Radiative Heat Transfer
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Lecture – 08
Inside Sphere and Monte Carlo Method

Hello friends, welcome to this course on radiative heat transfer, in the previous lecture, I discussed some of the methods to evaluate view factor between 2 surfaces bounded by vacuum, in this lecture we will study few more methods to find out view factors between surfaces, the methods that we are going to discuss in this class are for special configurations of plates.

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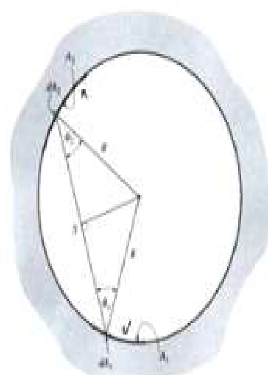
The Inside-Sphere Method

The view factor between two surfaces part of the same sphere depends only on the size of the receiving surface, and not on the location of either one.



We will discuss the inside sphere method, the unit sphere method and the Monte Carlo method, as you see that the Monte Carlo method is generic and you can apply to any configuration while the inside sphere method and the unit sphere method are specially designed and developed for special class of problems. So, first we will discuss the inside sphere method now, in this configuration, we have a special case when the 2 surfaces can be represented as part of the same sphere.

That means, we can fit these surfaces inside a sphere having some radius R, So for example what you see on this slide is basically 2 surfaces A_1 and A_2, okay and these two surfaces they are curved surfaces and they are part of the same sphere, the radius of the sphere is R and we are interested in finding the view factor between these 2 surfaces F_{12}. What we will do is; we will start with small element area dA_1 on surface A_1.

And we will take small element surface area dA_2 on surface A_2, the angle between the line joining dA_1 and dA_2 and line joining them with the centre of the sphere gives you an angle of these two surfaces, θ_1 is the angle of dA_1 and θ_2 is the angle of surface dA_2.

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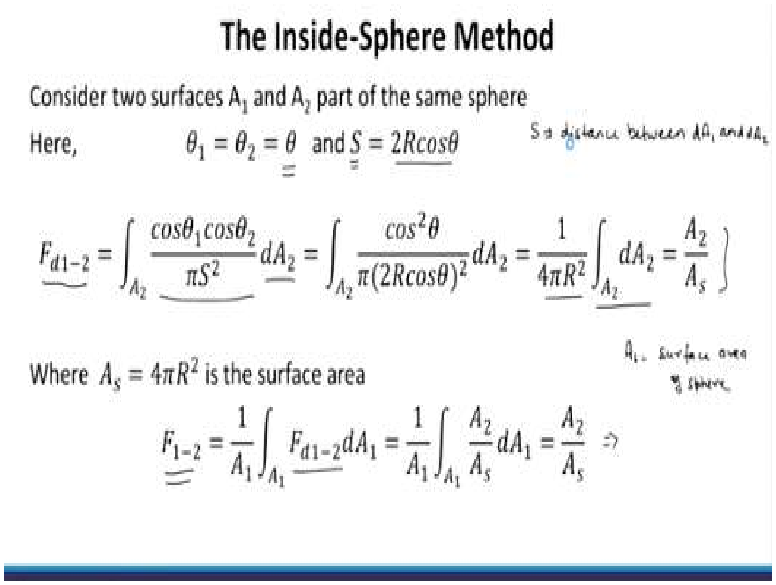

Now, form just purely geometrical considerations, we can say that θ_1 and θ_2 are same and let us say the value

$$\theta_1 = \theta_2 = \theta$$

And this angle is basically is given by; is decided by the radius R and the chord length, so we can write down the distance between the elements dA_1 and dA_2 that is

$$S = 2R\cos\theta$$

from the triangle given on this slide. So, from this triangle, we can basically apply the trigonometric theorem and calculate the distance between the elements dA_1.

So, S is distance between dA_1 and dA_2, the small elements on A_1 and A_2, now, we defined the view factor between these two infinitesimal elements dA_1 and dA_2, okay, so we define F_{d1-2} that is the entire surface 2 which will be then integrated over the small element view factor F_{d1} to F_{d2}, so

$$F_{d1-2} = \int_{A_2} \frac{cos\theta_1 cos\theta_2}{\pi S^2} dA_2$$

is the small element view factor between dA_1 and dA_2.

And when we integrate over the entire surface A_2, this will be the view factor F_{d1-2} and we can just substitute for θ_1 and θ_2 in the expression, so this becomes $cos^2\theta$ and we substitute for S, so

$$S = 2Rcos\theta$$

and this becomes now

$$F_{d1-2} = \int_{A_2} \frac{cos^2\theta}{\pi(2Rcos\theta)^2} dA_2 = \frac{1}{4\pi R^2} \int_{A_2} dA_2 = \frac{A_2}{A_s}$$

where R is independent of the integral, so it can be taken out ,and then integrated over the surface area A_2, dA_2 is simply the surface area A_2.

So, we get basically the view factor between small element dA_1 to the entire surface $A_2 = A_2/ A_s$, okay, where A_s is the surface area of the sphere, so this will be equal to

$$As = 4\pi R^2$$

so we put F_{12} that is the total view factor from surface A_1 to A_2 =; now, we have to integrate over surface area A_1, so we write

$$F_{12} = 1/A_1$$

as per the definition of the finite areas view factor, integrate it over F_{d1-2} and dA_1.

$$F_{1-2} = \frac{1}{A_1} \int_{A_1} F_{d1-2} dA_1$$

Now, we substitute the value of $F_{d1-2} = A_2/A_s$ in this expression and because A_s does not depend on area A_1, this can be taken out, so what we are basically left with is

$$F_{1-2} = \int_{A_1} \frac{A_2}{A_s} dA_1 = \frac{A_2}{A_s}$$

So this gives you an idea that when 2 surfaces are part of the same sphere, the view factor is independent of the position of the 2 surfaces, so the θ does not affect the view factor, the orientation of the 2 services does not affect the view factor.

The view factor is just depends on the radius of the sphere in which we are fitting the surfaces A_1 and A_2 and the surface on which the radiation is basically falling that is A_2 and that is basically the definition, so in this inside sphere method, the view factor between 2 surfaces does not depend on the location of either surfaces, is independent of the location, so for this type of configuration it is very handy to calculate view factors using this simple inside sphere method.

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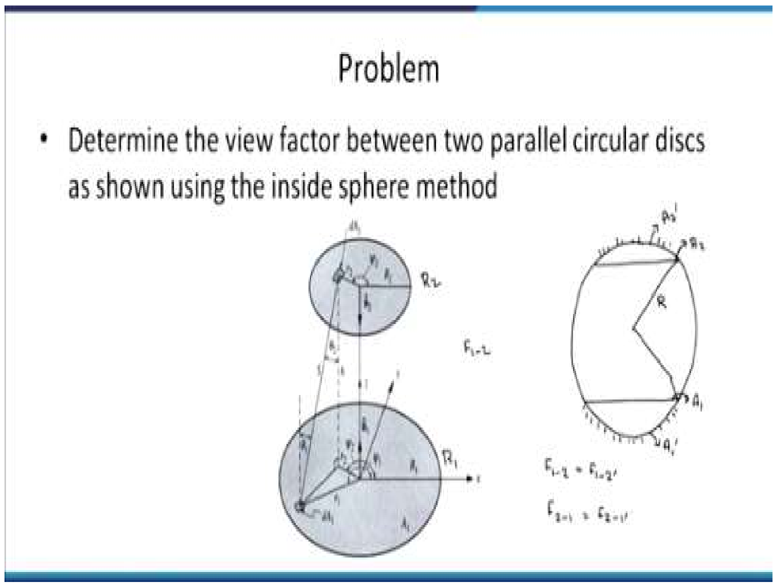

Let us see how this method can be applied to solve problems, so we will take a problem that we have solved in the previous lecture also, we have 2 parallel circular discs, one disc has radius R_1 and another disc has radius R_2, the distance the separation between these disc is h, okay and we will apply the inside sphere method to find out the view factor F_{1-2}, so this view factor we have to find out.

Again to apply the method what we have to do is; we have to fit these plates inside a sphere okay, so let us first draw sphere of radius R and then we will apply; we will fit the plates, so this is going to be first sphere, this is a sphere that we are going to fit our plates in, so we have this plate A_2 and we have this plate A_1, this is the centre of the plate okay, so we will just write, this is radius R of the sphere, okay.

And we can just connect this also, okay so, we will write some geometric relations and find out the view factor between these 2 surfaces, so one thing before we start, you should, this is the surface A_1 and this is surface A_2, which is not part of the sphere, we have just fitted this plates inside the sphere, so let us write down or find out the areas, let us call this $A_{2'}$ and this surface is basically $A_{1'}$, these are spherical caps basically, okay.

So, these are part of the sphere on which we will apply the inside sphere method, we cannot apply directly the inside sphere method to the circular plates but we can apply the inside sphere method to the spherical caps, the areas $A_{1'}$ and $A_{2'}$, so one thing you should just observe is F_{1-2} that is view factor from plate 1 to 2 is basically same as $F_{1-2'}$, all the radiation that basically emitted by surface 1 and falling on surface 2 will actually go to surface 2' also.

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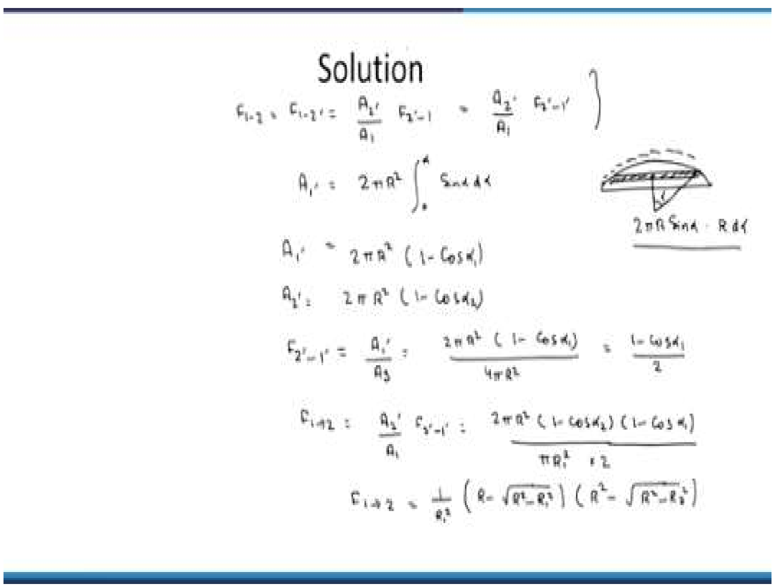

And similarly, we can also write

$$F_{2-1} = F_{2-1'}$$

Just by similar arguments. Now, let us start this, so we have

$$F_{1-2} = F_{1-2'}$$

okay as already said is equal to; now, we apply the reciprocity theorem, so we write, just by applying the reciprocity theorem and this becomes equal to

$$F_{1-2} = F_{1-2'} = \frac{A_{2'}}{A_1} F_{2'-1} = \frac{A_{2'}}{A_1} F_{2'-1}$$

just look at this.

So, 2’ to 1 same as 2’ to 1’, okay, so we can also write here as

$$F_{2'-1} = F_{2'-1'}$$

okay so we get this relation, now let us final out these areas, so area 1’, so we need areas to calculate the view factors, so $A_{1'}$ is basically given by

$$A_1' = 2\pi R^2 \int_0^{\alpha} \sin\alpha \, d\alpha$$

let us see in the geometry what this basically represents, how are we calculating?

So, we have to find out this area, so let me just draw the spherical cap on of which we have to find out the area, we have to find out the area of this spherical cap, so how do we calculate this; we take a annulus, this annulus we take,

the surface area of this annulus $=2\pi R \sin\alpha \; R d\alpha$

so this is going to be the area; surface area of this small annulus on the spherical cap.

And then we want to integrate over the entire surface, we have to just integrate α form; so we have to integrate from 0 to α, okay so

$$A_1' = 2\pi R^2 \,(1 - \cos\alpha)$$

okay and we are using A_1', so we will just use $\cos\alpha_1$ and similarly,

$$A_1' = 2\pi R^2 (1 - \cos\alpha_1)$$

$$A_2' = 2\pi R^2 (1 - \cos\alpha_2)$$

so this is how we can calculate the 2 surface areas; A_1' and A_2'.

Now, we define the view factor

$$F_{2'-1'} = \frac{A_1'}{A_s} = \frac{2\pi R^2 (1 - \cos\alpha_1)}{4\pi R^2}$$

so this basically becomes

$$F_{2'-1'} = \frac{(1 - \cos\alpha_1)}{2}$$

Now from this equation we have view factor

$$F_{1-2} = \frac{A_{2'}}{A_1} F_{2'-1} = \frac{2\pi R^2 (1 - \cos\alpha_2)}{Area\ of\ disc} F_{2'-1} = \frac{2\pi R^2 (1 - \cos\alpha_2)}{\pi R_1^2} F_{2'-1}$$

So, we have basically, we are talking about this area of this disc, so this disc has radius R_1, so R_1 and R_2 are the radius of the discs while R is the radius of the sphere, so $2\pi R^2(1-\cos\alpha_2)$ okay and then we have to multiply by this $F_{2\prime,-1\prime}$ prime, so

$$F_{1-2} = \frac{2\pi R^2(1-\cos\alpha_2)(1-\cos\alpha_1)}{\pi R_1^2 \times 2}$$

so this is our area, so we have, we can substitute for $\cos\alpha_1$ and $\cos\alpha_2$ and relation becomes

$$F_{1-2} = \frac{1}{R_1^2}\left(R - \sqrt{R^2 - R_1^2}\right)\left(R^2 - \sqrt{R^2 - R_2^2}\right)$$

So this is our expression for the view factor now, we have to find out just the value of R, okay, so how do we find the value of R?

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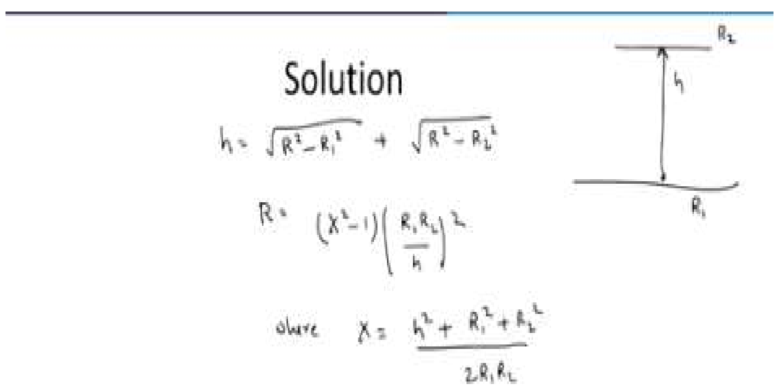

So, we know the separation distance the between the two spheres, let us say this was our original configuration, these are the 2 parallel discs separated by a distance h, the disc is having radius R_1, this disc has radius R_2, so we have to find out the value of h and we can write down

$$h = \sqrt{R^2 - R_1^2} + \sqrt{R^2 - R_2^2}$$

so this is our value of h.

Now, we have to square this equation twice and solve for the value of R, on solving this, we get the value of R to be

$$R = (X^2 - 1)\left(\frac{R_1 R_2}{h}\right)^2$$

Where

$$X = \frac{h^2 + R_1^2 + R_2^2}{2R_1 R_2}$$

so, this is how we can calculate the view factor using the inside sphere method, all we have done is basically fitted these 2 flat plates inside a sphere, calculated the radius of the sphere and based on the formula that the view factor = the area of the surface 2 divide by the area of the sphere.

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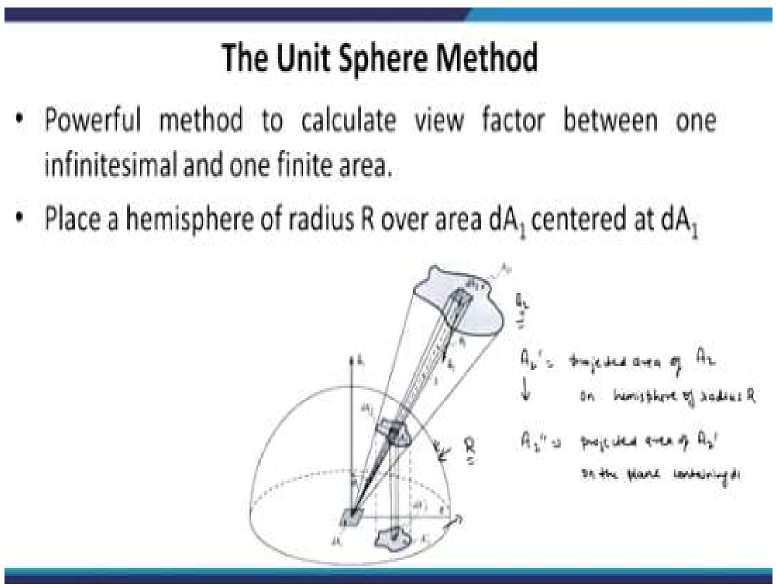

We have easily calculated the view factor between these 2 parallel disc, the next method is the unit sphere method, again this is a special class of method that is for used to find out view factor between two surfaces; the one surface out of the two is infinitesimal that is the small area dA_1 as I shown in this image and the other surface is a finite area, the surface is A_2, the area of the surface 2 is a finite while the area of the surface one is infinitesimal.

For this special category of problem, we can use this method of unit sphere, now this surface A_2, we have to basically fit a hemisphere around the sphere; the surface A_1 that is dA_1, okay this sphere of radius R, we have to put and we have to put this hemisphere in such a way that the surface dA_1

is at the centre of this hemisphere, surface A_2 maybe inside the sphere or it may be outside the sphere and then this surface makes a solid angle, okay.

Now, what we have to do is; we have to project this surface A_2 on to this sphere of radius R, so $A_{2'}$ is basically the projected area of A_2 on hemisphere of radius R, so we have projected this surface onto the hemisphere of radius R, now R can be arbitrary, okay, the choice of R is arbitrary, we will see with the help of an example, there is no restriction on the choice of R, okay, so we have projected surface A_2 on this sphere and $A_{2'}$ is the projected area.

Now, what we do in the second step is; we project this $A_{2'}$ onto the surface in the plane of the area dA_1, so this is the plane of the area, we have projected $A_{2'}$ onto this plane and the area is$A_{2''}$, so this is the projected area of A_2 prime on the plane containing dA_1, so we have projected A_2 twice, in the first projection, we have projecting it onto the sphere of hemisphere of radius R.

And in the second step, we have projected the projected area $A_{2'}$ onto the plane of the area dA_1, okay.

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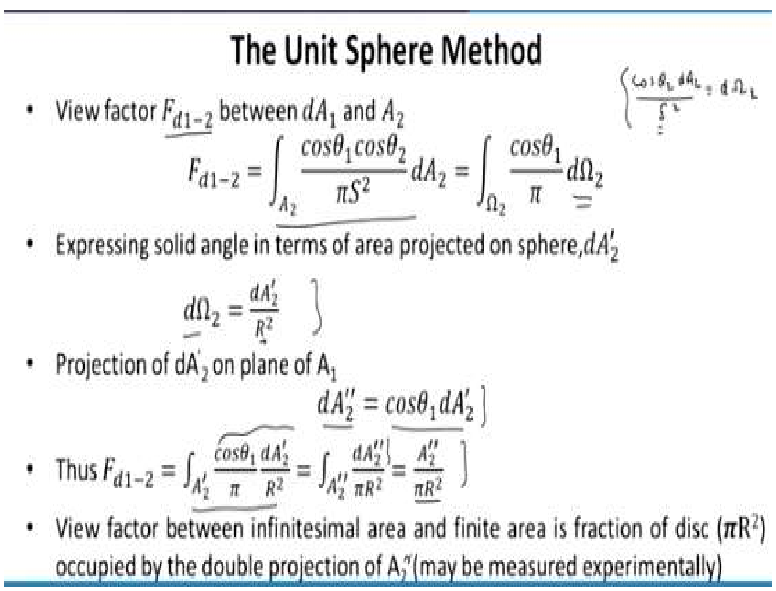

So, let us see how the mathematics of this method comes up, so we are trying to find out the view factor between a small area dA_1 and surface A_2, so F_{d1-2} is the view factor between dA_1 and dA_1 and mathematically, we can define this view factor as integral over the entire surface

$$F_{d1-2} = \int_{A_2} \frac{cos\theta_1 cos\theta_2}{\pi S^2} dA_2$$

where S is the distance between the small element area dA_1 and a small element area dA_2, okay.

So, now this $\frac{cos\theta_2}{S^2} dA_2$ is basically the solid angle, $d\Omega_2$, so we can also write this view factor as integral over the solid angles through which surface A_2 is seen by surface of infinitesimal area dA_1 and we have

$$F_{d1-2} = \int_{\Omega_2} \frac{cos\theta_1}{\pi} d\Omega_2$$

Now, $d\Omega_2$ is solid angle and it can also be written in terms of area of the projected surface.

So, we can also write down $d\Omega_2$ in terms of the projected area

$$d\Omega_2 = \frac{dA_2'}{R^2}$$

Gives you the same solid angle which this formula gives okay, so in the other sense, $cos\theta_2 dA_2$ is the projected area; dA_2' and the distance of the original area was as S, while when we are projecting it on to first hemisphere of radius R, the distance will become R.

so we replace $d\Omega_2 = \frac{dA_2'}{R^2}$ in the expression for the view factor, so

$$F_{d1-2} = \int_{A_2'} \frac{cos\theta_1}{\pi} \frac{dA_2'}{R^2}$$

now the second projection area dA_2 prime is related to the projected area on the hemisphere by this angle $cos\theta_1$, so,

$$dA_2'' = cos\theta_1 dA_2'$$

where theta 1 is the polar angle of surface A_2'. So, we substitute this again, so

$$F_{d1-2} = \int_{A_2''} \frac{dA_2''}{\pi R^2} = \frac{A_2''}{\pi R^2}$$

so this term when integrated over this area dA_2'' just simply becomes $\frac{A_2''}{\pi R^2}$, so this we see the view factor between these two surfaces, one finite and one infinite decimal is equal to the fraction of the area of the disc.

So, the disc that contains this hemisphere that contains this one infinitesimal area dA_1, the disc area is πR^2 where R is the radius of the hemisphere and A_2' is the projected area of A_2 onto this plane, so we can say that this is a faction of the disc occupied by the double projection of A_2, and this is applicable to any arbitrary shape of A_2, it does not have to be just a plane surface, it can be an arbitrary shape.

So, this method gives you a powerful way to calculate the view factor of any arbitrary shape, the only condition is that the other surface should be infinitesimal, okay and we can measure this A_2'', experimentally or for some simple configuration, it can be calculated using analytical formulas, so we will see, let us see how this method can be applied to solve a simple problem.

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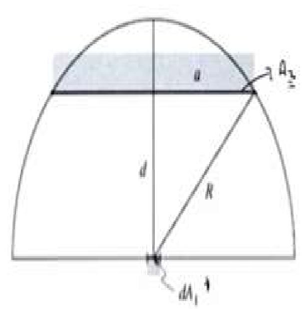
Problem

- Determine the view factor for the configuration as shown



$$F_{dA_1 \rightarrow A_2} = \frac{\pi a^2}{\pi R^2} = \frac{a^2}{R^2}$$

$$R = \sqrt{a^2 + d^2}$$

$$F_{dA_1 \rightarrow A_2} = \frac{a^2}{a^2 + d^2}$$

In this case, we are trying to find out the view factor between infinitesimal area dA_1 located at this centre of this hemisphere now, as we said that radius of the hemisphere is arbitrary, so let us see how we have chosen this radius, the surface; the second surface A_2 is basically a circular disc that is having the radius a, so this circular disc has radius a and the separation distance is basically = d, okay.

So, we have to find out the view factor between these 2 surfaces, and what we have done is; we have chosen our hemisphere in such a way that the hemisphere basically, passes through the edges of a disc, so as I said there is no condition the disc can be inside the hemisphere or the disc can be the outside the hemisphere, the radius of the hemisphere can be arbitrary, so we have chosen to make things simple analytically in such a way that the hemisphere passes through the edges of the disc, okay.

So, as per this definition, the view factor $F_{dA_1-A_2}$ is simply equal to the protected area, so we have to project this area onto the plane, okay so the projected area will be = πa^2, so it does not matter whether you projected it first on hemisphere or you projected on the plane, so the projected area of the disc will be simply equal to πa^2 divided by the area of the disc on which dA_1 is lying and that is basically = πR^2, so this becomes.

$$F_{dA_1-A_2} = \frac{\pi a^2}{\pi R^2} = \frac{a^2}{R^2}$$

Now, what is the value of R; so, R from this triangle

$$R = \sqrt{a^2+d^2}$$

so our view factor

$$F_{dA_1-A_2} = \frac{a^2}{a^2+d^2}$$

so this is the view factor you can calculate very easily using this method called unit sphere method, okay.

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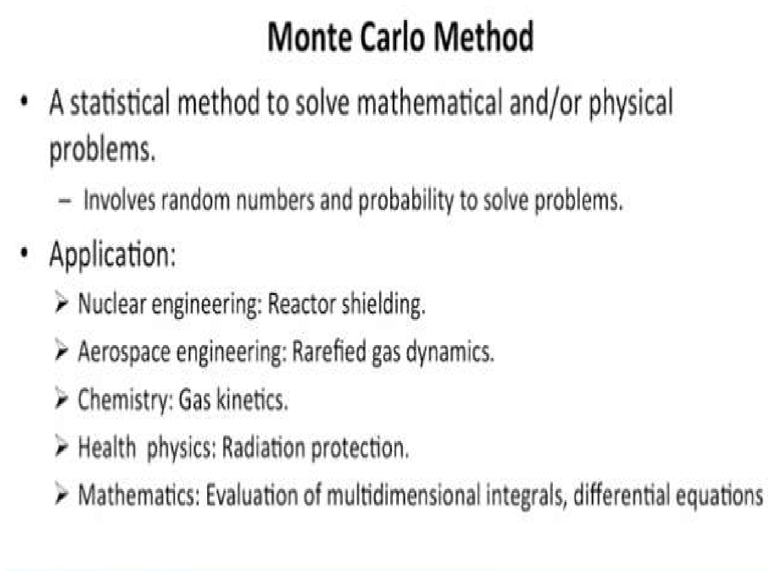

The last in the series of view factor calculation is the method called Monte Carlo method, now Monte Carlo method is different from all other methods in the sense that it is basically based on statistical methodology, it requires random numbers and the method is not just used in view factor calculation, the method is basically used in large number of problems be it in nuclear engineering, aerospace, chemical kinetics, health physics or mathematics.

So, there are large number of problems, so any complicated mathematical or physical problem can be solved using the Monte Carlo method, all you have to do is basically defined the solution domain then randomly generate the solution; possible solutions and then find out the exact solution using the conditions applied on the solution domain, we will understand this method with the help of a simple example.

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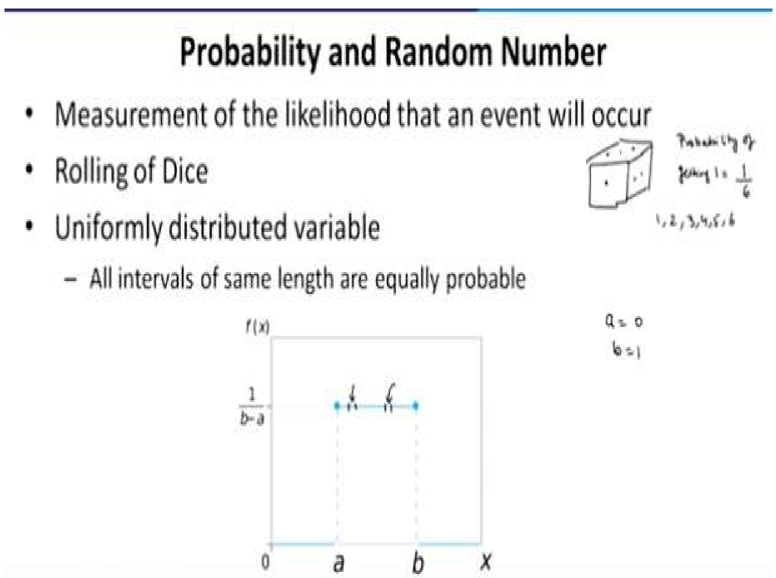

But before going to that example, let us look at the basic logic of probability and random number, because the method of Monte Carlo is based on probability theory and random number, so probability is basically a measure of likelihood that an event will occur, so for example if we have a dice, so let us say we have a dice, okay numbered from 1 to 6, it has 6 faces and we throw this dice, okay then the probability of getting 1 or any other number is uniform that means, the probability of getting 1 = probability of getting 2 = probability of getting 3.

So, it is basically uniform and this probability is 1/6, okay so there are 6 possible outcome either we can have 1, 2, 3, 4, 5, or 6, so the total number outcomes is 6 and the desired outcome either 1, 2 or 3 will be unique 1, so the probability of getting 1 or 2 = 1 / 6, so this is how we basically define probability. In Monte Carlo methods, we use different type of probability distribution but here I will talk about uniformly distributed random variable.

So, uniformly distributed random variables is basically a special kind of a probability distribution, where we say that the probability of finding a number between any 2 intervals is same throughout the distribution variation, so if our distribution varies from a to b, then the probability of finding a number in this range is same as probability of finding the number in this range, okay, so that is basically, we mean by uniformly distributed variable.

As opposed to other distributions like random number like a Gaussian distribution and gamma distribution or Poisson distribution, uniform distribution means that the probability of finding a number is uniform throughout the interval, so here the interval is a and b, so the minimum value of the random number that you will get from this is a and the maximum value that you will get is b. So, normally in this Monte Carlo calculation, we take a value of a to be 0, and b to be 1.

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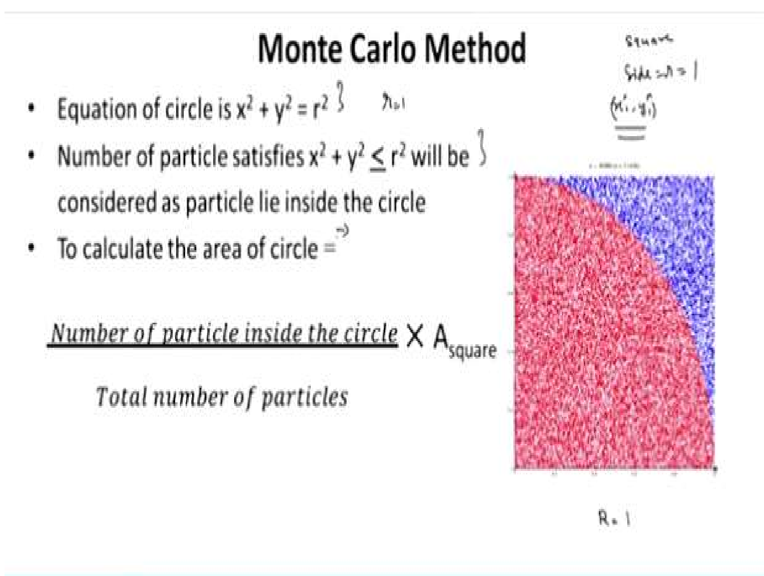

So we are more interested in finding the random number between 0 and 1, okay and it says that the probability of getting any number between this interval is same that is the uniform distribution.

So, let us see how this Monte Carlo method, how powerful it is and how different type of problems can be solved using this method, so in this slide I give you a special example which is nothing to do with the radiative transfer or view factors, here we are trying to find out the area of quarter circle, okay, so we have a circle defined in the first quadrant, the radius of the circle is 1 and we all know that the equation of circle is given by

$$x^2 + y^2 = R^2$$

here in this case, $R = 1$, so we have to find out the area.

Now, suppose we do not know the area, although we know it is going to be $\pi R^2/4$. But suppose, we do not know it and we want to find out this area using the Monte Carlo methods, so what we will do is; we will first define a bounding box in this case, we have defined a bounding box as a square passing through the edges of the quarter circle, so we have defined this square with side $R = 1$, the square has side = the radius, so we have defined this. Why we have done it; because we know that we are going to use uniform distribution, okay.

So, we want points to be generated using random numbers uniformly over this surface of the square okay, so what we will do is; we generate let us say 100 or 1000 or maybe a million number of points, the points will be x_I and y_I, okay. Now, x_I and y_I represents the coordinate of the points okay, now this x_I and this y_I is basically is selected from a uniformly distributed random number.

So, x_I is uniformly distributed random number, it should vary from 0 to 1 anywhere it can come, y_I is uniformly distributed random variable, its value can be anywhere from 0 to 1 and x_I, y_I is the point in side this square of radius of side $R = 1$, okay and when we generate sufficient number of points, we will see that the points occupied the entire space of the square, okay, the inside of square, entire space is occupied.

So, we have to make sure that the selection process of these points is purely random okay, so many computer languages come up with the random number generator, so we use these random number generator on a computer to generate random points x_1 and y_1, now we apply this condition,

$$x^2 + y^2 \leq R^2$$

that is all the points that lie inside the circle should be considered and we have pointed this using red dots.

And all the points that do not satisfy this condition represented by blue dots or although inside the square but they are not inside the circle and they do not contribute to the area of the circle, so the area of the quarter circle is then given by a number of particles inside the circle that is the red dots divided by total number of particles that we have generated multiplied by the area of the square which is R square or 1, okay.

So, this method very easy to implement on different type of problems can solve complicated problems, it can give you view factors also, we will use this formula to; this method to calculate view factors for complicated geometries, okay.

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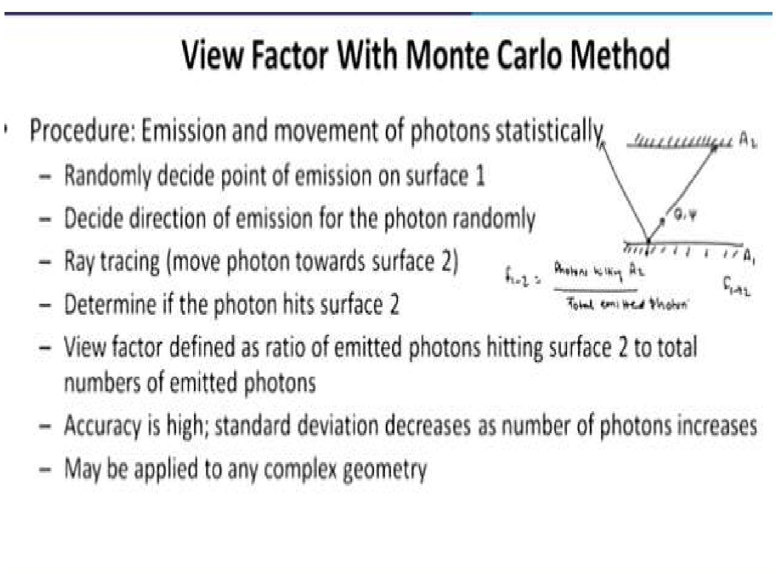

So, how do we calculate view factors using the Monte Carlo method, so let us just understand the basic suppose, we have 2 plates, let us call this A_1 and plate 2, let us call this A_2 and we want to find out the view factors F_{1-2}, okay now the plates may have arbitrary geometry, in this case, we have taken flat but this can taken have arbitrary geometries, so how do we solve for the view factor?

So, the first point in the Monte Carlo method to calculate view factor using Monte Carlo method is to find out a point of emission, so we randomly select a point on this surface A_1, okay so let us call this point as location of emission or point of emission, okay now, this point of emission we will basically emit a photon because radiative transfer is governed by the movement of photons, so this point of emission basically gives you photons.

Now, photons may be emitted in the entire solid angle of 2π, so we have to select a arbitrary direction; a random direction using the polar angle theta and azimuthal angle ψ, okay so this direction of motion; motion of this photon should be purely random, okay and we have choose the polar angle and azimuthal angle randomly, then we have to move this photon until it hits the surface A_2, okay that is called ray tracing.

So, we moved the photon until it hits the surface 2 or it does not hit, some photons may hit, some photons may not hit the surface A_2, so we count how many photos actually hits surface A_2, okay and we also count how many photons, total number of photons we have emitted from A_1, okay, the view factor is basically defined as so,

F_{1-2} = photons hitting A_2 / total emitted photons.

So, in this way you can calculate the view factor very simple algebra using random numbers, the method has very high accuracy although you have to simulate a large number of photons as with any statistical method, the accuracy depends on the sample size, so the sample size should be large enough if you want good accuracy, it can be applied to any complicated geometry, as you increase the sample size, the standard deviation will also decrease, okay.

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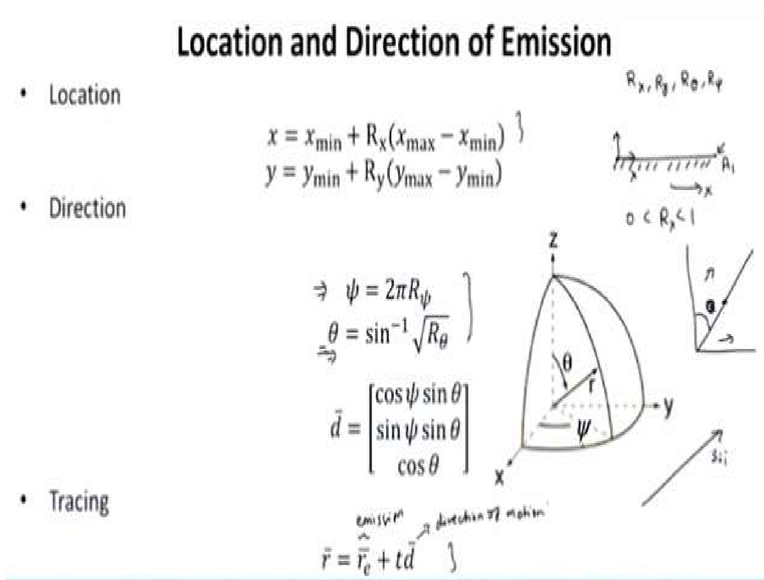

Now, how to calculate the location and direction of emission using random numbers, so let us say we have taken 4 random numbers; R_x, R_y, R_θ and R_ψ, so we have selected 4 random numbers, okay this 4 random numbers are all uniformly distributed between 0 and 1, okay so, let us say we have this flat surface okay, so point of emission will have 2 coordinates x and y, okay, this is the x direction.

So, we write this formula

$$x = x_{min}$$

which is in this case may be 0, if you have a coordinate system located at the end of the plate, if $x_{min} = 0$, our expression is simply $x = R_x x_{max}$

if R_x value is 0, so the value of R_x will vary from 0 to 1, it is uniformly distributed, if value of R_x is 0, then we have x to be 0 that means the point of emission lies at the end of the plate, left end of the plate.

And if R_x happens to be 1, then our point of emission will be at the other end of the plate and this value mostly will be between 0 and 1, so we get uniformly distribution of x coordinates and similarly, we get uniformly distribution of y coordinate, okay and then we get 2 more random numbers; R_θ and R_ψ now we have θ as defined as azimuthal angle and azimuthal angle has we know vary from 0 to 2π.

So, any value between 0 to 2π will give you a direction of emission in the azimuthal direction, so we define a random number or random directions

$$\psi = 2\pi R_\psi$$

if R_ψ happens to be 0, then basically we are emitting the photon in x direction, if R_ψ happens to be 1, we are basically emitting the photon in; again in x direction because of the 2π angle, its complete angles is2π. Now, θ angle is little tricky because of the spherical coordinate system.

We basically are measuring θ with respect to normal, so this direction has more capacity than this capacity, so more number of photon should be emitted between normal and the direction at an angle θ and that is why we use basically a sinusoidal distribution, so the polar angle is basically calculated, so it is not a uniform distribution, photons are not emitted uniformly over the polar angle, this thing you should keep in mind.

Because if the effect of solid angle in spherical coordinates, so photons are not emitted uniformly over the polar angle, they follow basically a sine distribution, so sin theta is basically under root of R_θ, okay, so based on this relation, we can calculate the polar angle and then we can define the direction of motion and as

$$\bar{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} \cos\psi\sin\theta \\ \sin\psi\sin\theta \\ \cos\theta \end{bmatrix}$$

so this is going to be our direction of emission, S_{ij} okay.

And we can trace the photons using this vector addition, where R is the emission location and this is basically, the direction of motion okay, so we can basically move the photons in this direction and then we can see whether the photon is hitting the surface 2 or not and we can count how many photos are basically hitting the surface, okay. Thank you very much, so here we end our study on view factors, we have discussed many methods.

And some of the methods are applicable to general geometry, while some methods are developed for very specific geometries like the inside sphere method and the unit sphere method, in the next lecture we will study about radiative energy exchange between black surfaces using the view factors that we have already derived, thank you.