Mechanics and Control of Robotic Manipulators
Professor Santhakumar Mohan
Department of Mechanical Engineering
Indian Institute of Technology, Palakkad
Compound rotations part 1

Welcome back to Mechanics and Control of Robotic Manipulators. So far, what we have seen, we started what is robot and we have come across so far what we call transformation matrix. This particular lecture we are going to talk about transformation matrix as a recap, and then we would be talking about individual rotation matrix for example, if it is only rotated with the X or Y or Z, how individual rotation matrix look like.

And finally, we said in the last lecture itself, we gave a break say that this rotation matrix if in real. So, there is a 3 orientation how we can put together as a composite rotation or compound rotation. So, we will be introducing the compound rotation in this particular lecture, and we will see how it goes one by one.

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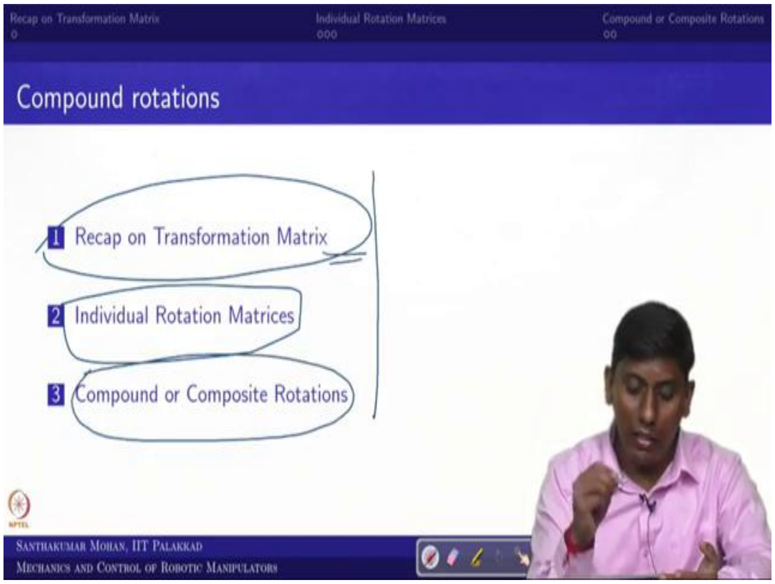

So, in that sense, we can see like, so, the lecture how it goes, so we will be recapping the transformation matrix, will not be seeing in detail, but we will be giving a brief then we will be taking the individual rotation matrix for example, X axis rotation, Y axis rotation and Z axis rotation. Then finally, we will end up with what you call compound or composite rotation.

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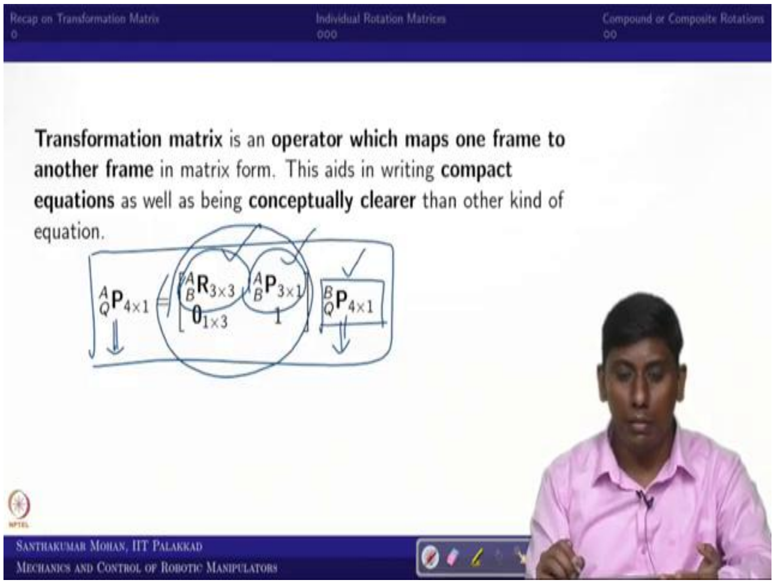

So, if that is the case, we can see like we will take the recap on transformation matrix. So, transformation matrix we know like it is an operator, which maps one frame to another frame in a compact matrix form. So, this aids writing a compact equation as well as the conceptually clear than the other kind of equation, in the sense, you can make one body to another body like that you can stack each box would be represented as a transformation matrix.

For example, this is transformation matrix of 1 with respect to 0, this is 2 with respect to 1 and this is 3 with respect to 2 like that, you can keep stacking and inside this you would be having both, you call rotational and positional information, this is what we did. So, based on that what we can see like the position of Q with respect to A whereas, the position of Q with respect to B is given and if we know the transformation matrix of B with respect to A, we can get it the final end.

So, now, we can see that the position vector here the generalized position vector we have increased to 1 dimension. So, that is what we call it is homogeneous coordinate or homogeneous position vector. So, here we have added the scaling factor. So, in simplicity we take the scaling factor is 1, otherwise we can bring the original scale.

So, now, you can see inside there is a rotation matrix and the position vector where the rotation matrix we keep it as it is like 3 cross 3 and similarly, the positional information between two frames also we keep it as a 3 cross 1 as a vector. So, now, this is what we are writing as a transformation matrix.

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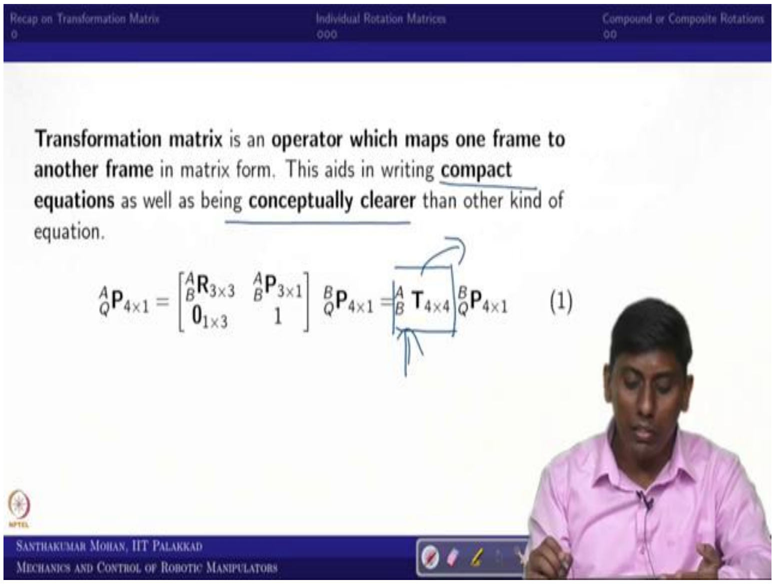

So, now, based on that what you can see the transformation matrix is four cross four, and this is what we are say saying that compact equation or compact form and it is conceptually clear, it would be having a homogeneous where the rotational and positional information is inbuilt.

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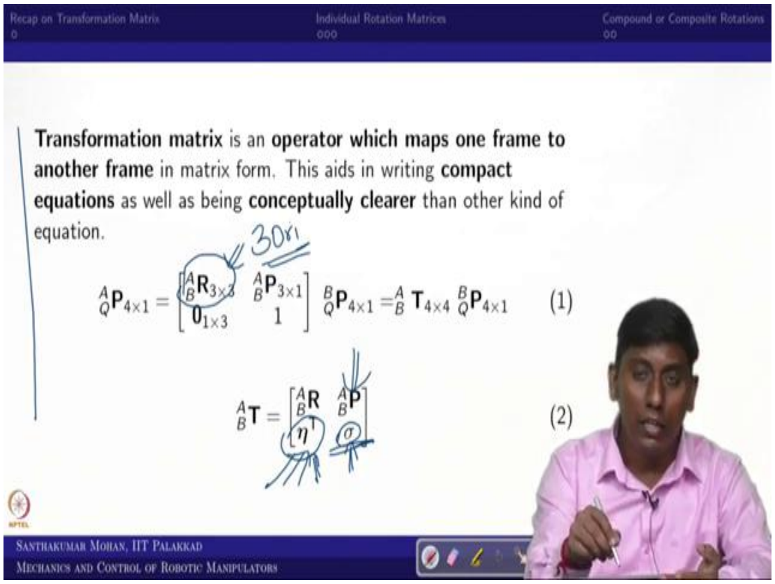

So, I hope now, you are clear even then we have said when the prospective will come, if your projection or when you are looking the position in a different view rather than orthographic projection or orthographic view. So, then this perspective vector will come in reality most of the camera will have a perspective view.

So, this perspective vector will come, similarly, for making sure that the scaling of the position so, we can use the scaling factor. So, for example, you are taking a camera based positional information, so, 1 pixel equal to how much we use that unit that you can convert, this position vector still we can keep it in pixel and the scaling factor will taking care of all those other conversions.

So, now, we are clear on this. So, we said that how this rotational information would be obtained in 3D case. So, in this sense, it would be having 3 orientational information how we will get? So, that is what we call compound rotation, but, we will see how that can be obtained in this particular lecture.

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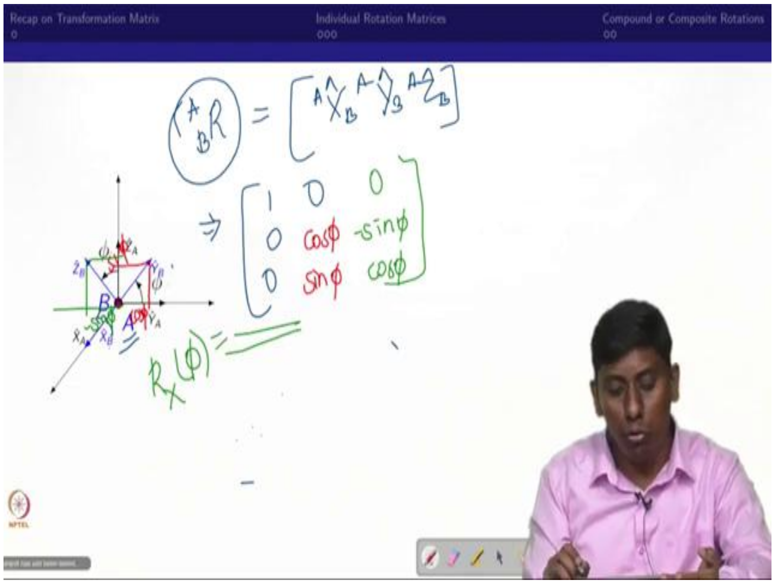

For going to that so, we will first of all talk about individual transformation, in the sense, individual rotation matrix. For example, I have a frame A. So, now, there is a body which is associated with the A in such a way that initially the X axis coincide here, I bring the B frame where the XB unit vector and the XA vector coincide.

Now, I am rotating with respect to this sum, so, what happened this, so, YB cap and ZB cap would be moved. So how much it moves or how we can get the rotation matrix that is what the idea here. So, now you can see like there is a phi angle rotated with respect to XB axis. So, now you can see that A frame and B frame are having only 1 transformation which is nothing but the rotational.

So, now, we have to find this rotation matrix of B with respect to A where the rotation happened with respect to X axis. So, now, you can look at it in that case, so, what would be the projection? So, you know this can be written as XB transformed to A, so, then YB unit vector projected on A and ZB unit vector projected on A. So, this is what we did and we have taken this is simple direction cosine or we call the dot product.

So, XB projected on XA that would give 1 because both are coincide So, then what happened, so, they XB cannot be projected on YA or ZA because this is coincide or parallel with XA. So, in that sense that projection would be 0. So, now, what else you can see, the YB cap can be projected on XA that would be 0 because that is in the other plane YZ plane, but if I project this.

So, if I project this into here, so, what that would be, that would be cos phi and if I project on the ZA that would be sin phi. So, now, that is what we are writing. So, similarly, you can project this is ZB unit vector on the YA frame so, that would be cos 90 minus phi. So, this is sin phi, but that would be the opposite direction and this would be cos phi.

So, what happens, so, this is, so minus sin phi and this is cos phi. So, like that we found the individual rotation matrix. So, now, what we have taken, so, this is equal to RX of phi angle. So, this is what we call individual rotation matrix.

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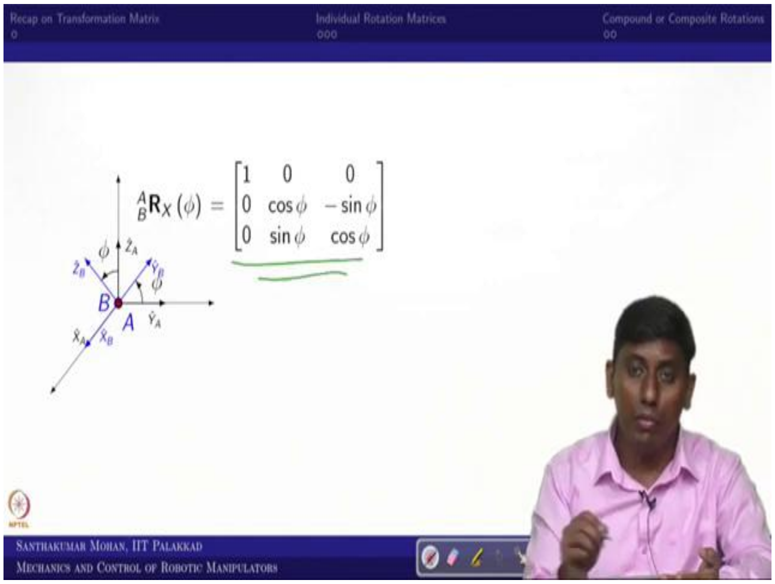

So, now, I hope this is what we have written so, the rotation matrix of B with respect to A where the rotation happened only with respect to X axis within a phi angle. So, then we can write the rotation matrix in this form, this is what we have seen.

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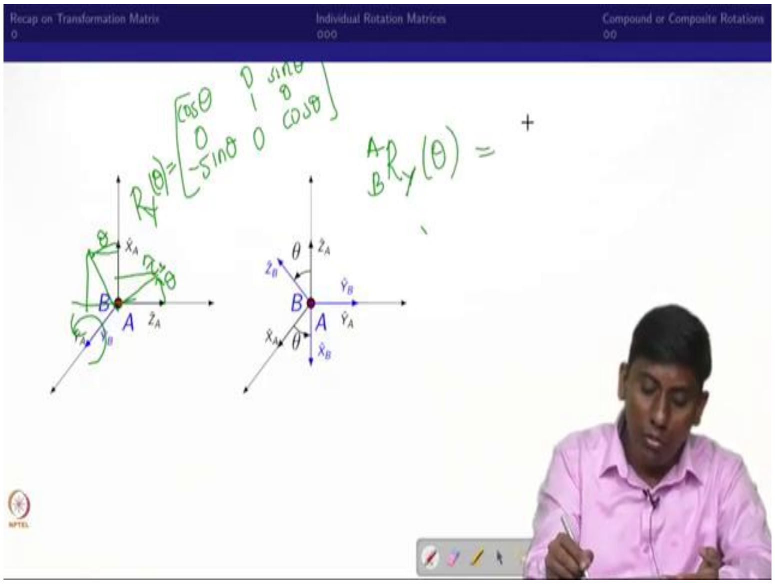

So, in the same continuation, we will do it for the other axes. So, now, we can see that the A frame has given, now the B frame has come in such a way that YB axis coincide with YA axis. So, now, in that case if I rotate with respect to this, so what happened, this would move like this or rotate like this.

So, now, I call this is some angle. So, for example, this is I call theta, so, this theta angle I can write RZ, sorry RY of theta can I write it? Yes. So, for betterment or for better clarity what we are taking it, so, we are taking it YA coming out of the screen, so, that what happened this ZX plane can be seen in the board.

So, now, what happened this is coincide with YB cap, YB cap is coincide now you are rotating with respect to this. So, what happened this ZA moved to ZB cap, so, this is what we call theta angle. So, similarly, this is also rotated as a theta angle. So, now, the same way you can project it, but what you can see the XB. So, this XB unit vector projected on XA so, that would be cos phi in this case cos theta.

And on YA cap that would be 0 and on the ZA cap that would be minus sin theta. So, what it will come, so, this would be cos theta 0 minus sin theta is the first. Similarly, Y, so, YB projected on A frame that would be 010 because it is coincide. So, now we take the ZB. So, ZB projected on XA frame that would be sin theta.

So, ZB projected on YA cap that would be 0 and the other way around it is projected on A frame that would be cos theta. So, now this is what you can see this is equivalent to the rotation matrix of Y with respect to or Y at an angle of theta. So, now if I write, so, RB with respect A where Y frame, or Y axis is used for rotation. So, this is what the matrix, so that is what we have written here.

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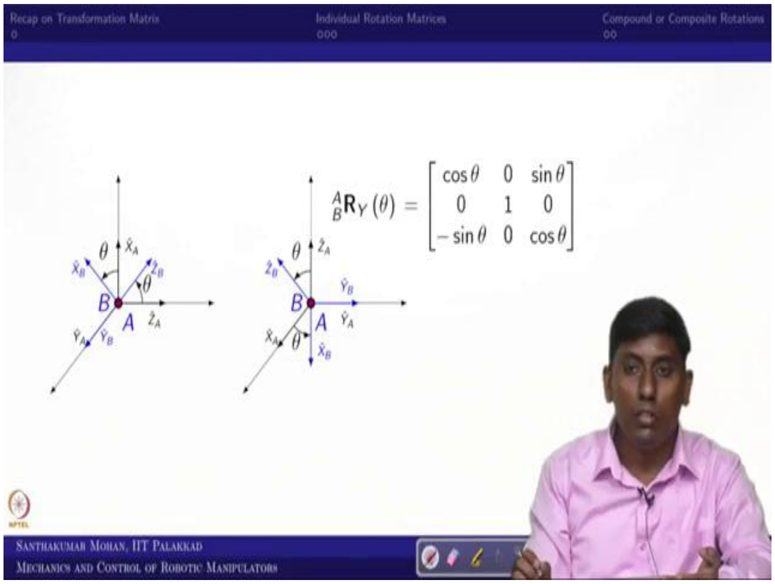

So you can see, so this is what we obtained. So, th is is another individual you call rotation matrix.

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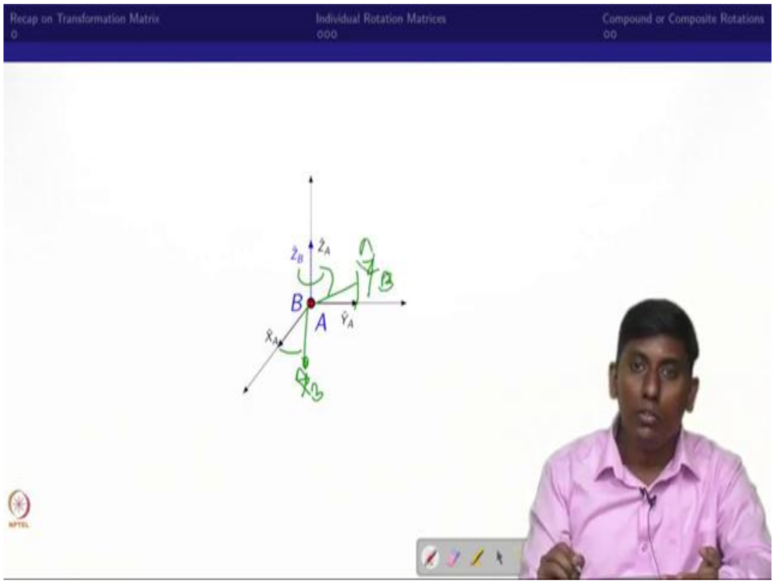

So now we will see the last one or with respect to Z axis. So Z axis easily, we can release here still, we will bring it back. So, now, we brought the B frame, the B frame where ZB coincide with ZA. So now we are trying to rotate. So, what happens this XA would be rotated that would be XA become XB cap and this would be rotated. So that would be so YB cap.

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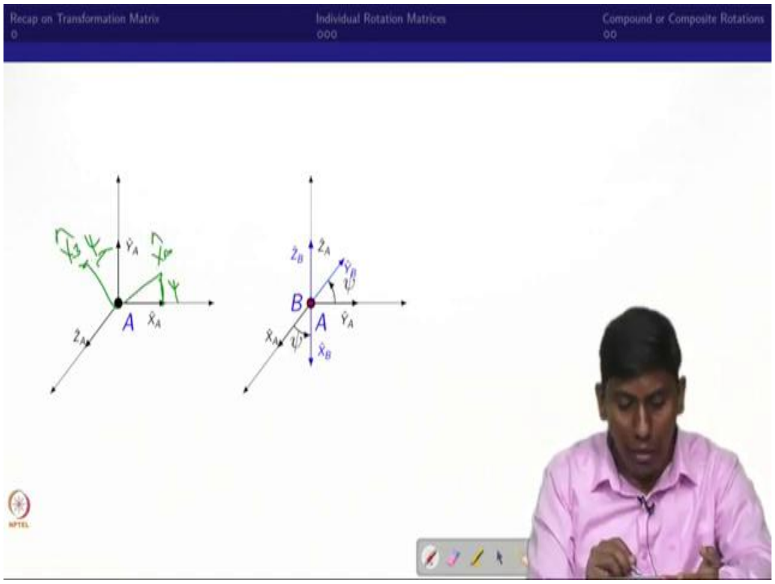

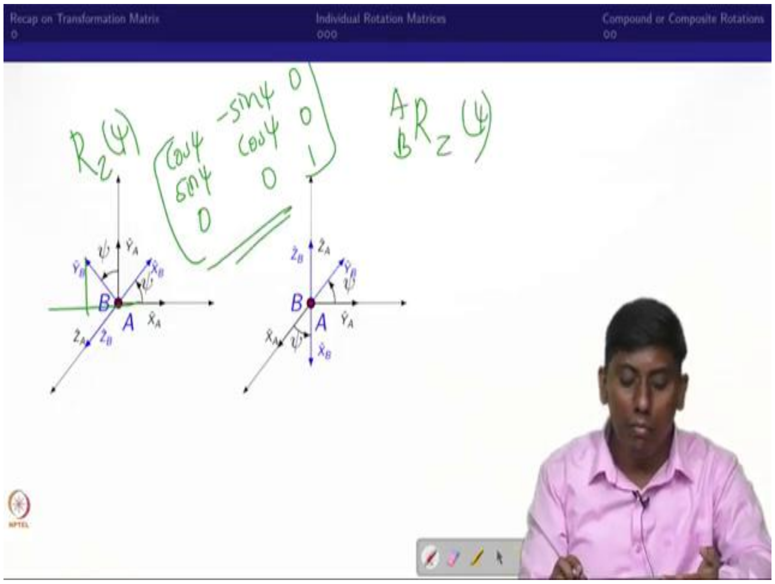

So, this is what we did here. So now the angle we call psi. So again, you can see like it is not so very clear. So we will take the same way. So, we will put ZA cap is coming out of the screen and we use the X and Y on the screen. So, now in that sense, if I rotate what happened, this would rotate and this also would rotate.

So this rotate and a psi angle this is also rotate and a psi angle. So, now this is YB cap and this is XB cap. So, now we project XB cap to frame A. So that is what we are doing it. So, now if we project XB cap on XA. So that would be cos psi and the XB cap projected on Y. So that would be sin psi. And XB cap projected on ZA frame that would be 0.

So, similarly YB if you project so, that would be sin psi, but that would be the other side. So minus sin psi and this would be cos psi and this is 0 and ZB cap projected on X and Y that would be 0, but in Z it would be 1. So, this is equivalent to what RZ of psi. So, now if I rotate, if I use the rotation matrix of B with respect to A, so, this is like this, so this is the final matrix.

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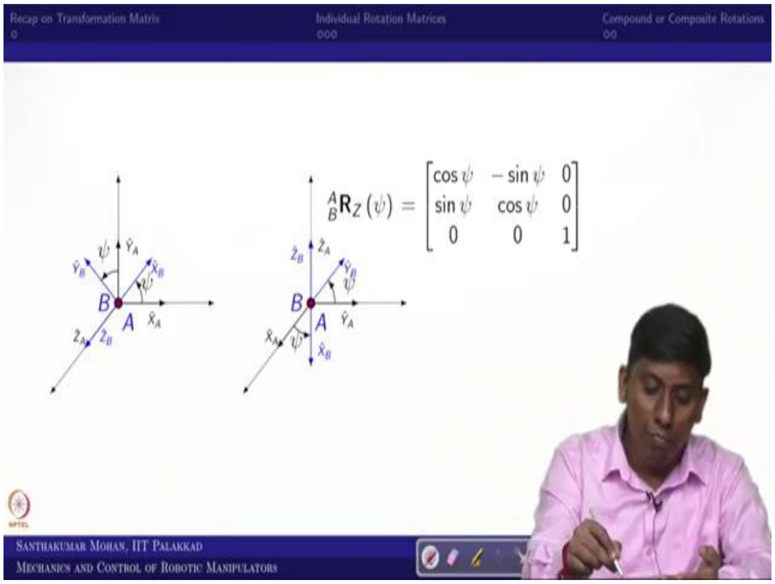

So, now you can see this is what we obtain. So, this is going to be useful for the composite our compound rotation. So, that is why we have done this. So, let us move to the compound rotation what exactly we want to do it there.

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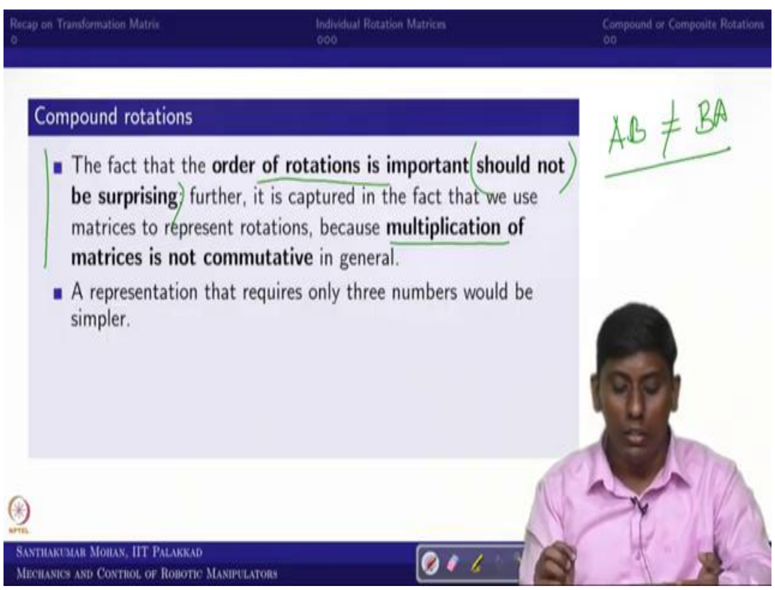

So, what we are doing it there. So, the compound rotation, the order of rotation, because you have 3 axis rotation. So, the order of rotation is very important, because it is not supposed to be surprising, because if you take the order interchange what happened. So, your matrix multiplication will give different answer that is what we have written. So multiplication of matrixes is not commutative. So, what happens, so, if you have AB matrix, so, this product is not same as this.

So, if you take the order is different than the final answer would be completely different. So, that is why we need to have some standard representation. So, that we can keep that representation as a uniform and anybody who uses this robotic system. So, they can say this is what I use, they will understand, this is the representation, this is the way we can recall.

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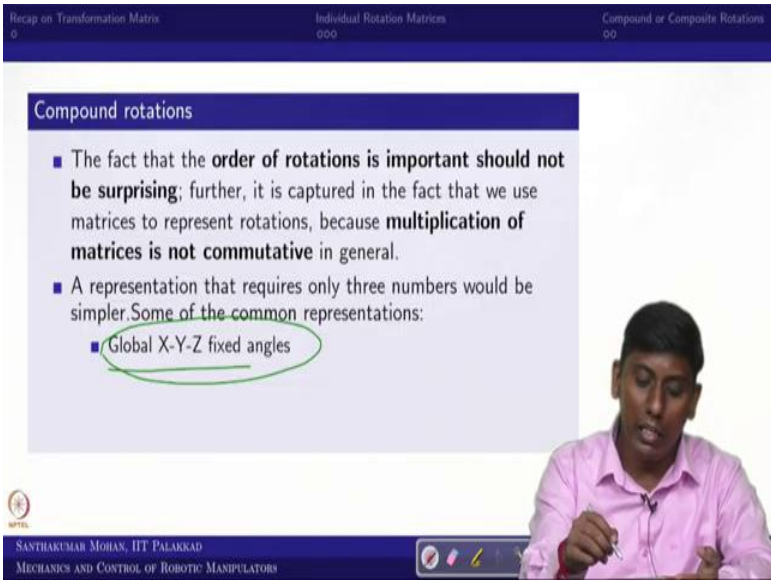

So, if that is the case, so, what one can see there are several common representations, we would be using two of them and we would be addressing that here. So, before that we will see what are the general compound or composite rotation representations. So, one would be, we call global X-Y-Z, global Roll-Pitch-Yaw, because Roll-Pitch-Yaw usually we call it with respect to mobile frame, but here what we call fixed angle.

So, you would be rotating with respect to the inertial frame. So, inertial frame is rigidly fixed with respect to Earth. So, that is why we call it is a global X-Y-Z fixed angle. So, this was come from one of the popular researchers Winkel. So, the Winkel angle also some of the people used to call especially if you go to Europe community, European community they call this Winkel angle rather than global X, Y or global RPY angles.

So, I hope now you are clear there is one representation with respect to X axis rotation, Y axis rotation and Z axis rotation of fixed frame. So, first we will rotate with respect to fixed frame X axis, then Y axis and Z axis. In the next lecture we will see that little.

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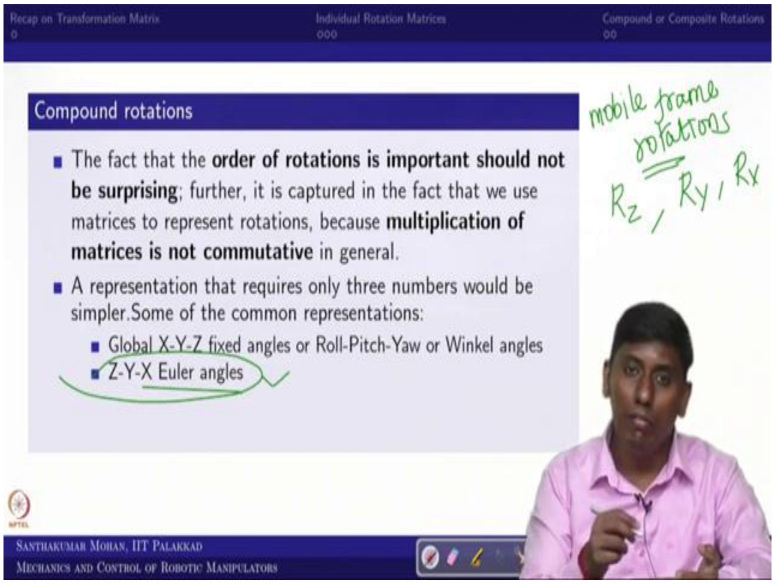

So, let us move the next representation. I already said this globally we call Roll-Pitch-Yaw or RP Y angle or some people call this is Winkel angle. So, let us move what would be the other representation. So, Euler gave there are 12 convention, there are 12 representation. So, but we are going to use one such, so, we call Z-Y-X Euler angle, even there is other representation but this is universally used especially in the mobile robot community and the manipulator community. So, which is what we call is that Z-Y-X Euler angle. So, what that mean?

So, here Euler angle all talk about the mobile frame rotation. So, mobile frame rotations were the consideration. So, in that case, there are several configurations or conventions can come in that one such convention will take. So, first we will rotate Z, then we will rotate Y and finally, we rotate about X. So, this some people call local R P Y, but local R P Y is a slightly different, we will see that in the next convention.

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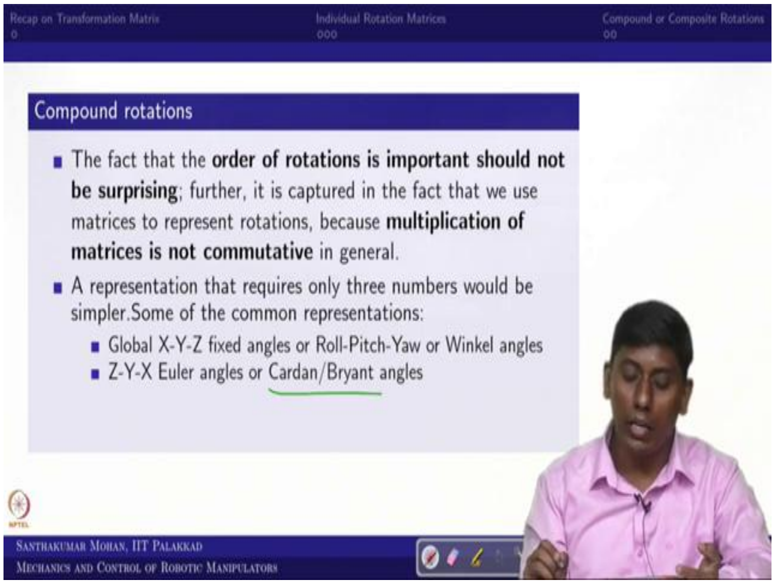

So, this is given by Cardan or Bryant. So, even some people in European community they call it is a Cardan angle or Bryant angle, these are same as Z-Y-X Euler angles.

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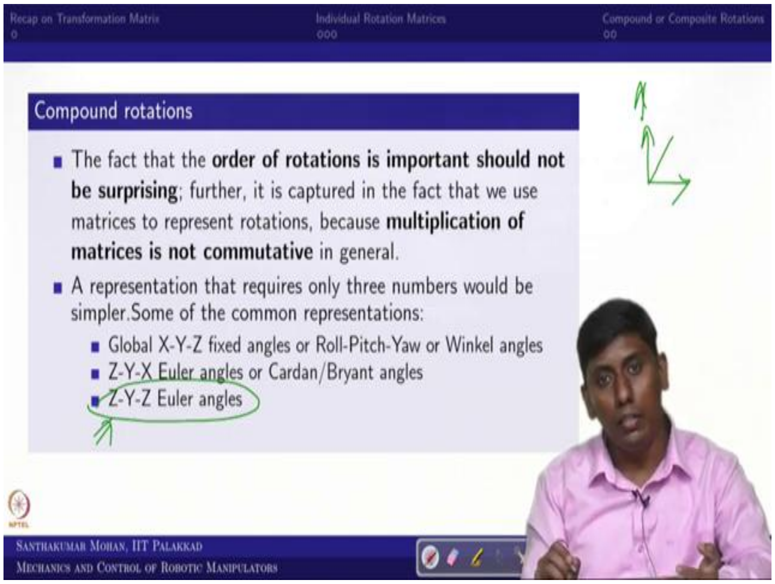

So, let us move to the next one I said even now, some of the aerial robot community or aerial robotics community they use Z Y Z as Euler angle. So, this is another popular but this is mainly popular in aerial and space robots and even the manipulator some of the manipulator via the spherical joint, so that cannot be done. So, what they do first Z axis rotation, then Y axis rotation, again Z axis rotation that is what they use. So, in that sense, you can write as Z Y Z Euler angle.

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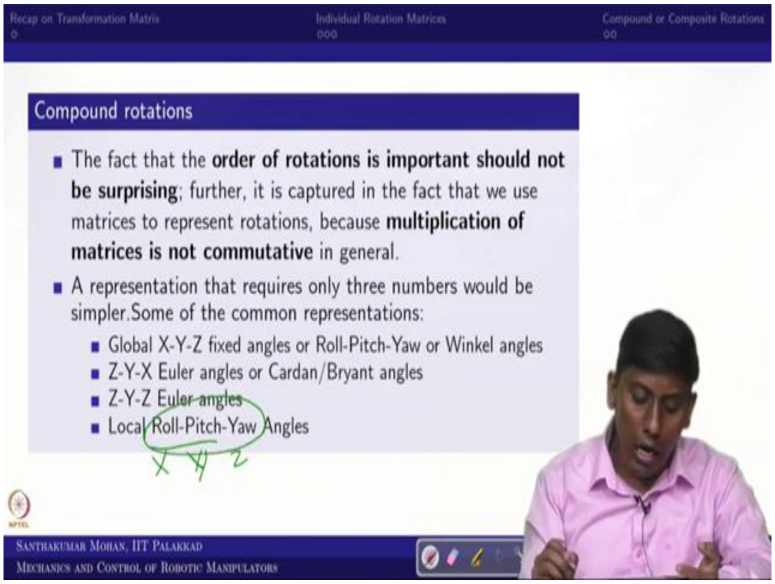

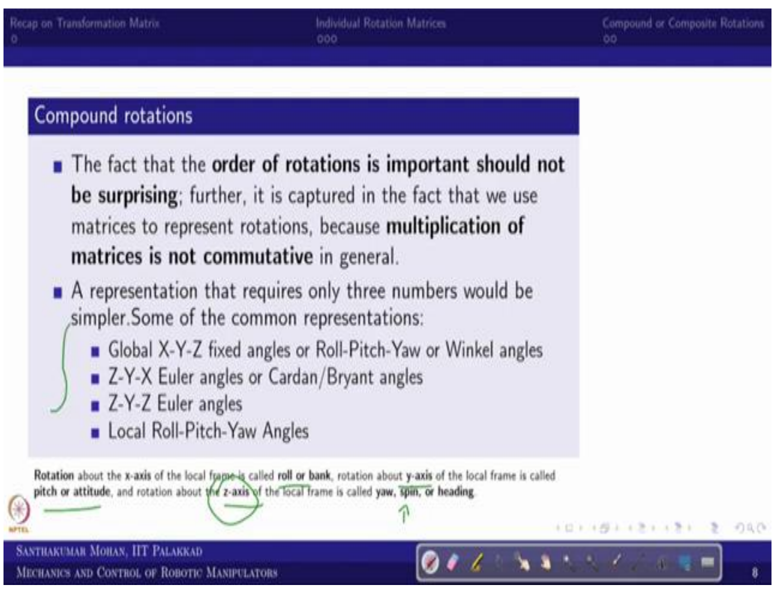

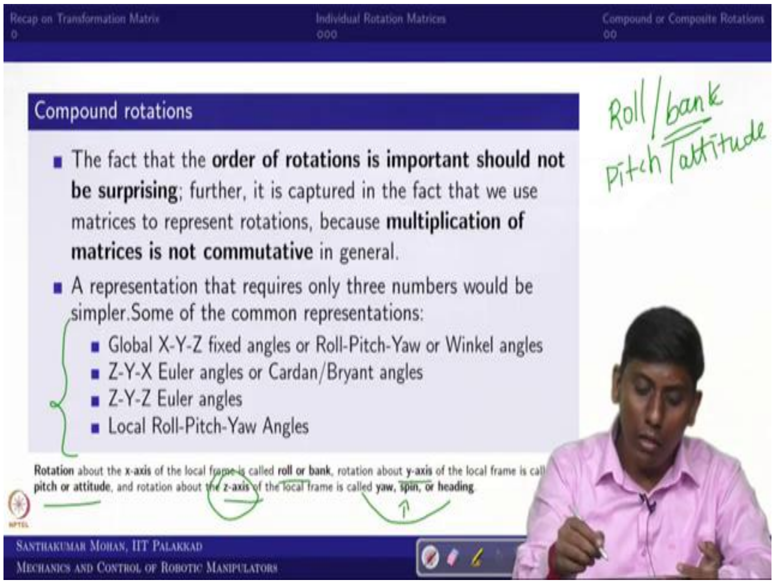

So, then there is another representation which I said local R P Y, so, where it is X Y Z. So, but this is on the mobile frame so that is why it is called Roll-Pitch-Yaw, but it is a local this local R P Y angle we called slightly different, the rotation about a local frame of X axis we call roll or bank, similarly, rotation about local X axis. So, we call roll and bank, local Y axis we call pitch and attitude. So, some people call attitude control, attitude control means nothing but pitch control. So, please be clear.

So, similarly, rotation about is Z axis of the local frame call Yaw, some people call heading, some people call spin, spin is more commonly used for the roll, but the local frame maybe represent the spin would be on the Z axis. So, now, you got some clarity, so, there are several representations. So, how we can bring it these all into a picture.

Similarly, you can see like several keywords we commonly use roll or bank I think if you follow some of the aerial robot or you can say aerial vehicle based on Hollywood movie and all say that I am banking, I am banking they are rolling. Similarly, people call attitude control. So, this attitude control is not human attitude, this attitude the pitching, so the pitching of the aerial vehicle or pitching of the mobile robot are very important, that is what we call pitch or attitude.

So, then the final one is very common, we usually call heading control or Yaw control, very rarely we use spin, but Yaw or heading is common. So, which would be about the local Z axis rotation. So, now we will let go one additional slide. With that we will close this particular lecture. So, what that.

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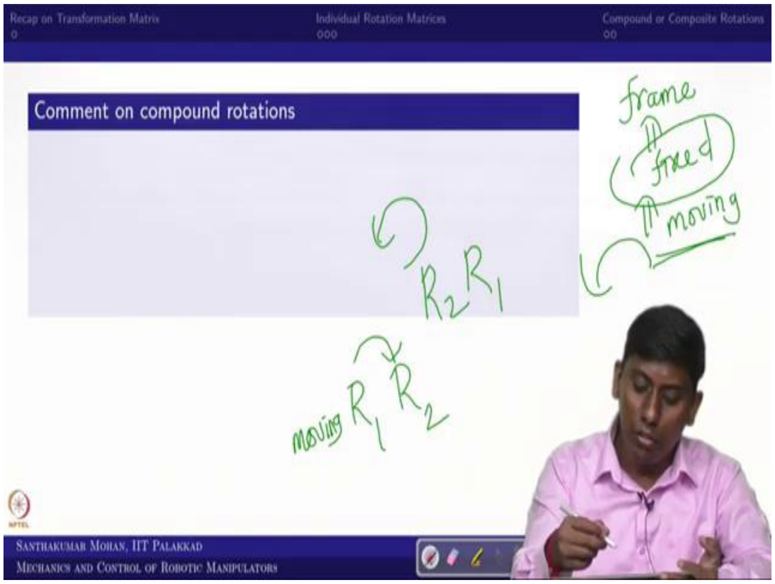

So, you have several representations and several form, so you have rotated the axis with respect to frame, if you refer from the fixed frame, so, you have certain format, if you are moving or rotating with respect to moving frame or local frame, that is set of another regulations or some kind of set of procedures, what that?

So, if you are rotating with respect to fixed axis, then you have to premultiply that rotation matrix with respect to your position vector or your predefined rotations. However, if you are using the mobile frame, so what happened you have to rotate as post multiply. So, if you have a moving frame, then you have already one rotation.

So, you have another then this is post multiply. So, this is with respect to moving frame and whereas this, so this is R1 and R2, where this if you are using a fixed frame, so R1 and R2 is pre multiplying. So, this is something I will show you in one slide then you will get idea. So this is what we call the comment.

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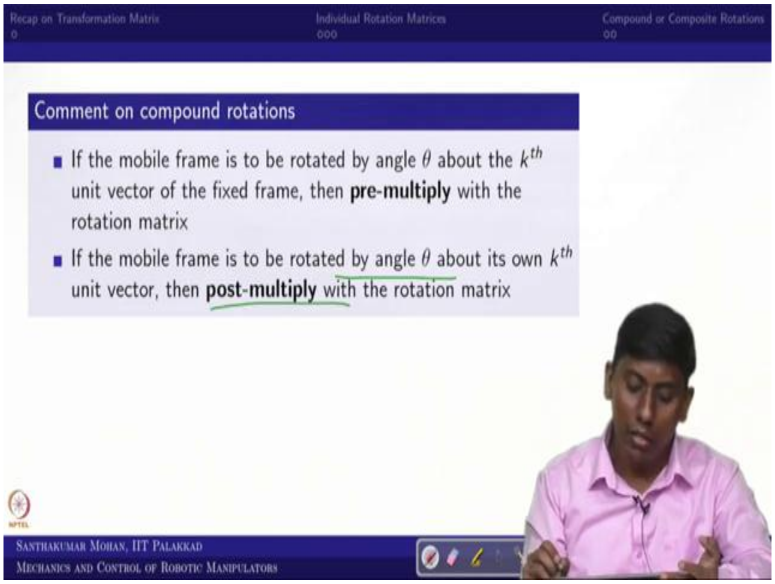

So, what comment is if the mobile frame is to be rotated by an angle theta, about kth unit vector of the fixed frame, then you pre multiply with the rotation matrix whereas if it is rotated by an angle about its own kth unit vector, then you post to multiply with the rotation matrix. I hope you are clear on that.

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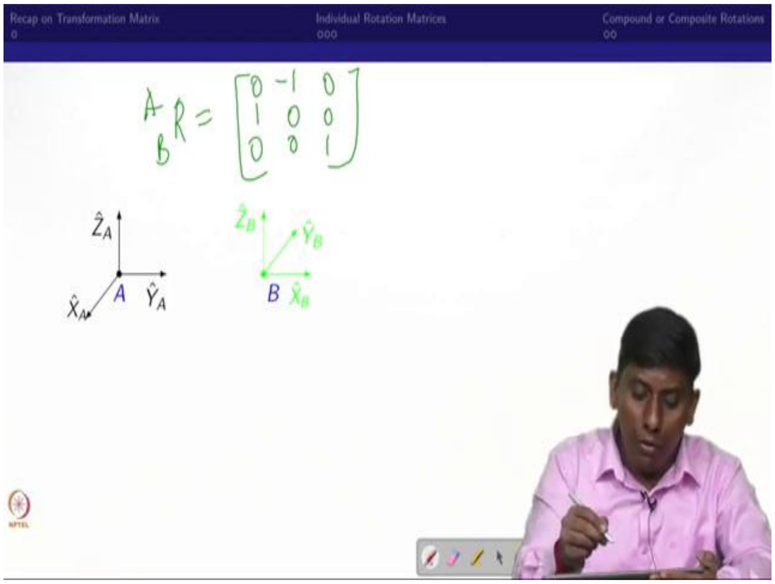

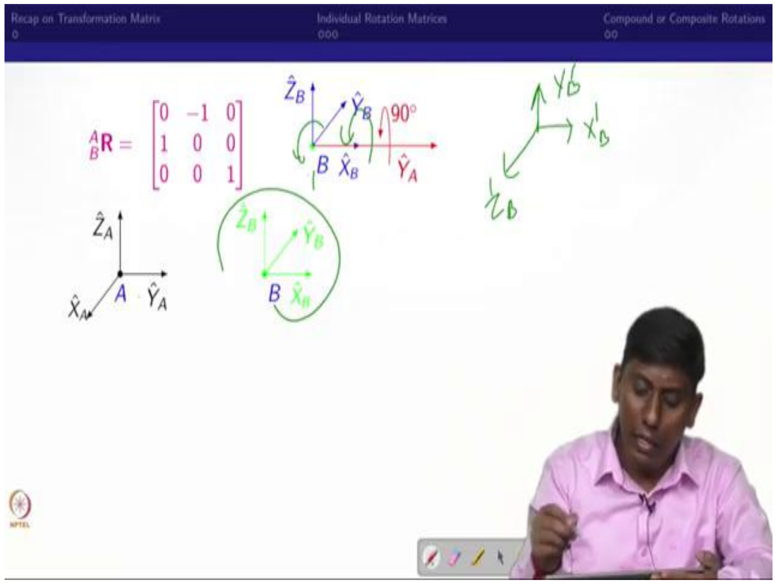
Recap on Transformation Matrix
Individual Rotation Matrices
Compound or Composite Rotations
$^{A}_{B}\mathbf{R} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
$\hat{Z}_B$ $\hat{Y}_B$ 90° B $\hat{X}_B$ $\hat{Y}_A$
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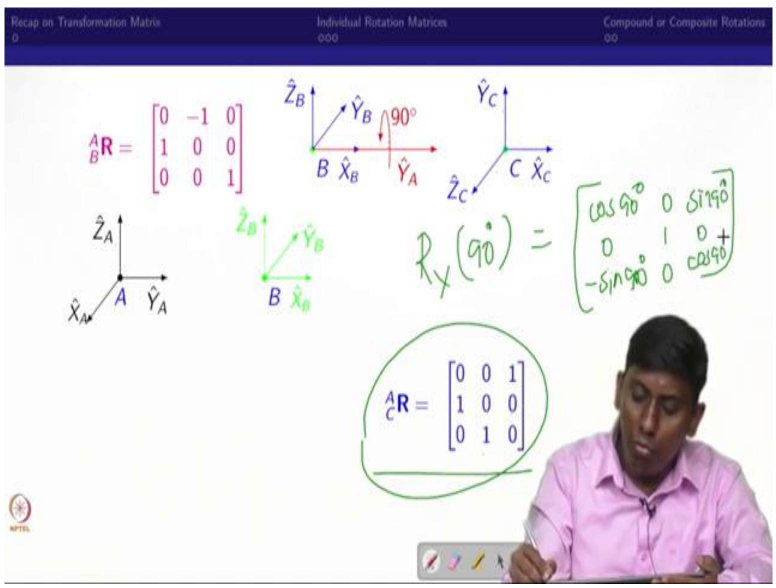
Recap on Transformation Matrix
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$\hat{Z}_B$ $\hat{Y}_B$ 90° B $\hat{X}_B$ $\hat{Y}_A$
$\hat{Y}_C$ $\hat{Z}_C$ C $\hat{X}_C$
$\hat{Z}_A$ $\hat{X}_A$ A $\hat{Y}_A$
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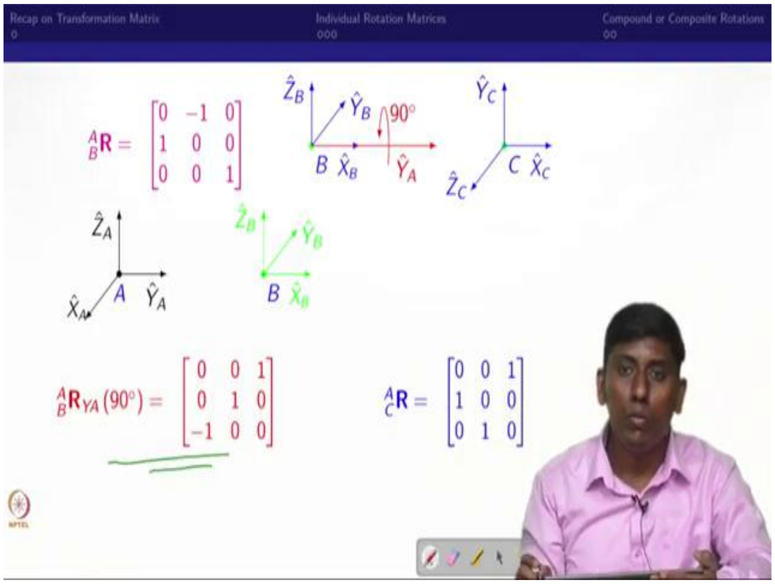
$^{A}_{C}\mathbf{R} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$



So, for understanding that more clarity or more clarity manner. So, I brought 2 frames. So, just to give a clarity, so, what clarity says that there is a frame A and B which are put it here. So, now if you look at it here, the rotation matrix of B with respect to A I can simply write like XB can be projected on A frame. So, in that case what happened this would come 010.

So, YB axis projected on A frame that would be minus 1 because YB is opposite direction of XA and this is 00 and ZB is projected on the C. So, this is the rotation matrix. Now, I hope you are clear how it has come. So, this is the rotation matrix. Now, what I am taking is there is a rotation about YA as a 90 degree you are rotating this 90 degree. So, you are taking a B frame here and you are rotating.

So, what happened 90 degree if you rotate, so YB dash will come and ZB dash will come because you are rotated 90 degree like this. So, then this would be you XB dash. So, now, what we have done, we have taken the rotation with respect to YA fixed so, we can write that rotation matrix. So, this is the case, so, if you write the rotation matrix with respect to A. So, what that would be XC is projected on YA, so, in this 010 and YC is projected on ZA.

So, 001 and is ZC projected on A frame which is coincide with the XA, so, 100. So, this is the final matrix which we obtained, but what we did we have rotated with respect to YA. So, what that would be? So, this is what I can write, so, 90 degree, so, what that, so, that would be you recall, so it is written cos 90 degree, so 0 then this is minus sin 90 degree that is why I derive the individual matrix.

So, 010 then this is sin 90 and 0 cos 90. So, this is what we did it in the earlier lecture. So, I hope that you are recalling, so or you can go back that particular slide. So, now you substitute this, so, this becomes 0, this become minus 1, this is 010, this is 100, sorry 100. So, that is what I am showing it here.

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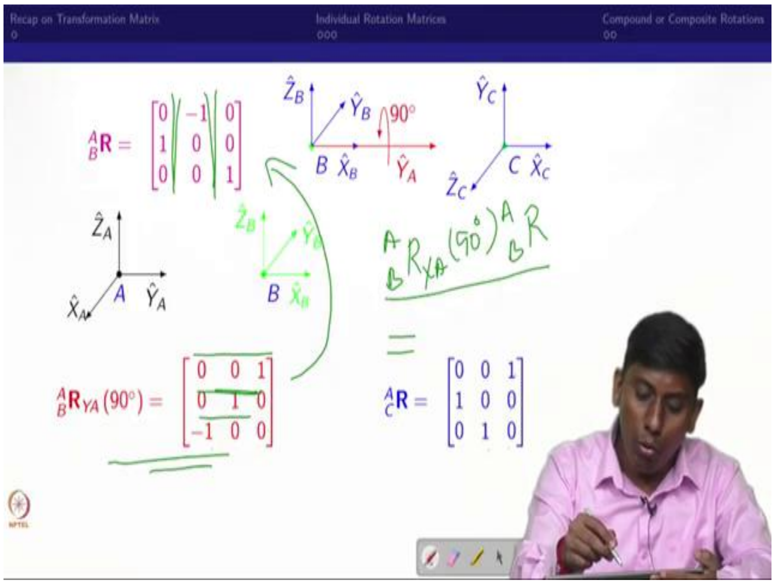

So, now, so if I take these 2 matrixes if I use the rule, since it is rotated with respect to the fixed frame, what I had to do, I have to pre multiply this. So, RBA YA rotated 90 degree multiply with RBA whether it is giving this value or not. Fortunately, you can see so, this is first and you multiply you can see this 0. So, this taken here that is also 0, this and the third row you can see it is giving 1.

So, similarly here you take this and multiply 1 and this multiply with this 0 and 0 like third, like that, you can take it so this multiply with this 0, this multiply minus 1 and minus 1. So it becomes 1 and 0. So, you can see the pre multiplication is giving the right answer. However, the post multiplication can be do it or not.

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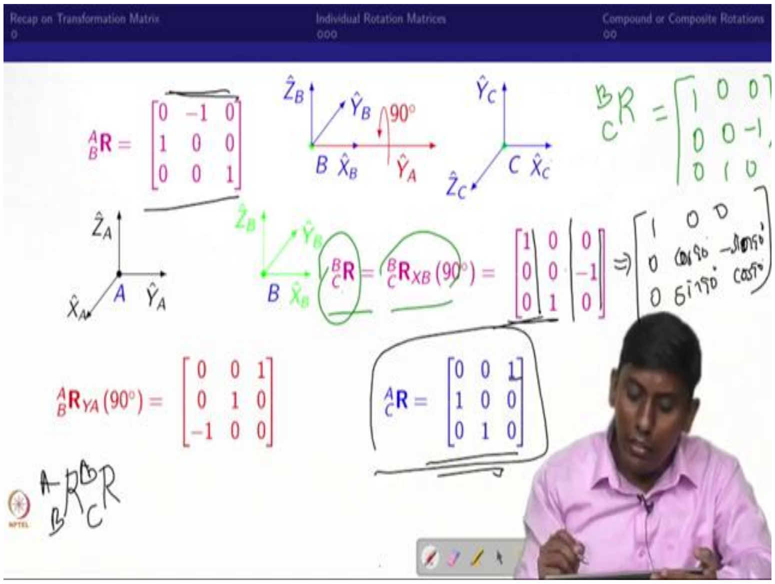

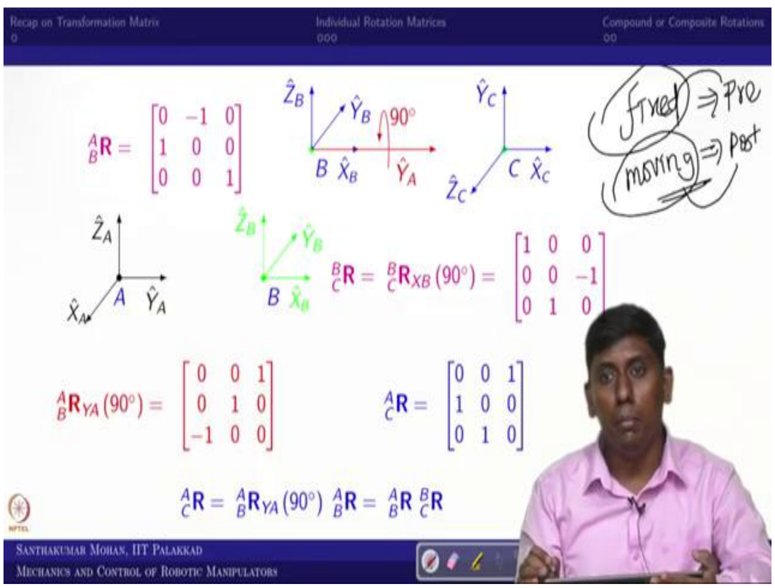

So, for that what we are taking it, we are taking it so the RB, so I am taking RCB, so that I can see. So, XC is projected on B frame where it is coincide with XB so 100 so where this YC is projected on ZB 001 and where this ZC is projected on Y, but opposite, so it is 0 minus 1 0. So, this is what we have written as RC with respect to B rotation matrix of C with respect to B.

But what rotation we did here with respect to XB? So, you can recall what would be that. So that would be if I write it correctly. So that would be 100, then cos 90 degree minus sin 90 degree 0.

Okay, sorry, this is 0, cos 90. This is I will just erase it and write it. So, this we are writing. So, 100, this 00, this is cos 90 degree, this is minus sign 90.

So then this is sin 90 and this is cos 90. So, obviously, this becomes 0 and this becomes minus 1 and plus 1 this is what matrix. Now, you need to get this. So, already you have this. So, what you can do, you can multiply RB with respect to A and post multiply that is what we have seen. So, if you are multiplying this post multiply with this, so, what you can see 0 minus 1 0 multiply here that becomes 0 and this so, you can see again that would be 0 this multiply with this minus 1 minus 1, so, it become 1.

So, similarly, you can see this is coming, what one can see, so, this equal to so R, so I will write it RBA and RCB. So, that is what we are written as post multiplication and pre multiplication. So, I hope why we are doing pre multiplication and why we are doing post multiplication you would be clear, so, if you have a fixed frame rotation, so, then you can pre multiply, if you are rotating with the moving axis rotation then you can post multiply.

So, with that I am closing this particular lecture. So, the next lecture would be talking about, more about this compound rotations. So, I hope that would be clear in the next lecture. So, right now, you clear that, so, why we are doing a fixed rotation, why we are doing the moving rotation as post and pre multiplication, that you are clear.

And why we are going with the compound rotation also you are must be clear, and in that two of the representation we are going to see one is a Winkel angle, the other one is a Cardan or Bryant angles. So, with that see you then, take care, bye.

$$C(v) = \begin{bmatrix} 1 & 0 & 0 & 0 & 168.57w & -133.04v \\ 0 & 1 & 0 & -168.57w & 0 & 96.30u \\ 0 & 0 & 1 & 133.04v & -96.30u & 0 \\ 0 & 168.57w & -133.04v & 0 & 8.10r & -9.26q \\ -168.57w & 0 & 96.30u & -8.10r & 0 & 4.47p \\ 133.04v & -96.30u & 0 & 9.26q & -4.47p & 0 \end{bmatrix}$$