Mechanics and Control of Robotic Manipulators
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Transformation Matrix

Welcome back to Mechanics and Control of Robotic Manipulator, the last lecture what we have seen is mapping between 2 different frames where both translated and oriented, this is the mapping between 2 frames we have seen in different way.

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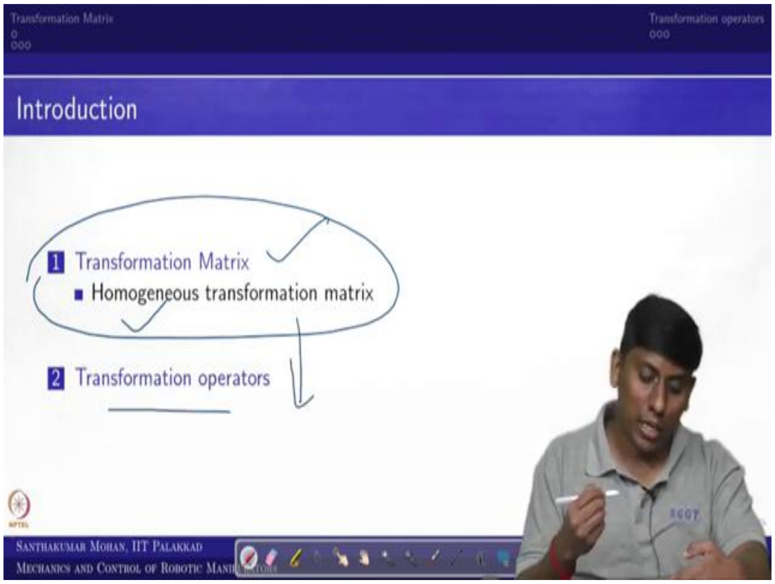

So, what we have seen that we would be continuing, but what exactly here we are continuing is basically 1 additional aspect called transformation matrix, we will bring it, why this transformation matrix and why it is called homogeneous and how this can be used as an operator further, this is what we are going to see in this particular lecture. So, if that is the case, we will agree call where we stopped.

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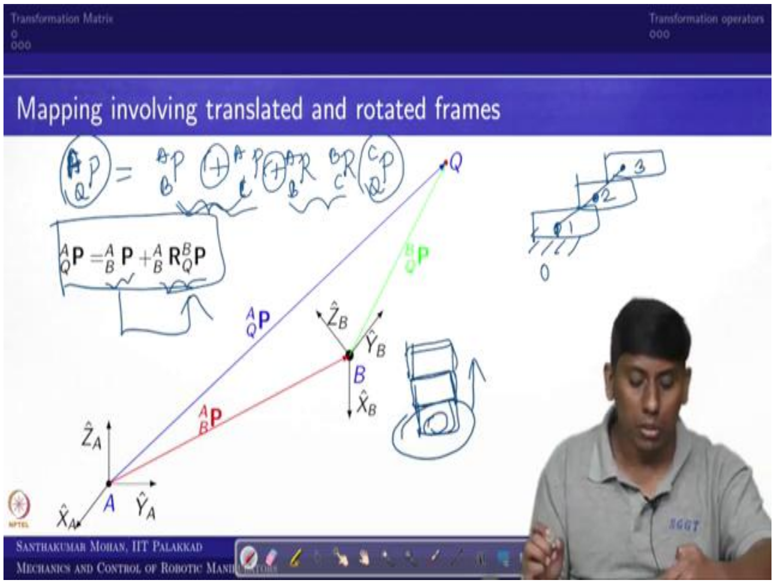

So, we have seen that there is a position vector of Q with respect to B is given, if the positional information of B with respect to A and the orientation information of B with respect to A is given, we can find this position vector. So, this is what we have seen in the previous lecture. So, now, we imagine there are several bodies associated, like this. So, for example, this is a body 0, 1, 2, 3 that keep going.

So, here is shown it is something in series, but imagine this transformation to this transformation, can I get information? Yes, I can do, similarly can I get this? So, if I keep getting, so what this is the relation I will get for example, now I want to find the C, so, what I have to do so, position vector of Q with respect to C, I have to write position vector of B with respect to A then plus position vector of you can write position vector of C with respect to A, this is the position vector.

Then you can write fast you out or rotate about you call a C with respect to B then you can make another rotation. So, what that rotation, so, you can rotate about B with respect to A then that would be giving position vector of Q with respect to C is known we can write as A. So, this way you can write.

So, now, if you write this equation, it is complex, right? So, there is 1 addition, so there is another addition, but it is having a multiplication even here this is initially multiply, and it is added. So, that there are several constraints will come if you keep increasing the number of bodies, instead of that can I make a single entity or in the sense.

So, I have 1 body, another body, another body that, I am using as a briefcase, the sense inside the briefcase, I would be having information about orientation and the position, but 1 briefcase would be simple box. So, another briefcase is equal dimension I keep it in the sense I can make a stack and I can operate easily.

So, now, in order to do this what first of all you have to see, here that operation is heterogeneous. So, what kind of operation, 1 is addition the other 1 is multiplication. So, now you have to make a homogeneous operation. So, for what we are trying to do, we are going to do a simple multiplication. So, that is what we are trying to do.

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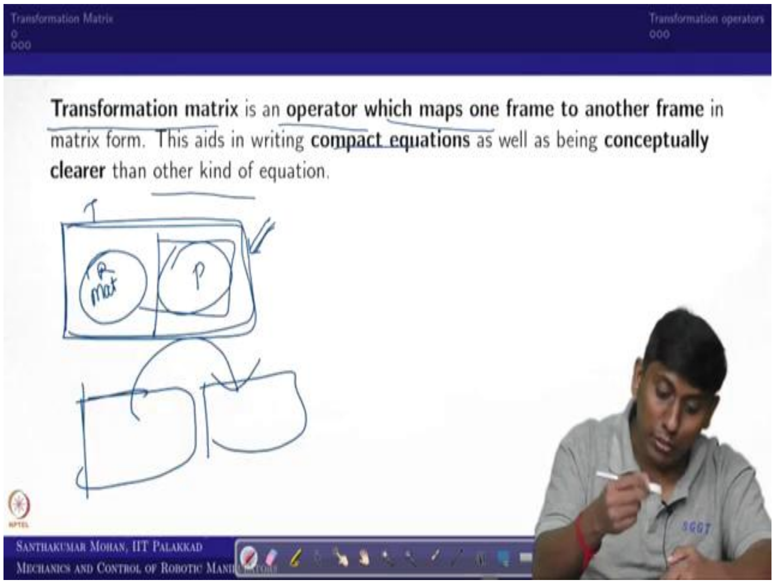

In that sense. We will come back here. So, we are bringing a transformation matrix. So, what that, that would be equivalent to an operator, it would be equivalent to a rotation matrix, but not straightforward, but what that, that maps 1 frame to another frame in a matrix form, which will give a simple aid of compact equation, that is what I said it is something a briefcase.

So, as well being a conceptually clear, so, inside the briefcase, you can see that there would be a positional information and orientational information. So, if that is the case, I will take a very simple example, which is probably a crude example. So, inside the college or inside the class, so you can obviously you can find 2 different class of students, in fact, there are 3 but I will say that majority will have, so girls and boys.

So, what if I call that group as a general as a student, they are homogeneous, but if I see inside class with the gender wise, so girls and boys are heterogeneous, a similar way so the matrix rotational matrix and the positional information. I take it as a girl and boy, but instead of that I am calling as a transformation matrix as a single entity, as a student.

If I do, I will get what? So, I can pass easily 1 information to another information, for example, I am calling probably 1 another institute I am saying that this information you should pass to students. But they can see whether this information should pass to girl or boy that separate but the student, if I say the student, the information should pass to student it is homogeneous in the sense 1 institute to another institute or 1 class to another class, it is a student-to-student information I do not bother.

So, the mode of information pass probably simple circular or simple email or anything I can do, but inside so the classes would be having girls and boys. So, which is heterogeneous. So, now, that is the same way we are trying to do although your transformation is consist of 1 positional information, which is in the vector form and the orientational information in the matrix form, but we are combining. So, for that what 1 can do? So, that is why I have returned the dimension also in the previous slide.

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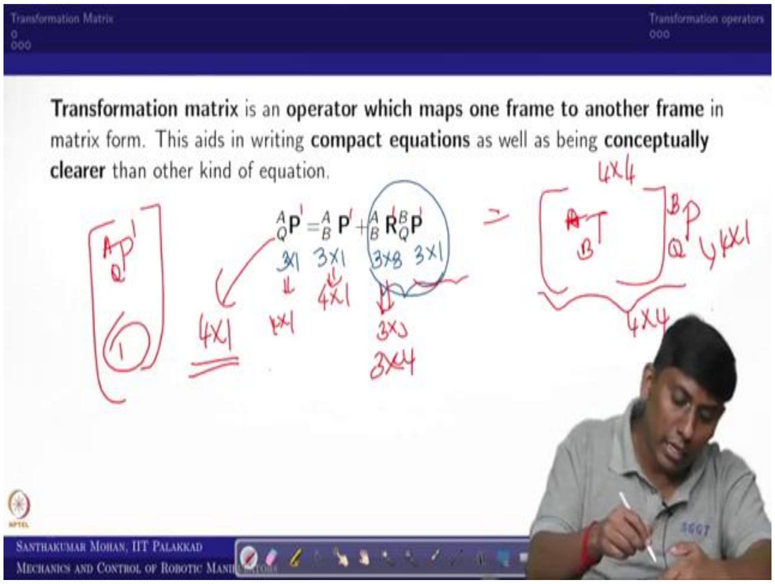

So, this is 3 cross 1 and this is also 3 cross 1 and this is 3 cross 3 and this is 3 cross 1, I cannot do it because this is something I cannot stick with the same dimension. So, then what I can do? So, I can increase the dimension instead of this. So, if I write this is 4 cross 1 where this I keep it as a dash. So, I keep it as a dash. So, in the sense P Q A dash here something I am writing the sense 1 additional dimension is coming.

So, then what I can see, I can bring it as 1 additional dimension which bring it 4 cross 1. So, then this is also supposed to be 4 cross 1 later on. So, then this is supposed to be retained as what, so 3 cross 3 then this cannot be multiplied, then it should be so 3 cross 4. So, then how I can do it, it is very complex.

So, instead of that, can I make it a single matrix, which is transformation matrix of B with respect to A. So, this would be multiplied with the outside information, which is the position vector of Q with respect to B. So, I can do it. So, now, this dimension should be not 3 cross 3, it should be 1 dimensional extra, which is supposed to be 4 cross 1, then this would be 4 cross 4 and this would be 4 cross 1. So, this is what we are interested.

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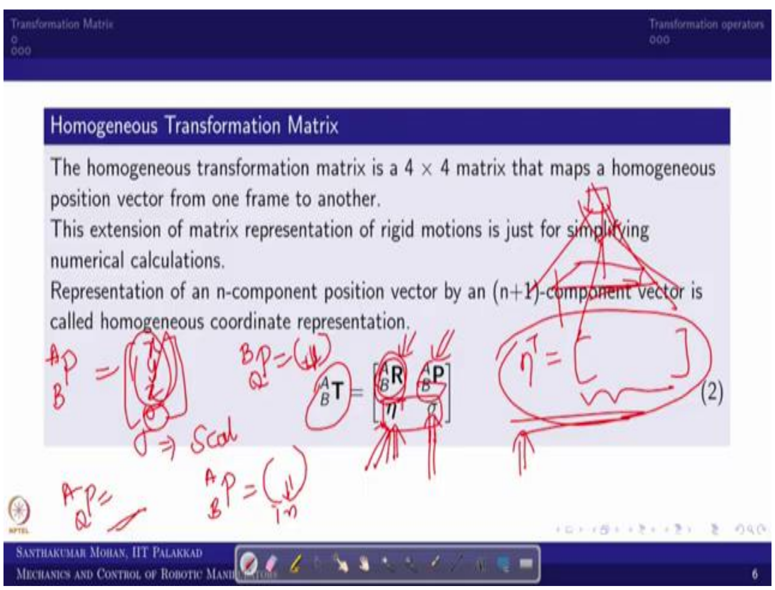

So, if I do this, what I can, what I supposed to do it. So, this is what I am trying to write. So, now, the straight forward this equation is not consistent. Because in the previous lecture, I said this is 3 cross 1, this is 3 cross 1 and this is 3 cross 3 and this is 3 cross 1, this cannot be consistently said this is not unit wise good. So, then this is not possible, then what we can do.

So, that is what we are trying to do. So, we are trying to bring a 4 cross 4 matrix and in order to make this 4 cross 4, so the position vector also would be increased by 1 dimension. So, that is what we are trying to show. So, the representation of n component position vector would be increased n plus 1 component vector.

So, this is what we are calling homogeneous coordinate representation. So, what that, so, instead of calling girls and boys, we are calling students, the same way we are bringing it. So, in the sense 1 simple difficulty will come that we will see. So, now the transformation matrix, I am writing into 4 subcomponents, 1 is rotational information, another 1 is positional information. In addition to that, there are 2 things that are coming, strictly speaking, this is eta transpose.

So, eta transpose is written as in this way, it is a prospective vector, sigma is scaling factor. So, why the scaling is important, for example, if you write your position vector B with respect to A, so, I can write x, y, z, I am writing as a sigma. So, the sigma is a scaling. So, the scaling will give 1 additional benefit to us, what benefit would be given to us is you can use any kind of unit as long these 3 are same entity.

So, your x position, y position and z position all represented in the same unit, then the scaling can be used. Now, for example, I am using the position vector of B with respect A in millimeters. Okay, but position vector of Q with respect to B in inches, but I want the information of position vector of Q with respect to A in meters. I can do it, here I can use the corresponding scaling factor sigma and here corresponding scaling factor sigma then it would be done.

So, that is the advantage of using the scaling factor. This prospective vector when it would be used, so, this prospective vector would be used if we are not using orthogonal projection are orthogonal view. In the sense for example, you are keeping a camera, so you are keeping a camera and this camera is giving a proper perpendicular view of us, you do not need to worry, but this camera is inclined.

So, it is giving your view in a prospection way, so then this prospective vector would be used. Mainly if you look at some of the computer algorithm people would be very much encourage this or appreciate this because they use this prospective actor everywhere when they do graphics and all. So, that is what the whole idea here also. So, we will be having a prospective vector. So, then you have scaling factor, then your position vector and your rotational or rotation matrix. So, this is what you are bringing as the transformation matrix.

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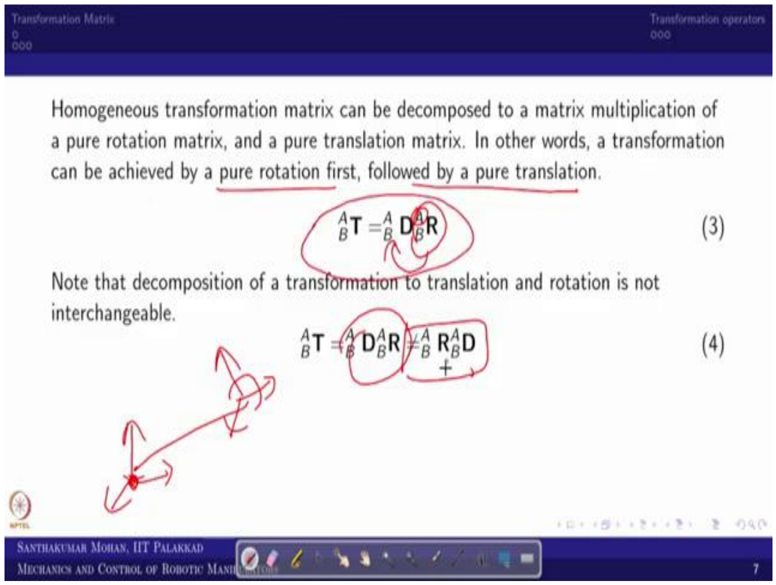

This transformation matrix can be divided into 2 parts. So, are decomposed into 2 part, what part, first you rotate and then translate, since we are talking about A frame is fix, so, you have to pre multiply but the same way you cannot do the other way around, first translate and rotate cannot be done, you know matrix.

So, the matrix multiplication is not commutative, in the sense so, these 2 are not equal and as per the transformation matrix first rotate and then translate. So, in a sense you have 2 coordinate frames. So, first you have to rotate this parallel to this then you translate that is the meaning of this. So, now, rotation is first then translation. So, I hope you are clear what exactly the matrix decomposition, not this.

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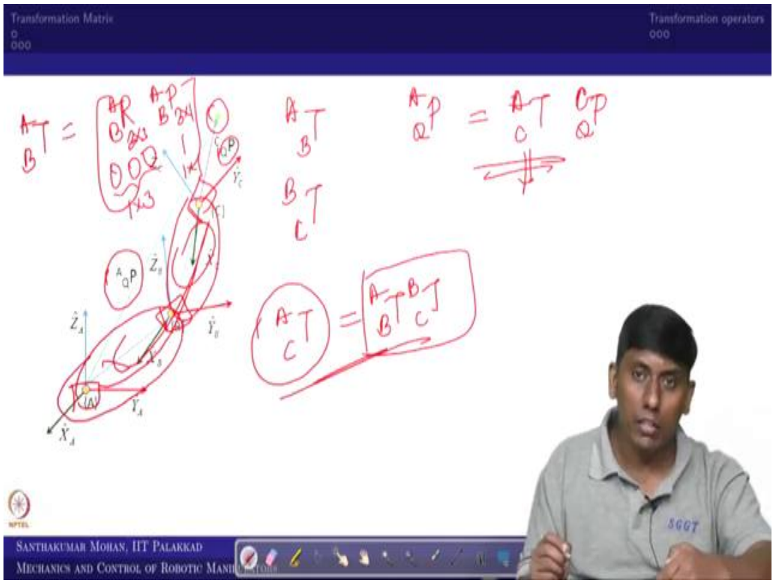

So, now, we will comeback, how this can be used as the transformation operators. So, there are several options. So, I have taken 1 simple idea to understand, this is 1 body, and this is another body and this is the third body as I already said can I write the transformation matrix as a box and I can pass the information? Yes, I can do.

So, in the sense what you would be knowing, so, you would be knowing the transformation matrix of B with respect to A, you know the transformation matrix of C with respect to B. So, even if I know the transformation matrix of C with respect to A. So, what I can do? I can simply I commute, in the sense I just multiply, so, this will be obtainable. So, this is what we call the transformation.

Now, for example, this position vector of Q with respect to C is known and the information of these 2-transformation matrix is known what I can do I can get this very straightforward, the position vector of Q with respect to A can be written as the transformation matrix of C with respect to A multiply with the position vector of Q with respect to C.

But this can be written as this form. So, if I write this, so, what you know the transformation matrix general, it would be a rotational matrix and position vector and conventionally we assume that there is no prospective in the sense the prospective vector would be we consider as 0 in the sense it is always orthogonal view and we always assume that it is for lecture, we always assume that everything is in a single unit in the sense the scaling factor is 1. So, this is what I want to say. So, this is 3 cross 3 and this is 3 cross 1 and this 1 cross 1 and this entire thing 1 cross 3.

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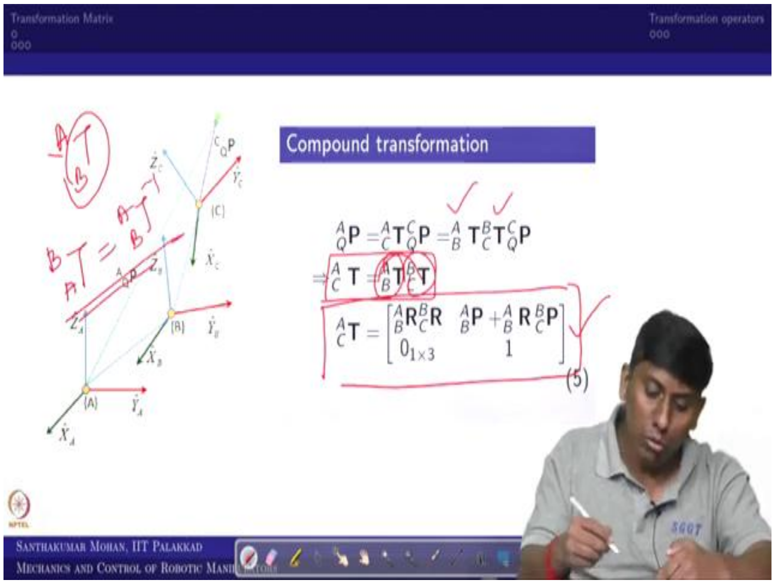

So, if I use that, so, what I will get the rotational information compound rotation can be written in this form and this can be expanded in this form, once we know these 2 can be written or transformation matrix of C with respect to A in this way and this is what the final matrix. Now, it is easy. So, once you know this box and this box you just post multiply and you get it.

So, now, this is what the final matrix you can get it. Now, this can be used as a simple compound transformation tool. So, that is what we have used. Now, the next case is if you know the transformation matrix of A or transformation matrix of B with respect to A, can I get transformation matrix of A with respect to B. So, this is nothing but this inverse, can I get? So, this is what the second tool which we are going to use.

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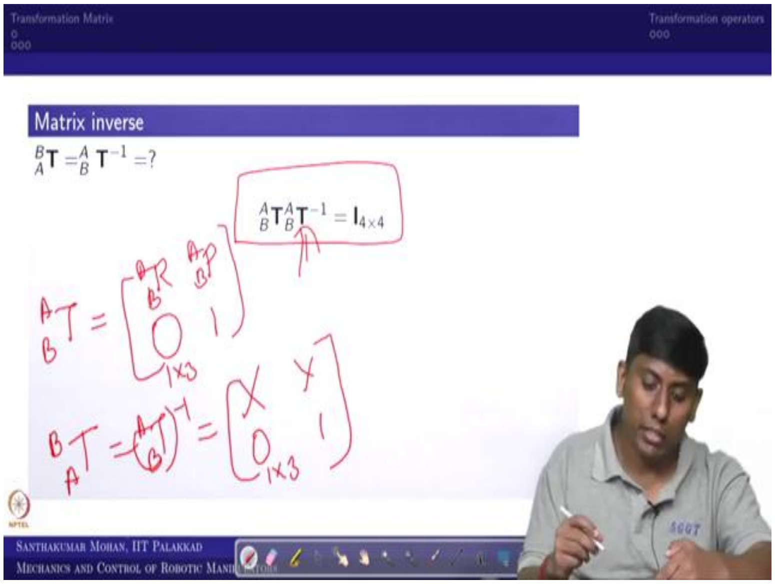

So, that is what we are going to use, for that what 1 supposed to know. So, this product would be giving identity matrix. So, that is the algorithm or that is the logic we are taking. So, this product would be identity matrix. So, if we assume that, so, if we assume that this information is unknown in the sense that transformation matrix of A with respect to B is unknown, but transformation matrix B with respect to A can be written this way.

And this is big 0 which is 1 cross 3 and this is 1 and transformation matrix of A with respect to be in the sense transmission matrix of this inverse would be having X to the bigger fact and Y as another fact and this is 0 which is 1 cross 3 and this 1. So, what I can do, I can multiply these 2 matrices and equate to identity matrix.

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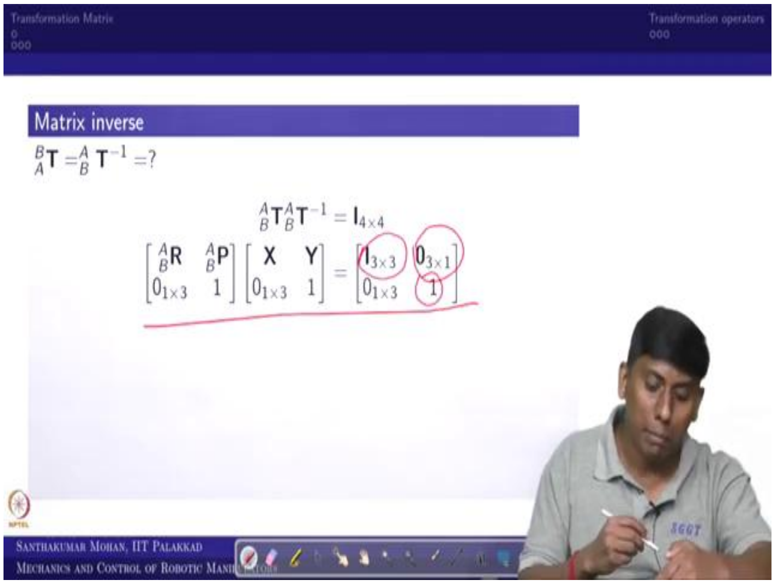

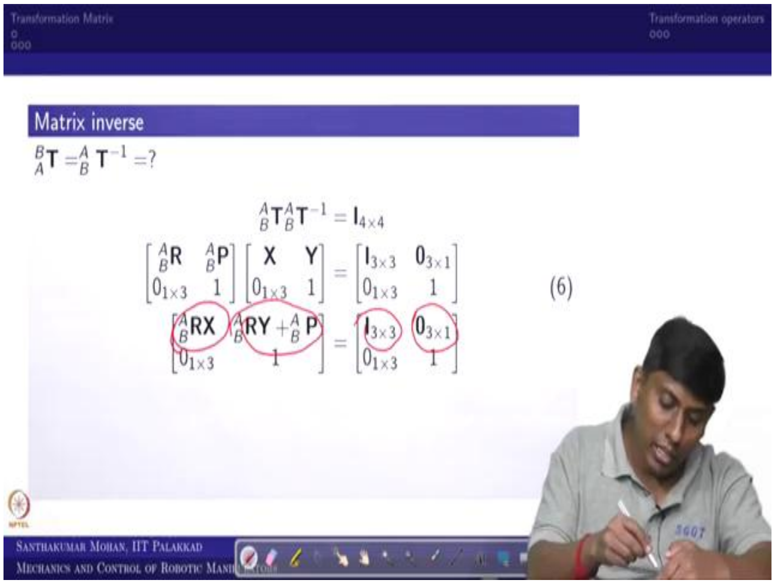

So, that is what we are going to do. So, this is another subset of identity matrix, and this is 1 scalar, so, and this is 0 vector can I multiply. If I multiply what I will get, this is the first term which is equivalent to identity matrix and this the second term this is equivalent to 0 vector. So, in this sense I can rewrite this.

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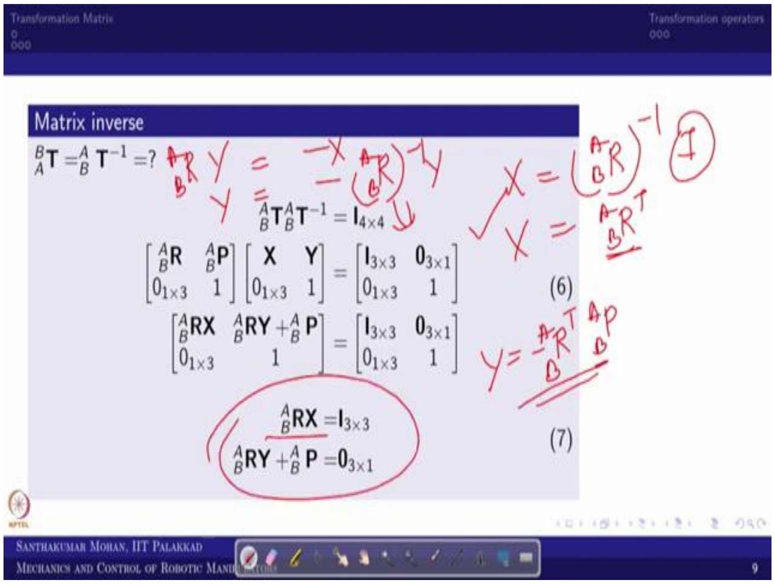

So, from this, so, what 1 can see x can be written as, so, R B A inverse into identity matrix, but identity matrix no need to inform because this is a matrix. So, then what you can write this is R B A transpose because we know that the determinant is 1, so, in that since the X can be get like this. So, similarly Y can be obtained. So, how?

So, you can write, so, this is R B A equal to minus X then Y is minus R B A whole inverse into Y and is equivalent to what? So, I am writing Y is, so, R minus transpose into P B A. So, this is the case. So, now, these 2 obtained. So, now X and Y.

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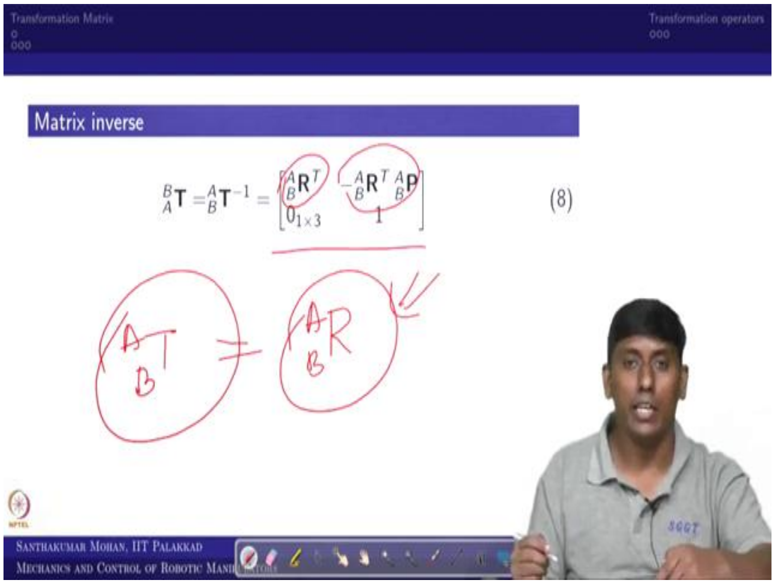

So, what 1 can see, this can be as the operator. So, this is what the idea. So, now, that we substitute that Y and the X. So, now, it is straightforward in the sense the transformation operator where we can use for compound or matrix inverse is obtainable. So, in the sense what we obtained in this particular lecture, we have obtained the transformation matrix of B with respect to A, if there are frames A and B are given.

So, further in the next lecture what we can expect how to find this rotation matrix this? This is consist of compound rotation, in reality, what reality in the sense in the reality you have the 1 body to another body would be in space. So, obviously, there are 3 orientational information is known or given, how we can write this rotation matrix of B with respect to A for a given you call 3 orientational information, which is very tricky.

So, there are several compound transformations are coming in the sense it should not be called a compound transformation although it is, but we have to call composite rotations. So, there are several notations are there, we are going to use that in the next lecture. And till then see you bye, take care. I hope you are clear about the transformation matrix. Thank you.