Mechanics and Control of Robotic Manipulators
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Compound rotations part 2

Welcome back to mechanics and control of robotic manipulator. So last class we have seen that so what is individual rotation matrices. And then we have given an introduction to what you call compound rotations. Even in that class itself I told that we can do two things if you are rotating any frame with respect to you can say fixed axis then you are you can say pre multiply that rotation matrix.

Whereas if you are rotating with respect to you can say moving access, so then you have post to multiply. So that we have seen and then we have seen that there are several common representations are available in that two of them we are going to see in this particular lecture. So one is we call cardan angle the other one is we call the Winkel or Bryant angle. So simply we call one is global fixed so the other one is Euler angle.

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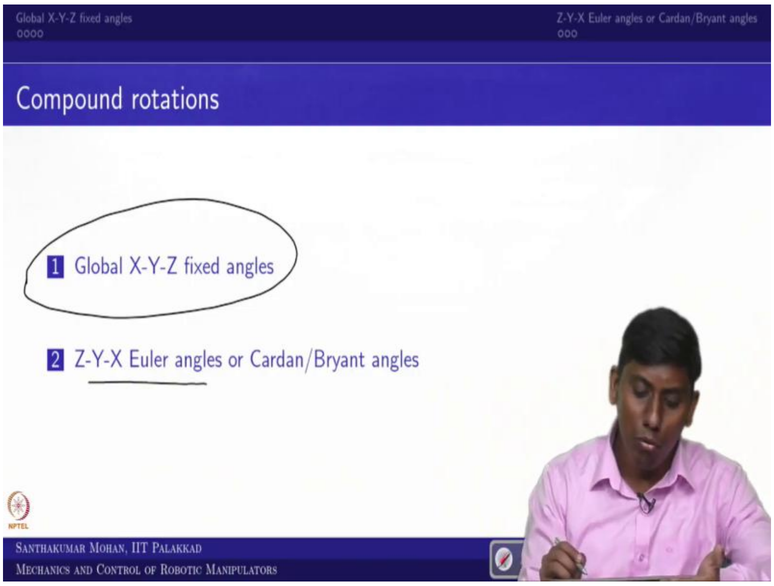

So, let us move to that into the lecture. So what we are actually going to see is one is what you call global X Y Z fixed angle which we simply call R P Y in the sense roll pitch yaw or some people call it is a you can save in Winkel angles. So whereas the other one is Z Y X Euler angles or simply we can call Cardan or Bryant angle. So let us move one by one.

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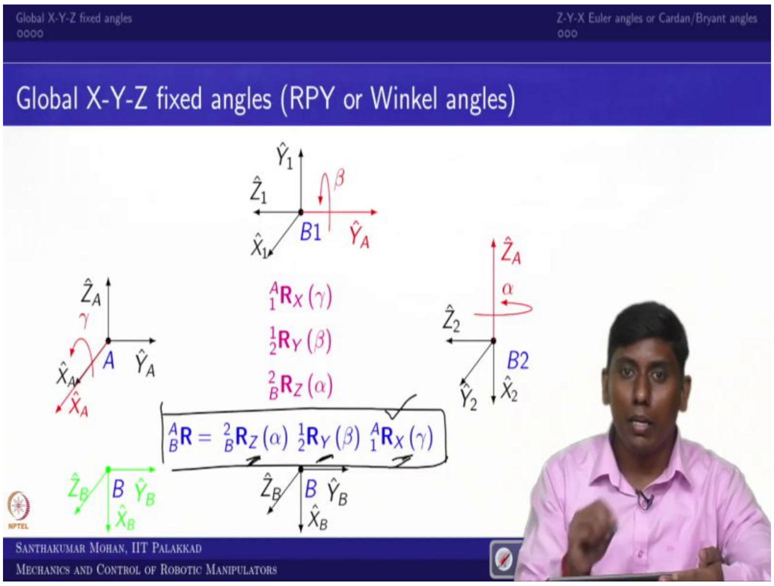

So, for understanding that we have taken two frames, so where A is the fixed frame and B is the final body frame, so now you can see that these two are having different representation. So at least we can see like three orientational information is required. Straight forward you cannot get simple with one or two rotation. So in that sense what we can try to do we will start rotating with respect to x axis.

So certain an angle then we will go to y axis certain angle then the Z certain angle. So here I make it inconveniently it is make 90 degree each. So, for example, if I take X A axis rotate the frame. So then I will get a new frame that would be I call rotated with respect to fixed X A axis. So then I will get one rotation matrix then I take that corresponding frame again I will rotate with respect to Y A frame then I will get another frame.

So finally I will rotate that with respect to Z A which is fixed frame then that will finally end up with the B matrix this is what we are trying to do. So in that sense we will see first what we are trying to do we are trying to rotate gamma angle with respect to you call X A cap. So now what it will do it will take this here and this goes here. So I will get a new coordinate so that I am calling as a B 1.

So what I did so this would be X A cap I taken as the reference I rotate that. So now I got a new frame that would be X 1 so Y 1 and Z1. So this is one new coordinate. So now what I am trying to do the second one I take Y A cap as the comment and rotate this again with some other angle called beta. So now I am first rotate it so rotation matrix of 1 this is the B 1 with respect to A is rotated about x axis and a gamma angle.

So now we will go to the next one. So I am rotating this with respect to the fixed axis. So Y A so I am rotating and beta angle so this is the rotation angle. So this much angle I taken. so now I will get a new thing for simplicity I have taken beta as 90. So what I will get I will get a new frame called B 2. So now you can see like what I will get I will get R Y beta which is a in right rotation matrix of B 2 with respect to 1 B 1 so this is the second one.

So that is what we did. So now finally what we are trying to take we are taking the Z A cap and rotate you can say alpha angle I am trying to see what would be the frame representation. So now if that is the case what we can see. So I take this and rotate alpha angle this is the positive rotation. So I take this for convenient I take a 90 degree what I will get I will get this you can see.

So now what I did, I did Z axis rotation of alpha where that will give so B 2 so B. So this is what I get. So now what we did so we did that one simple example say that if you rotate everything with respect to fixed axes then you have to pre multiply then what you are final rotation matrix supposed to be so your rotation matrix R you can say B with respect to A should be so first you have rotated you can say like X axis as a gamma angle.

Which give rotation matrix of B 1 with respect to A then you are taking the second one then you pre multiply so then you will get R Y beta. So this will give 2 to 1 B 2 to B 1 then final you have rotated about z as alpha. So that would be given what you can say So B 2 you call B 2. So this is what you get it. So that is what we are also writing. So now if you see that this is the final rotation matters of B with respect to A. So I hope this is clear to you.

$$ {}_{B}^{A}\boldsymbol{R} = {}_{B}^{2}\boldsymbol{R}_{Z}(\alpha)\, {}_{2}^{1}\boldsymbol{R}_{Y}(\beta)\, {}_{1}^{A}\boldsymbol{R}_{X}(\gamma) $$

Now we will put the individual rotation matrix if it is X axis rotation. So you know what if it is Y axis rotation what will be the individual matrix. If it is Z axis rotation what would be the individual matrix. Now you are clear why we have attempted this in the previous lecture. So now we will substitute those values directly.

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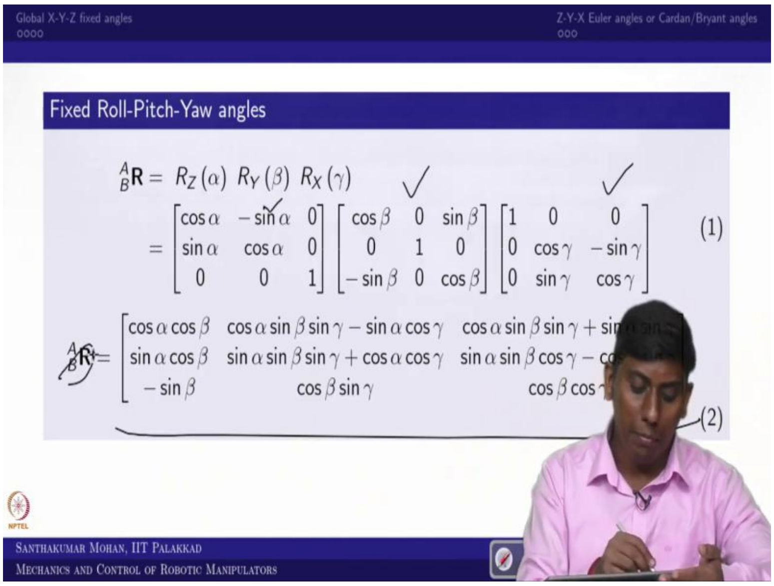

So this is the x axis rotation and this is the y axis rotation and this is the z axis rotation you multiply this end. So you will get this bigger matrix. So this bigger matrix is what so this is what the rotation matrix of B with respect to A. So, now if you are taking this into consideration, so what would be this equal into?

$$
{}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} cos\alpha cos\beta & cos\alpha sin\beta sin\gamma - sin\alpha cos\gamma & cos\alpha sin\beta sin\gamma + sin\alpha sin\gamma \\ sin\alpha cos\beta & sin\alpha sin\beta sin\gamma + cos\alpha cos\gamma & sin\alpha sin\beta cos\gamma - cos\alpha sin\gamma \\ -sin\beta & cos\beta sin\gamma & cos\beta cos\gamma \end{bmatrix}
$$

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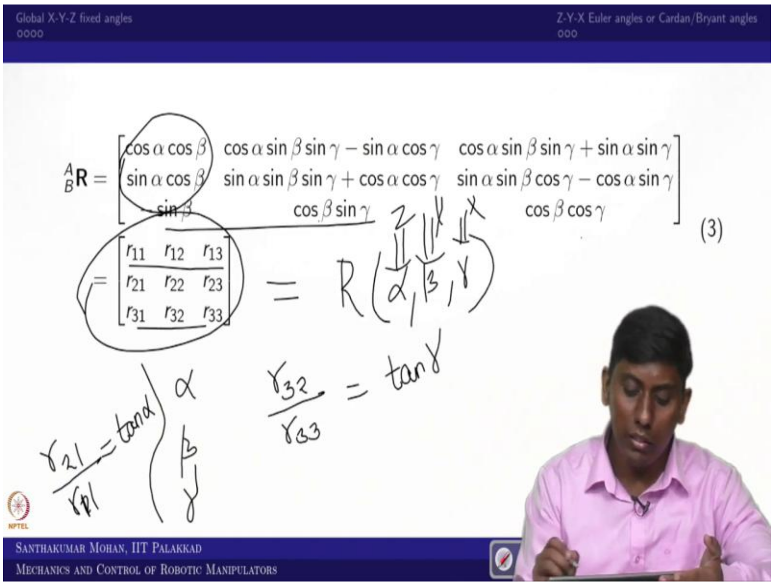

So this is the bigger matrix. So I have taken the same matrix but I have taken a conveniently safer place where you can see. So now this is equal into what I am writing as r 1 1 to r 3 3. So now what this? This is equal into R you can say alpha, beta; gamma where alpha is a rotation about Z axis. So beta is rotation about y axis this gamma is rotation about x axis. So I want to find out what is alpha what is beta and what is gamma if I know this value.

If I know this final end as this, can I find this? So for that what we are trying to do we are trying to use this relation. So if I take these two. So for example if I take r 3 3 and r 3 2 for example r 3 2 divided by r 3 3 what that will give that will give tan gamma. So similarly I take these two.

So what that will give so that will give something like so I say like r 2 1 divided by r 1 1. So that will give tan alpha. So like that I will get one by one. So once I get this so then I can use it for this and I can use by substituting this I can find the beta as well.

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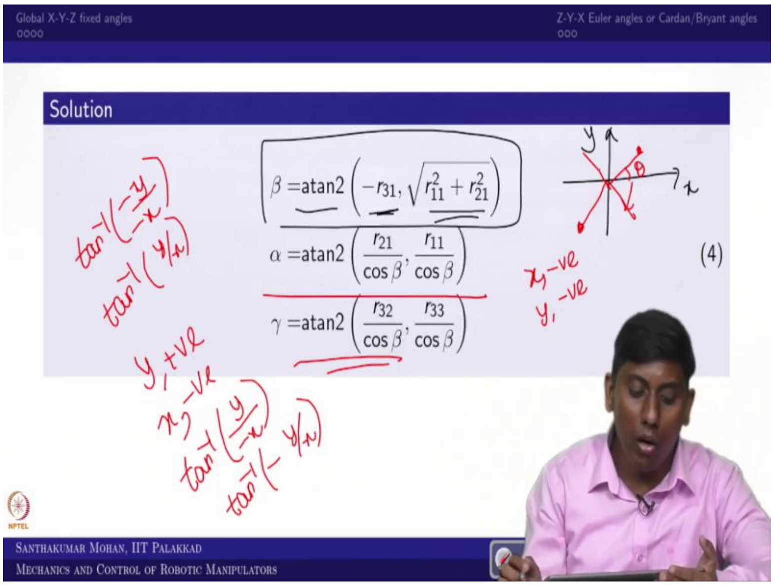

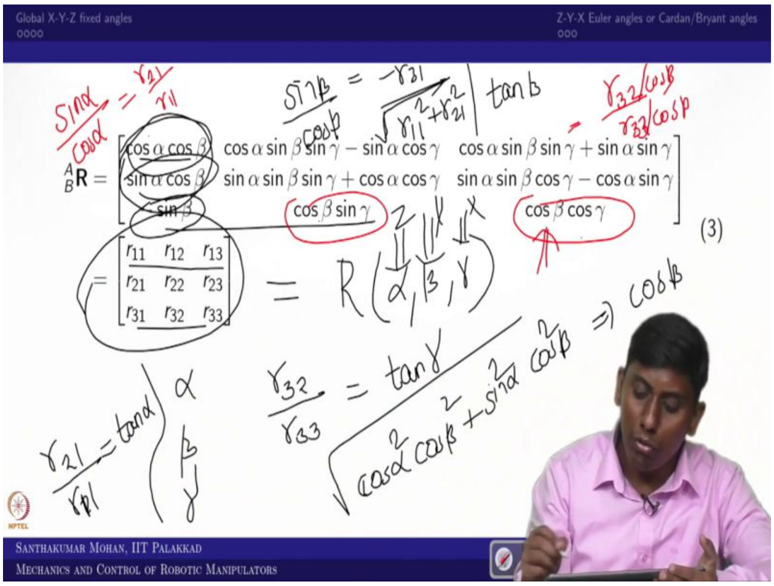

So that is what I am writing as the equation. So you can see like if I take if I square and add and take the square root what I will get that would be get it equal into cos beta. So that is what you get as a cos beta. So then what this is will give you can say sine beta equal. So then that is actually going to give so beta. So you can look at it so r 3 1. So previous so r 3 1. So this is r 3 1 which is minus sine beta.

So, r 1 1 and r 2 1, so if I square what happened? So, cos alpha, so squared, so cos beta squared plus so sine alpha squared and cos beta squared. If I take the square root what it will give that will give cos beta. So then you can find so the beta by taking this tan beta how I can write. So tan beta I can write in the way. So sine beta divide by cos beta. So here is sine beta is a minus r 3 1.

So this we taken as so r 1 1 squared plus r 2 1 squared square root. So this is what we have taken as tan beta that is what we have used. And here we are using a two argument tan what that mean. So you know the tan will give a quadrant basis. If I take you can see x positive y also positive. So what angle for example this is the location. So what angle will give this is the theta?

$$\beta = atan2(-r_{31}, \sqrt{{r_{11}}^2 + {r_{21}}^2})$$

$$\alpha = atan2(\frac{r_{21}}{cos\beta}, \frac{r_{11}}{cos\beta})$$

$$\gamma = atan2(\frac{r_{32}}{cos\beta}, \frac{r_{33}}{cos\beta})$$

So now you assume that x is negative and y also negative. So then what if you substitute with a single argument. So tan inverse of so minus y divide by minus x. So it becomes so tan inverse of y by x. So then it says that although that is there somewhere here. It is a third quadrant but it shows that it is in the first quarter. So that is why we are taking a tan to argument in the sense it is actually quadrant dependent.

So based on that you can say the sine of x and y it differ. Similarly if you take this, so in the sense of y is positive and the x is negative. So what it is supposed to give. So tan inverse of, so y minus x but this is equal into tan inverse of so minus y by x then it will use this value rather than this angle. So in the sense you have to take each quadrant and do it that is why we are using the arc tan to argument.

So then you are clear that it is quadrant dependent. So similarly we will do the, what you call the other angle. So you can see like alpha can be obtained again with respect to the same thing. So you can take these two so where that would be equal into what it is equal to tan you can say alpha. So you can see like a sine alpha divided by cos alpha. So in this case what it will come it will come like r 2 1 divided by r 1 1.

So that we can use it but the cos beta would come. So we put divided by cos beta. So that is what we are using it. So now you can see that this will give alpha the similar way you can take these two equation and substitute once you know the cos beta. So then you can take r you can say 3 2 divided by r 3 1 whole divided by cos beta.

So 3 3 so then that would be equal into tan you can say gamma. So then you can find it. So this is the way we can find for the global x y z which we call you can say Winkel angle we can find it.

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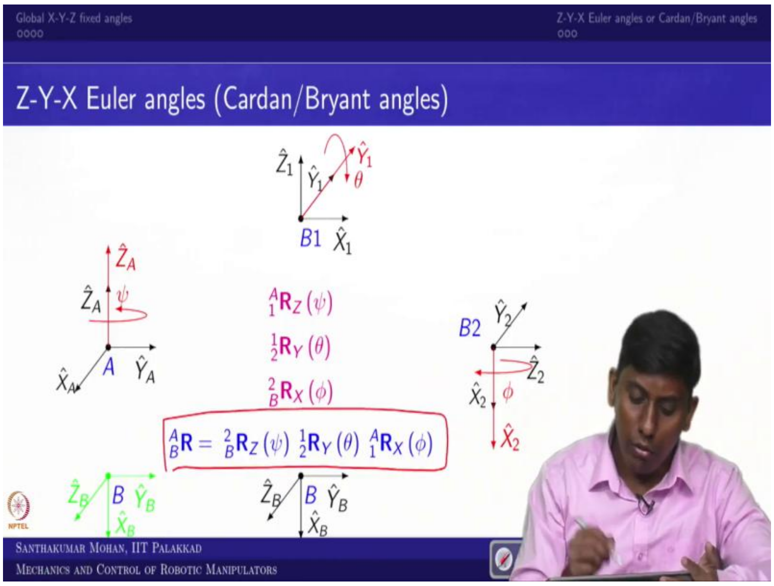

So let us go to the next one which is we call the same thing but we are writing in a moving frame which we call cardan or Bryant angle where we have taken this and we are trying to find. So in this case what first so first z axis rotation so then y axis rotation finally will come with x axis rotation. So, but what with respect to the moving frame? So in the sense right now we assume this is a moving frame so I rotate about this.

I call probably psi angle so then what it is coming as a new coordinate frame from that I will take why I rotate then that will give a new coordinate frame from there I will rotate again x then finally I will end up with this. So we will see that so first I rotate psi angle here. So I like to like get a new matrix sorry new coordinate frame this become 90 I taken so this become X A become you can see here and Y A become here.

So that will give a new coordinate frame which I call B 1. So now what the second stage second stage how to rotate with respect to this. So before that what we did, we did this so we will go the next one. So next one I am rotating about Y 1 theta degree theta angle. So now what happened this would go here and this come here for 90 degree rotation. So you will get a new coordinate frame. So now what we did we did R Y.

So theta angle so which will give 2 1. So now again we are rotating with respect to this. So we are rotating against some other angle called phi. So that will get here so what you can rotate this 90 degree you will get it. So that is what we are doing it. So phi angle and positive rotation all about you can see based on the right hand.

So you are rotation axis you keep it your thumb finger and where these four fingers are coming that is a positive rotation in this case this is the thumb finger where it is coming out toward is the positive rotation in the sense Y 2 become you can say parallel to Z and Z 2 become parallel to Y but opposite direction. That is what here. So now we got the new coordinate which is equal to your original indented.

But what angle we did so x with the phi angle. So this gives B 2 2 but what we did here we did everything with respect to moving axes. So then what we have to do so first we did 1 to A Z psi. So, the second one is Y that is supposed to multiply. So in the sense R Y theta which gives this then finally you did X. So which will give B 2 2.

So this is what you can say like z y x Euler angle which we simply call cardan or Bryant angle. So now you can see like what you obtained this is what the final matrix.

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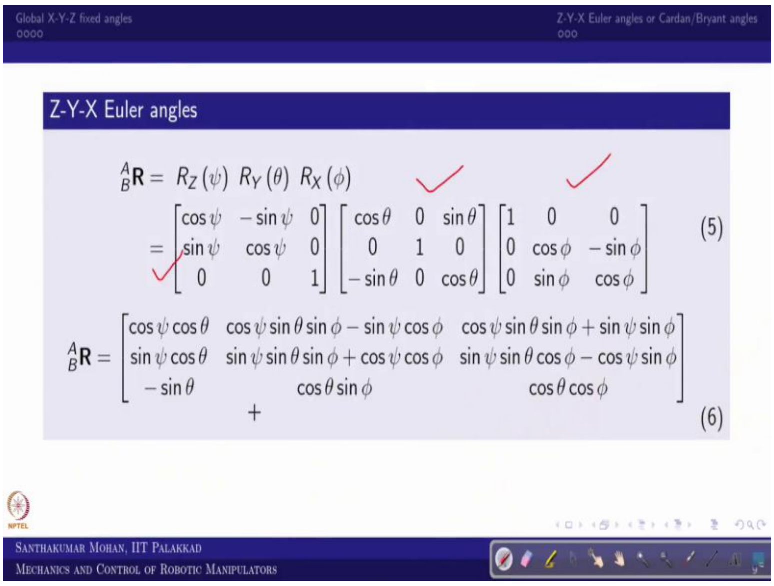

So now this matrix again you can substitute the individual rotation matrices and you multiply this. So what you will get this bigger matrix.

$$ {}^{A}_{B}\boldsymbol{R} = \boldsymbol{R}_Z(\psi)\boldsymbol{R}_Y(\theta)\boldsymbol{R}_X(\varphi) $$

$$ {}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} cos\psi cos\theta & cos\psi sin\theta sin\varphi - sin\psi cos\varphi & cos\psi sin\theta sin\varphi + sin\psi sin\varphi \\ sin\psi cos\theta & sin\psi sin\theta sin\varphi + cos\psi cos\varphi & sin\psi sin\theta cos\varphi - cos\psi sin\varphi \\ -sin\theta & cos\theta sin\varphi & cos\theta cos\varphi \end{bmatrix} $$

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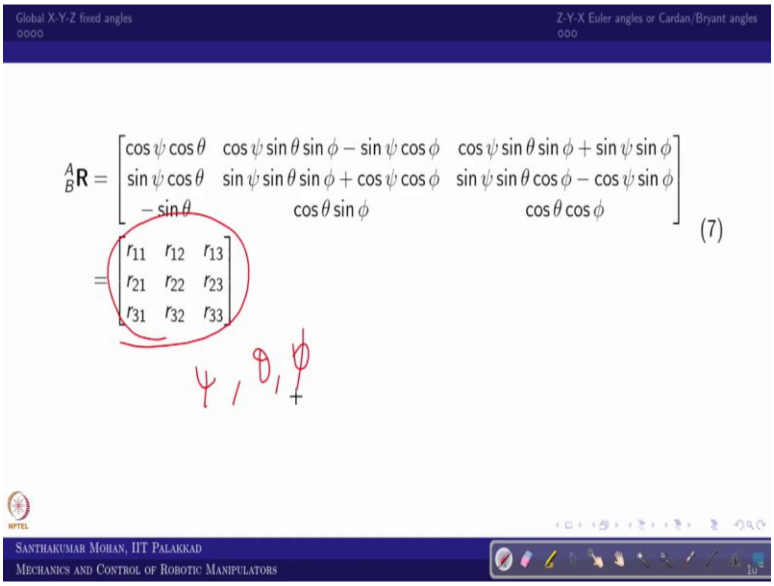

So this bigger matrix again we can equate to this. So then what you can find you can find the psi theta and phi what we did in the earlier case the same way we can try to find. So what that?

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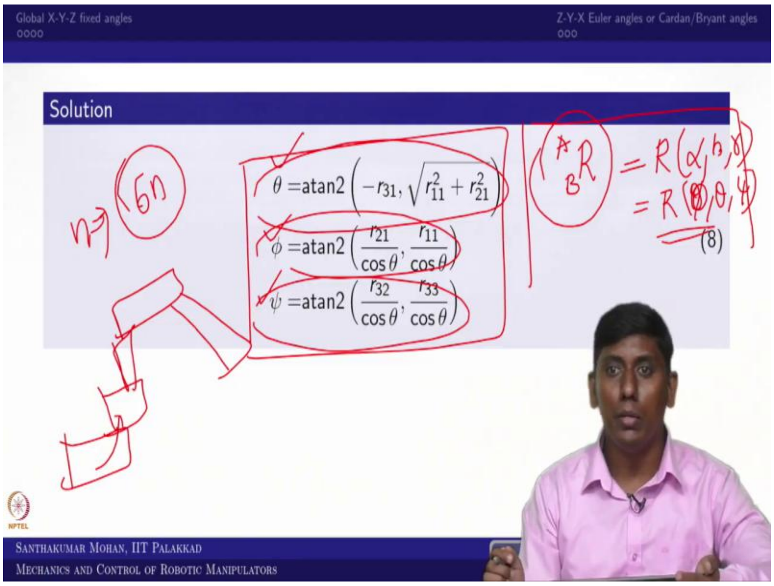

So you can see like the same way we have done and this is theta is a pitch angle local pitch. So this is a local role and this is local you can say yaw. So in the other way around you can say that this is this is heading angle. This is a bank and this is the attitude. So these all we have obtained. So now there would be one bigger questions will come one bigger question will come what that. So sir you have used the compound rotation and that to two different representation.

So but what is the relevance into this course. So there is a relevance because we are trying to find out one frame to another frame with respect to you can say and you can say the orientational information what you call rotation matrix. This rotation matrix would be consist either alpha, beta, gamma or it would be in the form of you can say like phi, theta and psi. So either way you have to write but is it very simple? no!

If it is rigid body that to like only one body or maximum you are using two body system you can easily do it this. However if you are using multi body system for example you have a mobile robot, sorry manipulator which is having a several body. So one body to another body information would be having you can say three translational information and three orientational information.

The translational information is very straightforward because it is a given as a position vector. However the orientational information is not straightforward, you cannot directly write alpha, beta, gamma. So you have to get some rotational matrix. So for finding rotational matrix how will you get this alpha beta gamma. So this is one of the question and there is no straightforward technique to use this.

So that is why people are bringing some other you call convention. So especially for you can say manipulator either it is serial manipulator or parallel manipulator. The general representation may not be explicitly used because the constraint is it is composite. The composite is giving some complexity. This complexity arises so there is a huge question. So in addition to that if you have n bodies. So, how many informations you should have.

So one body to another body information you should have 6 valuable variables less than three translational and three orientation you have n bodies how many variables required to describe the entire system in a you can say kinematic form. You should have 6 n variables although the total variable to describe the overall system although in the spatial it would be having only 6 which is we call degree of freedom.

But the individual you can say representation of one body to another body required 6 if you have n bodies you have 6 into n. So can we reduce it or can we use some simplification or the rotation matters can be modified as some kind of simplified form. So there are certain forms which we people use in robotic community. So one is one of the popular method called Denavit-Hartenburg. So the other popular method is a screw algebra.

So unfortunately this particular course you would not be talking about screw algebra anymore. So here we would be talking about Denavit-Hartenburg. So in the next lecture we will be talking about that. So how to represent how the parameters are simplified or reduced or how we are simplify the rotation matrix all it would be same. So until then see you so I hope you are clear about what is Winkel angle and what is Cardan and Bryant angle.

In general you know like what are the representation. So the Euler would be moving frame representation. And the other one is global fixed, you can say x, y z which is most of the manipulator community people uses and a mobile robot community or rigid body dynamic people uses the Euler angle especially z y x. And very few you can say aerial robot or spatial robot community people use is that z, y, z or you can say z, y, z convention.

So with that I am closing even some other convention also there but that and all will come into the upcoming lectures. So with that I am saying thank you. See you then. Bye. Take care