru

Lecture – 1
The Longitudinal Wave in Vibrating Spring

(Refer Slide Time: 00:30)



Welcome to you all to this first lecture on sound and structural vibration. So, the name of the course is "Sound and Structural Vibration". As was said in the introduction we will be interested in how sound influences structure and how structural vibrations cause sound. So, to begin with, we need to know the wave equation that governs the behaviour of sound and it is given by (for a one dimensional case)

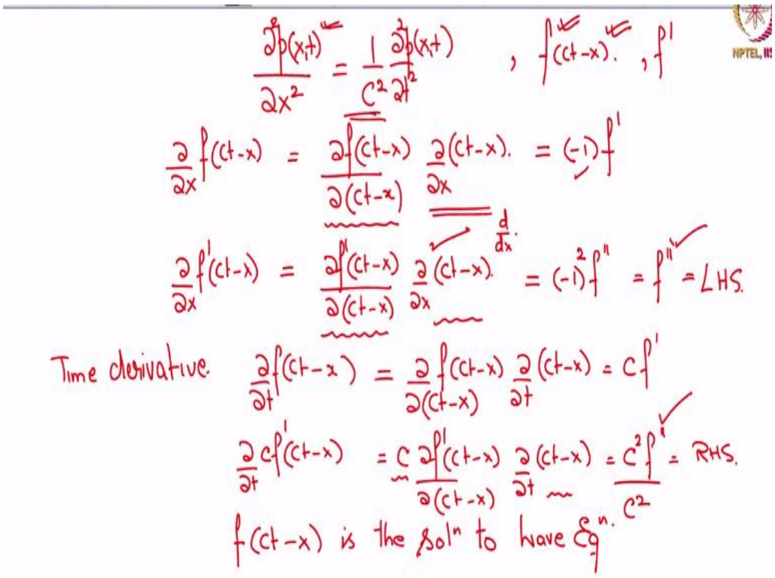
$$\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2}\ .$$

You are expected to know this equation. So, the general solution to this equation is

$$p(x,t) = f(ct - x) + g(ct + x).$$

These are two functions, one has the argument $\mathrm{ct} - \mathrm{x}$, the other has the argument $\mathrm{ct} + \mathrm{x}$. So, we will just see how $f(ct - x)$ happens to be the solution to the wave equation. So, let me use the next page.

(Refer Slide Time: 03:59)



So, let me rewrite the equation we have $\frac{\partial^2 p(x,t)}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 p(x,t)}{\partial t^2}$ and we are looking at this function $f(ct - x)$. So, I am going to substitute $f(ct - x)$ in place of p. So, we will need $\frac{\partial f(ct-x)}{\partial x}$ which happens to be

$$\frac{\partial f(ct-x)}{\partial x} = \frac{\partial f(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial x},$$

we have used the chain rule. So, this entity $\frac{\partial f(ct-x)}{\partial(ct-x)}$ the derivative of f with respect to its own argument I will write it as f' and here the derivative of $(ct - x)$ with respect to x gives me -1. So, I get here

$$\frac{\partial f(ct-x)}{\partial x} = \frac{\partial f(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial x} = (-1)f'.$$

Then I need to take the $\frac{\partial f'(ct-x)}{\partial x}$. So, that gives me again

$$\frac{\partial f'(ct-x)}{\partial x} = \frac{\partial f'(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial x},$$

So, now, what we get here again is -1 from the first effort, then the derivative of f' with respect to its argument which is f'' and then one more -1 from $\frac{\partial(ct-x)}{\partial x}$, so it is $(-1)^2$. So, this gives me

$$\frac{\partial f'(ct-x)}{\partial x} = \frac{\partial f'(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial x} = (-1)^2 f'' = f''.$$

So, the left side gave me f'' which is the left hand side spatial derivative, now I have to do the temporal derivative (time derivatives). The time derivative gives me $\frac{\partial f(ct-x)}{\partial t} = \frac{\partial f(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial t}$. So, this gives me

$$\frac{\partial f(ct-x)}{\partial \mathrm{t}} = \frac{\partial f(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial \mathrm{t}} = \mathrm{c}f',$$

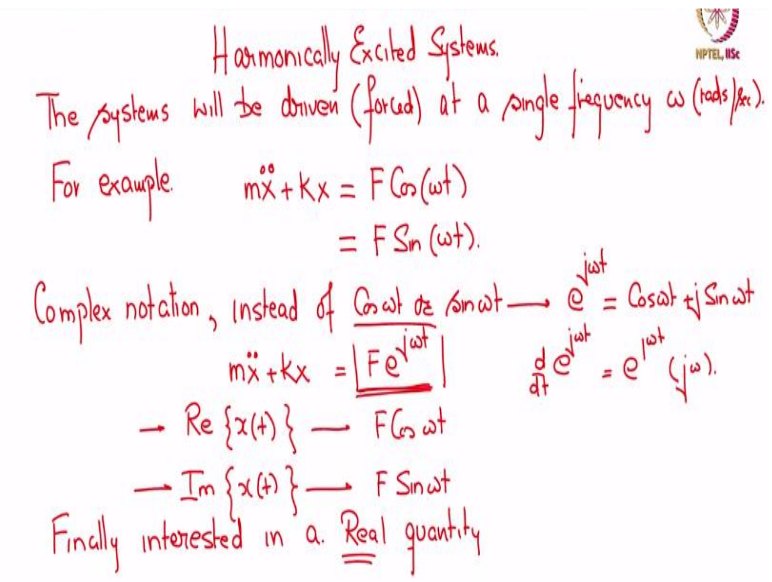
and I have to take the second derivative. So, $\frac{\partial\, cf'(ct-x)}{\partial \mathrm{t}} = \frac{c\ \partial f'(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial \mathrm{t}}$.

So, this gives me

$$\frac{\partial\, cf'(ct-x)}{\partial \mathrm{t}} = \frac{c\ \partial f'(ct-x)}{\partial(ct-x)}\frac{\partial(ct-x)}{\partial \mathrm{t}} = c^2 f'',$$

one more c comes from $\frac{\partial(ct-x)}{\partial t}$, there is already a c in $\frac{c\,\partial f'(ct-x)}{\partial(ct-x)}$, so, I get $c^2 f''$. So, this is right hand side. Now, the right hand side also has $\frac{1}{c^2}$. So, $c^2 f''$ has to be divided by c^2. So, I have if I divide this by c^2 , I get f'' which is the same as left hand side. So, what does that prove? It proves that indeed $f(ct-x)$ is the solution to the wave equation.

(Refer Slide Time: 10:03)



The next idea is this, “Harmonically Excited Systems”. So, in this course we are going to deal with harmonically excited systems and what do we mean? What we mean is, the systems will be driven, what we mean is forced, driven or forced at a single frequency ω. ω is in the units of $rads/sec$ that is what we mean by harmonically excited. So for example, you must have seen the spring mass system.

The equation of motion is

$$m\ddot{x} + kx = F\cos(\omega t),$$

hence driven harmonically. The harmonic force is $F\cos(\omega t)$. So, it is driven at ω or the forcing could be $F\sin(\omega t)$, here also it is driven at the same ω, ω is in $rads/sec$. However, I would also like to introduce the idea of a complex notation which means that instead of $\cos(\omega t)$ or $\sin(\omega t)$, I will use $e^{j\omega t}$.

There is a lot of convenience in here, $e^{j\omega t}$ as you know is $\cos(\omega t) + j\sin(\omega t)$. So, which means suppose I force it with the complex notation $m\ddot{x} + kx = Fe^{j\omega t}$ and I solve it, so I find my solution $x(t)$, then if I take the real part of this then I get the solution to the forcing $F\cos(\omega t)$. And if I take the imaginary part of this, then I get the solution to $F\sin(\omega t)$ force.

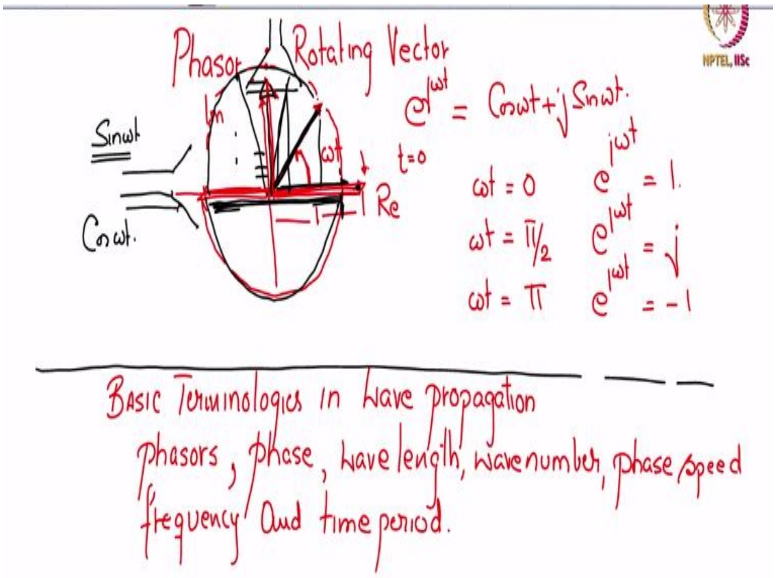
So, with one stone I hit two birds that is one advantage. The other advantage of course is that derivatives are very easy to take. If I take a derivative of this, I get

$$\frac{de^{j\omega t}}{dt} = e^{j\omega t}(j\omega).$$

So, the derivative becomes very simple, so that is why we use the complex notation. It should be remembered always that we are finally interested in a real quantity, not a complex quantity, in a real quantity.

Displacement or velocity or acceleration is a real quantity. So, $e^{j\omega t}$ notation is a convenience. So, finally the answer is either the real part of this obtained solution or the imaginary part of the obtained solution. However, we will use this notation which is very convenient. So, now let us see.

(Refer Slide Time: 15:23)



The next idea, I am just building ideas that I need. So, the next idea is one of a phasor. Phasor what is that? It is a rotating vector. Now, let us take an axis to explain this and I have a phasor. It is given by $e^{j\omega t}$ with this as the real axis and this as the imaginary axis. So, now this angle is ω times t, ωt, that angle is ωt provided my $t = 0$ begins here, $t = 0$ begins over here so this angle is ωt and this phasor rotates. This vector rotates.

So, at any instant of time it subtends $\cos(\omega t)$ the projection on real axis and it has $\sin(\omega t)$ the projection on the imaginary axis. So if ωt happens to be 0 where the $t = 0$ and let us say $t = 0$ on the real axis, let us say, then my $e^{j\omega t} = 1$. So, this value is 1 and if ωt happens to be $\frac{\pi}{2}$ that means the phasor is here at the vertical position.

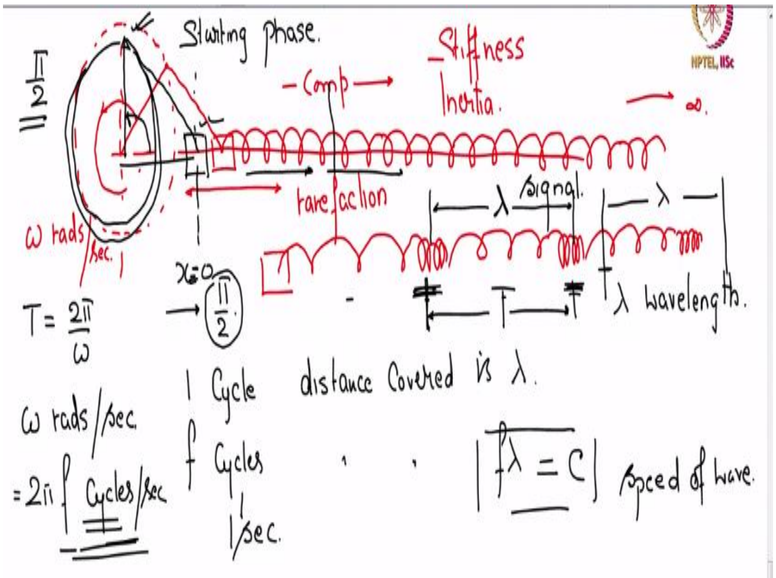
Then my $e^{j\omega t} = j$ is equal to j. Along the same lines when ωt is equal to π my $e^{j\omega t} = -1$, so I am here -1. So as this phasor rotates you can see that the projection on the real axis, if I start here moves as $\cos(\omega t)$. So, suppose this was actually made of material, a solid rod and I show a light over here, I put a torch light over here so there is a beam of light coming out and then you track the shadow.

If you track the shadow, let us say that you put a paper here and you track the shadow then it begins with the value 1 when this phasor is dead on the real axis and as it starts to move, it drops its value from 1 and I track the projection. As I track it, this oscillation on the real axis is that of $\cos(\omega t)$, the shadow or the tip. Similarly, if I show a light in the horizontal direction and this arrow is made of solid material, then the shadow starts at 0 and starts to grow.

Starts to grow and reaches its peak when this arrow is aligned with imaginary axis. So that moves as $\sin(\omega t)$. So, the phasor gives me the advantage of thinking about either $\sin(\omega t)$ or $\cos(\omega t)$ simultaneously, at the same time and I can choose towards the end what I want, what sort of forcing I want I can choose at the end, I do not have to choose it right in the beginning. So we will use this phasor notation.

Now the next topic here is this. I would like to introduce you to some basic terminologies in wave propagation. So in order to do that let me change the page here, let me say what I am going to do then we will use this page. So we will talk of again phasors. We will talk of a phase, we will talk about wavelength, about wavenumber, this will be a new idea wavenumber and then phase speed, then frequency and time period. So, let us change the page.

(Refer Slide Time: 22:44)



So in order to do this I will devise a little system. Here is my system. Let us say it is made of a crank slider mechanism and the crank is a phasor, it is going to rotate at frequency ω. At ω $rads/sec$ it rotates. And this mass is attached to a spring aligned along x axis and it goes off to ∞. We do not know where the other end is, it would go off to ∞.

So, now as this phasor or rod rotates, as it completes the cycle after cycle, so this mass will oscillate back and forth and therefore pulses of compressions will move along the spring, pulses of rarefaction will move along the spring. This is a wave bearing system. This has stiffness, the spring has stiffness, the spring has inertia; so together it is a wave bearing system. So, this repeated pulses or repeated movement of the crank will be transmitted along the spring.

So how will spring look like at some point? Somewhere the mass may be here at some point and then there may be a very expanded region then there will be a compressed region. Then again there will be an expanded region, again a compressed region, again an expanded region and so forth. So we will set the the reference to be vertical. For convenience let us the reference be vertical. I start over here that is my starting point and the starting phase also.

So, this phase is a very confusing idea, used in many meanings. So, we will consider this angle to be the phase. So, the reference phase is vertical. So reference phase is $\frac{\pi}{2}$. The reference is unimportant in some sense because we will talk of differences, but the reference starting point is vertical. So now how does this slider crank look like? It looks like this. The mass may be shifted because it is tied. That is how it looks like.

So, this point, the starting point is for me the $x = 0$ and its phase is that of the phasor. It has the same phase as the phasor let us say. As I said this particular value is not important, however we will keep track of it, we got it $\frac{\pi}{2}$. And so as this crank now goes around and comes back to the same vertical position, this spring has been pulled and brought back. It has gone through one cycle.

So, one cycle of information gets transmitted along the spring. When this phasor goes a second time, two cycles of information have been transmitted. That means what you will have rarefactions, compressions, rarefactions, compressions and as it goes more and more cycles this information is moving. So, now there will be two regions on the spring which are doing the same type of motion, because in one cycle the same information was transferred.

When it started a certain information was given to the mass, when it goes around once that information has been transferred. So, some other region is seeing it, so there are regions which are seeing similar movement. They are λ apart. And because this phasor is rotating at ω, the time period T, the time period for one rotation is $\frac{2\pi}{\omega}$. So, one time period has gone by and in one time period, λ distance has been covered by the signal.

The signal has covered λ distance and λ is the wavelength. The distance covered by the wave in one time period is the wavelength. It is also true that one wavelength apart, two particles of the spring will be seeing the same motion. Any two regions you take, this region and λ apart you take another particle, both of them will be seeing the same type of motion. Now, what is the use of this?

First of all, let us say in one cycle, so in one cycle, the distance covered is λ and the rod is rotating at ω $rads/sec$ which you know is equal to $2\pi f$ (f in $cycles/sec$). So, the rod is doing f rotations or f $cycles/sec$, so $\omega = 2\pi f$. So, we see, in one cycle the distance covered is λ, in f cycles the distance covered is $f\lambda$. But f cycles are covered in one second, f cycles are covered in one second.

So, this is the distance covered in one second so that has to be the speed c, this is the speed of the wave. So, let me close the lecture here for today. We will continue with this in the next class. Thank you.