Mechanics and Control of Robotic Manipulators
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Lecture No. 08
Kinematic Parameters

Welcome back to mechanics and control of robotic manipulator. So, this particular class would be really going to be useful, I am really happy to say that because we are going to talk about one of the important concept called Denavit-Hartenberg representation. So, mainly we will start with the kinematic parameters now.

So, we have seen in the last lecture like what is compound rotations in that two of the compound rotation representations we have seen, commonly used and then we have raised a question how we can actually like get these compound rotations in a simplified manner? And how we can actually like you can say transform or how we can actually like use a information from the specifically the orientational information from one frame to another frame in a very easiest way?

So, for that actually like what we have come up. So, there are several attempts made. So, we are going to see one simple attempt to call Denavit-Hartenberg approach. So, before going to see that we will see like why we are going that and how we are actually like attempting.

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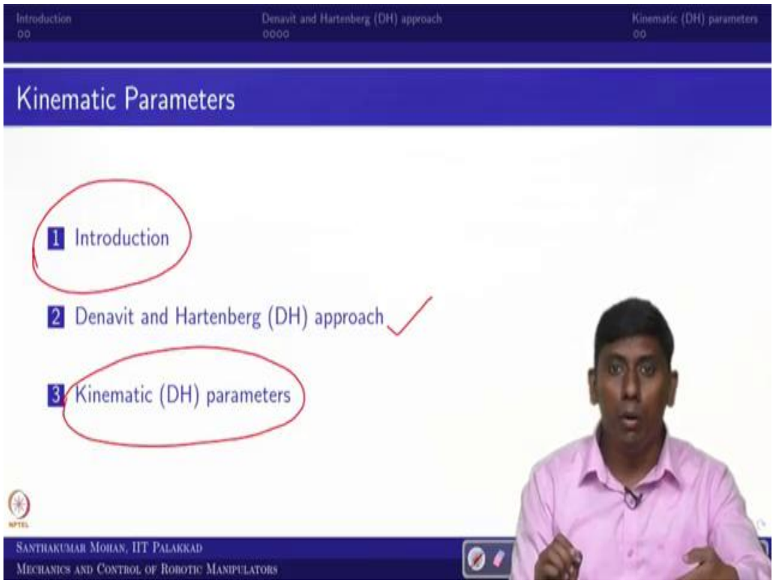

So, in the sense we would be introducing why the Denavit-Hartenberg approach? And what is Denavit-Hartenberg approach? And what are the parameters they have used? So, they simply call Denavit-Hartenberg parameters. So, how this all came into a picture, so that is what we are trying to see here.

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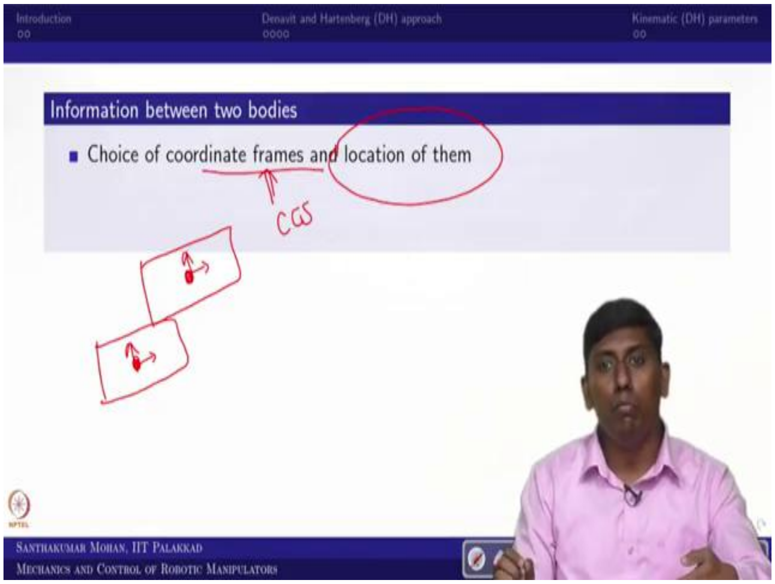

So, if that is a case, so first thing when you are actually like trying to give information between two body for example, this is one body and this is another body. So, what is the easiest way so you are to fix the coordinate? So, now, if you are actually like locating the coordinate, the choice of coordinate is actually like infinite, And location of them also like infinite.

So, one of the easiest thing we have use a Cartesian coordinate system, so that is okay, but location of them is actually like still quite open. So, one of the easiest way people always do in rigid body, so they take the centroid and the centroid they will fix the frame. But that may not be useful for the manipulator or the mechanisms, why? That we will be see.

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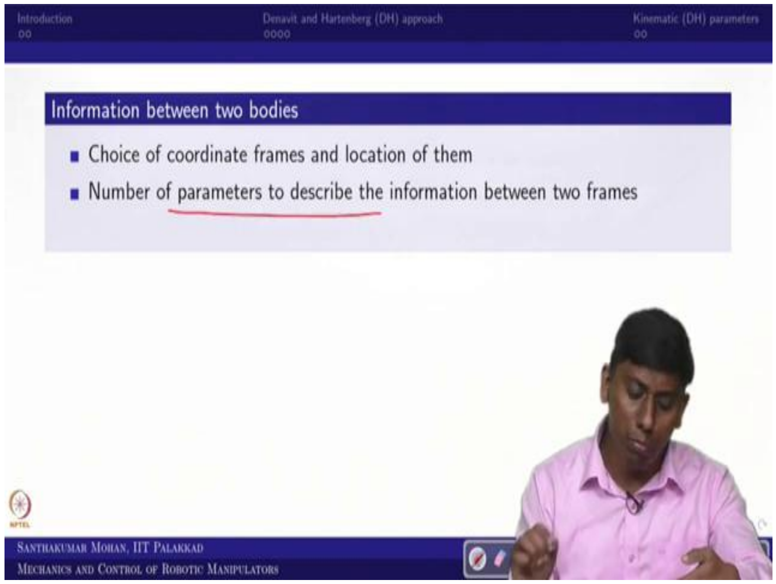

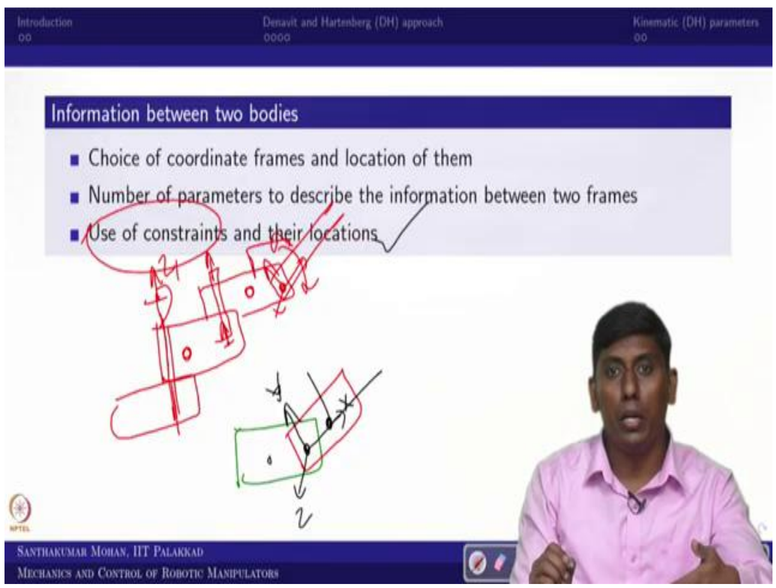

So, that is what I say the number of parameter to describe the information between two frame is not so easy, if you are actually like represent with centroid, it is probably easy if you are talking about only multi body dynamics in a rigid body approach. But in a manipulator approach it may not be that quite easy. So, that is what we are actually like trying to say.

So, in that sense use of constraints and their location would be benefit rather than using centroid for example, now I assume that these are the two body and these two bodies actually like having one rotary joint and instead of using these two CGs, can I use one frame here, another frame here. Similarly, I can actually like go across, so how a number of bodies so I can actually like use these constraints, which is nothing but the joints and their locations.

For example, now the joint is actually like having only vertical rotation. So, in the sense I can keep Z axis here, so that I can relate the rotation with respect to that axis rather than having a normal way of representing with respect to CG, I can actually like modified. For example, now there is another link which is actually like rotate with respect to this in this form. So, now I can keep the coordinate in such a way that I can write this form.

So, if that is the case, very detail I just show. So this is one body so and this is another body. So, this two bodies actually like rotating. So, now instead of having this, I can actually like make it my z like this, x like this, y like this. But if I do the centroid, so centroid would be coming and the centroid would be along this like.

So, instead of that I can use that you can say joint or constraint and their location for you can say transferring the information between one body to another body. So, this is one of the important aspects. So, once you do this, what benefit you will get it?

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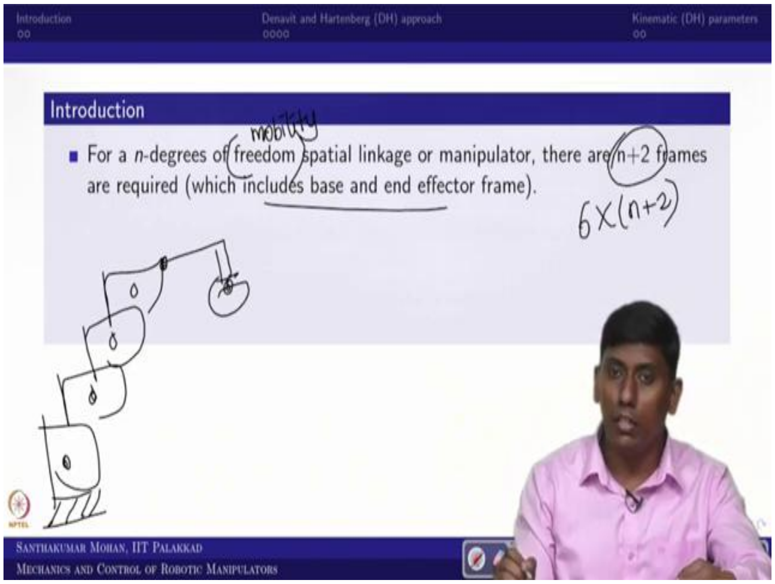

So, there are several benefits so before going to see the benefit will start. So, if you have n-degree of freedom spatial linkage. So, in the sense n degree of actual like mobility I can write. So, so mobility you have a number, so how many actual like bodies involved? So, obviously n bodies would be involved. So, in inbuilt so what you can see like this is actual like number of bodies.

So, now how many frames required so one body each, in addition to that unit you need to actually like fix the inertial frame. So, in the sense you need one additional frame and similarly you are going to use as a useful outcome on the end. So, end-effector frame. so in the sense if you have n axis spatial robot serial robot, so then you have n plus 2 frames are required. So, which includes base and end effector.

So, in that sense how many information's you have to derive, so you have to actually like derive 6 into n plus 2. Can I reduce it? That is the biggest question that is what we are actually like trying to say.

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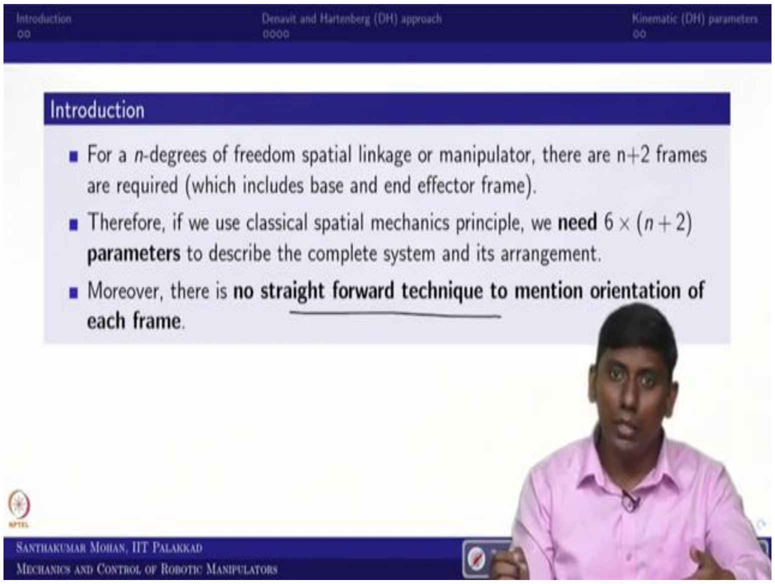

So, therefore, if we use a classical spatial mechanics principle, we need 6 times of n plus 2 parameters. So, to describe the complete system and its overall arrangement. So, if that is the case what additional complexity will come, so there is no straightforward technique to mention the orientation of each frame. So, if that is the case can we bring it some simplified method.

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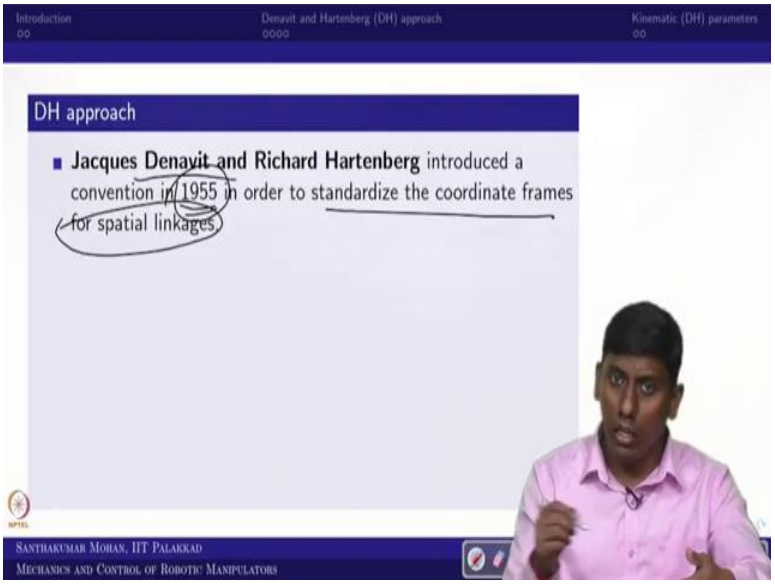

So, there are several methods, in that one of the popular method we are going to take. So, that as given by you can see Denavit and Hartenberg jointly written a paper you can see on the kinematic parameters. So, this published in 1955 in order to standardize the coordinate frame for a spatial linkage.

So, that is what the whole idea, it is not intended to do the you can say serial manipulator or serial robot, it is mainly intended to us spatial linkages. In fact, it was actually like decide for open chain. So, that is what we are going to use. So, how are what they did that we have to see.

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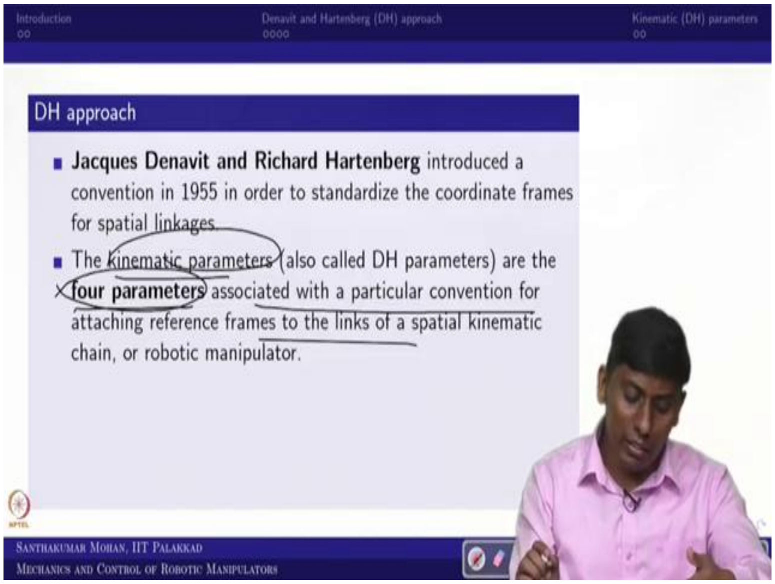

So, what they said so they said we can use only 4 kinematic parameter, so they use a kinematic parameter only 4 instead of 6. But what they said it would be associated with a particular convention, so that you can actually use only 4 parameter, that is sufficient to represent instead of 6. So, then there is supposed to be some you can say pseudo or passiveness. So, that is what we are going to see in the next slide. So, but before that what are the parameters they said?

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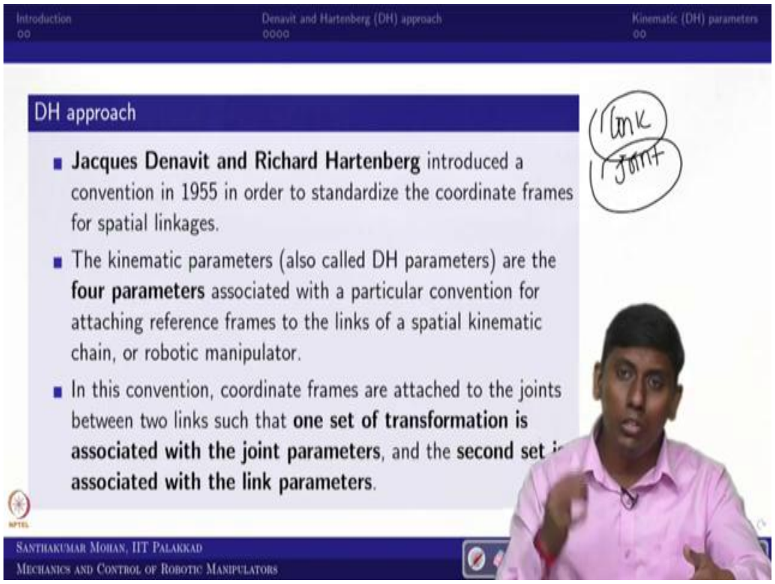

So, they said there are actual like four parameters, but these four parameters would be associated with what you call link and joint. So, they said so two each on link and joint can be associated in the sense the relative information of links can be represented from joint and relative information of joint can be represented with a link. So, in that sense it is actually like somewhat tricky, so something is missing. So, for that what we are actually like bringing it. So, the original idea of what they have given in a detailed manner.

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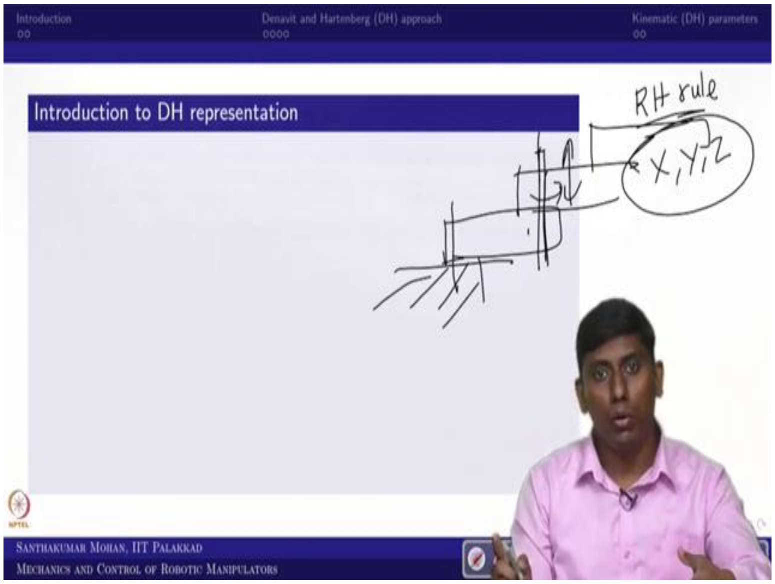

So, what they have given? So, they said there are two sets, the first set would be you can say joint information, the second set would be the link information. But how they did? So, they did not do anything beyond us. So, what they said we would be using a hand rule the first thing, so in the sense x, y, and z, axis would be coming as per your hand. So, in the sense if you have two axes, so one axis would be easily fixate based on the right hand.

So, similarly, the rotation direction again hand rule, the thumb finger would be associated with your first two direction of axis. So, then the rotation direction positive would be how the remaining forefinger coming outward, this is very straightforward, in addition to that what they said. So, any link you take that link would be having only two joints, so which we mean to say is binary links.

So, binary link means the link would be having only two connections. So, any body you take that would be having only two connections. So, in that sense the ground would be excluded, because the first link you take that would be attached with the ground based on this. So, then the second link to third link, third link to fourth link like that keep going with the two joints.

So, because of that what happened, so one link to another link is constraint with only one joint, further what they constraint this one joint would be having provide or it alone only one motion, in the sense this can actually like rotate with respect to this axis, or translate with respect to this axis in the sense. So, Denavit-Hartenberg indirectly they said use binary link and each joint would be one DOF joints. So, these are the constraints which they put it.

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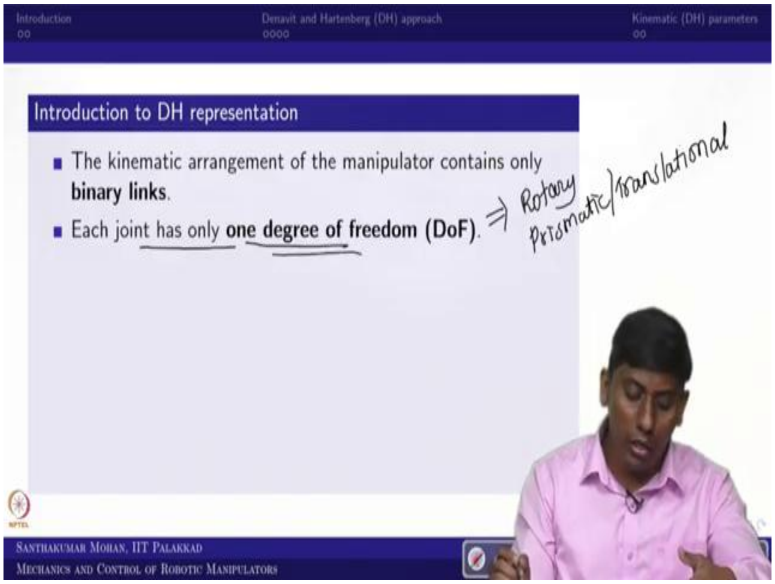

So, if I actually I put that way, so the kinematic arrangement what they said it contains only binary links. The second point they said each constraint or joint has only one degree of freedom in the sense, the joint can be either rotary, or prismatic, or you can call translation joint. So, this is a way.

Then you can ask if I have a spherical joint, then you have to actually like virtually take three bodies, because it is actually like spherical joint as a three degree of freedom, then you have to take it is actually three bodies with the three joints, so then only it would be fulfilled. So, this handicap is there, but this is the I call pseudoness, but what we have got? So, one degree of freedom joints, all the joints.

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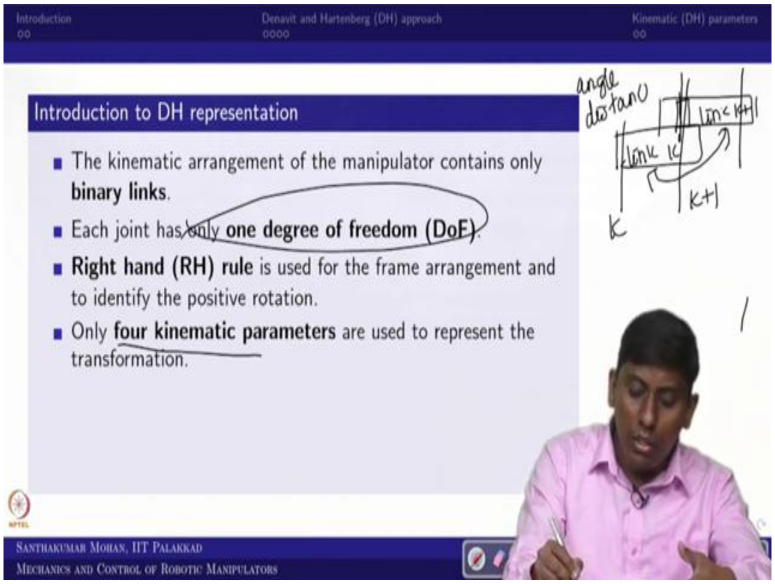

So, further what they have done as I already mentioned they have used the right hand rule. So, in that sense if you have two axes, you can always take the third axis, further they said four kinematic parameters. So, they say two associated with the link, two associated with joints. So, for example, now as per your like as per their conditions, so this is link k and this is link k plus 1. So, this would be k to k plus 1 frame, this is k plus 2 frame like that the joint.

So, this is a one joint, so if I take this joint, this joint is giving what the relative information between link k and k plus 1 and based on the constraint, it is only one DoF system, the information would be actually like having only two forms . So, what that would be? One would be angle, the other one is distance, so that is what they said.

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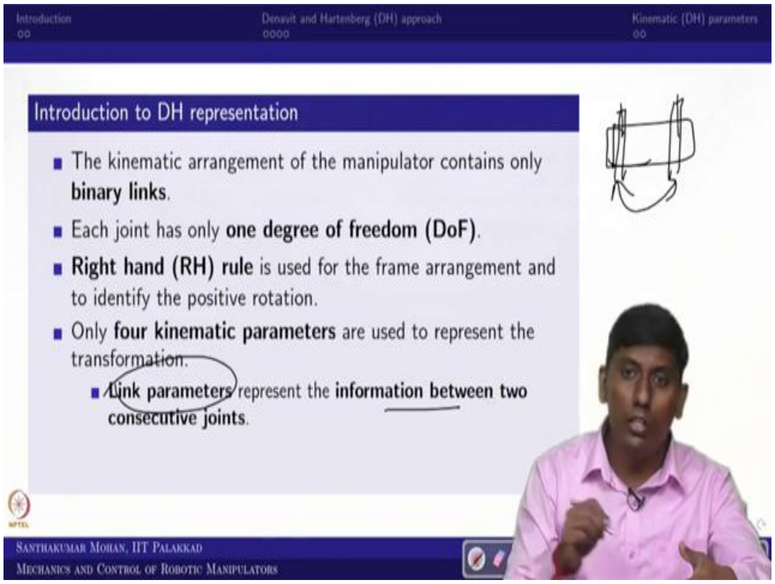

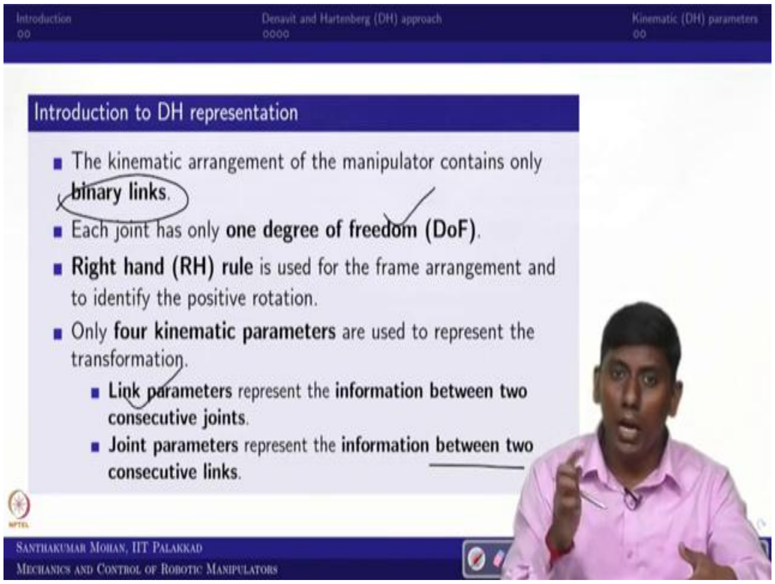

So, in the sense what they said so the link parameter again, so you take this is actual like you call one link, this link would be having two joints. So, if I take this link, this link would be giving a relative information between one joint to another joint. So, that is what we are actually like trying to say. So, in the other way around you can say the link parameter giving information between two consecutive joints whereas, the joint parameter is giving information between two consecutive links, you are clear on this.

So, first thing it is a binary link so if you take any link that would be having two joint. So, the joint information can be given as link parameter, we take any joint that is one DoF system, then that would be having two consecutive link information, so we will see in detail.

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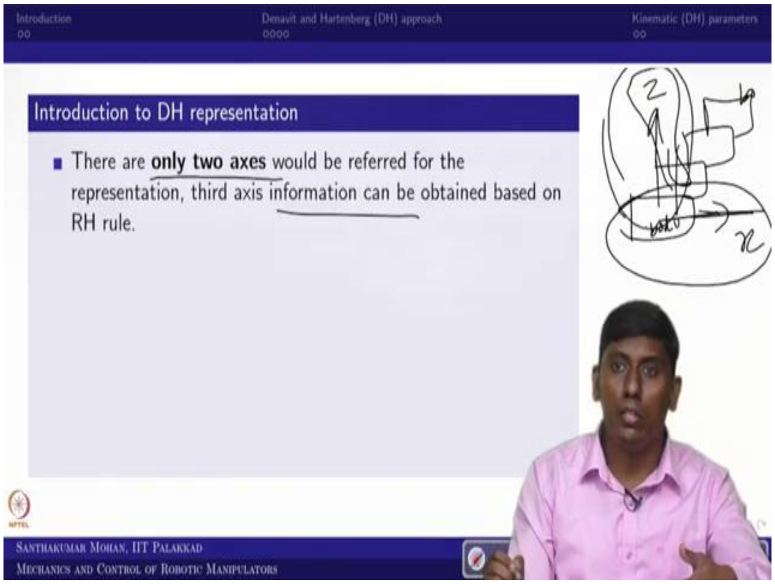

So, in that sense what we can see only two axes are sufficient to represent this what you call Denavit-Hartenberg. But, what they have done you can see like there are several bodies associated. So, I am actually like saying this is body 0 to body n. So, how I have accumulated? I accumulated one by one, in the sense I am stacking it. So, what axis in general it is stacking? It is stack in a z axis.

So, in the sense one body attached with another body is joint. So, now, I associated the joint associated with the joints all with z axis, in the sense z axis associated with the joint axis. So, then you can see one axis associated with the joint, the joint will give one link to another link information, in addition to that what you can see this link is actually like expanding laterally.

So, in the sense you can see expand left to right, so what we usually write it in that way, so there is a two option, but what we usually say this we would be write x. So, in the sense the link information we will express in x axis although, we have choice of x and y, but we take x or the Denavit-Hartenberg given as x. So, in the sense the link information always represented with the x and the joint information always represented with z.

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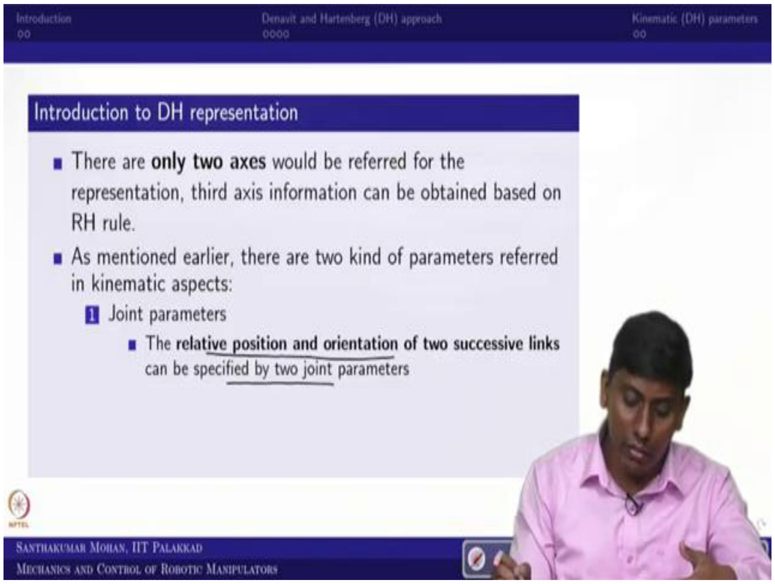

So, that is what we have actually like represented here. So, the joint parameter is actually like relative position and orientation of two successive links can be specified by two joint parameters. So, that we will be seeing explicitly in the next coming slides.

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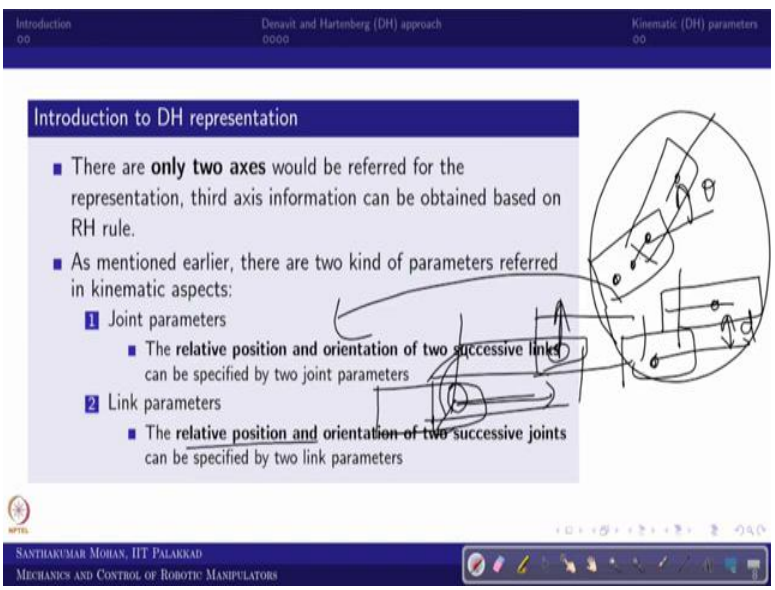

Then the link parameter is actually like the relative position and orientation of two successive joints. So, now you can see this is the three you can say the top view, so this is one link and this is another link. So, the joint is actually giving what so this giving and the frame to frame the distance also it is giving.

So, that how I can see in the top you can see, so this is this actual like I call the distance and this I call the orientation. So, this is what we call the joint parameters which is given where as you can see like the link. So, I am saying that this is one of the link, so that another link is connecting like this. So, the previous link is connecting. So, if I take this, so this is one axis and this is another axis.

So, these two are having angular information and as well as the position information. So, that is what we call link parameter, which actually like giving two successive joints. So, we will actually like go to in deep, I said further information of this.

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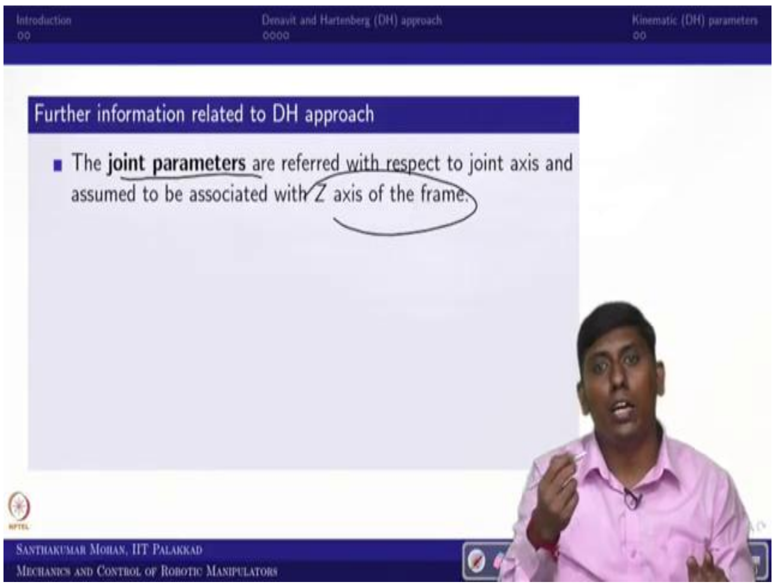

So, what said that the joint parameter always referred with z axis, because the joint information is given with respect to the body propagation, this is one body attached with another body like this, this is actually like propagated in the vertical axis, we always take the vertical axis z axis. So, that is why the joint parameter always related with z axis.

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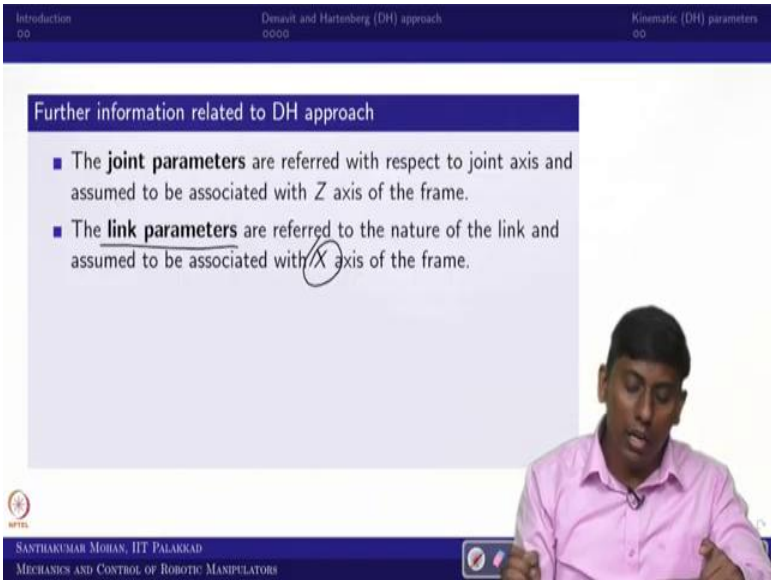

So, in that sense what would be the other side? The link parameter should be represented either y or x. So, x would be the best because you are linked would be actually like connecting two joints, that would be expanding you can say one direction. So in this sense, what you can see left to right or to left whatever we consider, but that we can write it in x axis, that is easiest? So, that is what the idea. So, now this is actually like a link parameter, now we will actually like see further.

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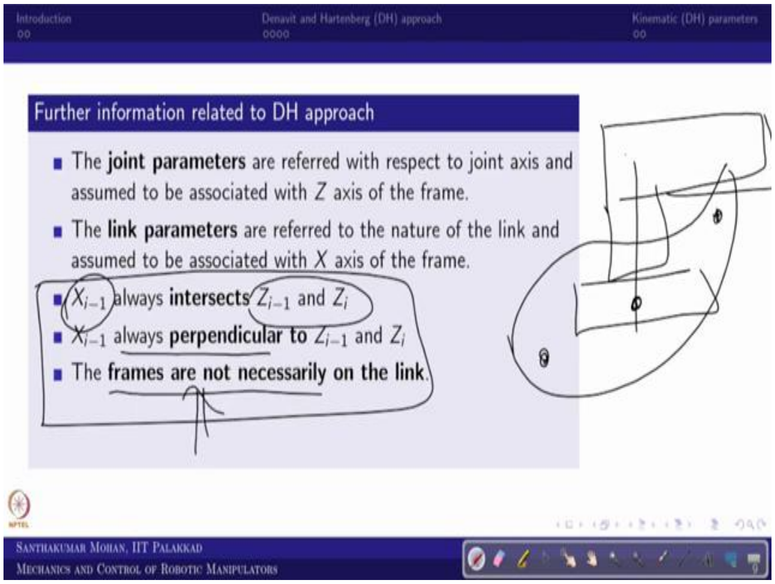

So, in the sense what we can actually like take it further on the X i minus 1 axis always intersect Z i minus 1 and Zi if it is not intersecting then it should be perpendicular to Z i minus 1 to Zi. So, in the sense the frames are not necessarily on the link. For example, so this is the link and here one joint and this is another joint. So, now you can actually like see that although it is actually like, like this.

So, even I can actually like see, so I actually like to take it this is that you can say link. So, now this is actually like attached with another link. So, you can see like even the frame can be fixed here rather than here. So, like that we can actually say that, says, say that the frame may not to be necessarily on the link even it can be outside. So, that is what the whole idea here, so let us actually like move further.

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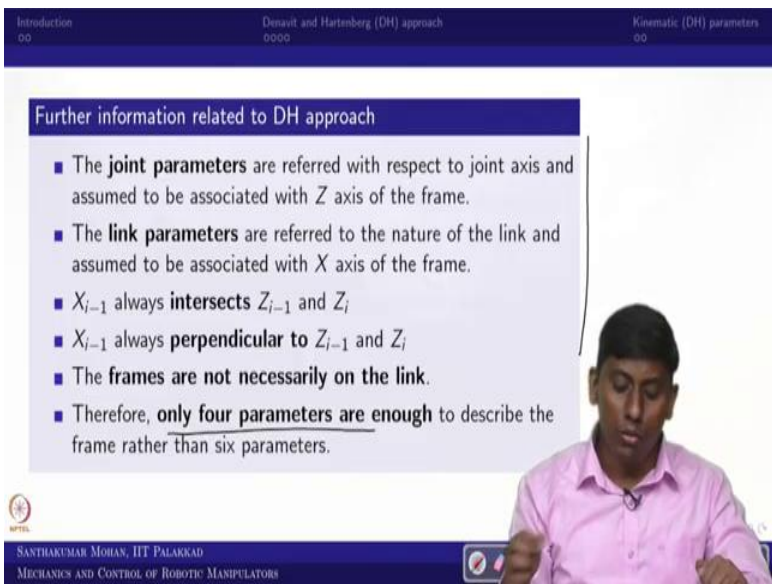

So, because of these all constrain all you can say simplification what you can see like we can use 4 parameter that is enough to describe the frame to frame instead of 6. But what we did? Already we have actually like make it a handicap, or the passivness in the sense the y axis information we completely ignored, because y axis also would be having one rotation and one positional information, that we have actually ignored by considering that the link is binary and the joint is one DoF system.

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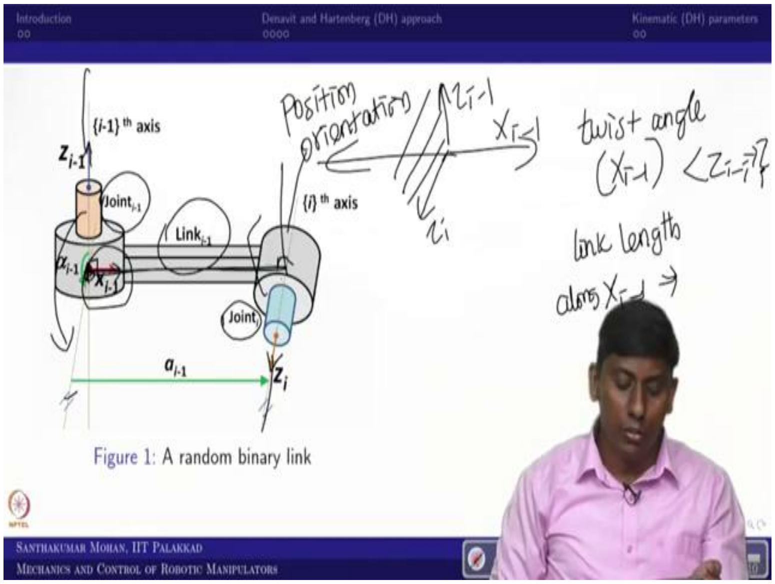

So, let us actually like move to the original case how we can actually like realize? So, for example, I am taking a link k minus 1. So, the link k minus 1 is having joint i minus 1, and joint i. So, as per our understanding this would be having you can say one joint axis which is i minus 1 axis and this would be having i th axes. So, now, these two axes are actually like there and as per our convention, the joint axis supposed to be associated with what you call z axis. So, this is Zi and this is Zi minus 1.

So, now what we are actually like interested? We are interested to find how these two joints are actually like orient and distance apart, in the sense we are trying to find out the position/ positional information and orientational information. So, in order to get that, so what we can do? We can actually like take the previous slide information what it says, so they xi always intersect or perpendicular.

So, in this case, so you can see that this Zi and Z i minus 1 making a plane this. So, now the intersection is not directly happened, but you can dry draw that perpendicular line. So, that would be go either inside or outside for betterment, because the link is propagating the site. So, I am taking X i minus 1 here. So, in the sense xi minus 1 here so even if it is not the way. So, you draw the common normal between these two axes and you fix the Xi axis that way.

So, now you draw the common normal and thinks you are xi axis in the propagation side that i to i minus 1 to i. So, now, what you will get, so you get 1 axis, now with respect to this you rotate

Zi minus 1 with some angle. So, that make parallel to Zi and you translate along Xi minus 1, you will reach from this point to this point.

So, now these are the two information, so what that mean? So, you have actual like this is 1. So, you can see like one of my finger is actually like coming towards you and another finger is actually like coming vertically up. So, these are the two you can say joint access. So, when these two would be parallel, you twist this. So, in other way round it is like this, it is actually to straight like this.

So, now this is actually like what you can call the link twist or twist angle. So, it is actually like the link was actually like to straight in this form. So, that is what you can see, if I actually like kind of twist. So, what happened this would be parallel. So, that is the first information, which we call twist angle, the twist angle is actually like with respect to xi minus 1, what angle is the angle between Zi minus 1 to Zi . So, that is what we can actually like write it.

So, now the same way what would be there, so there would be a distance, so that you can call length. So, this is actually like corresponding to the link. So, it is a link length, so along you can say xi minus 1. So, what the distance between this point to this point, so that is what the link length. So, by nature itself link length cannot be negative. So, you always put xi minus 1 in the propagated side. So, that is the idea.

So, I actually like to explain that even with some kind of PVC pipe in the next lecture, but let us actually like, see what exactly the definition.

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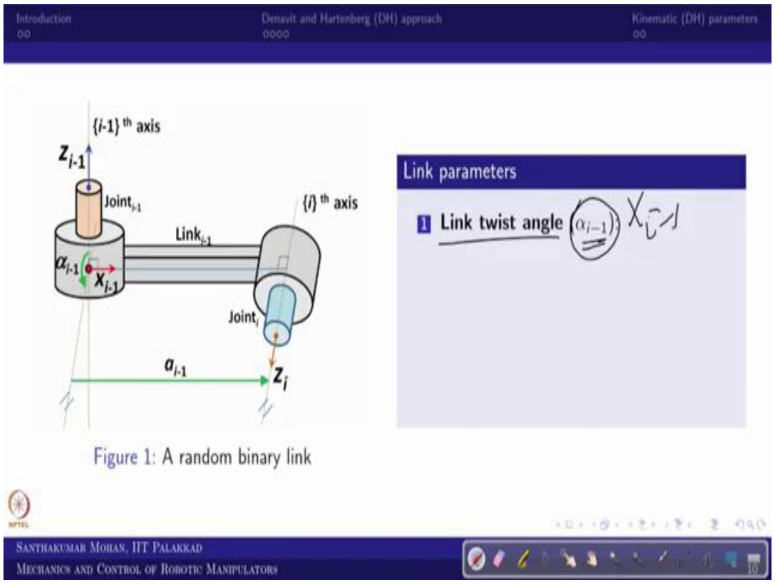

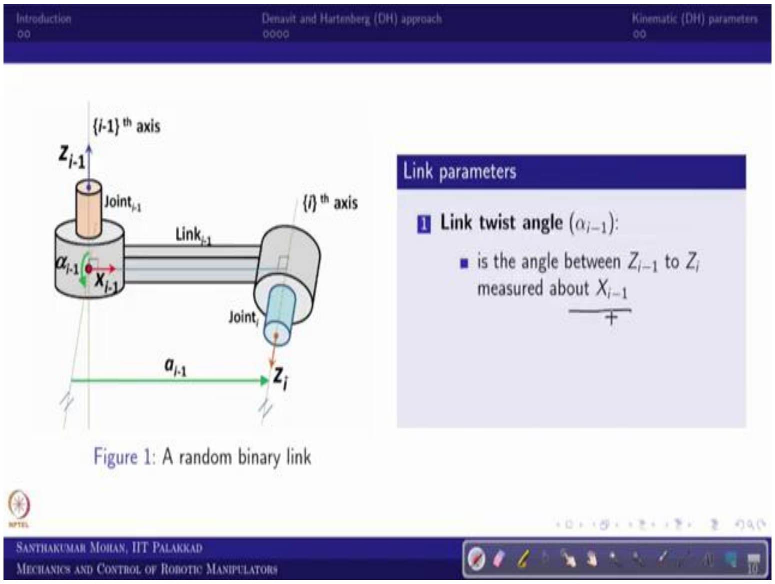

So one is link twist angle, which we write as alpha, so it is with respect to i minus 1 frame. So, it is actually like alpha i minus 1, because we are taking rotation about X i minus 1 that is why we have written as alpha i minus 1. So what that? So, this is actually like angle between Z i minus 1 to Zi with respect to X i minus 1 frame, or X i minus 1 axis, So, measured about X i minus is 1, you are clear I hope.

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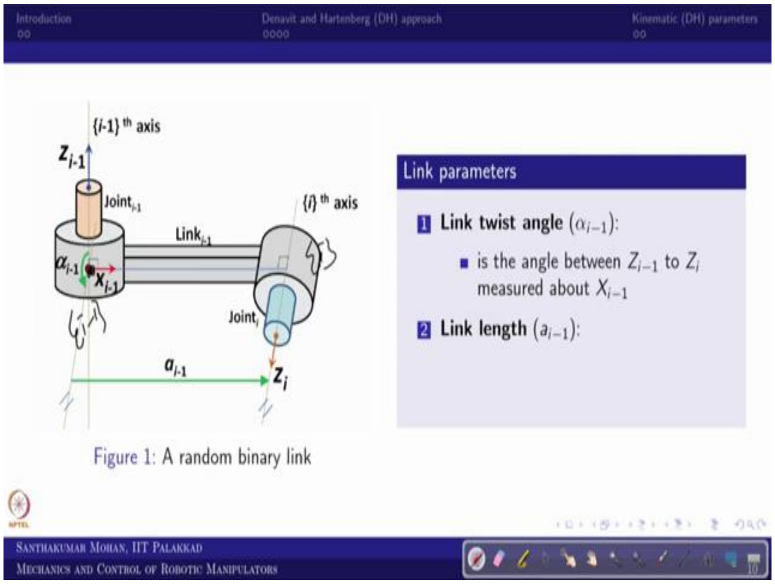

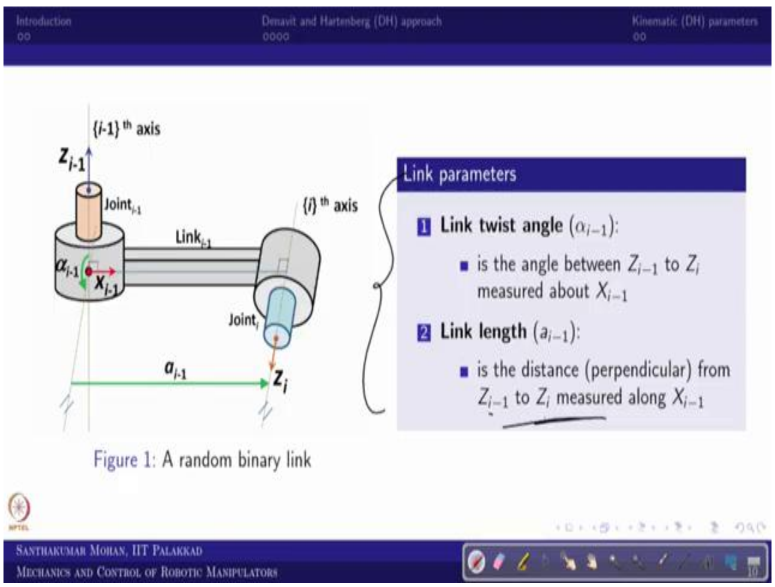

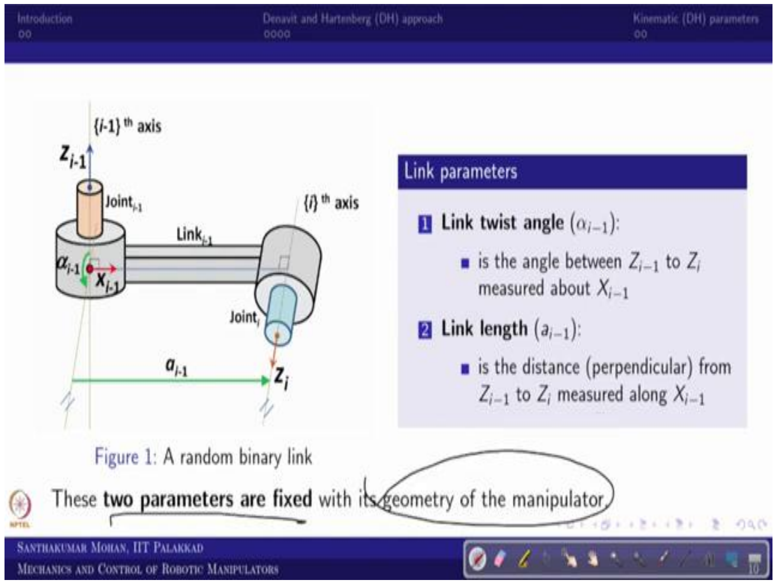

So, now next thing is link length, so it is a distance between you can say the point which is actual like intersect, or you can say i minus 1 frame to i frame I can write. So, this is the distance along Xi minus 1. So, that is what you call link length, you can actually like see.

So, the distance between Zi minus 1 and Zi measured along you can say Xi minus 1 you call link length. Now, as per our understanding link length cannot be negative, so that be clear and further as per our understanding, so the joint axis always associated with Zi or Z. So, in that sense these two parameters are constant. So, in the sense these two are fixed, so these two parameters are fixed, or simply you can call these are geometric.

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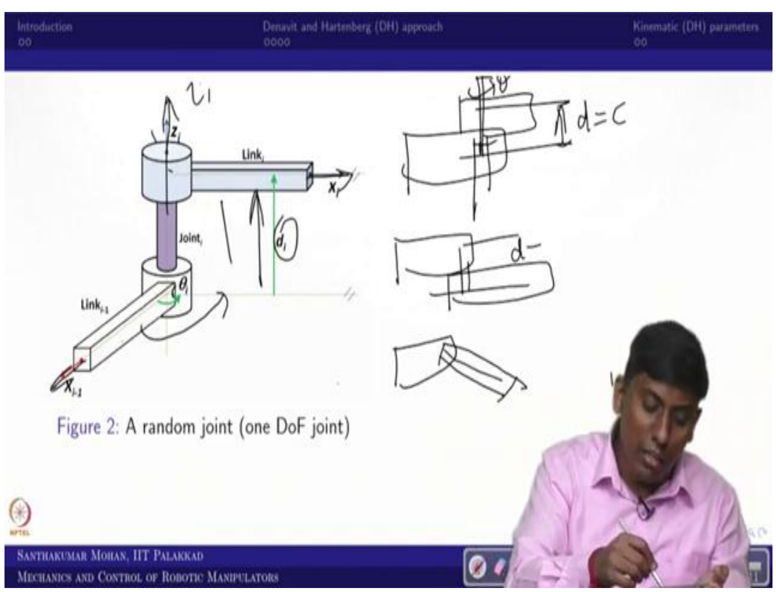

So, let us move to the next one, we call joint parameter for that I am taking joint i which is connecting link k i minus 1 and link k. So, now you can see like this is your i minus 1 frame Xi and this is actually like ith X. So, this is Xi and Xi minus 1. So, you can see like this is the joint access we fixed it Zi. So, with respect to Zi you can see that there is one angle and one distance

So, what this is giving? So, link i k minus 1 to ki, so r so link i minus 1 to link i there is a position and you can see orientational difference. So, that is what we are trying to give. So, first you will actually like rotate about Zi try to make it parallel Xi minus 1 to Xi. So, this angle call joint angle. So, this can be variable if you are taking a rotary joint, if you are taking Zi or you can say this to this average distance, if you are using a translation joint.

So, this would be as a piston cylinder kind of thing this would be a variable. So, for example, you have this is the case this is you assume as a rotary motor you attach. So, now what happened this offset distance would be a constant, but there is offset distance but this theta would be variable. So, the other case where you can see like this is you can say expanding as translation joint.

So, in this sense what you can see this distance is variable, but if you look at in the top view, if you are having some kind of angular information. So, this would be constant for you because it is going to translate only up and down. So, that is what we are actually like trying to say, but here what we are referring everything with respect to Zi.

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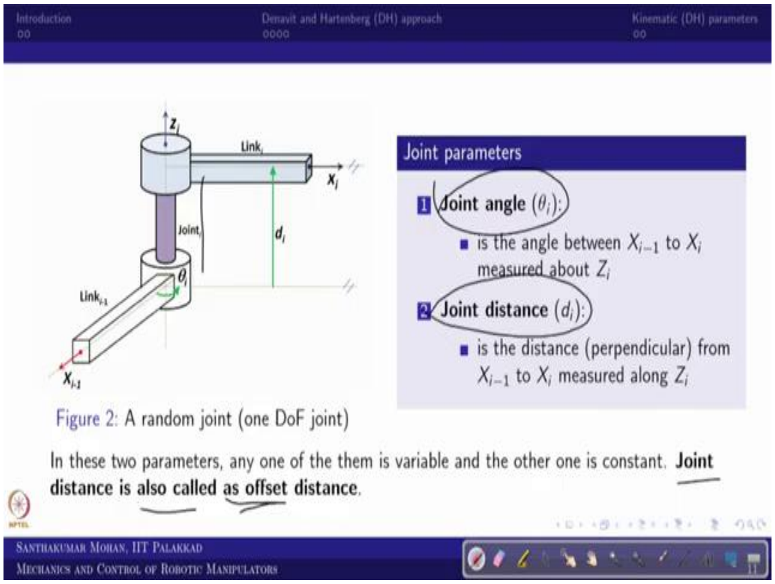

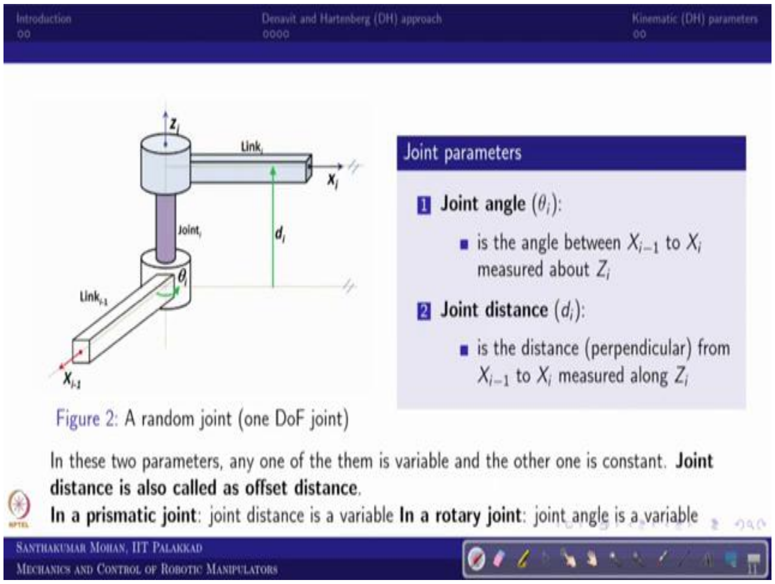

So, in the same joint parameter one is joint angle just joint angle is nothing but the angle between X i minus 1 to Xi measured about Zi, so which is actually like given here. So, the second one is actually like a joint distance. So, which is actually like again the distance between you can say xi minus 1 to Xi measured along Zi. So, now these two are actual like important so in that as per Denavit-Hartenberg one of them is variable, and sometimes the joint distance people call offset distance because it is offsetting the planes.

So, that is why it is called offset distance. So, further what you can actually like see that these two are actually like any one is variable, if it is a rotary joint, so the joint angle is variable, if it is a prismatic joint the joint distance is variable. So, that is what we have given. I hope now you are clear what Denavit-Hartenberg has done.

So, he has given only four parameter, which we call link twist, link length, joint angle and joint distance, we will see a little more explanation with respect to you can say a small PVC connection as a manipulator. I will explain on you can see screen. So, with that we will actually like conclude the Denavit-Hartenberg parameters.

And then we will go to the Denavit-Hartenberg representation, how to bring you can say the frame information in terms of arm or transformation matrix. So, that would be seeing in the next lecture. So until then, thank you and see you, bye. Take care.