

Steam and Gas Power Systems
Prof. Ravi Kumar
Department of Mechanical industrial engineering
Indian Institute of Technology - Roorkee

Module No # 08
Lecture No # 38
Axial Flow Compressor Characteristics

Hello I welcome you all in this course on Steam and Gas Power Systems today we will discuss axial flow compressor characteristics.

(Refer Slide Time: 00:30)

- Dimensionless parameters
- Losses in axial flow compressor
- Choking flow
- Stalling
- Surging
- Worked examples

We will start with the dimensionless parameters, which are used for comparing the performance of axial flow compressors then losses and axial flow compressors, choking flow, stalling, surging and we will do some worked example. There are certain dimensionless parameters which are used in axial flow compressor in order to judge the performance of axial flow compressor.

(Refer Slide Time: 01:00)

$$\begin{aligned}
 1. \text{ Flow Coeff } (\phi) &= \frac{m}{m_{tip}} \\
 &= \frac{V_f}{V_f (\tan \alpha_1 + \tan \beta_1)} = \left(\frac{V_f}{u} \right) \\
 \phi &= \frac{1}{\tan \alpha_1 + \tan \beta_1}
 \end{aligned}$$

Number one is flow coefficient of an axial flow compressor is the mass flow rate and mass low rate and tip of the blade and this is VF where flow velocity. This is flow velocity Tan Alpha 1 + Tan Beta 1, it is nothing but VF by U because rest of the things for calculating the mass flow rate are going to remain same.

Only there will be a change in velocity actual mass flow rate and mass flow rate corresponding to thrift velocity. So we will be getting this expression, so flow coefficient is one by Tan Alpha 1 + Tan Beta 1, it is the dimensionless quantity.

(Refer Slide Time: 02:13)

$$\begin{aligned}
 \text{Head coeff } (\lambda) &= \frac{\Delta h}{\frac{1}{2} u^2} = \frac{u (V_{w2} - V_{w1})}{\frac{1}{2} u^2} \\
 \text{Pressure coeff } (\psi) &= \frac{\Delta h_{is}}{\frac{1}{2} u^2} = \frac{2 (V_{w2} - V_{w1})}{u} \\
 \psi &= \eta_i \lambda = \frac{2 V_f (\tan \alpha_2 - \tan \alpha_1)}{V_f (\tan \alpha_1 + \tan \beta_1)}
 \end{aligned}$$

Another is head coefficient is denoted by λ and it is enthalpy rise in a stage divided by enthalpy corresponding to peripheral velocity. This is kinetic energy at the tip of the rotor. Now enthalpy rise is $U(V_{w2} - V_{w1})$ and kinetic energy is $\frac{1}{2} U^2$ then $\lambda = \frac{U(V_{w2} - V_{w1})}{\frac{1}{2} U^2}$ and this will be cancelled out and two $V_{w2} - V_{w1}$ divided by U .

This is head coefficient and we can further write $2V_f V_{w2} = V_f \tan \alpha_2 - V_f \tan \alpha_1$ divided by $V_f \tan \alpha_1 + \tan \beta_1$ or this V_f will be cancelled out it will become dimensionless and after the head coefficient there is a pressure coefficient. In pressure coefficient the difference between these two is pressure coefficient the ΔH isentropic enthalpy rise is taken into the account okay.

And we can always say, that the pressure coefficient is isentropic efficiency multiplied by head coefficient like steam turbine. So impulse reaction, steam turbine axial flow compressors also do have degree of reaction.

(Refer Slide Time: 04:28)

Degree of Reaction
 $R, D, A.$

$$R = \frac{W_s - \frac{(V_2^2 - V_1^2)}{2}}{W_s} = \frac{W_s - \frac{(V_f^2 + V_{w2}^2 - V_f^2 - V_{w1}^2)}{2}}{W_s}$$

The diagram shows a velocity triangle with vectors V_1 , V_2 , V_f , and V_w .

It is denoted by R , in some of the books it is denoted by Ω or capital λ . So the degree of reaction is again the temperature rise in rotor divided by temperature rise in stage or if you multiply by CP then energy imparted in rotor divided by energy imparted in stage or work in a stage or we can write work in a stage minus $V_2^2 - V_1^2$ divided by work in a stage okay.

Now $V_2^2 - V_1^2$ is work in a stage minus, now in a velocity triangle velocity diagram, this is V_{R1} V_1 and this is U and this is V_F and this is V_W right. So V_1^2 is nothing but $V_F^2 + V_{W1}^2$ and V_2^2 is $V_F^2 + V_{W2}^2$ and V_1^2 is $V_F^2 - V_{W1}^2$ divided by work in a stage.

(Refer Slide Time: 06:13)

The image shows a handwritten derivation of the degree of reaction R . It starts with the definition of R as the ratio of static enthalpy rise to total enthalpy rise:

$$R = 1 - \frac{V_{w2}^2 - V_{w1}^2}{2U(V_{w2} - V_{w1})}$$

Next, it shows the static enthalpy rise W_s as the difference in total enthalpy minus the change in kinetic energy:

$$R = 1 - \frac{V_{w2}^2 + V_F^2 - V_{w1}^2 - V_F^2}{2U} = \frac{W_s - \frac{(V_2^2 - V_1^2)}{2}}{W_s}$$

The final result is circled:

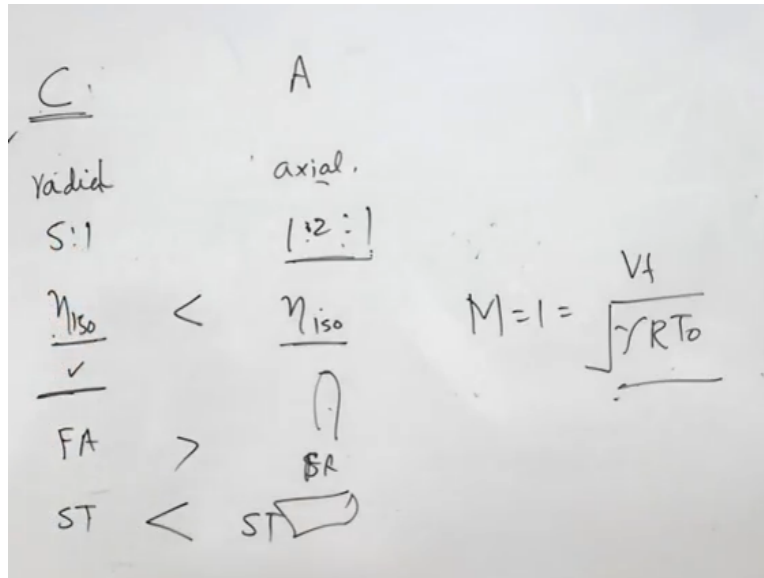
$$R = 1 - \frac{V_{w2}^2 - V_{w1}^2}{2U}$$

To the right, a velocity triangle is drawn. It is a triangle with a vertical base of length U . The left side is V_1 , the right side is V_2 , and the top side is V_F . The vertical height is V_{w1} and V_{w2} respectively. The angle at the top is β .

Now degree of reaction are is equal to, now instead of doing it here, we can write this is sorry $V_{w2}^2 + V_F^2 - V_{w1}^2 - V_F^2$. So this will be cancelled, so R is $1 - V_{w2}^2 - V_{w1}^2$ square divided by work in a stage. And the work in a stage is $UV_{w1} + V_{w2} - V_{w1} V_{w2} - V_{w1}$ and this multiplied because this is 2 here this is divided by 2 and 2 will come here.

So R is $1 - V_{w2} + V_{w1}$ divided by 2 multiplied by U or $R = 1 - V_{w2} + V_{w1}$ divided by UV_{w1} is mean will component. So this is how we can find the degree of reaction of an axial flow compressor. Now if we compare the centrifugal compressor with axial flow compressor

(Refer Slide Time: 08:05)



And axial flow compressor and centrifugal compressor, the flow is radial in the flow is axial as it implies from the name itself. Pressure ratio per stage is high 5 is to 1 or 6 is to 1. Here the pressure ratio per stage may be 1.2 is to 1. But in axial compressor we can have number of stages, it is very easy to add on stage in axial flow compressor. So per stage pressure is low but we can have number of stages in axial flow compressor.

Isentropic efficiency in centrifugal compressor is less than isentropic efficiency of axial flow compressor, the reason I have already told you because in centrifugal compressor there is change in direction okay which imparts losses in the during the flow of fluid, that is why isentropic efficiency of centrifugal compressor is less than isentropic efficiency of axial flow compressor.

We are not discussed here this choking and surging, so but we will discuss after this. So choking and surging the gap is quite substantial in case of centrifugal compressor but choking and surging stage, here the gap is not much the choking and surging. I will discuss after this, this as large frontal area, this is small frontal area so because axial flow compressor have small frontal area that is why they are very useful for jet propulsion or in aircraft applications right.

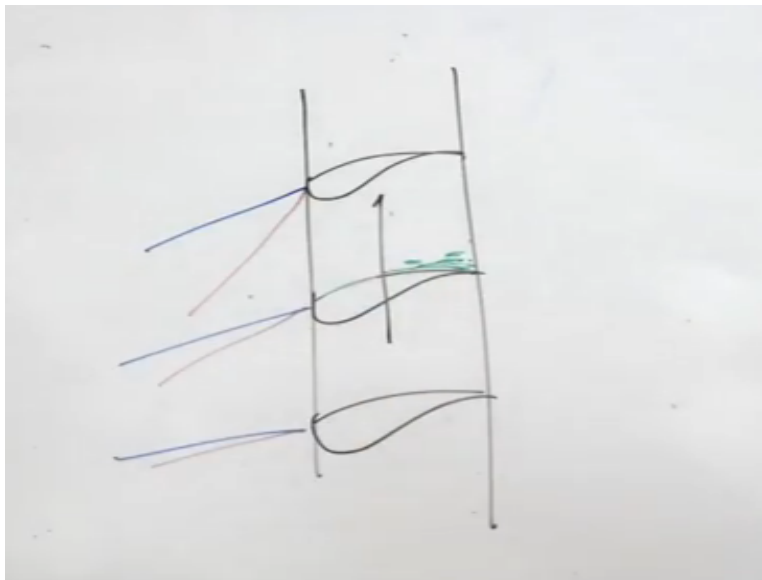
But when working with contaminated fluid, when in centrifugal compressor if the fluid is contaminated we can use centrifugal compressor but that is not the case of axial flow compressor. In axial flow compressor, the working fluid should not have any contaminations it

should be clean starting tort to start the compressor. The starting tort in centrifugal compressor is less than the starting tort in axial flow compressor.

And this centrifugal compressor construction cost is less, it is less complicated. Here construction cost or fabrication cost is more and is a riddle, more complicated. If you compare with the centrifugal compressor, now before we start with the numerical, we will very short we will discuss the choke flow we have already discussed for centrifugal compressor during flow inside the compressor.

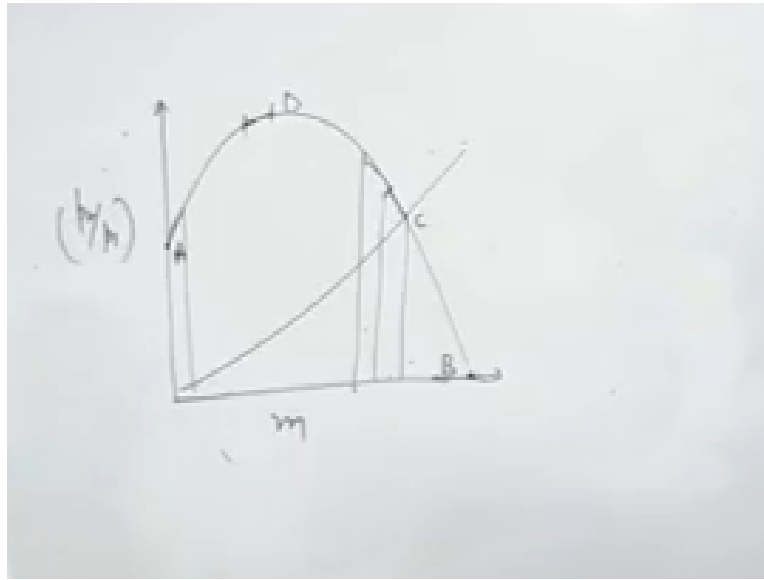
The velocity should not exceed $M1$, that is $\frac{VF}{\sqrt{\gamma R T_0}}$. So this should not shock or flow should not become choke flow, otherwise if it becomes supersonic in the later stage some shock may take place and that will incur energy losses during the flow through a the compressor and it will affect the efficiency and it will also physically damage the compressor right. So it is ensured that choke flow does not take place inside the compressor there is a term stalling, now stalling it is due to the change in the direction of inlet.

(Refer Slide Time: 11:58)



Suppose in a cascade there are number of blades let us take three blades and there is a different angle of incident changing and here at particular angle, the p and fag end he flow separation may take place. This is known as Stall and this is stall is not stationary it moves in opposite direction of the flow with half of the speed right and that is known as Rotating Stall.

(Refer Slide Time: 12:57)



Now another phenomena is surging, let us draw the mass flow rate and pressure ratio and this is the surging curve. This is a characteristic curve and in this position there is a pressure ratio. But there is no mass flow rate it means the output valve is closed. Now in this position, mass flow rate is very high but pressure ratio is 0 it means the valve is fully opened, full throttle.

It is imaginary situation because if pressure ratio is not there will not be any flow so practically some pressure ratio has to be there right. Now there is a characteristic curve from here which cuts, this at state let us say this is A, this is B, this is C. Now when the flow is taking place at state C and we partially close the wall, when we partially close the wall the mass flow rate will reduce we further close the wall the mass flow rate will reduce.

When the valve is fully closed here when we slightly open the wall the pressure ratio will increase and mass flow rate will also increase it is reverse of this why it is happening the moment? We open the wall the flow will come with the certain velocity and this velocity will be converted into the admission pressure and that is how the pressure ratio is increasing but up to certain point only.

Now we are closing valve, we cross this point D and come to this side. When we come to this side, we further close the valve in that case pressure ratio will decrease. Initially when we are

closing the valve the pressure ratio was increasing, now we are closing the valve the pressure ratio is decreasing it means pressure in the pipe at the exit of the compressor is more than at the inlet of the pressure at the pipe so reverse flow will take place, so fluid will start flowing it is a surging type of flow.

The fluid will start flowing backwards towards the compressor but the moment it enters the compressor the pressure will be neutralized and there will be interrupted flow or oscillating flow type of phenomena. So this is known as the surging in the flow and it is witnessed in both in centrifugal compressor and axial flow compressor as well.

(Refer Slide Time: 16:00)

In an axial flow compressor, the overall stagnation pressure ratio achieved is 4 with overall stagnation isentropic efficiency 86%. The inlet stagnation pressure and temperature are 100 kPa and 320 K. The mean blade speed is 190 m/s. The degree of reaction is 0.5 at mean radius with relative air angle of 30° and 10° at rotor inlet and outlet respectively. The work done factor is 0.88. Calculate the stagnation polytropic efficiency, number of stages, inlet temperature and pressure, blade height in the first stage if hub-tip ratio is 0.4 mass flow rate is 20 kg/s.

Now after this we will do a worked example now this numerical states that in an axial flow compressor the overall stagnation pressure ratio achieved is 4.

(Refer Slide Time: 16:15)

$$\frac{P_2}{P_1} = 4$$

$$\eta_{is} = 0.86$$

$$P_1 = 100 \text{ kPa}$$

$$T_1 = 320 \text{ K}$$

$$U = 190 \text{ m/s}$$

$$R = 0.5$$

$$T_{02} = 475.7 \text{ K}$$

$$T_{02'} = 501 \text{ K}$$

$$\eta_p = 0.884$$

$$T_{02} = T_1 \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} = 320 (4)^{0.286}$$

$$T_{02'} = T_1 + \frac{T_{02} - T_1}{\eta_{is}}$$

$$\eta_p = \frac{\ln \left(\frac{P_2}{P_1} \right)}{\ln \left(\frac{T_{02'}}{T_1} \right)}$$

So pressure ratio is stagnation pressure ratio P_{02} by $P_{01} = 4$ and isentropic efficiency is 86 percent. The inlet stagnation pressure and temperature are $P_{01} = 100$ kilo Pascal and T_{01} is 320 Kelvin, the mean blade speed is 190 meter per second. So $U = 190$ meters per second, the degree of reaction is 0.5. The mean radius with radius angles air angle of 30 degree and 10 degree and rotor inlet and outlet respectively.

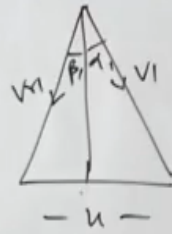
Work done factor is 0.88. Calculate the stagnation polytropic efficiency. So here we will start simply $T_{02} = T_{01}$. Now Z we are assuming, Z number of stages in axial flow compressor and in each stage is certain degree of pressure rise will take place and cumuli effect will be P_{02} by P_{01} .

So T_{02} is T_{01} P_{02} by P_{01} is to power $\gamma - 1$ over γ and that = 320 stagnation pressure at temperature at inlet is 320 pressure ratio is 4 and this is .286 and this $T_{02} = 475.7$ Kelvin. This is the temperature at the exit of the compressor not at the exit of the stage T_{02} dash will be calculated as $T_{01} + T_{02} - T_{01}$ divided by isentropic efficiency.

Now we have the value of T_{02} T_{01} isentropic efficiency T_{01} is already with us. So T_{02} dash will be is going to be 501 Kelvin. Now we will take polytropic efficiency, small stage efficiency is natural log P_{02} by P_{01} raise to power 0.86 $\gamma - 1$ over γ divided by natural log of T_{02} dash by T_{01} .

This is polytropic efficiency, we have already done earlier this one right and now we will be putting the values. We have all the values with us and from here the polytropic efficiency is going to be 0.884 okay. Now we will go for velocity diagram for the compressor now velocity diagram for the compressor, we can always take from the velocity diagram.

(Refer Slide Time: 19:53)



$$\frac{u}{V_f} = \tan \alpha_1 + \tan \beta_1$$

$$= 0.7536$$

$$V_f = \frac{u}{0.7536} = \frac{190}{0.7536} = 252.12 \text{ m/s}$$

From compressor U upon $V_f = \tan \alpha_1 + \tan \beta_1$. If we draw the triangle then this is V_1 V_{R1} this is U α_1 , β_1 . Now α_1 and β_1 are known to us calculate the relative air angles of 30 degree and 10 degree so for α_1 and β_1 are known to us okay.

So $\tan \alpha_1 + \tan \beta_1$ that = 0.7536. So $V_f = U$ by 0.7536 = 190 by 0.7536 = 252.12 meters per second. So V_f , we can note down because we will be frequently reading this 252.12 meters per second.


(Refer Slide Time: 21:07)

$V_f = 252.12 \text{ m/s}$
 $V_{W2} = 145.56 \text{ m/s}$
 $V_{W1} = 44.45 \text{ m/s}$

$V_{W2} = V_f \tan \alpha_2$
 $= 252.12 \tan 30 = 145.56 \text{ m/s}$

$R = 0.5$
 $\alpha_1 = \beta_2$
 $\alpha_2 = \beta_1$

$V_{W1} = V_f \tan \alpha_1$
 $= 252.12 \tan 10 = 44.45 \text{ m/s}$



Now $V_{W2} = V_f \tan \alpha_2$ $V_{W1} = V_f \tan \alpha_1$. Now here since, here since degree of reaction is 0.5. So $\alpha_1 = \beta_2$ and $\alpha_2 = \beta_1$ right. So now $V_{W2} = V_f \tan \alpha_2$ and that is $= 252.12 \tan 30$ and that is $= 145.56$ meter per second.

So this is CW $V_{W2} = 145.56$ meters per second. Similarly we can calculate $V_{W1} = V_f \tan \alpha_1$ and that $= 252.12 \tan 10$ and that is going to be 44.45 meters per second. So $V_{W1} = 44.45$ meters per second.

(Refer Slide Time: 22:36)


$V_f = 252.12 \text{ m/s}$
 $V_{W2} = 145.56 \text{ m/s}$
 $V_{W1} = 44.45 \text{ m/s}$

$V_{W2} = 145.56 \text{ m/s}$ $V_{W1} = 44.45 \text{ m/s}$

$R = 0.5$
 $\alpha_1 = \beta_2$
 $\alpha_2 = \beta_1$

$V_{W2} = V_f \tan \alpha_2$
 $= 252.12 \tan 30 = 145.56 \text{ m/s}$

$V_{W1} = V_f \tan \alpha_1$
 $= 252.12 \tan 10 = 44.45 \text{ m/s}$



So work done per stage is $UVW2 - VW1$ multiplied by some work done factor is also given, so multiplied by λU is with us, U is 190 $VW2$ and $VW1$. We have already calculated λ is 0.88 and this gives work done per stage as 16.9 kilojoules per kg right.

(Refer Slide Time: 23:18)

Handwritten calculations on a whiteboard:

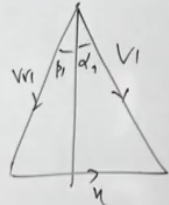
- $V_1 = 252.12 \text{ m/s}$ $Z = 11$
- $V_{W2} = 145.56 \text{ m/s}$
- $V_{W1} = 144.45 \text{ m/s}$
- $W_s = 16.9 \text{ kJ/kg}$
- $R = 0.5$
- $\alpha_1 = \beta_2$
- 75.7 K $\alpha_2 = \beta_1$
- 501 K
- 0.884
- $W_c = 181.9 \text{ kJ/kg}$
- $W_{\text{comp}} = C_p (T_{02}' - T_{01})$
- $Z = \frac{181.9}{16.9} = 10.76 \approx 11 \text{ stages}$

Work in compressor is work done in stage and total work done during compression is $C_p T_{02}' - T_{01}$. This is total compression and C_p is already with us, 1.005. T_{02}' we calculated 501 Kelvin. T_{01} is with us, this is 320 kelvin and from here the work of compression is calculated as 181.9 kilojoules per kg. So compression work is 181.9 stage work is this much if we take the ratio.

We can find out the stages, so number of stages $Z = 181.9$ divided by $16.9 = 10.76$ or approximately eleven stages. So number of stages $Z = 11$ because we have to take the integer. We cannot take 10.76 stages. So eleven stages during compression stage axial flow compressor now once the stages are calculated, then inlet velocity V_1 .

(Refer Slide Time: 24:50)

$V_0 = 252.12 \text{ m/s}$ $Z = 11$
 $V_{w2} = 145.56 \text{ m/s}$
 $V_{w1} = 144.45 \text{ m/s}$ $V_1 = 256 \text{ m/s}$
 $W_s = 16.9 \text{ kJ/kg}$
 $R = 0.5$
 $\alpha_1 = \beta_2$
 75.7 K $\alpha_2 = \beta_1$
 501 K $W_c = 181.9 \text{ kJ/kg}$
 0.884
 2 K



$$T_1 = T_{01} - \frac{V_1^2}{2C_p}$$

$$= 320 - \frac{256^2}{2 \times 1005}$$

$V_1 = V_1 \cos \alpha_1$ so V_1 is $V_1 \cos \alpha_1$. So V_1 , we are getting from here. We are getting $V_1 = 256$ meters per second and temperature at inlet because we have only stagnation temperature. So temperature at inlet is going to be $T_1 = T_{01} - \frac{V_1^2}{2C_p}$.

This is stagnation temperature, so inlet temperature is going to be $320 \text{ K} - \frac{256^2}{2 \times 1005}$. I have converted Kilo joules into Joules in specific heat and then this gives the value of T_1 is, 302 Kelvin . Stagnation temperature is 320 and absolute temperature is 302 Kelvin .

(Refer Slide Time: 26:16)

12 m/s $Z = 11$
 145.56 m/s $p_1 = 81.6 \text{ kPa}$
 144.45 m/s $V_1 = 256 \text{ m/s}$
 16.9 kJ/kg
 $R = 0.5$
 $\alpha_1 = \beta_2$
 $\alpha_2 = \beta_1$
 $W_c = 181.9 \text{ kJ/kg}$

$$\left(\frac{p_1}{p_{01}} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{T_1}{T_{01}} \right)$$

Now after this P_1 by $O_1 = T_1$ by T_01 raise to power γ over $\gamma - 1$ right. Now here we have the value of stagnation pressure at inlet 100 kilo Pascal T_1 . We have calculated T_01 is also with us this will give the value of P_{1e} as 81.6 kilopascal.

(Refer Slide Time: 26:56)

$V_{t2} = 252.12 \text{ m/s}$ $Z = 11$
 $V_{w2} = 145.56 \text{ m/s}$ $P_1 = 81.6 \text{ kPa}$
 $V_{w1} = 44.45 \text{ m/s}$ $V_1 = 256 \text{ m/s}$
 $W_s = 16.9 \text{ kJ/s}$ $P_1 = 0.941 \text{ m}^3/\text{kg}$
 $R = 0.5$
 $\alpha_1 = \beta_2$
 475.7 K $\alpha_2 = \beta_1$
 $= 501 \text{ K}$ $W_c = 181.9 \text{ kJ/kg}$
 $p = 0.884$

$$P_1 = \frac{P_{1e}}{R T_1^{0.286}}$$

Now we need to calculate the density at inlet ρ_1 is $P_1 / R T_1$ right. Now P_1 is these values are with us $R = 0.286$ and this will give the ρ_1 as 0.941 meter cube per kg okay. Now the mass flow rate is 20 kg per second and blade to hub ratio is .4.

(Refer Slide Time: 27:37)

$V_{t2} = 252.12 \text{ m/s}$ $Z = 11$
 $V_{w2} = 145.56 \text{ m/s}$ $P_1 = 81.6 \text{ kPa}$
 $V_{w1} = 44.45 \text{ m/s}$ $V_1 = 256 \text{ m/s}$
 $W_s = 16.9 \text{ kJ/s}$ $P_1 = 0.941 \text{ m}^3/\text{kg}$
 $R = 0.5$ $r_t = 17.87 \text{ mm}$
 $\alpha_1 = \beta_2$
 475.7 K $\alpha_2 = \beta_1$
 501 K $W_c = 181.9 \text{ kJ/kg}$
 0.884

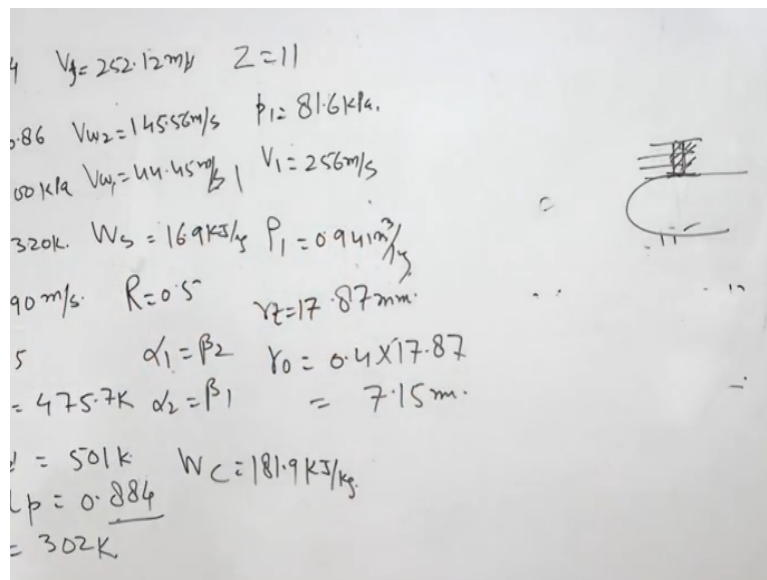
$$m = \frac{0.941 \times \pi r_t^2 (1 - 0.4^2) \times 252.12}{\gamma_t} = 20$$

Blade to hub ratio means in axial flow compressor, this is hub the blades are mounted to the hub and this ratio blade to hub ratio is .4. So 0.941 density multiplied by $P_1 / R T_1^2$ tip square 1 -

0.4 square. This is area this is density area and velocity 252.12. This is the velocity of flow we will give the mass flow rate and that = 20.

This is density of air at inlet we have calculated Rho by RT 1, this is because mass flow rate is taking place between the hub and the tip. So this cross section area we have calculated here multiplied by the velocity and we are getting mass flow rate. From here we are getting the value of RT and the RT is 17.87 MM and when we multiply this 17.87 MM by .4, we will get the diameter of the hub.

(Refer Slide Time: 29:09)



Handwritten calculations and a diagram of a blade cross-section. The diagram shows a curved blade profile with a hub diameter indicated by a horizontal line at the base.

$$\begin{aligned}
 &4 \quad V_f = 252.12 \text{ m/s} \quad Z = 11 \\
 &0.86 \quad V_{w2} = 145.56 \text{ m/s} \quad P_1 = 81.6 \text{ kPa} \\
 &0.0119 \quad V_{w1} = 14.45 \text{ m/s} \quad V_1 = 256 \text{ m/s} \\
 &320 \text{ k} \quad W_s = 169 \text{ kJ/kg} \quad P_1 = 0.941 \text{ m}^3/\text{kg} \\
 &90 \text{ m/s} \quad R = 0.5 \quad r_t = 17.87 \text{ mm} \\
 &5 \quad \alpha_1 = \beta_2 \quad r_0 = 0.4 \times 17.87 \\
 &= 475.7 \text{ k} \quad \alpha_2 = \beta_1 \quad = 7.15 \text{ mm} \\
 &2 = 501 \text{ k} \quad W_c = 181.9 \text{ kJ/kg} \\
 &L_p = 0.884 \\
 &= 302 \text{ k}
 \end{aligned}$$

So tip diameter and RO = 0.4 into 17.87 = 7.15 MM. So now we have the diameter of the blade tip, diameter of the hub and rest of all the values. So that is all for today in the next class we will start with Jet Propulsion.