Mechanics and Control of Robotic Manipulators
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Lecture 4
Description of Position and Orientation

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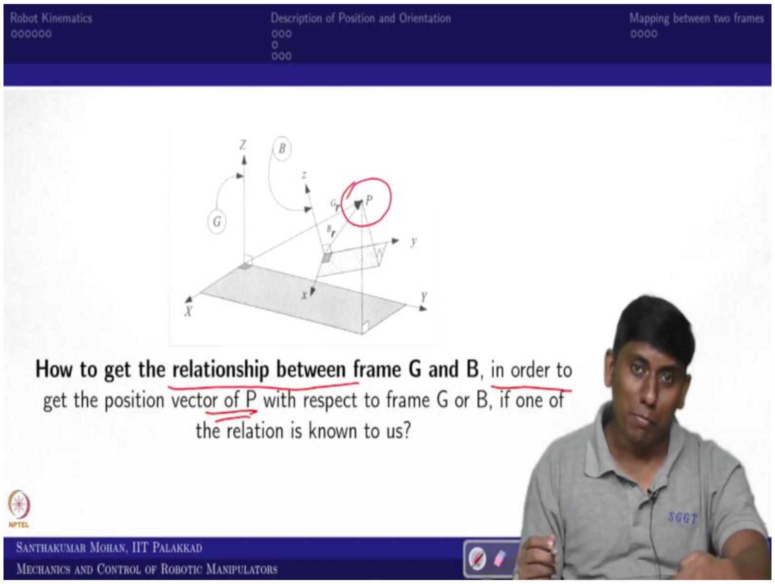

Welcome back to the course on Mechanics and Control of Robotic Manipulator. If you recall the last lecture we stopped this particular slide, so where we can say that how to get the relationship between the frame G and B. This is what our main interest. So why we are taking this?

In order to get the position vector of P with respect to either G or B if one of the relationship is known to us. So in that sense what I said in last lecture itself we will be talking about description of position and orientation in the upcoming lecture. So let us actual like move that.

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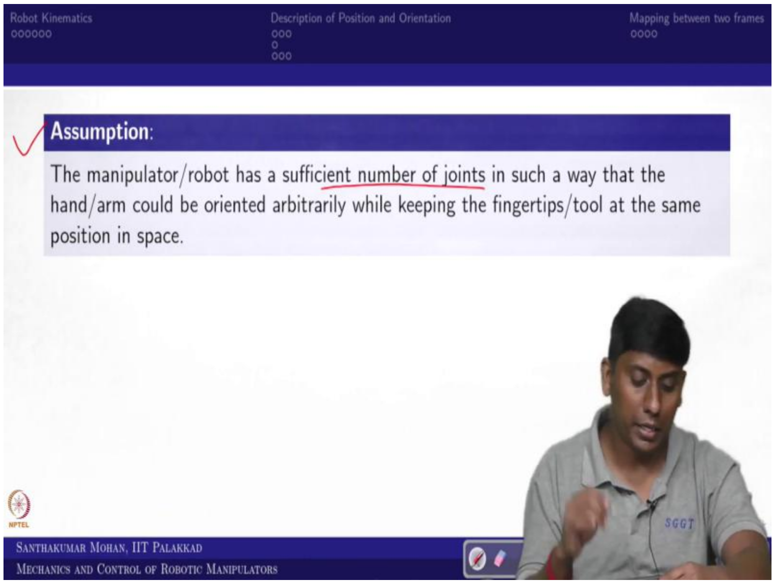

So for that what one simple aspect we are bringing, the assumption that the manipulator or the robot has sufficient number of joints in such a way that the hand or arm could be orient arbitrarily while keeping the finger tip or tool at the same position in space. In the sense when I derive the equation or when I derive the relationship we assume that these all actual like exists in the sense. So the robot constraint we have not incorporated in this particular description. That is what the whole idea.

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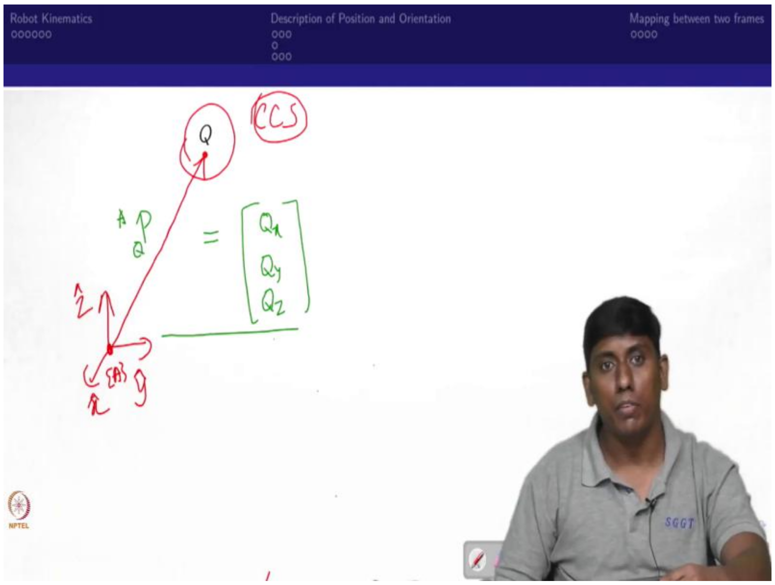

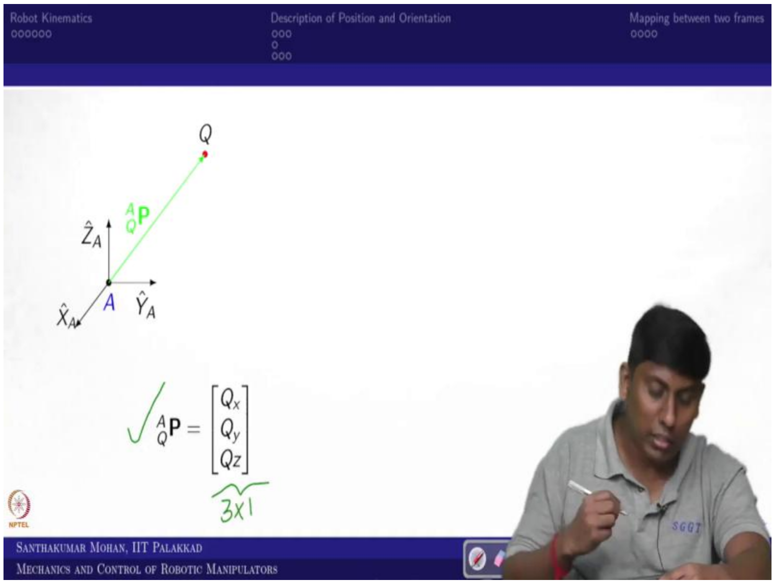

If that is the case what we can see? The position vector description we can do. For example now there is a point Q in a; you can say three dimensional space is given, can I relate this with respect to one of the frame, which is very easy? Because we are using the Cartesian co-ordinate system, we simply call CCS. So in the sense Cartesian co-ordinate system, which means it would be having so, so X, Y and Z.

So we usually denote in the, you can say, unit vector which is, I wrote it as A cap. So what that means? So this is one of the frame called A. So the A frame would be having zx cap, zy cap and, you can say sorry, Ax cap, Ay cap and Az cap all would be there. So further what we can see like since it is a Cartesian so this position can be directly written as a position vector.

So what we can write? This is position vector of Q with respect to A as straightforward as 3 quantity which I can write Qx, so Qy and Qz. This is very, very straight forward, right. So the position vector description with respect to Cartesian coordinates system is not big deal, right. So we can see that is what I have shown. So there is a frame A and A to Q, I have seen there is a position vector. So now you can look at it.

So capital P denote with the bolded, this denotes as a vector which is the position vector. So whatever the, you can say, suffix which is relate to the body or the system. Now position vector of Q, so whatever I have written on the top that is with respect to. So now this I suppose to write as, so position vector of Q, so with respect to frame A.

So now you are clear, right? What we would be using as the denotion or what would be our representation. This is what we have writtem. I hope this is not, you can complicated. You can get the position vector of Q with respect to A as 3, you can say, scalar quantities which denote as 3 cross 1 as a vector which is very easy.

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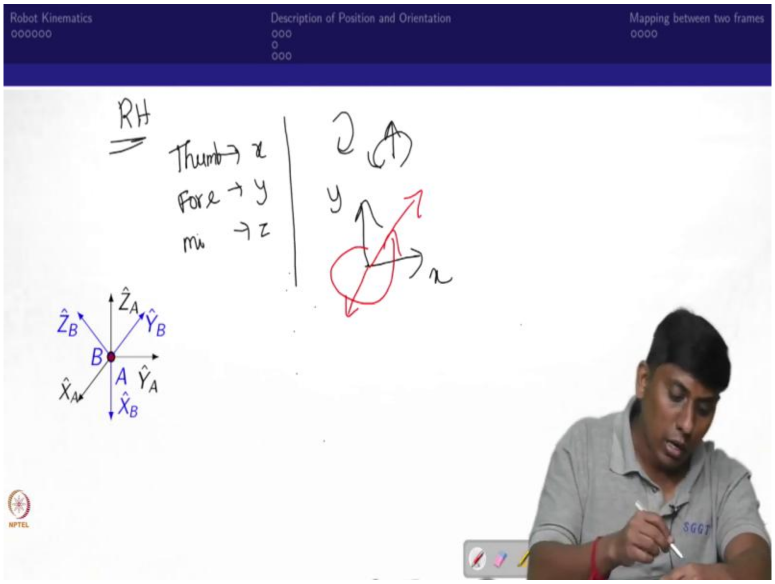

Let us go to the next one which is what you call little complex, which you call description of orientation. When there would be a description of orientation will come? If the position is a simple point or the body which you consider as a point it won't come. Now there is a two body so one body which is I considered as a ground.

There is another body which is rotating. So what I can see? This body would be having XB as the frame and the ground which would be having XA as the frame. So now I can see that this is orient with respect to this axis, so and so angle I can say. This is what I can say, right. But now this is in the three dimensional space.

So how many angle it would be there? So three orientation will come. Can I write it straight away? So for understanding that what we are doing? We are taking the frame A first. Then I have another body which is oriented with respect to A in this form. So the body which is having B as the frame, the B frame is orient XB YB and ZB. Now this is the case.

So before going to talk so I already mentioned that we are going to use only right hand rule in the sense, so the thumb finger would be related to what you call x axis, then the forefinger is denote to y axis and the middle finger is denoted to z axis, right. And you can take for the orientation or the rotation positive direction.

So you can say that the thumb direction you keep your thumb you can keep it in the positive direction of that particular axis and how the remaining fingers will come, the forefingers, right, so that is the positive direction of rotation. So in that sense if I say that this is the screen where this is x axis and y axis and I know that the z axis will come outside the screen.

So now what would be the positive rotation? I keep the thumb finger in this way. So how it is coming? So it is coming like this, right, in the sense anticlockwise direction is your positive direction. So now why there is a positive direction in counter clockwise if I draw in the screen, so this is the way. If I use this z axis this way then I would say that is opposite. Now I hope what denotions we are using or what representation we are using, that I hope you are clear.

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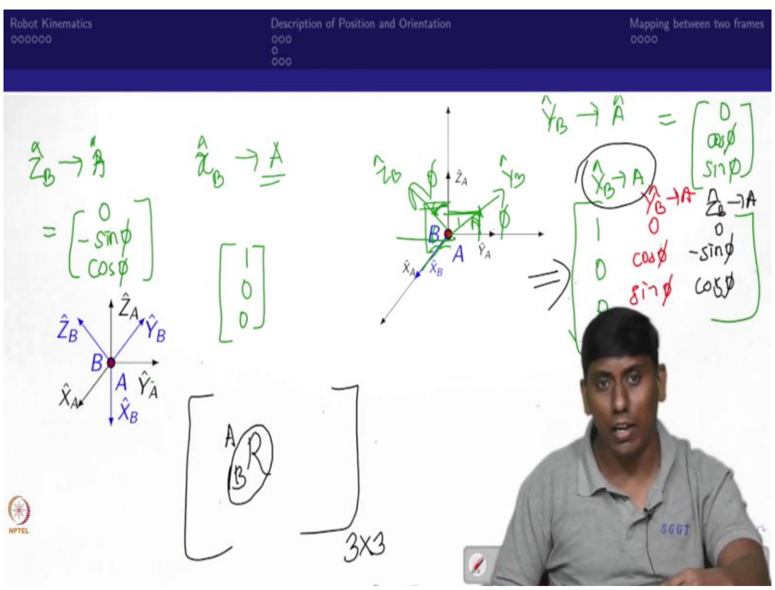

So let us move further. So what that, so we can see like if these are there can I find some kind of idea to give the orientation of B with respect to A? So obviously you can, one can see that the A to B there is no straight forward positional information which we did in the earlier. So then what one can see? So one can see, can I do some other way?

So what other way? For understanding that I take one another example. So there is a body which is having you can see XB and XA as projected in the sense, it is collinear. So only we will see you call the orientation with respect to, you can say, XB frame, it is actually oriented as phi angle. So now one easily you can get. So what one easily get?

So these all unit vectors, if I have a unit vector value, can I actually project it on the frame A? In the sense can I project the XB unit vector projected on the A frame? What I will get? So in this case it will give a straight forward [1 0 0], right. If I project YB unit vector to A frame so what that would be? YB will not be projected on x. so that would be giving 0 but what would we projected on YA? So this is unit vector 1. This would be phi. So that would be giving cos phi.

So what would be the projection on the ZA? So that would be sin phi, right. The similar sense you can project your ZB unit vector project on the A frame. What would be the projection? Obviously that will not give a projection on x. So that will be 0. But this unit vector I project on YA frame. So what would be? That would be equivalent to, so this is phi.

This is 90 minus phi. This is cos 90 minus phi is sine and it is opposite direction. So this is minus sin phi and this is straightaway cos phi, right? So now what we get? So we will get three projection vectors which is simple cosines, right? So in that sense what, direction cosines that too, so now can I merge these three unit vectors? Yes, I can do.

So what that? So this is XB unit vector projection on A so that will give [1 0 0]. So then YB, so YB unit vector projected on A that would be giving [0 cos phi sin phi], right. So the third thing is you can see, so the ZB unit vector projected on A frame, that would be giving [0, minus sin phi and cos phi]. So you can see like what it is giving. So it is giving again another set of unit vectors.

But what it is giving? It is giving some kind of information of each unit vector of B projected on A. So now this is what we are going to consider as a combination of these three unit vector which is going to get 3 cross 3. This is what we are going to call as a orientational information which is simply called rotation matrix. So now the rotation matrix of B with respect to A can be written this way, like this.

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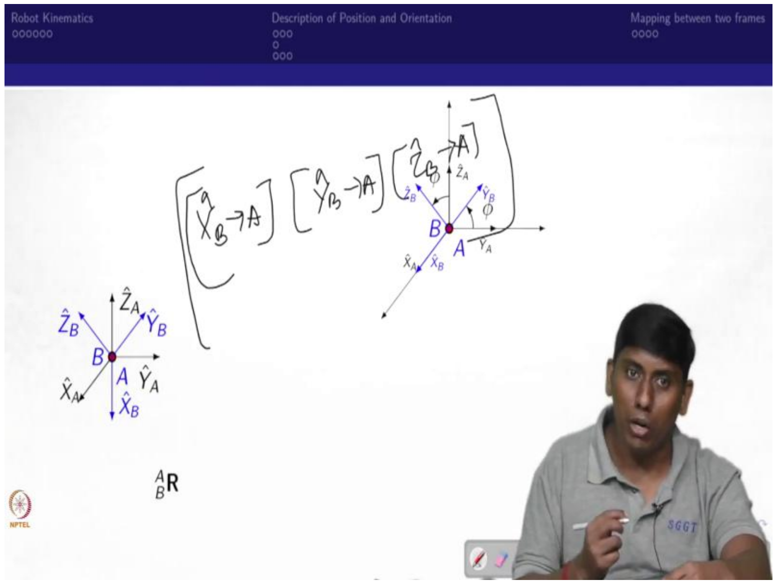

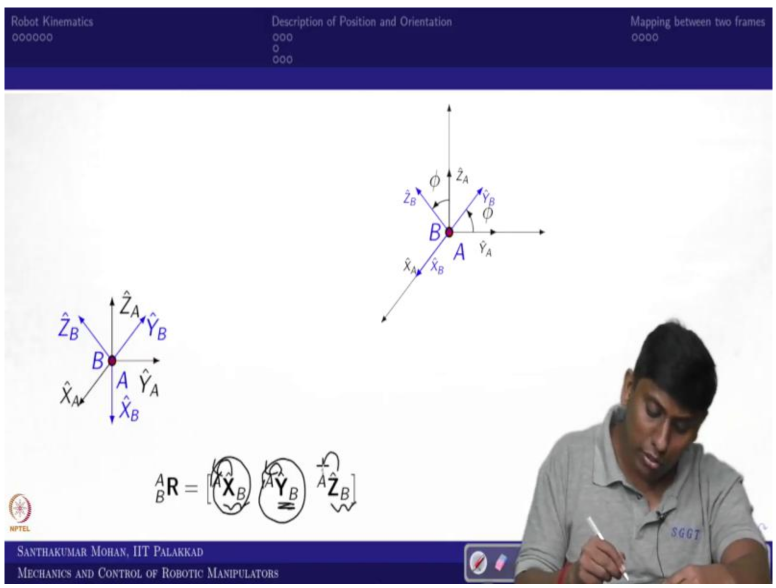

So now this is one specific case, right? How can I make it a generalized. So for generalizing what we can do?

We can make it some thing like the cosine, direction cosine or you can say a projection. So how I can do? I can always say that XB projected on A, this is one unit vector, then YB projected on A another unit vector, then ZB cap projected on A. So now these all make it. So now can take the projection? It is straight forward. So you can take simple dot product. This is what will be obtained. You can see in that.

So this is what we are rotating, in the sense YB frame or you can say YB unit vector projected on A, so XB unit vector projected on A, ZB vector projected on A, that is what the meaning.

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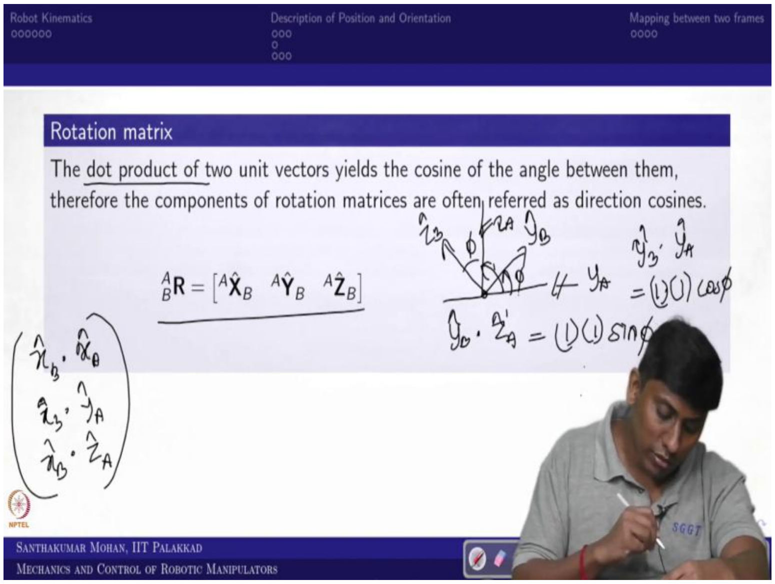

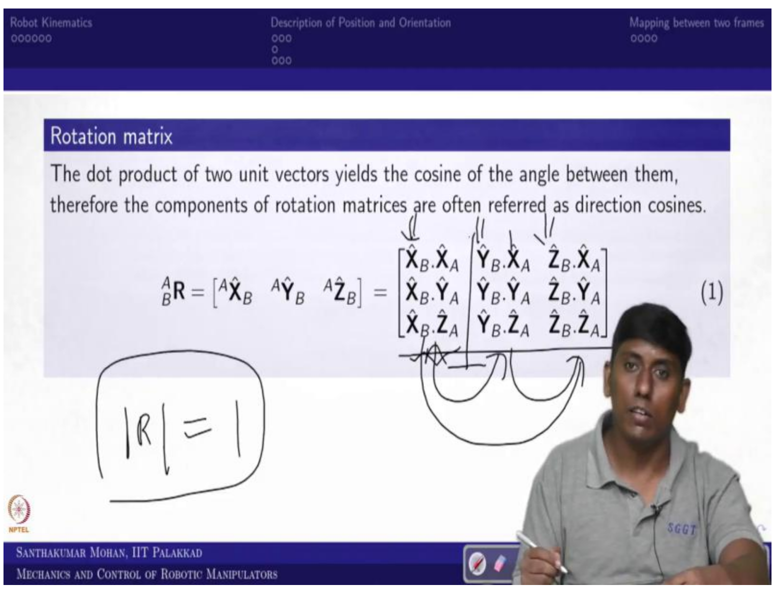

So now we will see. This is what we have written. But this how I can write? I can write as a dot product of two unit vectors is a cosine, right? So now what dot product I can take? So I can take the dot product XB cap to XA cap, XB cap dot product with YA cap and XB cap dot product with ZA cap. So now you can recall what we did in the last case.

So we say that this is YB, this is ZB cap, right and this is what you call phi and this is phi, equal right? So now this is you call YA and this is ZA, right? So you can recall what we did. So now this is 1 and this is 1 so the direction cosine what would be here? So YB cap YA cap. So what would be? This is 1 dot 1. Then what happen? So this is 1 1 and this is direction cosine cos phi. In this case so YB cap dot ZA cap should be 1 and 1.

This is what angle? So the angle is 90 minus phi. So you can say cos 90 minus phi is actually sine phi. So that would give sine phi, right? So this is also what we obtained, right? So that is what it is in this case. So in this case this is 0 angle so this is 1 dot with 1, 1. This is 0 and 0. So now we can write as a generalized. You can see this is the way we can write a generalized rotation matrix.

You can see this is XB projected on A frame, YB projected on A frame, ZB projected on A frame right? You got it. So now each, you can say, column would be a unit vector. So obviously you can see like this is also like giving a set of unit vectors. So now you can see even these unit vectors, how the nature? So nature is one perpendicular to another.

In the sense so these two unit vectors are orthonormal. These unit vectors are orthonormal. These two unit vector if you can take orthonormal. In the sense what you can see? This rotation matrix is consists of several orthonormal vectors. So obviously if you take the determinant of R, so what that would be? So the determinant would be 1. So now these kind of cases what we call are orthogonal matrix. So that is what we are saying.

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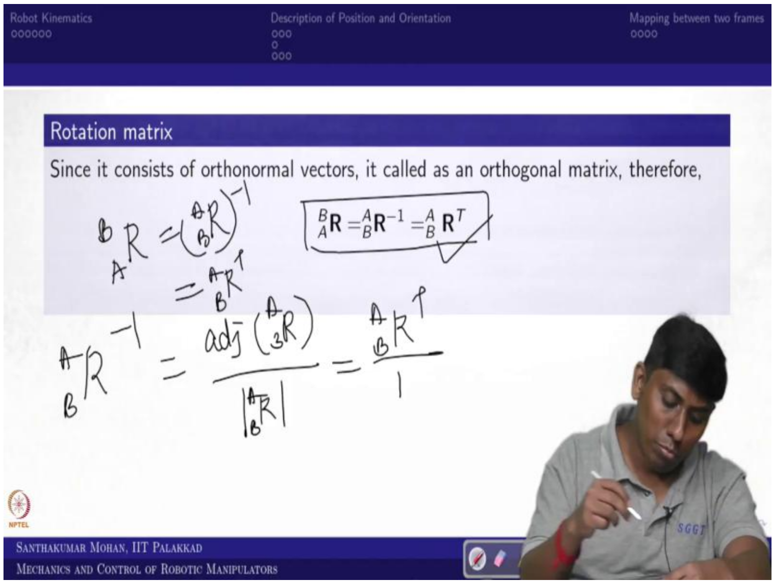

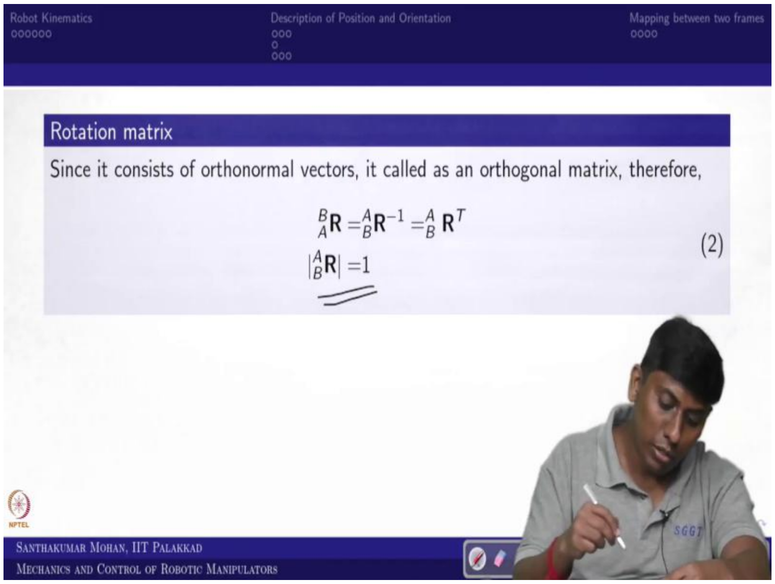

Since it consists of orthonormal vectors so it can be called as orthogonal matrix. Therefore so what you can see? So you know R B A inverse. You can write. So adjoint of a R B A which is equivalent to transpose with determinant of A, right? So what this? This is R B A transpose and this is equivalent to 1. So in that sense what we denote?

If you want to know the information of A with respect to B this is supposed to be equivalent to, so this whole inverse but this inverse is equivalent to simple transpose of this. So now we got some kind of very, very easiest, you can say, form. So now this form how we can use? This can be used as a operator. This can be used as a mapping tool.

This can be used as some kind of, you can say, information related aspect, right. So that is what we are actually trying to see in the next slide. So what we are trying to see? This is what we have found. So based on this condition what one can see?

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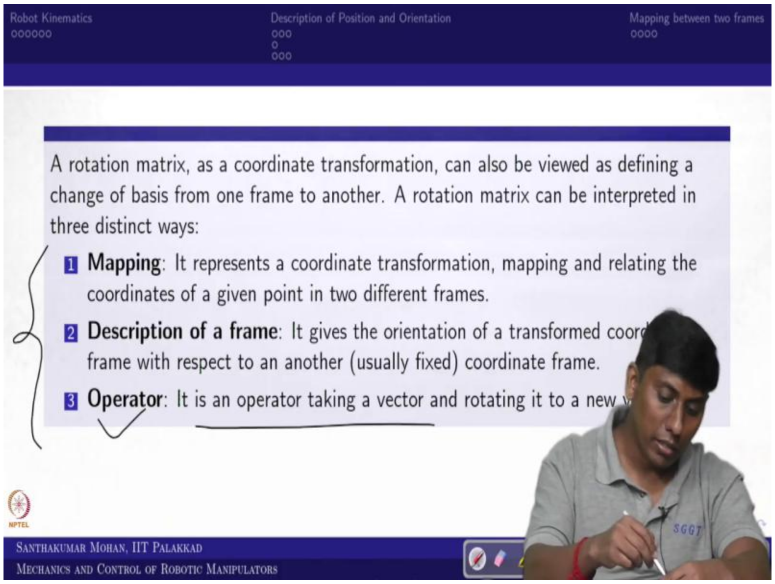

This rotation matrix can be used as, you can say, several form. So what are the forms? One is you can use it for mapping. In the sense it can represent the coordinate transformation. In the sense it can map and relate the coordinate of a given point to two different frames if there is a orientation axis. So what the next one can be use?

So it can be used as a description of a frame, simple. This is what we are going to use in further and further. So what, it gives the orientation of the transformed co-ordinate frame with respect to another, you can say, frame, or you can say co-ordinate frame. This is what we did. That is what we said description of orientation.

What would be the next thing? It can be used as a simple operator. So what that mean? It can be used for rotating existing vector to a new vector. It can be used as an operator. So now you can see that the mapping can be done and further what you can see? We have already defined what is description of position and orientation. So now the rotation matrix can be found which is one of the orthogonal matrix classes.

So I am not talking about Euclidean class and all. It is one of the special Euclidean class matrix. But we are not talking about that in here. But what one can see? We found one of the thing as a position vector, the other thing what we call as a rotation information what we call as a rotation matrix or you can say orientation information, we got it as a rotation matrix.

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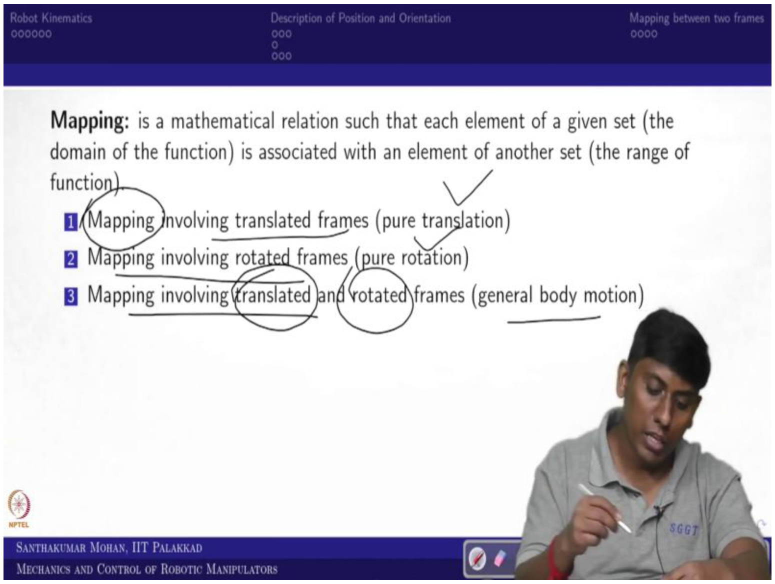

So let us move further. So we know like mapping means not only this. So what mapping means, which we have started at the very first slide. So mapping means we are actually trying to relate a mathematical relation from one body to another body or one set to another set, right. So if that is the case what are the forms which can come in the motion idea?

So it can be two translated frames or two oriented frames or two general frames in the sense it is, it consists of both translation and rotation. So that is what we are trying to see. We can map, involve the two translated frames which is we call pure translation mapping. So mapping involved rotated frame which is we call pure rotation.

And finally what we are interested here is the general body motion which can have both translation and as well as rotation. This is what we are interested.

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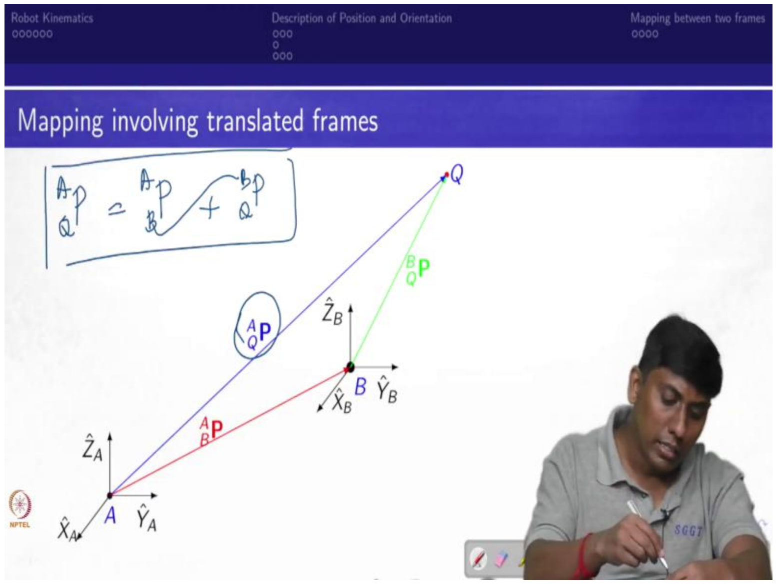

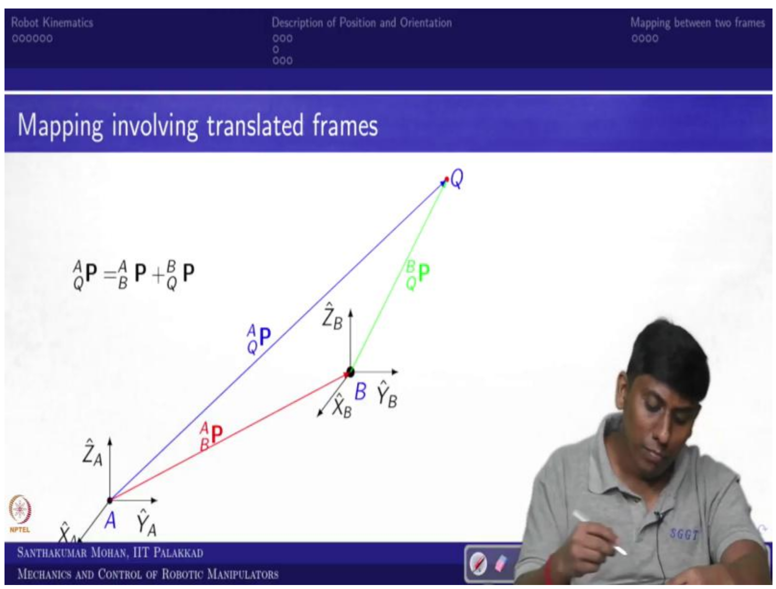

So if that is the case so what one can see from the generalized pictorial view from the, you can say, the mathematical relation? So what we can see? We can start with the frame A. So the frame A would be consist of three, you can say mutually perpendicular unit vectors. So now the body which is we call B. So the B is consist of what? Another body frame which is, you can say, the frame B.

So now I have one simple idea. If I have the positional information of Q with respect to B if I have, further I can derive this. Can I get this? Yes I can do. So that is what we are trying to do.

So you can see that if this information is available, further the translation information of these two are available we can find this. In the sense position vector of Q with respect to A we can find.

Provided what? The positional information of B with respect to A is known. So that is what we are trying to show here. So in the sense what would be? By looking the diagram itself you can see the Q can be written like this. But this can be simply written as the vector addition of this. In the sense so PQ with respect to A can be written as, so P B A plus plus P B Q.

In the sense position vector of Q with respect to B and position vector of B with respect to A. Some people even see that it this is matching then this is the transformation is done. So now this is what mapping between or mapping involved translated frame. It is straight forward, right? It is simple, what you call vector addition.

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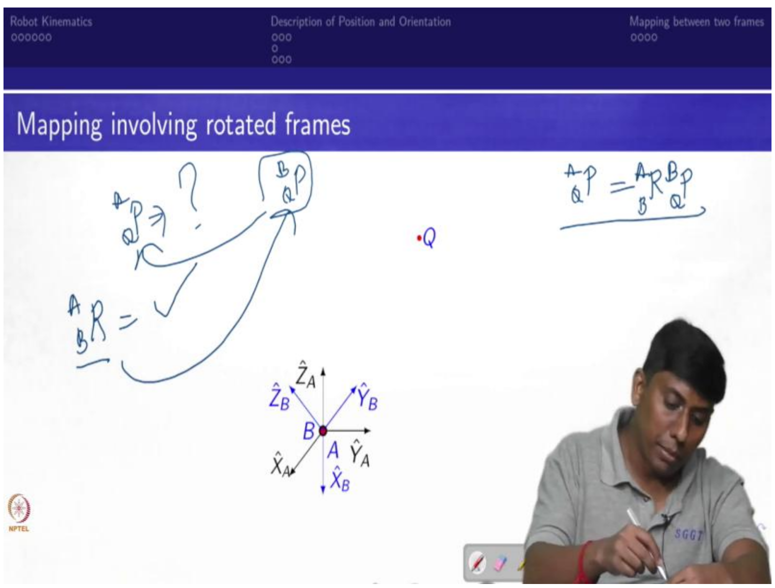

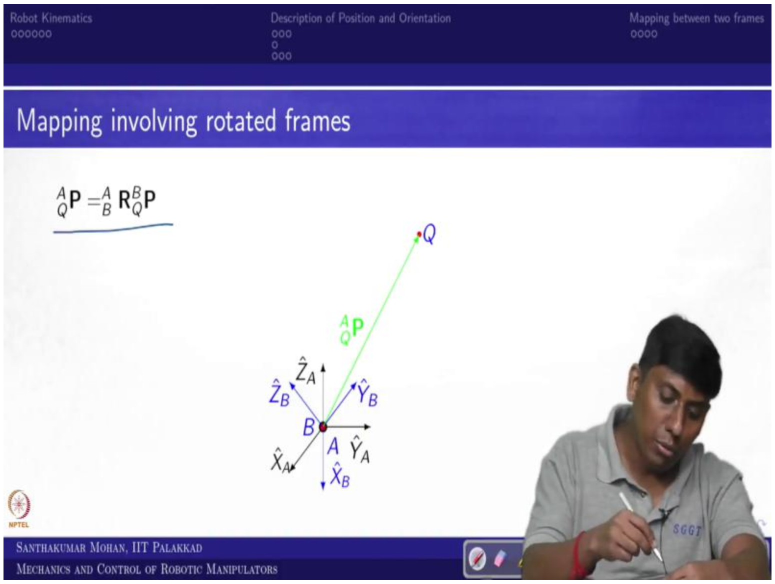

So let us go next one which is mapping involved rotated frames. Is it anything challenges? We can see. So obviously there are two frames we taking. So body 1 and body 2. So the body 1 we assume that initial frame which is A and the body 2 which is we call B frame. So now again, again there is a point Q which is one of the information is known with the sense, so P Q with respect to B is known.

So how I can find P Q with respect to A? So I need one additional information, right? So what information? How the frame B oriented with respect to A I should know. In the sense the rotation matrix of B with respect to A if I know I can bring this, right? With the help of this I can find this. What one can easily see? This is the rotational information.

If I rotate opposite direction of this so what I will get? I will get this Q with respect to A. In the sense what one can see easily? So we can see the position vector of Q with respect to A simply rotation matrix of B with respect to A multiply with the position vector of Q with respect to B, right? So this is another information which we obtain, right? So what that? This is mapping involving rotated frame. You can find that. So this is what we have obtained.

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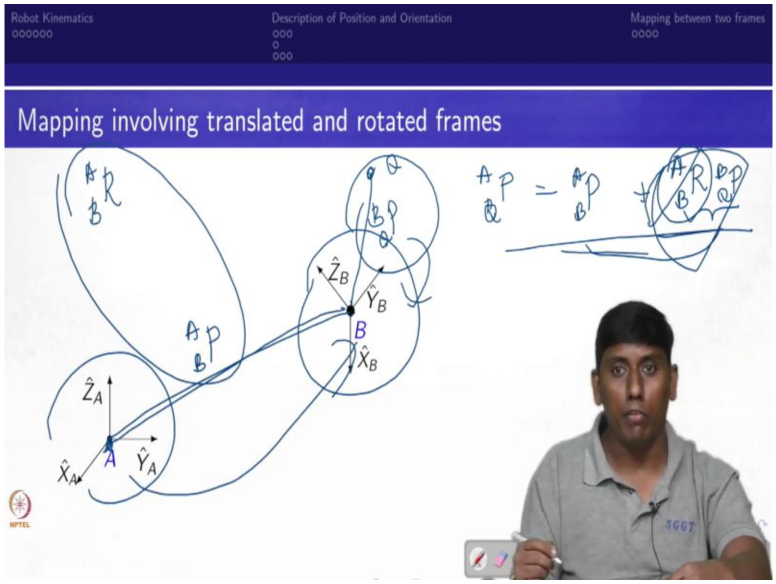

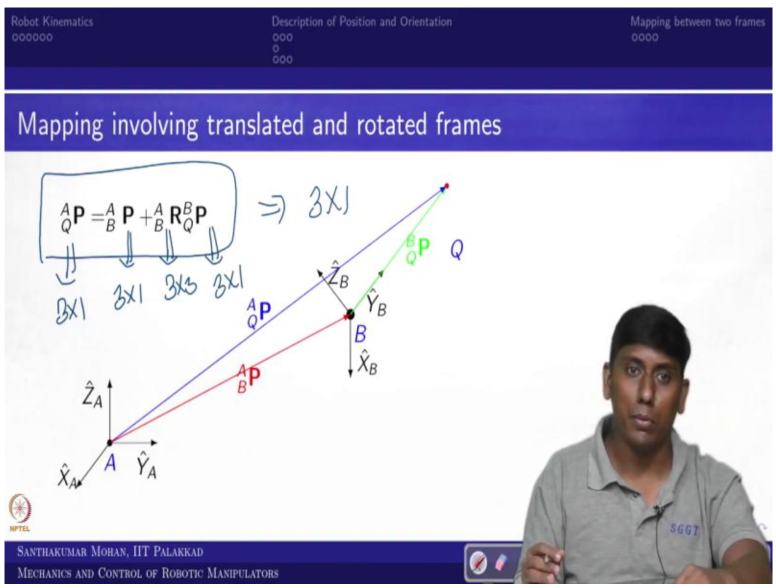

So now we will come to the general case. This is the last slide of us. So we can see that how it can be done this lecture. So we can see like this is A and the point you have frame, the coordinate system. And B also have a coordinate system, which is, you can see by looking itself, these two are oriented. So now what information I should know? The rotational information of B with respect to A and the translation information of B with respect to A should be known.

So now these two informations are known can I find any arbitrary point Q with respect to B I know? Can I find the position information of Q with respect to A? Yes. First what supposed to do? So first you have to orient this in such a way that that would be make it parallel. Then we can do the vector addition, right?

So then what we can expect? So we rotate the, you can say, the Q vector. So the Q vector, so we will make it first parallel to A. Then we do the vector addition. So now this is what we, you can say, mapping involving translated and rotated frame. So now you can see like if this B is associated with A in the sense these two are same point, this is what we do. Similarly if we assume that it there is no rotation so what we will do?

This assume that identity matrix, this is what we do, right? But in this case it is not, so first we rotate and then translate. You got idea, right? So this is what we have seen as mapping involved the translated and rotated frame. So now what we obtain? We obtained one straight forward relation. So this relation we are going to use further and we will bring some kind of relation further and further. So what this dimension?

This dimension is 3 cross 1, and this dimension is 3 cross 1, and this matrix dimension is 3 cross 3 and this dimension is 3 cross 1. In the sense overall the dimension is 3 cross 1. So now next lecture we will see how we can get into a single entity. So that is what we are identifying in the next lecture. So now with that I am closing this particular lecture. See you then, bye.