

Mechanics and Control of Robotic Manipulators
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Lecture – 39
MATLAB Simulation on Kinematic Control

Welcome back to mechanics and control of robotic manipulator. In the last class we have seen how to do a kinematic control, starting from inverse differential kinematics; that is also we can call as a kinematic control because it is an open loop. So, in this class we are trying to see the kinematic control including inverse differential kinematics in the MATLAB simulated form.

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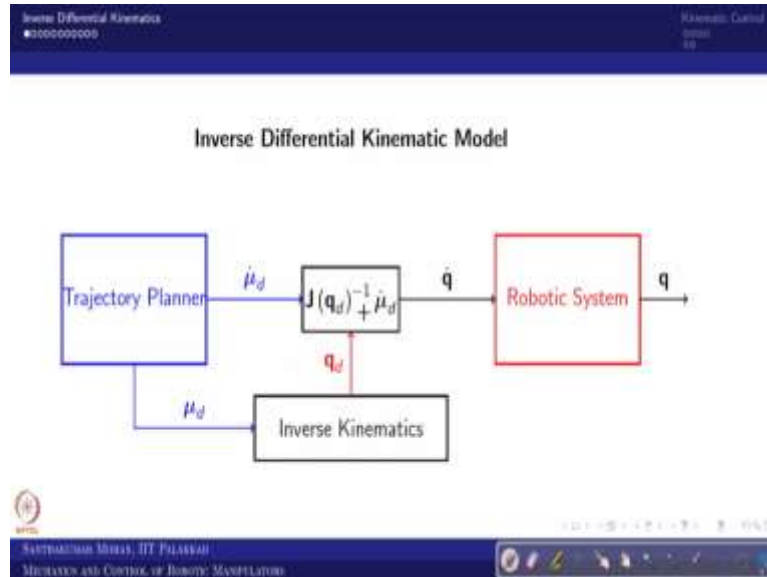


So, whatever we have derived the equation, those things we are trying to do. So, you can see like the inverse differential kinematics both joint space or task space; so, how one complement other that we can see. So, finally we will end up with a kinematic control; we will see joint scheme. So, there we talk about computed velocity control, where we are computing the velocity in the form of kinematic control scheme.

So, where feed forward term and feedback term will be there. So, even if we assume that the feed forward term is not there; so, then we can consider as a proportional control. Then there would be a steady state error, then the steady state error we can rectify with; so, you can see a

proportional integral control. These all we have seen in the last class; so, in this particular lecture we are trying to see that the same thing in MATLAB.

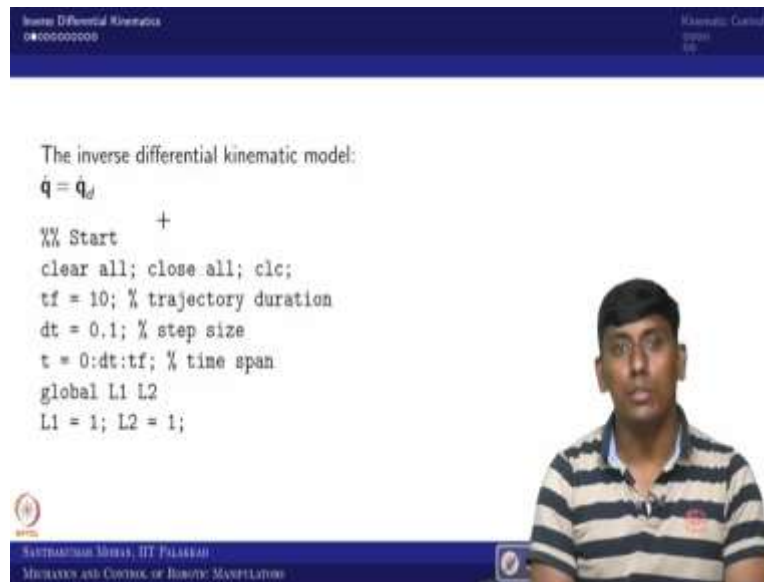
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So, for that we are taking a simple inverse differential kinematic model which we have seen in the last class. So, we would be assuming that the $\dot{\mu}_d$ and μ_d are known. Then what we can see, so we need the \dot{q} ; the \dot{q} we can write as $J(q_d)^{-1} \dot{\mu}_d$; where this q_d can be obtained from the inverse kinematics, so we can do it.

So, even if the trajectory planner is giving straight away the joint space; so, \dot{q}_d and q_d dot, then that is straight forward, where the \dot{q} can be written as \dot{q}_d . So, that is why that picture I have not plotted here; however, we can see in MATLAB, so how we can do it.

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```
The inverse differential kinematic model:  
 $\dot{\mathbf{q}} = \dot{\mathbf{q}}_d$   
  
%% Start +  
clear all; close all; clc;  
tf = 10; % trajectory duration  
dt = 0.1; % step size  
t = 0:dt:tf; % time span  
global L1 L2  
L1 = 1; L2 = 1;
```

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So, for that we are saying the first one is in the joint space, where \dot{q} can be considered as \dot{q} desired; this is the inverse differential kinematic model. So, here we assume that it is straight forward, so where the desired is known and we are assuming that is the actual; so, then we can write the MATLAB code in this way.

So, since it is a differential equation, so we are trying to solve the q of time by numerically integrating. So, again we will bring the Euler integration; so here you can see the 10 is the total duration of the trajectory planner; or probably you can say simulation. So, then the step size is here we have given us dt , and then the time span goes.

So, here I am doing only one addition because I am going to create a sub function. In fact, in last class itself I told we can use a sub function; so, any how I thought of introducing in this kinematic control itself. So, we are talking about global $L1$ and $L2$, where link length $L1$ and $L2$ I would be using in sub functions also. So, that is why we are taking it that way.

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The forward kinematic model:

$$x = L_1 \cos \theta_1 + L_2 \cos (\theta_1 + \theta_2)$$
$$y = L_1 \sin \theta_1 + L_2 \sin (\theta_1 + \theta_2)$$

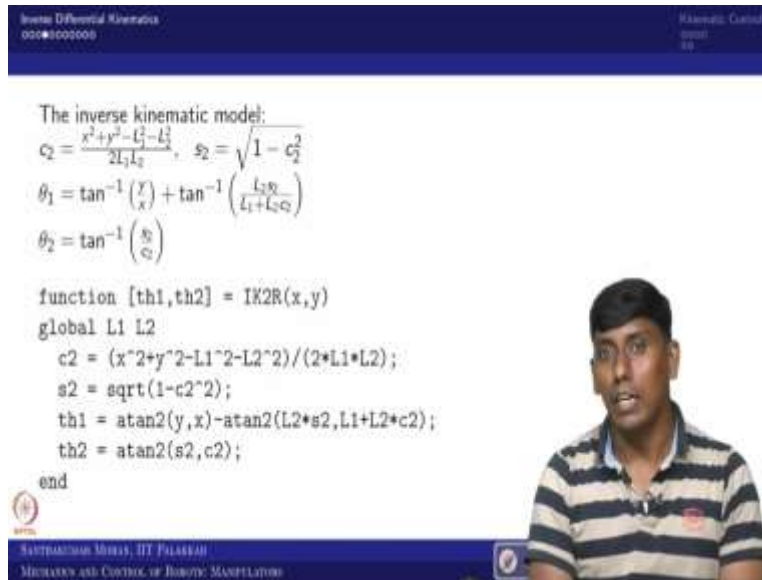
```
function [x,y] = FK2R(th1,th2)
global L1 L2
    x = L1*cos(th1)+L2*cos(th1+th2);
    y = L1*sin(th1)+L2*sin(th1+th2);
end
```

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So, what we are trying to do the forward kinematic model, we want to use it; so x and y we want to write it, so, the x and y write it in this form. So, the same thing I can write as a one of the sub functions, where function x comma y basically this particular function would be returning x and y , for given θ_1 and θ_2 .

So, even if you are going further and further, you can straight away write q and you can say q . So, that also we can do where q of 1 would be related to θ_1 , and q of 2 related to θ_2 ; and similarly, q of 1 is belongs to x , q of 2 is belongs to y that also we can use. Since, this is beginning, so we will use independent variable.

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System Differential Kinematics
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Kinematic Control
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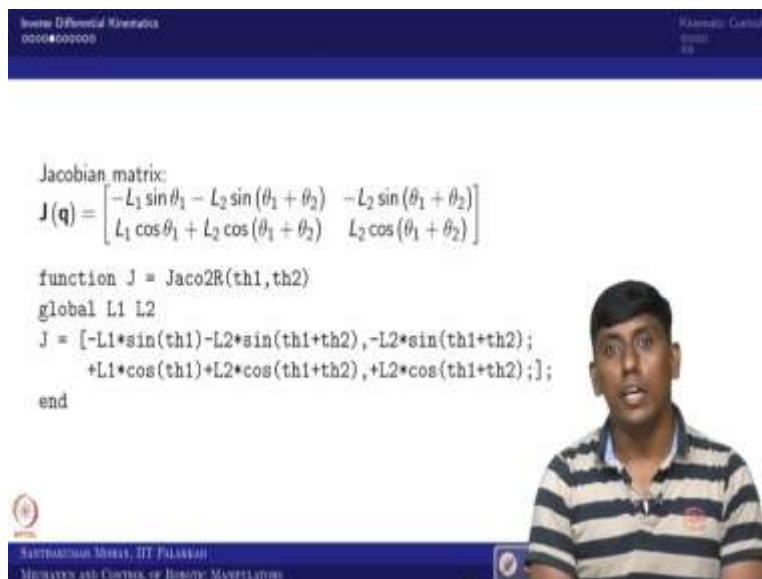
The inverse kinematic model:
$$c_2 = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}, \quad s_2 = \sqrt{1 - c_2^2}$$
$$\theta_1 = \tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{L_2 s_2}{L_1 + L_2 c_2}\right)$$
$$\theta_2 = \tan^{-1}\left(\frac{s_2}{c_2}\right)$$

```
function [th1,th2] = IK2R(x,y)
global L1 L2
c2 = (x^2+y^2-L1^2-L2^2)/(2*L1*L2);
s2 = sqrt(1-c2^2);
th1 = atan2(y,x)-atan2(L2*s2,L1+L2*c2);
th2 = atan2(s2,c2);
end
```

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So, the same way we want to know like what is inverse kinematic model; because any of q desired, we would be taking from the μ desired. So, in that sense we need to know this; so, we are taking an inverse kinematic model which we have derived in one of the lectures, the same model we are taking it in MATLAB function. So, you can see this is going to return θ_1 and θ_2 , for given x and y right. So, now these two functions are sufficient; so, then we need to know Jacobian.

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System Differential Kinematics
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Kinematic Control
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Jacobian matrix:
$$J(\mathbf{q}) = \begin{bmatrix} -L_1 \sin \theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 \cos \theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix}$$

```
function J = Jaco2R(th1,th2)
global L1 L2
J = [-L1*sin(th1)-L2*sin(th1+th2),-L2*sin(th1+th2);
     +L1*cos(th1)+L2*cos(th1+th2),+L2*cos(th1+th2)];
end
```

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So, the Jacobian matrix we derived 2 cross 2 in the given form; the same thing we can write as a sub function. So, Jaco2R is like Jacobian of the two are serial manipulator, for given theta1 and theta2 the Jacobian matrix would be written in this particular sub function; so, these all the sub function. So, further what we want? We want trajectory because so here we assume that the mu desired and mu desired dot would be there. So, in that case one of the easiest ways is we can take a cubic polynomial.

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Inverse Differential Kinematics
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Third order polynomial:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} x_0 \\ \dot{x}_0 \\ x_f \\ \dot{x}_f \end{bmatrix}$$

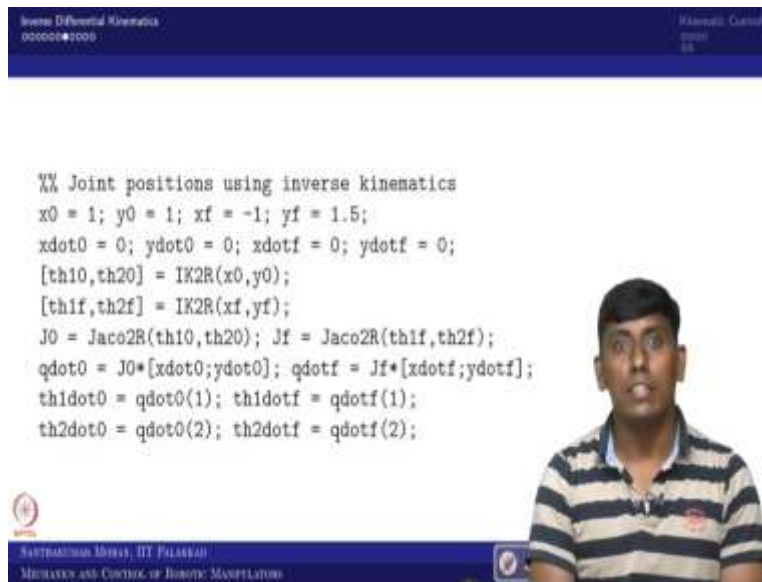
```
function tc = Cubic_TR(x0,xdot0,xf,xdotf,tf)
A = [1,0,0,0;
     0,1,0,0;
     1,tf,tf^2,tf^3;
     0,1,2*tf,3*tf^2];
b = [x0;xdot0;xf;xdotf];
tc = inv(A)*b;
end
```

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So, we are taking the third order polynomial or cubic polynomial; so, we assume that tf is given and t0 we have assumed as 0. So, in the sense general we take x as the variable; so, x of t I can derive based on this third order polynomial. So, I can find the trajectory coefficients tc for given input x naught, x dot naught, then xf, x dot of f and tf is given; then I can find the trajectory coefficient based on this.

Once I found the trajectory coefficient what I can do? I can go with x of t; so that is what we are trying to do here. So, first case what we are trying to do? So, only joint space, the q desired and q desired dot are given.

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```
%% Joint positions using inverse kinematics
x0 = 1; y0 = 1; xf = -1; yf = 1.5;
xdot0 = 0; ydot0 = 0; xdotf = 0; ydotf = 0;
[th10,th20] = IK2R(x0,y0);
[th1f,th2f] = IK2R(xf,yf);
J0 = Jaco2R(th10,th20); Jf = Jaco2R(th1f,th2f);
qdot0 = J0*[xdot0;ydot0]; qdotf = Jf*[xdotf;ydotf];
th1dot0 = qdot0(1); th1dotf = qdotf(1);
th2dot0 = qdot0(2); th2dotf = qdotf(2);
```

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So, in that case, so we assume that the initial position and final position of the end effector is given, and the time also given already t_f ; so, which is 10 second. So, now we assume that it is a 2R serial manipulator, where L_1 and L_2 also given.

So, now what we can do through the inverse kinematics? We can find θ_1 initial and θ_2 initial; similarly, θ_1 final and θ_2 final we can get. Some further extend you want to find the initial velocities, so we can use Jacobian; because here we have, we know the end effector velocities.

So, we can find the joint space velocity by the Jacobian matrix; so that is what we can do it. So, here we take inverse Jacobian so that anyhow, in this particular case would not be that beneficial; so, that we can correct it in the MATLAB code, so, what exactly we wanted. So, then $\dot{\theta}_1$ dot all those things we defined.

the two things we have modified. So, now we have taken L1 and L2 are one meter each or one unit each; so, then the t_f is 10 second.

So, the total simulation also like 10 second that is what we have taken. So, now this is the initial, so we are trying to do the inverse kinematics so that we can find θ_1 zero and θ_2 zero; so, based on that we have found trajectory coefficients. So, if you do not want straight away for example you are giving θ_1 initial and final, you have joint space coordinate straight away.

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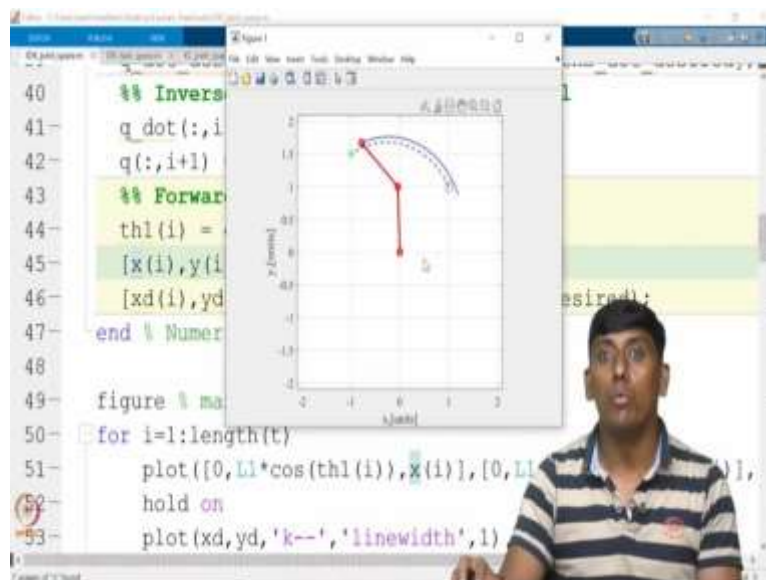
```
13- [th10,th20] = IK2R(x0,y0);
14- [th1f,th2f] = IK2R(xf,yf);
15- J0 = Jacob2R(th10,th20);
16- Jf = Jacob2R(th1f,th2f);
17- qdot0 = inv(J0)*[xdot0; ydot0];
18- qdotf = inv(Jf)*[xdotf; ydotf];
19- thldot0 = qdot0(1); thldotf = qdotf(1);
20- th2dot0 = qdot0(2); th2dotf = qdotf(2);
21- % th10 = pi/4; th20 = pi/4;
22- % th1f = 3*pi/2; th2f = pi/2;
23- % thldot0 = 0; thldotf = 0;
24- % th2dot0 = 0; th2dotf = 0;
25-
26- tcl = Cubic_TR(th10,thldot0,th1f,thldotf,th20,th2dot0,th2f,th2dotf);
```

```
37- th1_dot_desired = [0,1,2*t(i),3*t(i)^2]*tcl;
38- th2_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc2;
39- q_dot_desired(:,i) = [th1_dot_desired;th2_dot_desired];
40- %% Inverse differential kinematic model
41- q_dot(:,i) = q_dot_desired(:,i);
42- q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
43- %% Forward kinematics
44- th1(i) = q(1,i); th2(i) = q(2,i);
45- [x(i),y(i)] = FK2R(th1(i),th2(i));
46- [xd(i),yd(i)] = FK2R(th1_desired,th2_desired);
47- end % Numerical integration ends here
48-
49- figure % manipulator motion animation
50- for i=1:length(t)
```

So, then we can use this, so instead of using the inverse differential kinematics and inverse kinematics; we can directly use this, so let us go to the initial condition. Now, we are taking as an open loop control, so θ_1 desired and θ_2 desired we have derived; and $\dot{\theta}_1$ desired and $\dot{\theta}_2$ desired also we have derived. And then we are using this inverse differential kinematic model what it says?

So, \dot{q} is \dot{q} desired; so, we are trying to find out. So, we assume that x and y are the actual, and x desired and y desired are the desired task space or end effector positions. So, now we are trying to plot this. At the end we are trying to compare the results, or just I want to plot it this; this is what we have used. And these are the sub functions which we have seen in the slide also. So, now if I run this, now if I run this; so, I am using a shortcut F5.

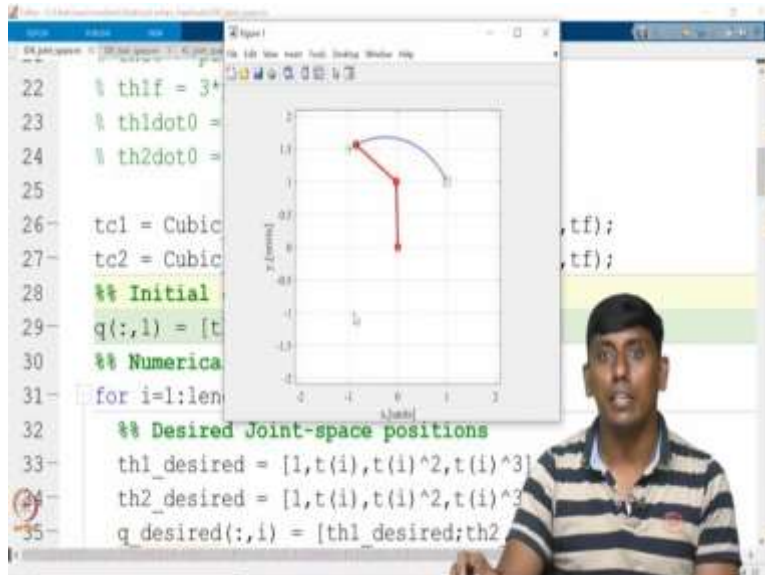
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```

22 % th1f = 3*pi/2; th2f = pi/2;
23 % th1dot0 = 0; th1dotf = 0;
24 % th2dot0 = 0; th2dotf = 0;
25
26 tc1 = Cubic_TR(th10,th1dot0,th1f,th1dotf,tf);
27 tc2 = Cubic_TR(th20,th2dot0,th2f,th2dotf,tf);
28 %% Initial conditions
29 q(:,1) = [th10;th20];
30 %% Numerical Integration starts here
31 for i=1:length(t)
32     %% Desired Joint-space positions
33     th1_desired = [1,t(i),t(i)^2,t(i)^3];
34     th2_desired = [1,t(i),t(i)^2,t(i)^3];
35     q_desired(:,i) = [th1_desired;th2_

```

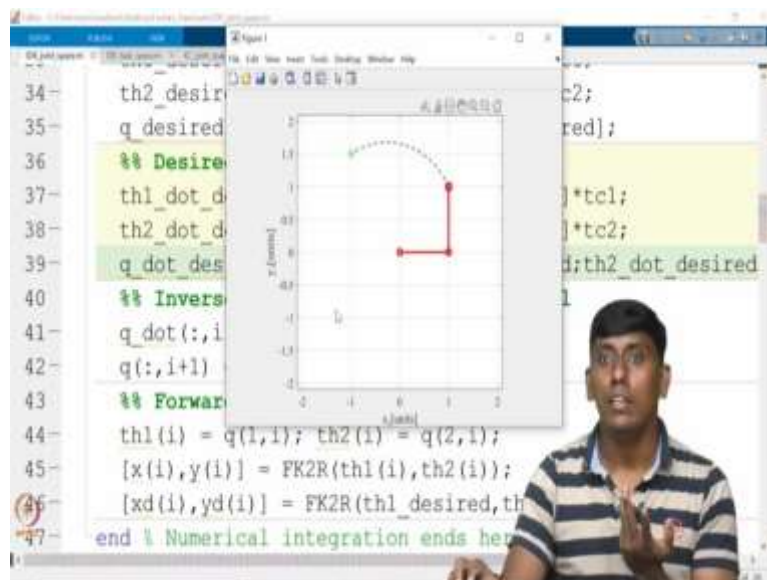


If I run this so what you can see this is the desired, and you can see like the actual is different; because it is an open loop control; it does not know how to correct it. So, in order to make it that clear for example, the initial conditions are same as the desired initial conditions. So, then you can see this would be giving the same trajectory following you can find it already; so, this is what we can see this particular plot is just for making more beneficial.

So, in fact this is not really required because we are trying to see whether the trajectory is following or not. So, now I already said so this is the case. So, now we assumed that the desired is like zero, or you can say \dot{q} desired is zero; so, this will not even propagate.

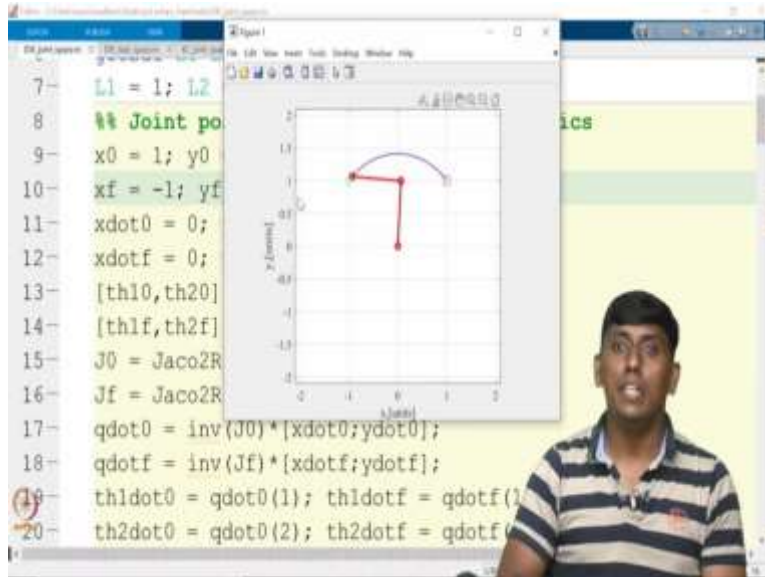
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```
34- th2_desired = [1,t(i),t(i)^2,t(i)^3]*tc2;
35- q_desired(:,i) = [th1_desired;th2_desired];
36- %% Desired Joint-space velocities
37- th1_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc1;
38- th2_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc2;
39- q_dot_desired(:,i) = 0*[th1_dot_desired;th2_dot_desired];
40- %% Inverse differential kinematic model
41- q_dot(:,i) = q_dot_desired(:,i);
42- q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
43- %% Forward kinematics
44- th1(i) = q(1,i); th2(i) = q(2,i);
45- [x(i),y(i)] = FK2R(th1(i),th2(i));
46- [xd(i),yd(i)] = FK2R(th1_desired,th2_desired);
47- end % Numerical integration ends here
```



So, I assume that this is like a zero; it one it actually like will not go. So, in order to understand this is the shortcut which I have used. So, it is not propagating why? Because we do not have any push. So, that is why I said this inverse differential kinematic model call open loop control will work or feed forward control will work when the q dot is desired is non-zero. And the initial conditions of both desired and actual are same; so that is what we have seen. So, now even you want to change this.

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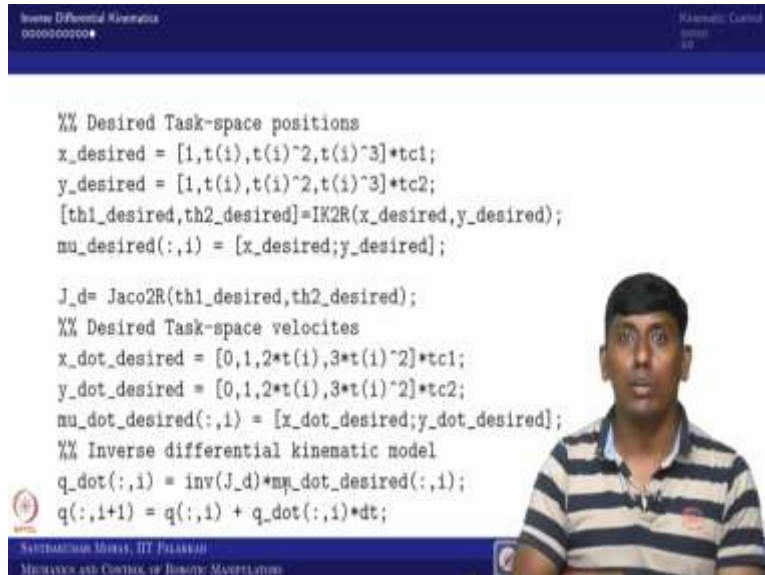


The image shows a MATLAB script window on the left and a plot window on the right. The script defines joint positions and velocities, and calculates the Jacobian matrices for the joint and task spaces. The plot shows a 2D coordinate system with a red line representing the end effector path and a blue arc representing the joint angle range.

```
7- L1 = 1; L2
8- %% Joint po
9- x0 = 1; y0
10- xf = -1; yf
11- xdot0 = 0;
12- xdotf = 0;
13- [th10,th20]
14- [th1f,th2f]
15- J0 = Jaco2R
16- Jf = Jaco2R
17- qdot0 = inv(J0)*[xdot0;ydot0];
18- qdotf = inv(Jf)*[xdotf;ydotf];
19- thldot0 = qdot0(1); thldotf = qdotf(1)
20- th2dot0 = qdot0(2); th2dotf = qdotf(2)
```

For example, I am trying to change this probably so 1; so, I am just changed this. So, you can see it is trying to follow; so, this is what we did in the joint space inverse differential kinematic. So, the same thing we can do it even in the task space. So, there would be a small change in the code. So, that is what we are trying to see here.

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The image shows a MATLAB script window with code for task space inverse differential kinematics. The script defines desired task-space positions and velocities, calculates the Jacobian matrix, and updates the joint positions and velocities. The plot window is not visible in this slide.

```
%% Desired Task-space positions
x_desired = [1,t(i),t(i)^2,t(i)^3]*tc1;
y_desired = [1,t(i),t(i)^2,t(i)^3]*tc2;
[th1_desired,th2_desired]=IK2R(x_desired,y_desired);
nu_desired(:,i) = [x_desired;y_desired];

J_d= Jaco2R(th1_desired,th2_desired);
%% Desired Task-space velocites
x_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc1;
y_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc2;
nu_dot_desired(:,i) = [x_dot_desired;y_dot_desired];
%% Inverse differential kinematic model
q_dot(:,i) = inv(J_d)*nu_dot_desired(:,i);
q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
```

So, this is the task space, so now what change you can expect? So, the theta1 desired and theta2 desired become x desired and y desired. So, now theta1 desired and theta2 desired you want;

then you can use inverse kinematics; so that is what we have done. And here mu desired and mu desired dot would be found; so, this is what we have get it.

You can see mu desired and mu desired dot we have obtained; so, then so the q dot is inverse of Jacobian multiply with mu dot desired, so, this is what we have obtained. So, the code; the prior of this and after this we are not going to change; so, only this content is going to change. In fact, I want to show it here.

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```
10- xf = -1; yf = 1.5;
11- xdot0 = 0; ydot0 = 0;
12- xdotf = 0; ydotf = 0;
13- tc1 = Cubic_TR(x0,xdot0,xf,xdotf,tf);
14- tc2 = Cubic_TR(y0,ydot0,yf,ydotf,tf);
15- %% Initial conditions
16- [th10,th20]=IK2R(x0,y0);
17- q(:,1) = [th10-0.1;th20-0.1];
18- %% Numerical Integration starts here
19- for i=1:length(t)
20-     %% Desired Task-space positions
21-     x_desired = [1,t(i),t(i)^2,t(i)^3]*tc1;
22-     y_desired = [1,t(i),t(i)^2,t(i)^3]*tc2;
23-     [th1_desired,th2_desired]=IK2R(x_desired,y_desired);
```

```
13- tc1 = Cubic_TR(x0,xdot0,xf,xdotf,tf);
14- tc2 = Cubic_TR(y0,ydot0,yf,ydotf,tf);
15- %% Initial conditions
16- [th10,th20]=IK2R(x0,y0);
17- q(:,1) = [th10-0.1;th20-0.1];
18- %% Numerical Integration starts here
19- for i=1:length(t)
20-     %% Desired Task-space positions
21-     x_desired = [1,t(i),t(i)^2,t(i)^3]*tc1;
22-     y_desired = [1,t(i),t(i)^2,t(i)^3]*tc2;
23-     [th1_desired,th2_desired]=IK2R(x_desired,y_desired);
24-     mu_desired(:,i) = [x_desired;y_desired];
25-     J_d= Jaco2R(th1_desired,th2_desired);
26-     %% Desired Task-space velocities
```

So, this is the inverse differential kinematics of the task space. You can see like this segment are same and the input whatever we have taken is same. So, only thing the trajectory coefficients we are calculating for x and y; so, because of that the x desired and y desired we are getting. Similarly, x desired dot and y desired dot we are getting. The same profile we have given and here also we have given induced this error.

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The screenshot shows MATLAB code for trajectory generation. The code includes comments for initial conditions and numerical integration. A plot in the background shows a red trajectory starting at approximately (-1, 1.5) and ending at (1, 1). The code is as follows:

```

13- tc1 = Cubic
14- tc2 = Cubic
15- %% Initial
16- [th10, th20]
17- q(:,1) = [t
18- %% Numerical
19- for i=1:len
20- %% Desire
21- x_desired
22- y_desired
23- [th1_desired, th2_desired]=IK2R(x_desired, y_desired);
24- mu_desired(:,i) = [x_desired; y_desired];
25- J_d= Jaco2R(th1_desired, th2_desired);
26- %% Desired Task-space velocities

```

The screenshot shows MATLAB code for calculating desired task-space positions and velocities. The code includes comments for initial conditions and numerical integration. The code is as follows:

```

13- tc1 = Cubic_TR(x0, xdot0, xf, xdotf, tf);
14- tc2 = Cubic_TR(y0, ydot0, yf, ydotf, tf);
15- %% Initial conditions
16- [th10, th20]=IK2R(x0, y0);
17- q(:,1) = [th10; th20];
18- %% Numerical Integration starts here
19- for i=1:length(t) I
20- %% Desired Task-space positions
21- x_desired = [1, t(i), t(i)^2, t(i)^3]*tc1;
22- y_desired = [1, t(i), t(i)^2, t(i)^3]*tc2;
23- [th1_desired, th2_desired]=IK2R(x_desired, y_desired);
24- mu_desired(:,i) = [x_desired; y_desired];
25- J_d= Jaco2R(th1_desired, th2_desired);
26- %% Desired Task-space velocities

```

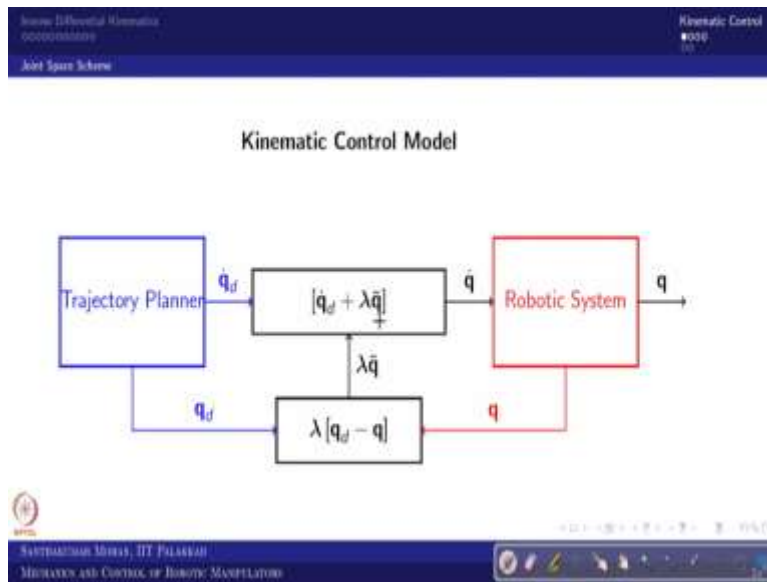
So, you can see that so now, so you can see like the profile is actually straight line; so earlier it was not. So, now if you are look at it this, you can see like it is non-zero or non-zero error I suppose to say; or you can say the desired initial position and desired or actual initial positions

are not same. So, this is the desired initial position, and this is the actual initial position; these two are same, so that is why the profile is not supposed to be followed here.

So, now in order to make it that the condition what you have seen is this. If the velocity is non-zero and the initial actual and desired are same; so, then you will find that this would be a good control. That is why it is actually we call open loop control, because it is a feed forward.

So, there is no compensation for the actual case. So, now even I introduce a small friction or some kind of uncertainty; this will not be following it. So, that is what we want to get it here; so, will go to the slide. So, then we can see how we can use this as a kinematic control.

(Refer Slide Time: 14:25)



So, for the kinematic control what we have taken? So, one of the conditions we have taken that. So, if the first order error dynamics is going to be stable; so, for that what we have taken? So, lambda is the one of the positive constants we have considered. So, then this is the kinematic control model; so, this is going to be a feed forward term, which we have seen inverse differential kinematics. Now, this is the feedback term, this is what we are going to call as a proportional control.

(Refer Slide Time: 14:56)



```
%% Initial conditions
q(:,1) = [th10-0.1;th20-0.1];
%% Control parameters
lambda = 4;

%% Joint position errors
q_tilda(:,i) = q_desired(:,i)-q(:,i);
%% kinematic control based on computing velocity control
q_dot(:,i) = (q_dot_desired(:,i)+lambda*q_tilda(:,i));
q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
```

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So, what code would be changing? So, the initial thing all are same. So, only thing we are adding the control parameter, where we have seen the lambda, that lambda is coming here; so I have taken as 4. And further these all same; so only added thing is the joint position error we have added. And you call the q dot change to q dot desired plus lambda into q tilde; the q tilde is q desired minus q.

So, now this is the change which we expected; so now based on that what we can look at it. So, the code would be getting change; so, we can see this is the, so kinematic control we are trying to see. So, what we can see here? So, these all same, whatever we have done earlier.

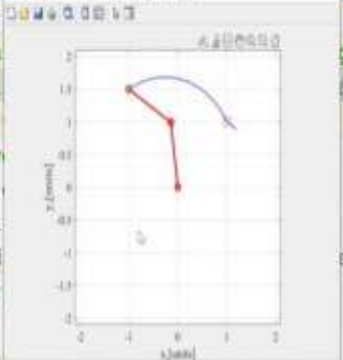
(Refer Slide Time: 15:49)

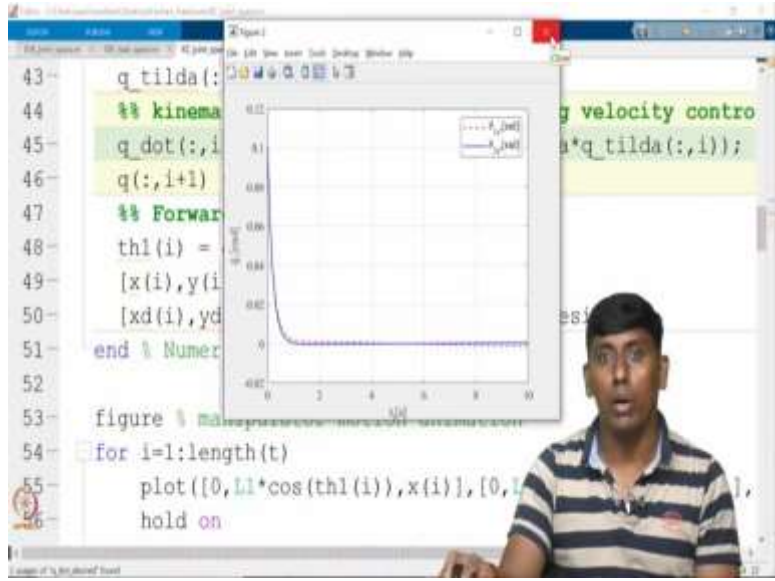
```
43- q_tilda(:,i) = q_desired(:,i)-q(:,i);
44- %% kinematic control based on computing velocity contro
45- q_dot(:,i) = (q_dot_desired(:,i)+lambda*q_tilda(:,i));
46- q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
47- %% Forward kinematics
48- th1(i) = q(1,i); th2(i) = q(2,i);
49- [x(i),y(i)] = FK2R(th1(i),th2(i));
50- [xd(i),yd(i)] = FK2R(th1_desired,th2_desi
51- end % Numerical integration ends here
52-
53- figure % manipulator motion animation
54- for i=1:length(t)
55-     plot([0,L1*cos(th1(i)),x(i)], [0,L1,1], 'r');
56-     hold on
```

So, even we have end up with the initial actual position is different from the desired position; and he lambda we have taken as 4. We will see if we change what will happen; and the joint position error is coming here, and the kinematic control is changed here.

(Refer Slide Time: 16:06)

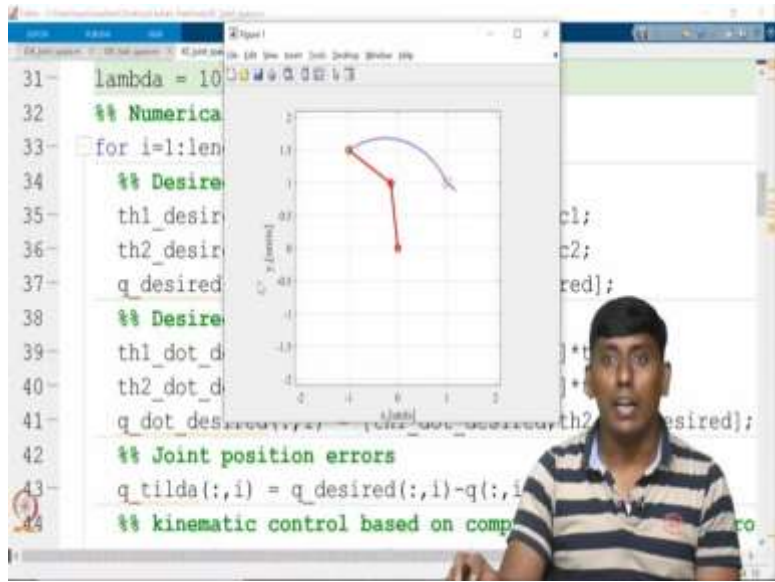
```
43- q_tilda(:,i) = q_desired(:,i)-q(:,i);
44- %% kinema
45- q_dot(:,i) = (q_dot_desired(:,i)+lambda*q_tilda(:,i));
46- q(:,i+1)
47- %% Forwar
48- th1(i) =
49- [x(i),y(i)
50- [xd(i),yd
51- end % Numer
52-
53- figure % ma
54- for i=1:length(t)
55-     plot([0,L1*cos(th1(i)),x(i)], [0,L1,1], 'r');
56-     hold on
```

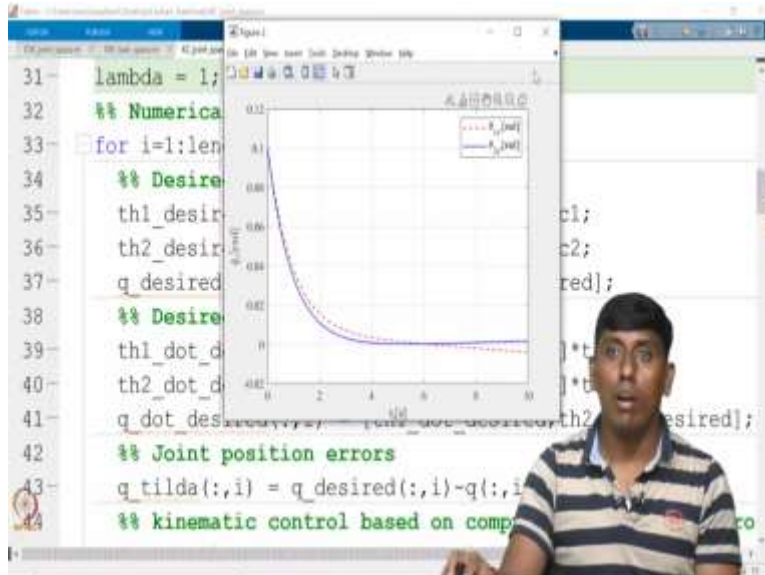




So, now if I run this, you can see that even though the initial is change; but you can see it is trying to follow it. So, that is what we can look at it. So, if you look at the error, initially it is 0.1 radian in both theta1 and theta2 that is converge to 0. But it is having a small error, so that error can be neglected by introducing some integral control. So, now if I increase this lambda or decreased my lambda, so what will happen if I increase?

(Refer Slide Time: 16:40)





```

43-   q_tilda(:,i) = q_desired(:,i)-q(:,i);
44-   %% kinematic control based on computing velocity contro
45-   q_dot(:,i) = (q_dot_desired(:,i)+lambda*q_tilda(:,i));
46-   q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
47-   %% Forward kinematics
48-   th1(i) = q(1,i); th2(i) = q(2,i);
49-   [x(i),y(i)] = FK2R(th1(i),th2(i));
50-   [xd(i),yd(i)] = FK2R(th1_desired,th2_desi
51- end % Numerical integration ends here
52-
53- figure % manipulator motion animation
54- for i=1:length(t)
55-   plot([0, l1*cos(th1(i)), x(i)], [0, l1
56-   hold on

```

So, you see it is like much faster; and as well as that the error would be eliminated very fast. Earlier it was going here, but now it is coming and error magnitude also very much reduced. So, like that you can take it, so instead of that lambda I consider as a 1. So, then also you can see the error would be taking; it is very significant, it is not followed.

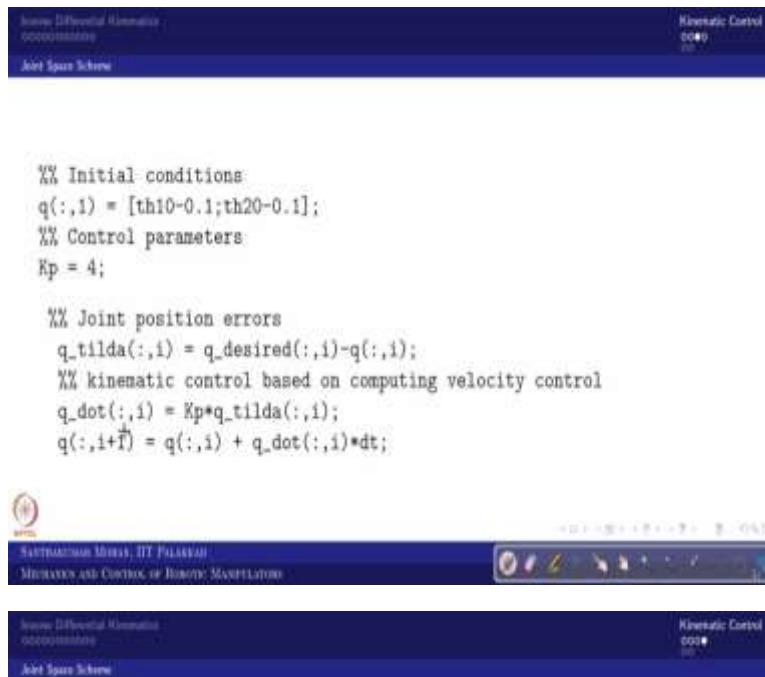
And the time taken for the error also converges; in fact, it is not even converged; if I by 10 second, some error is there. So, these all what we have seen as a kinematic control. So, what we have changed in the inverse differential kinematics? We have added the proportional control, where you have taken error multiplied with one of the positive constants; that is what we have done. So, now we will go to the slide; so, where we have considered only proportional control.

(Refer Slide Time: 17:40)

```
System Differential Kinematics
Kinematic Control
Joint Space Scheme

%% Initial conditions
q(:,1) = [th10-0.1;th20-0.1];
%% Control parameters
Kp = 4;

%% Joint position errors
q_tilda(:,i) = q_desired(:,i)-q(:,i);
%% kinematic control based on computing velocity control
q_dot(:,i) = Kp*q_tilda(:,i);
q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
```



```
System Differential Kinematics
Kinematic Control
Joint Space Scheme

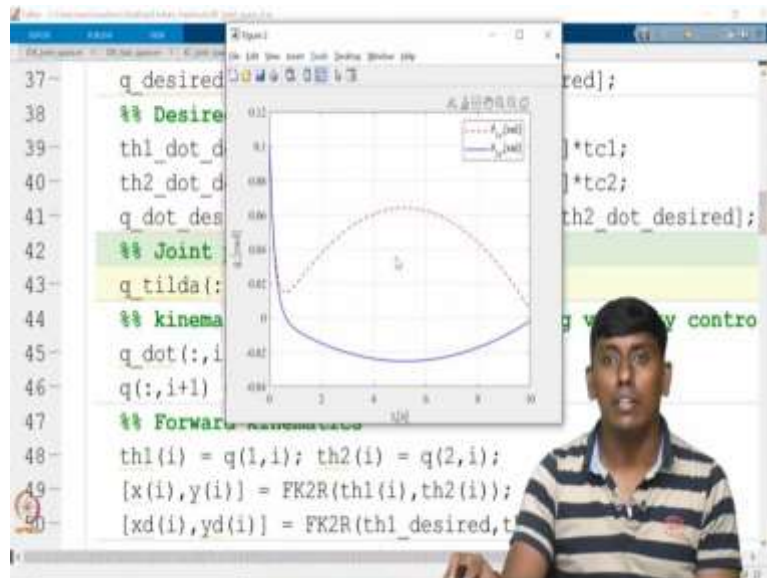
%% Initial conditions
q(:,1) = [th10-0.1;th20-0.1];
%% Control parameters
Kp = 4; Ki = 4; ei = [0;0];
+
%% Joint position errors
q_tilda(:,i) = q_desired(:,i)-q(:,i);
ei = ei + q_tilda(:,i)*dt; % integral error
%% kinematic control based on computing velocity control
q_dot(:,i) = Kp*q_tilda(:,i) + Ki*ei;
q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
```



You are assuming that the \dot{q} desired is unknown to us. So, then what happened the K_p we have taken; and $K_p q_{\text{tilde}}$ is the \dot{q} . So, the \dot{q} desired will not be coming into a picture; so, this would be having what we have seen in the last lecture. This would be having some steady state error; so that steady state error can be overcome with the integral control.

So, where we have brought the integral control term, and then you can see this is compensated with both the terms. So, here we need to know the integral error. So, initially we assume that the integral error is e_i equal to 0, 0; then it would be propagated. So, in order to get understand this will go to again MATLAB window.

(Refer Slide Time: 18:26)



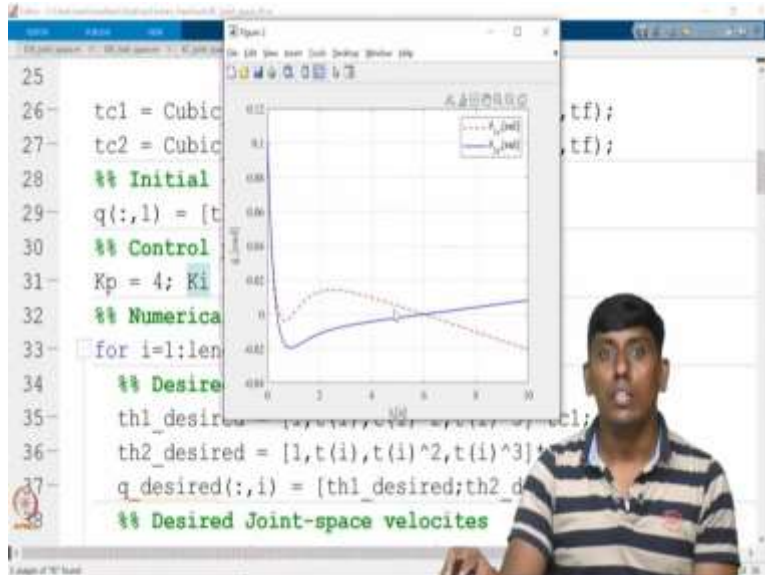
You can see this is the proportional control; so, where you can see K_p equal to 4. And this is the same thing what we did, only thing the q dot desired is taken away. So, now so since this is cubic polynomial, where q dot desired is there; but you have not considered. So, that there would be a error which would be some what visible to us.

So, since it is fast moving, so there is a error which is significant. So, this error you want to eliminate even without q dot desired; so, then we can go with the integral error. Or, you can say integral error with integral control. So, this is what we can see; so now what we have added.

(Refer Slide Time: 19:10)

```
43- q_tilda(:,i) = q_desired(:,i)-q(:,i);
44- ei = ei + q_tilda(:,i)*dt; % Integral error
45- %% kinematic control based on computing velocity contro
46- q_dot(:,i) = Kp*q_tilda(:,i) + Ki*ei;
47- q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
48- %% Forward kinematics
49- th1(i) = q(1,i); th2(i) = q(2,i);
50- [x(i),y(i)] = FK2R(th1(i),th2(i));
51- [xd(i),yd(i)] = FK2R(th1_desired,th2_desired);
52- end % Numerical integration ends here
53-
54- figure % manipulator motion animation
55- for i=1:length(t)
56-     plot([0, l1*cos(th1(i)), x(i)], [0,
```

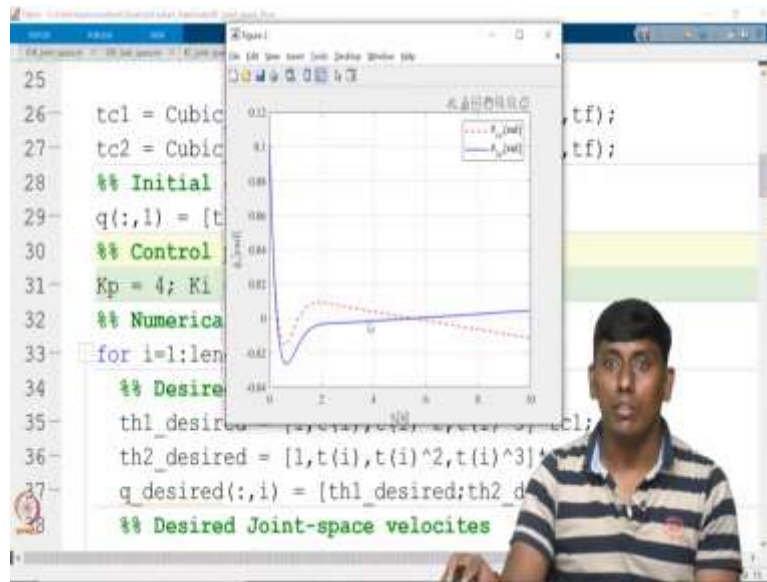
```
25-
26- tc1 = Cubic
27- tc2 = Cubic
28- %% Initial
29- q(:,1) = [t
30- %% Control
31- Kp = 4; Ki
32- %% Numerica
33- for i=1:len
34- %% Desire
35- th1_desired
36- th2_desired = [1,t(i),t(i)^2,t(i)^3]
37- q_desired(:,i) = [th1_desired;th2_d
%% Desired Joint-space velocites
```

So, we have added K_i into e_i ; so, for that we have taken e_i . E_i is e_i of previous plus q tilde integrated; so, this is the integral error. So, now if I add K_i some value, so here we have taken as 4. If I added this what you can see the same condition; so, you can see like the error would be reduced. So, when t tends to infinity, it would be like see.

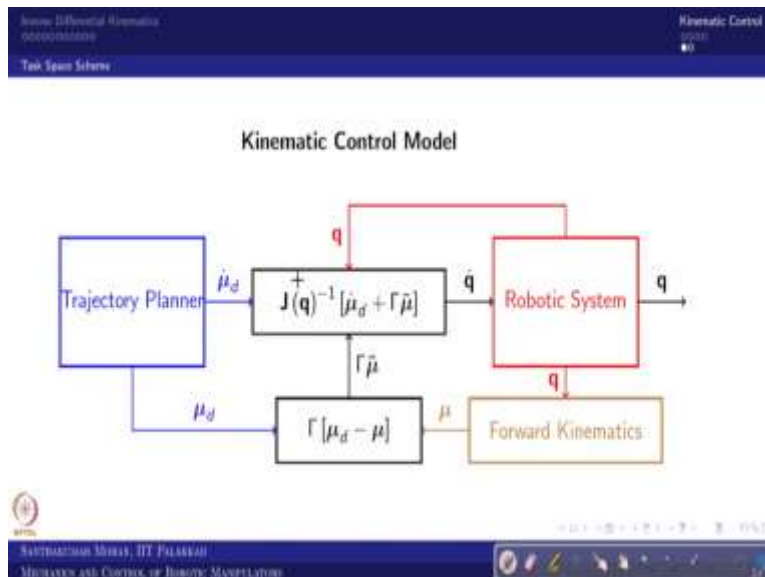
So, it is like reduced significantly; so earlier it was somewhere here; so now it is reduced. Even if we increase K_i further, so then this would be converged somewhere closer. So, if you want, we can check it; so I am just putting 8 just for understanding, just to see whether that you can see it is already somewhat it is faster. So, that is what we can look at it here.

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So, you can see it is trying to converge to 0; so, these all the benefit of proportional and proportional integral. So, in that sense what is desired and what is actual and then you can compensate. So, let us move the final part, where we are seeing the same thing, where the desired trajectory is given in the task space.

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So, then we can do the computed velocity control, where the J of q inverse is coming. So, this is one peculiar because some cases the J of q can be probably infinity; so, where for example in

two are serial manipulator. So, if the theta2 tends to be very close to 0 or 180 degree or even 0 and 180; so, this J of q inverse may be tends to infinity. So, that is why we always avoid this kind of cases; but this is what the one additional input, where this control law we can do it.

(Refer Slide Time: 21:05)



```
%% Initial conditions
[th10,th20]=IK2R(x0,y0);
q(:,1) = [th10-0.1;th20-0.1];
%% Control parameters
Gamma = 4;

%% end-effector position errors
mu_tilda(:,1) = mu_desired(:,1)-mu(:,1);
%% Kinematic control in task-space
q_dot(:,1) = inv(J_d)*(mu_dot_desired(:,1)...
+Gamma*mu_tilda(:,1));
q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
```

So, for that what we are doing? So, we are taking one simple addition. Just for comparison we are taking q ; so, then we are taking a gamma and mu dot desired plus gamma into mu tilde; and inverse of J_d . So, this is what we have taken, in fact instead of taking J_d , even we can take J so both will be the same result. Just to get that idea, we will go go to the kinematic control in the task space. So, this is the change which we were saying.

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The top screenshot shows MATLAB code for calculating desired velocities and kinematic control. The code includes:

```
34 x_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc1;
35 y_dot_desired = [0,1,2*t(i),3*t(i)^2]*tc2;
36 mu_dot_desired(:,i) = [x_dot_desired;y_dot_desired];
37 %% end-effector position errors
38 mu_tilda(:,i) = mu_desired(:,i)-mu(:,i);
39 %% Kinematic control in task-space
40 q_dot(:,i) = inv(J_d)*(mu_dot_desired(:,i)...
41 +Gamma*mu_tilda(:,i));
42 q(:,i+1) = q(:,i) + q_dot(:,i)*dt;
43 end % Numerical integration ends here
44
45 figure % manipulator motion animation
46 for i=1:length(t)
47 plot([0,L1*cos(q(1,i)),x(i)], [0,
```

The bottom screenshot shows MATLAB code for initial conditions and a plot of joint positions over time. The code includes:

```
4 dt = 0.1;
5 t = 0:dt:tf;
6 global L1 L2;
7 L1 = 1; L2 = 1;
8 %% Joint positions
9 x0 = 1; y0 = 1;
10 xf = -1; yf = -1;
11 xdot0 = 0;
12 xdotf = 0;
13 tc1 = Cubic;
14 tc2 = Cubic;
15 %% Initial conditions
16 [th10,th20]=IK2R(x0,y0);
17 q(:,1) = [th10-0.1;th20-0.1];
```

The plot shows the joint positions $q(1,i)$ and $q(2,i)$ over time t . The x-axis is time t from 0 to 10, and the y-axis is joint position q from -0.2 to 0.2. The plot shows two curves, $q(1,i)$ (dotted line) and $q(2,i)$ (solid line), both starting at approximately 0.1 and converging to 0 over time.

So, the mu tilde we calculate mu desired minus mu. So, then the q dot is inverse of Jd or inverse of J, multiply with mu dot desired plus gamma into mu tilde. So, now again we see that he initial is having error and this is a profile we want to follow, which is a straight line. We will see whether this is following it or not.

So, it is like following it, whereas in the simple inverse differential kinematics it was not followed; and the error is almost 0 in the mu tilde, x error and y error almost 0. So, now even we want to check whether you want to use this only q dot, sorry J desired; it can be even can be use J. So, in that case I will take it, so this like theta1 and theta2. So, or I have to like to use it.

(Refer Slide Time: 22:38)

```
22 %% Desired Task-space positions
23 x_desired = [1,t(i),t(i)^2,t(i)^3]*tc1;
24 y_desired = [1,t(i),t(i)^2,t(i)^3]*tc2;
25 [th1_desired,th2_desired]=IK2R(x_desired,y_desired);
26 mu_desired(:,i) = [x_desired;y_desired];
27 Jd= Jaco2R(th1_desired,th2_desired);
28 J= Jaco2R(q(1,i),q(2,i));
29 % Desired joint positions through IK
30 [th1_desired(i),th2_desired(i)] = IK2R(x_desired(i),y_desired(i));
31 %% Forward kinematics
32 [x(i),y(i)] = FK2R(q(1,i),q(2,i));
33 mu(:,i) = [x(i);y(i)];
34 %% Desired Task-space velocities
35 x_dot_desired = [0,1,2*t(i),3*t(i)^2];
```

```
34 %% Desired Task-space velocities
35 x_dot_desired = [0,1,2*t(i),3*t(i)^2];
36 y_dot_desired = [0,1,2*t(i),3*t(i)^2];
37 mu_dot_desired(:,i) = [x_dot_desired(i);y_dot_desired(i)];
38 %% end-effector position
39 mu_tilda(:,i) = [x(i);y(i)];
40 %% Kinematic chain
41 q_dot(:,i) = [mu_dot_desired(i,1);mu_dot_desired(i,2)];
42 +Gamma
43 q(:,i+1) = q(:,i) + Gamma*q_dot(:,i);
44 end % Numerical solution
45
46 figure % manipulator motion animation
47 for i=1:length(t)
```

So, I will just write J, so which means so I just see this; so I will just take it. This is a J, which is from the q; so, this is; so, q of 1 comma i comma q of 2 comma i; in the sense theta1 and theta2. So, now I calculated J, so J I calculated; and then this is going to give. So, now instead of this J if I do it, so the nature will not be changing; so that is what I wanted to show it here.

So, I hope now you are clear what is kinematic control and what is inverse differential kinematics, where we can use proportional control; and where we can use proportional integral. And here also we can take it simple proportional in task space, and proportional integral these all

same cases. So, I hope you are clear on the kinematic control and inverse differential kinematic of a serial manipulator.

The next class we will see what is dynamic control? and in the dynamic control extension we will see the dual loop; where the outer loop will be doing in a kinematic level and the inner loop would be in dynamic level. That is what we are going to see in upcoming lecture. So, this lecture is on kinematic control with MATLAB simulation. I hope you have enjoyed; and see you then thank you.