Mechanics and Control of Robotic Manipulators
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Lecture No 35
Trajectory generation for serial manipulators with workspace using MATLAB

Hi, welcome back to Mechanics and Control of Robotic Manipulator. I am really thinking that you might have enjoyed the last lecture where it was showing something like very close to the real system where you would have seen some kind of animated where the manipulator is moving. Although I made it restricted to only 2R serial manipulator. Which is easy to show in you can say on a screen that is why we have made it.

However, it is not restricted only with the 2R serial or planar manipulator. It can be extended to even spatial so, you can do it that extension by your own, this particular lecture we are trying to show the workspace computation. How to make the workspace in MATLAB. So, why I have taken MATLAB because you can generate a point and the point you can make it as a cloud in a 2D plot or 3D plot. So, which will give the workspace environment then we can see how th;e you can say end effector trajectory is going one to another. So, in order to get that idea.

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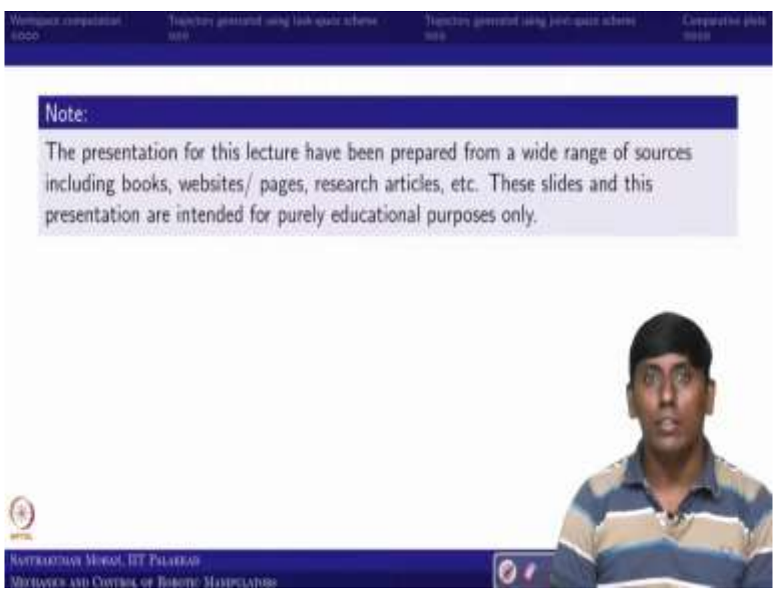

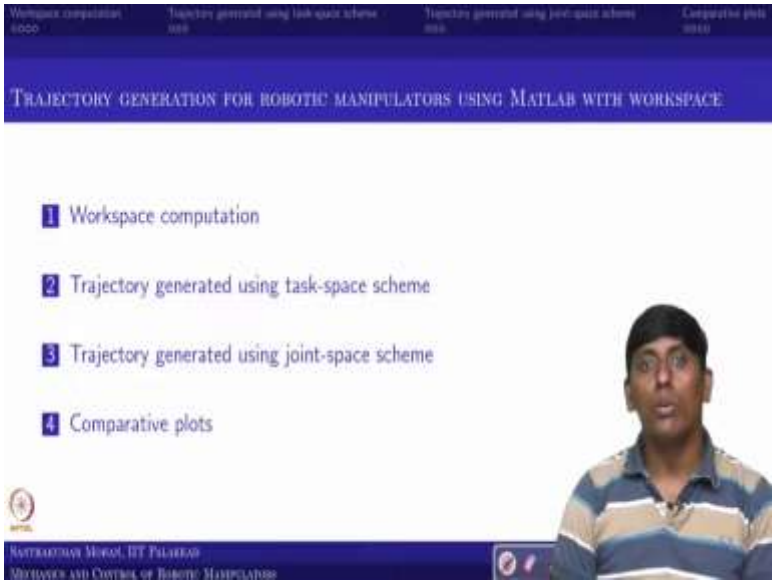

So, we will take this particular lecture. So, this lecture is going to talk about workspace computation, then the trajectory generated using task space scheme for same 2R serial manipulator including workspace. Then we can see like the trajectory generated using joint space scheme. Then we will compare one to another how it is happening. So, we will take the same case and compare it how it is happening one to another.

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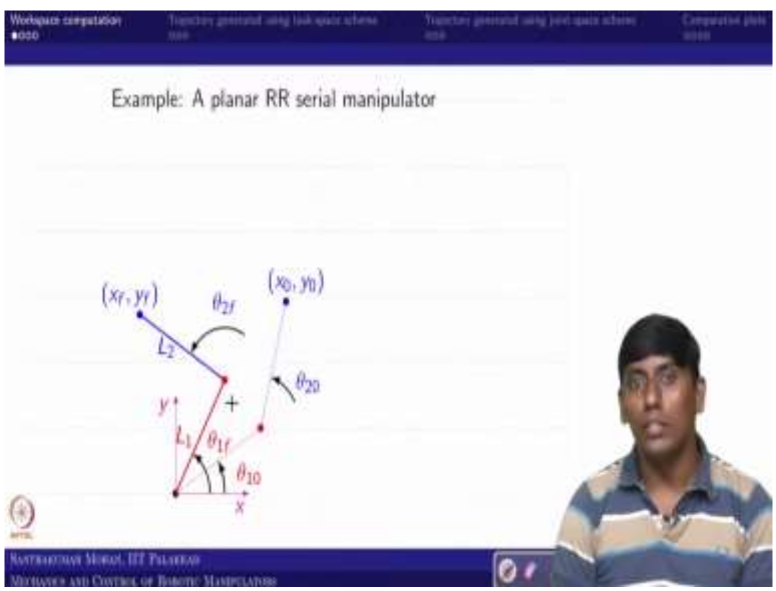

If that is the case. So, we will move to the generalize 2R serial manipulator situation where the initial position of the end effector and you can say final position of the end effector is given. In the other way if you have inverse kinematics solution. So, you know initial joint positions and final joint positions are known provided l 1 and l 2 are known. So, in that case, what one can see first we will try to see.

So, what is the workspace so for that the theta 1 minimum to maximum, you should know the joint limit. Similarly, the theta 2 minimum and maximum you should know the joint limits of the theta 1 and theta 2. So, once you know, we can a plot based on the forward kinematics, and we can generate the workspace.

So, provided you should know the forward kinematics solution. Since it is a 2R serial manipulator the forward kinematics solution is straightforward. The x can be written as l 1 cos theta 1 plus l 2 cos theta 2 or you can cross theta 1 plus theta 2. So, the y is l 1 sin theta 1 plus l 2 sin theta 1 plus theta 2. So, this is we know.

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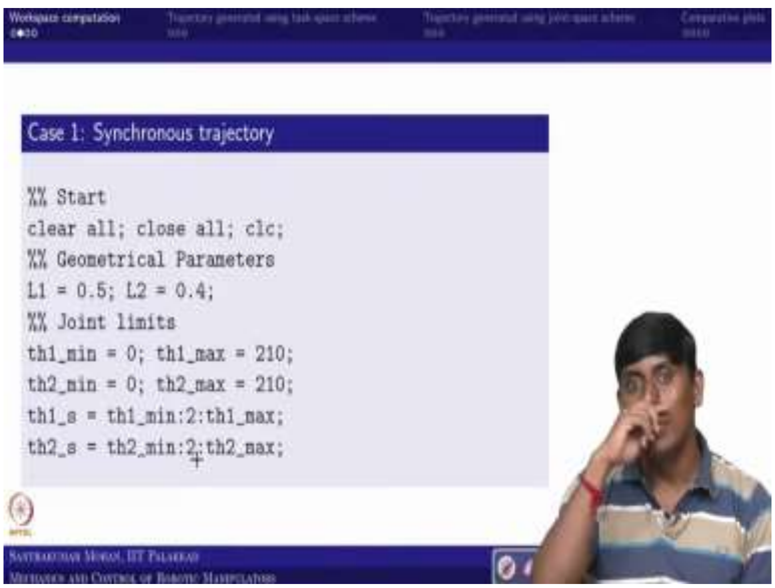

So, that is what we are trying to do. So, for that we are taking it you can say the synchronous trajectory all the time. So, it needs not to be explicitly given. So, that is given here. So, now coming to the general case so, here the theta 1 minimum and maximum. So, theta 2 minimum and maximum is given. So, in the sense the theta 1 you can say simulation which is I want to show where is the workspace.

So, which start from theta 1 minimum to maximum with the interval of 2 degree because I just want to show only 2-degree interval of the points it need not to be. Further you please remember if a here I have written as in degrees, so theta 1 and theta 2 all in degrees. So, whenever you write the equations, so you make sure that the unit as, units are matching.

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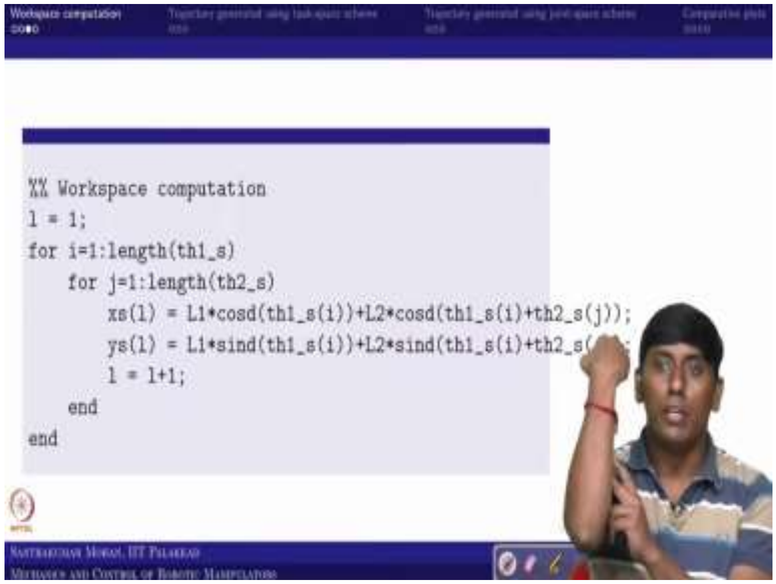

So, in that sense so, you can see the workspace I am trying to compute based on the forward kinematics solution. But what I am trying to do? So, I am trying to generate the x and y for the given range or the given range the theta 1 vary from 0 to or you can say minimum to maximum theta 2 also vary from minimum to maximum. So, that minimum to maximum I have already defined theta 1 s theta 2 s.

So, I am trying to run the loop where the theta 1 is running the first loop from minimum to maximum the second loop theta 2 which is running minimum to maximum. In the sense first I will keep the theta 1. So, you can look at it here. So, the theta 1 is coming here and the second is rotating from minimum to maximum once then you move another and again rotate from minimum to maximum like that you can do it in sequence. So, once this is done.

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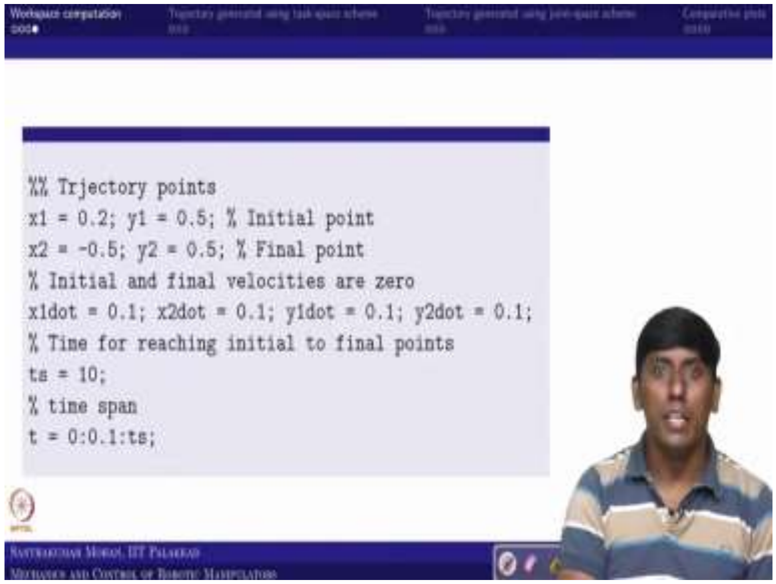

So, what do you want to you want to plot. So, for plotting, we will come back in the later but right now I am trying to generate the trajectory. So, for generating trajectory what I know. So, I know already that trajectory points which are given to us. So, I am taking the same but here l 1 and l 2 I have taken a slightly different value.

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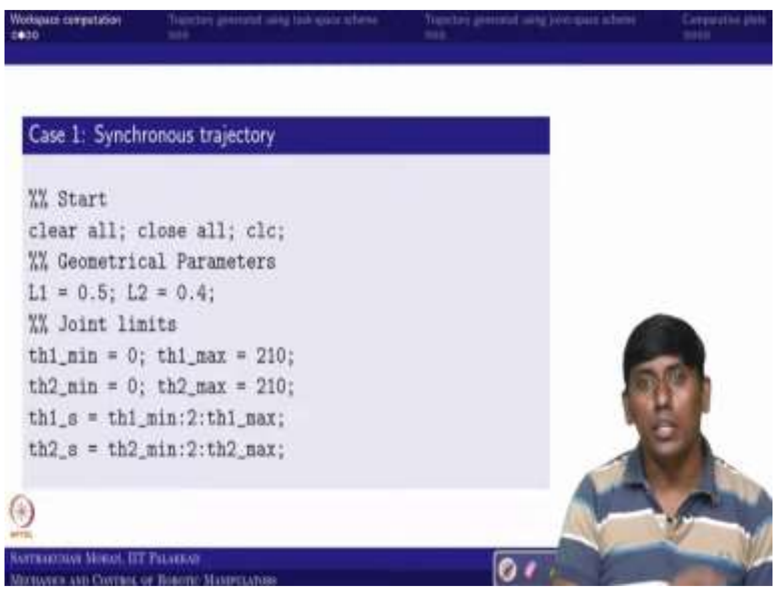

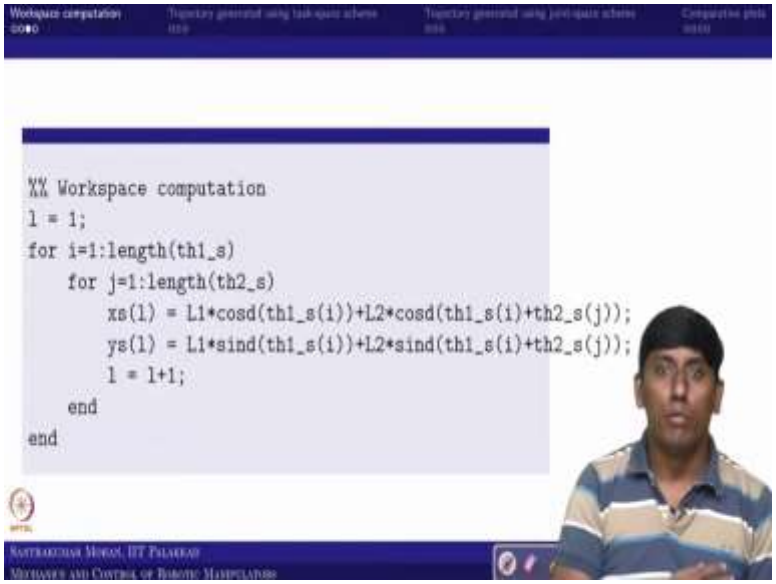

So, you can see it is 0.5 and 0.4. So, I did not take it is equal so that I can see the workspace even if it rotates 0 to 360 it would be donut. So, that donut meaning you can say internal diameter is 0.1 meter and the external diameter is 0.9 meter. So, I just want to show but here it is not going to be a donut it would be a small patch. So, we can see that so before going to see that we can take the trajectory variable.

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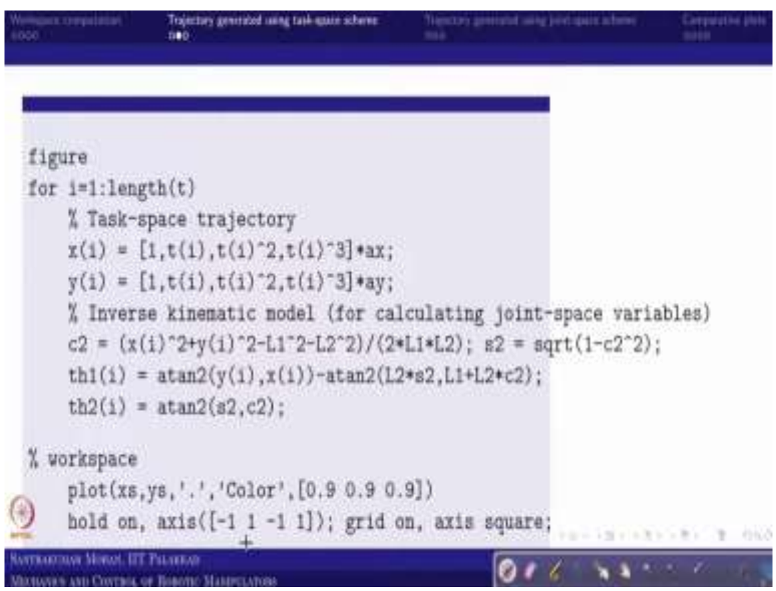

So, we will just take the trajectory generated in the task space. So, we will take the a as the same and bx and by as given an a x and a y can be calculated. So here we are doing it independent trajectory generation. So, in that sense, the trajectory is generated x of i and y of i like this and the inverse kinematic model we have taken so then we will go to the workspace. So here you can see the workspace I have plotted as a simple point across I hold on so that when I plot this all plot it is not that particular point.

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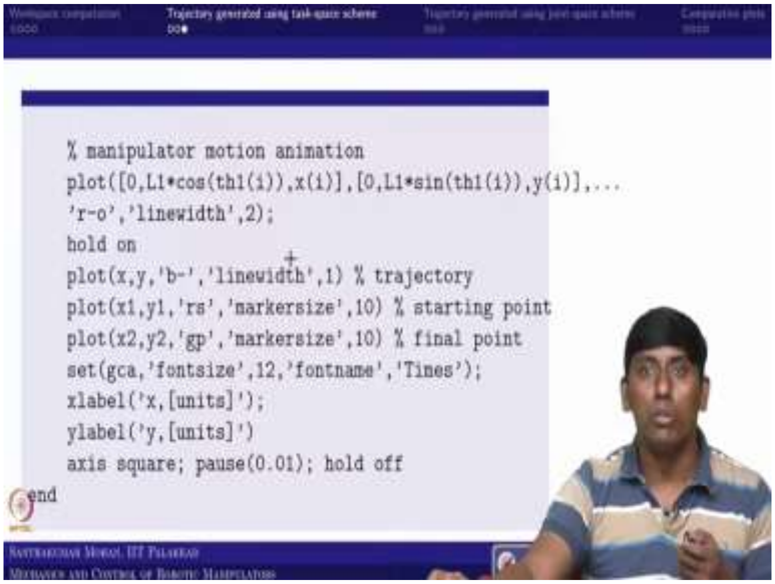

So, then I am ending the manipulator motion animation. So, I brought this you can say 0 x 1 x 2 0 y1 y2 and that I am making it as a line so that it would look like animation. So here it is 10 milliseconds as the delay and again the same thing. So, starting and ending point we, plotted and final trajectory also like we can generate. So, just to show that. So, we will first finish the entire thing then we will go to the MATLAB.

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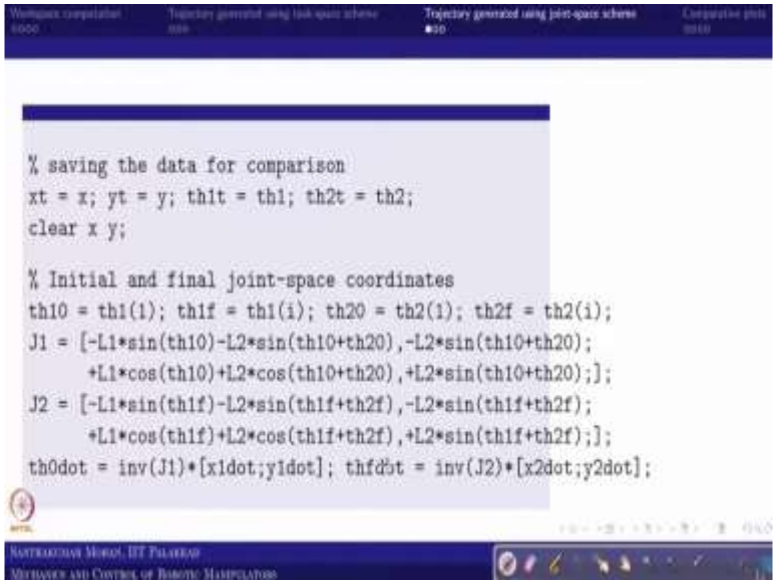

So, the second case, so, we are trying to save this data the second case is we are taking the inverse kinematics. So then inverse differential kinematics where the theta 0 dot and theta final dot of individual cases are found.

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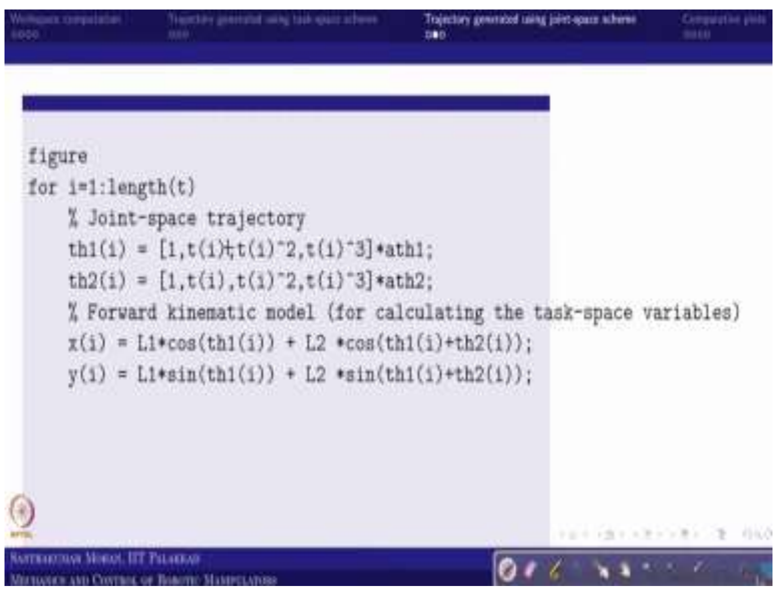

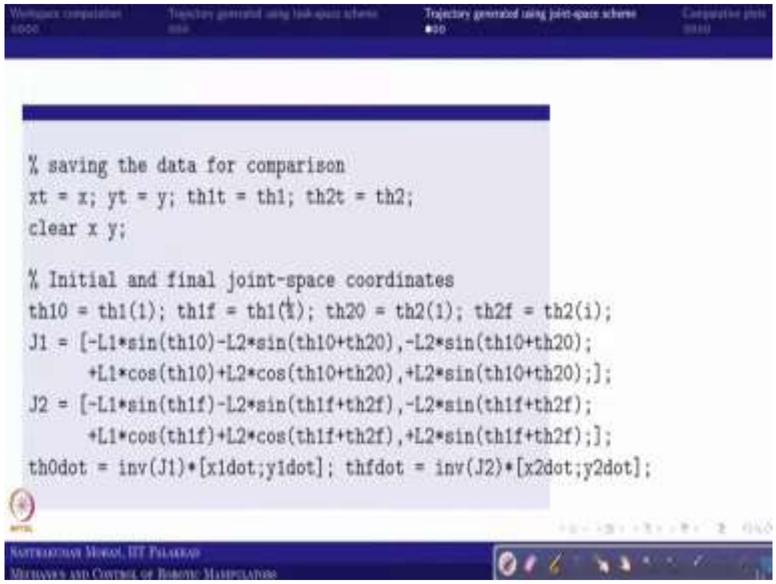

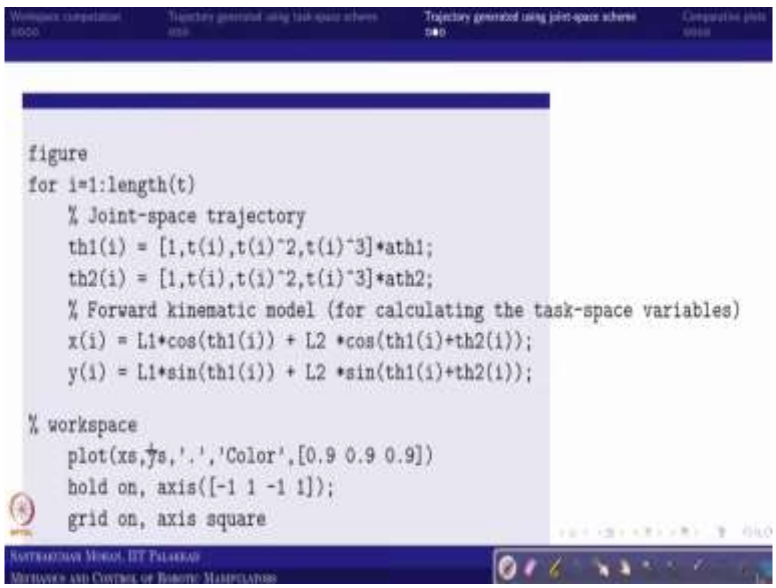

So, once these found we can generate the trajectory based on so and so thing. So, once these all found so you can see like here. So, once those things are found so we can calculate these all. So once these all calculated, so we can calculate the forward kinematic model just for comparison. So, then we can again plot the workspace.

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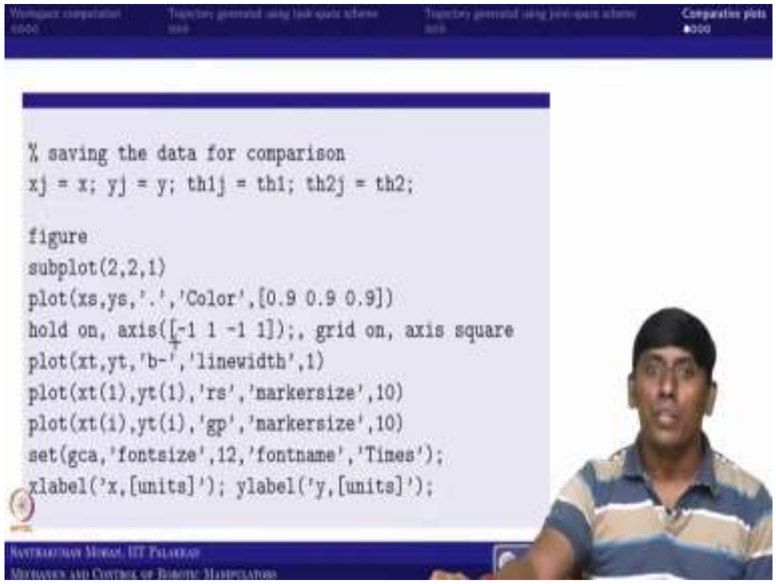

And the plot the individual animation we can try to show finally we will end with a comparative plot where you can say both trajectories how it looked like one is joint space, the other one is task space. How it looked like that we are trying to compare.

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So, for that we are trying to plot individual cases. So, and then we are ending it. So, for getting more clarity as it is a short video.

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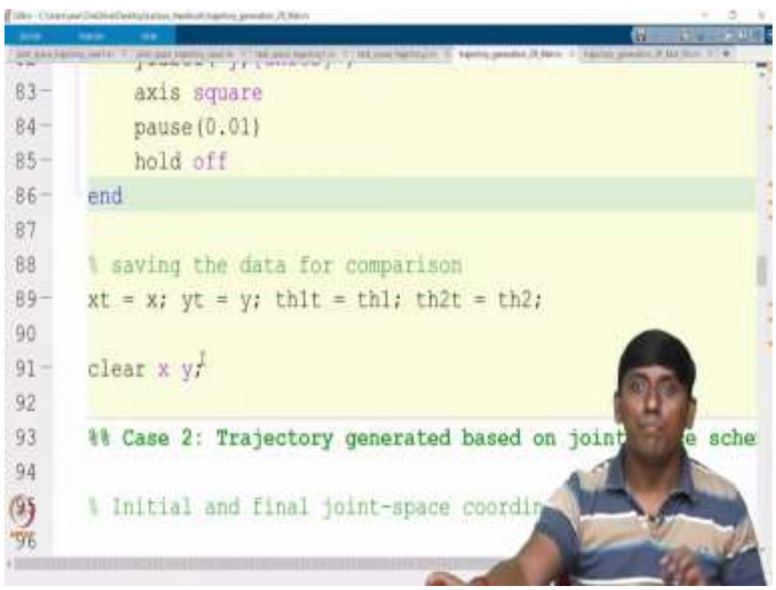
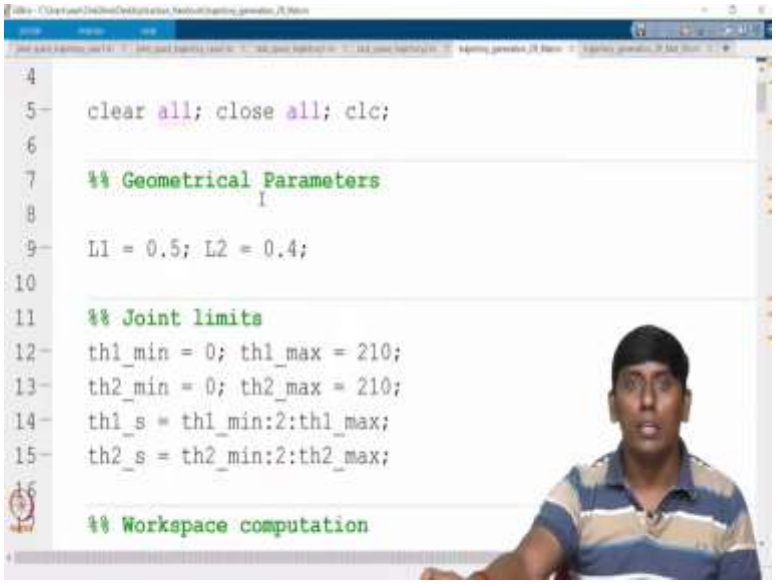
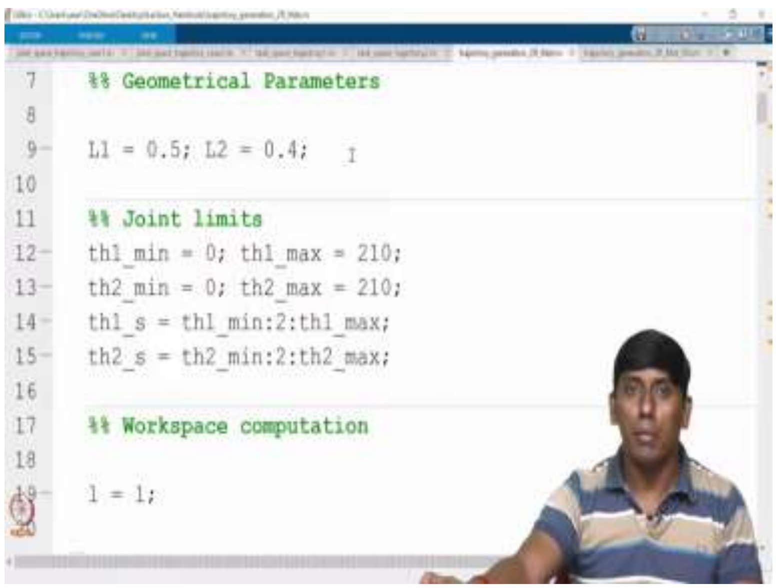
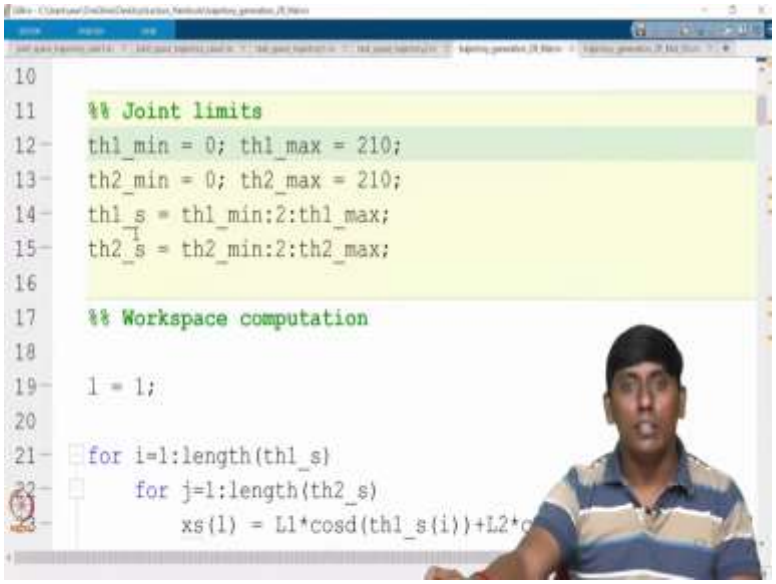


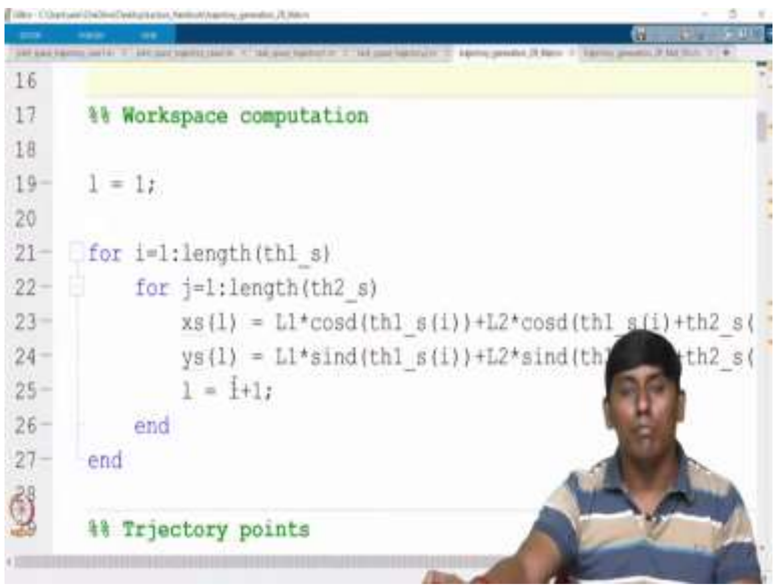


clear all; close all; clc;
%% Geometrical Parameters
L1 = 0.5; L2 = 0.4;
%% Joint limits
th1_min = 0; th1_max = 210;
th2_min = 0; th2_max = 210;
th1_s = th1_min:2:th1_max;
th2_s = th2_min:2:th2_max;
%% Workspace computation



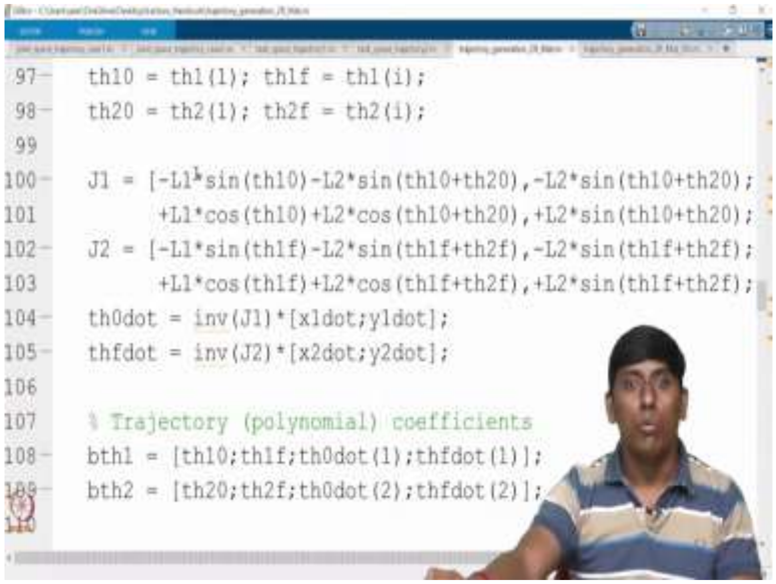
%% Geometrical Parameters
L1 = 0.5; L2 = 0.4;
%% Joint limits
th1_min = 0; th1_max = 210;
th2_min = 0; th2_max = 210;
th1_s = th1_min:2:th1_max;
th2_s = th2_min:2:th2_max;
%% Workspace computation
l = 1;


%% Joint limits
th1_min = 0; th1_max = 210;
th2_min = 0; th2_max = 210;
th1_s = th1_min:2:th1_max;
th2_s = th2_min:2:th2_max;
%% Workspace computation
l = 1;
for i=1:length(th1_s)
for j=1:length(th2_s)
xs(l) = L1*cosd(th1_s(i))+L2*c


%% Workspace computation
l = 1;
for i=1:length(th1_s)
for j=1:length(th2_s)
xs(l) = L1*cosd(th1_s(i))+L2*cosd(th1_s(i)+th2_s(
ys(l) = L1*sind(th1_s(i))+L2*sind(th
+th2_s(
l = l+1;
end
end
%% Trjectory points

```
        1,ts,ts^2,ts^3;
        0,1,0,0;
        0,1,2*ts,3*ts^2];
%% Case 1: Trajectory generated based on task-space schem
bx = [x1;x2;x1dot;x2dot];
by = [y1;y2;y1dot;y2dot];
ax = inv(A)*bx;
ay = inv(A)*by;

figure

for i=1:length(t)

    % Task-space trajectory
```

```
    th1(i) = atan2(y(i),x(i))-atan2(L2*s2,L1+L2*c2);
    th2(i) = atan2(s2,c2);

    % workspace
    plot(xs,ys,'.','Color',[0.9 0.9 0.9])
    hold on
    axis([-1 1 -1 1]);
    grid on
    axis square
    % manipulator motion animation
    plot([0,L1*cos(th1(i)),x(i)],[0,L1*sin(t     y(i)],
        'r-o','linewidth',2)
    hold on
    plot(x,y,'b-','linewidth',1) % tr
```



So, for getting more clarity we will go to MATLAB. If anything will explain, then there. So, you can see like here, this is the starting point. So, we are as general we are clearing the; you can say a workspace closing all the figure windows and clearing the command history. And the geometry parameter here link 1 is the length of 0.5 meter and link 2 is 0.4 meter. And the joint limits 0 to 2 and 210 degrees.

And this is the simulation, and I am computing the MATLAB using for MATLAB for finding the workspace the workspace I have computed then the trajectory point details are given. So, I have calculated all those things. So, then I calculated. So, you can see this. And again, I have calculated the; what you call the bth and this one. Then I calculated the individual joint trajectory coefficients and the joint space trajectory I have generated it is synchronous.

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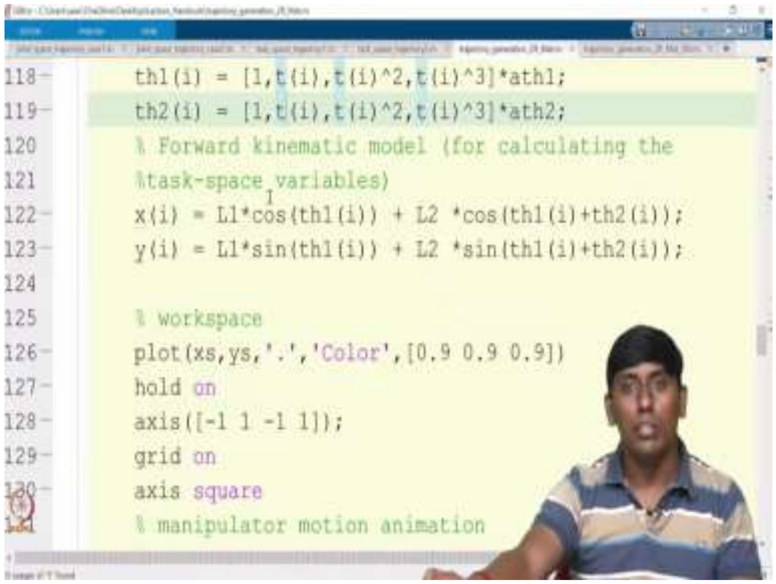
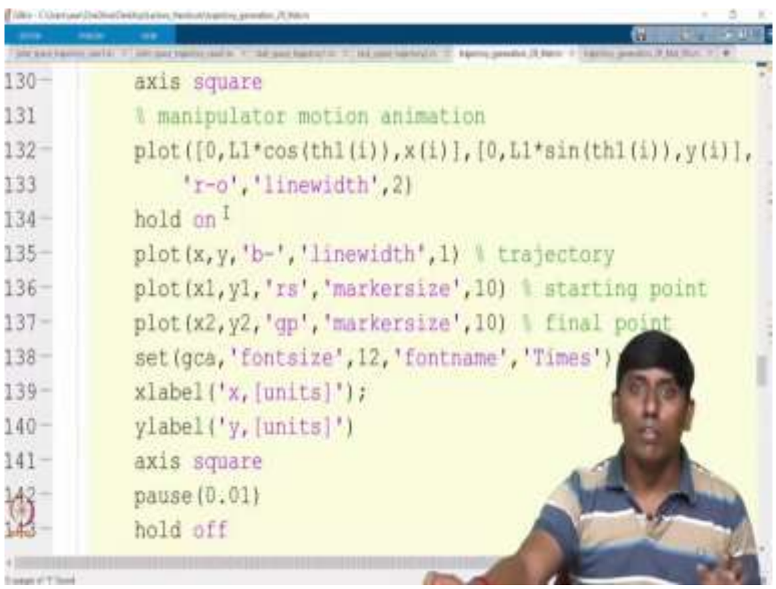
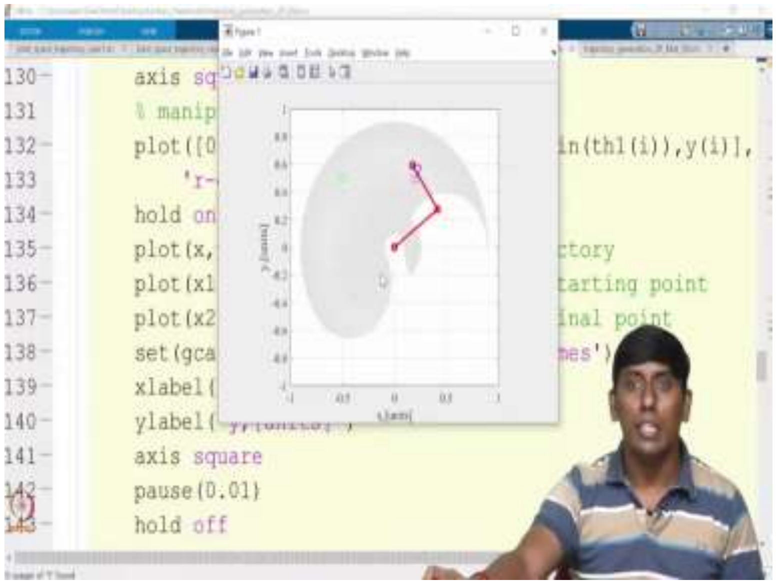
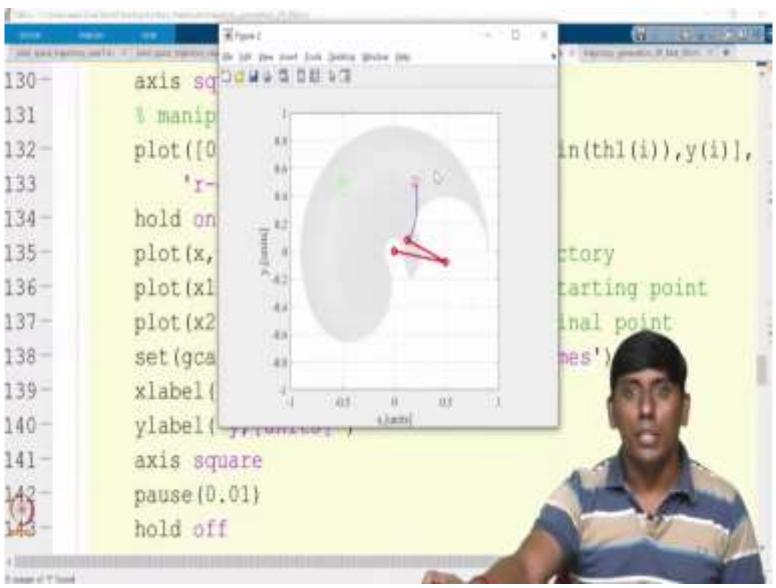
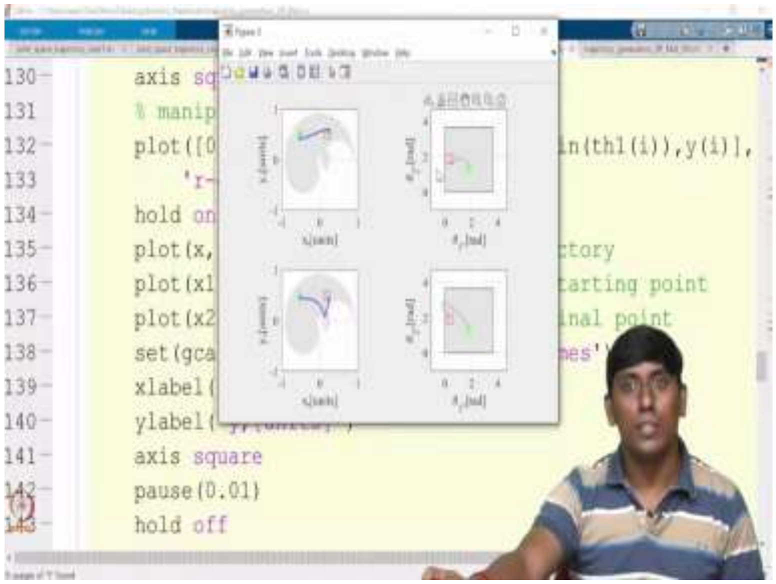





axis sq
% manip
plot([0
in(th1(i)),y(i)],
'r-
hold on
plot(x,
ctory
plot(x1
tarting point
plot(x2
inal point
set(gca
mes')
xlabel(
ylabel(
axis square
pause(0.01)
hold off


axis sq
% manip
plot([0
in(th1(i)),y(i)],
'r-
hold on
plot(x,
ctory
plot(x1
tarting point
plot(x2
inal point
set(gca
mes')
xlabel(
ylabel(
axis square
pause(0.01)
hold off



And the other one is independent this is a synchronous and then I have tried to plot it. First, I will run this then we will go back wherever there is a clarification required. So first I run this. So, you can see like this is the workspace and based on the given condition, you can see like the task space trajectory and joint space trajectory is giving some kind of peculiar patch. So that is what we are trying to see. So, you can look at it this.

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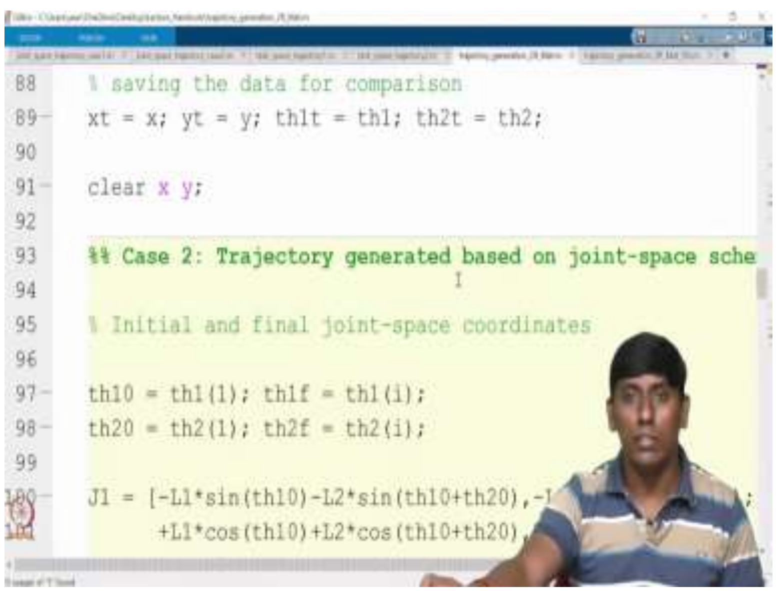
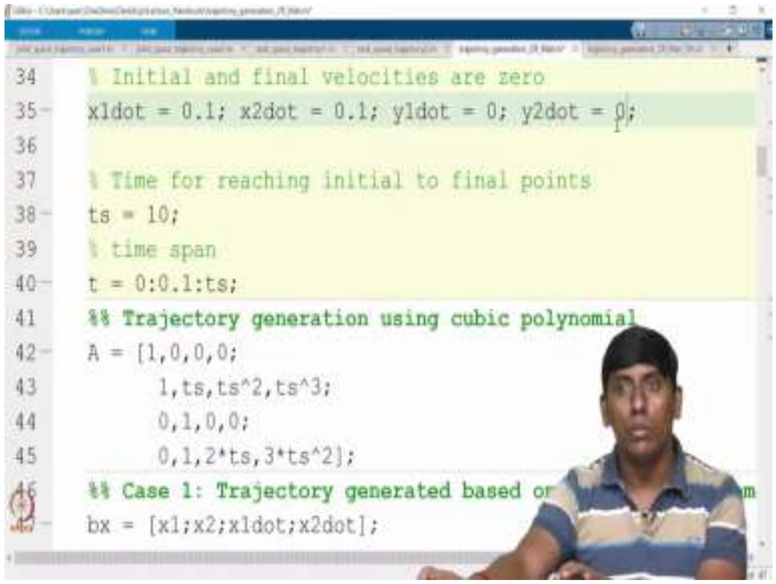
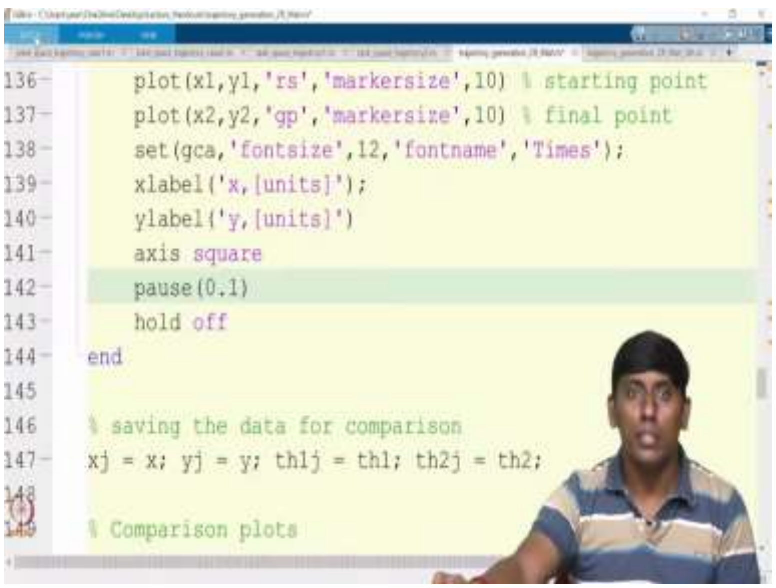
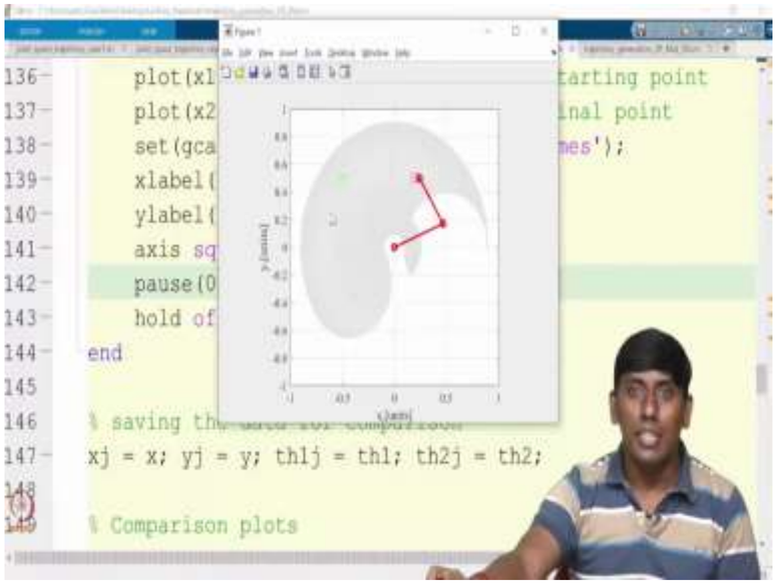
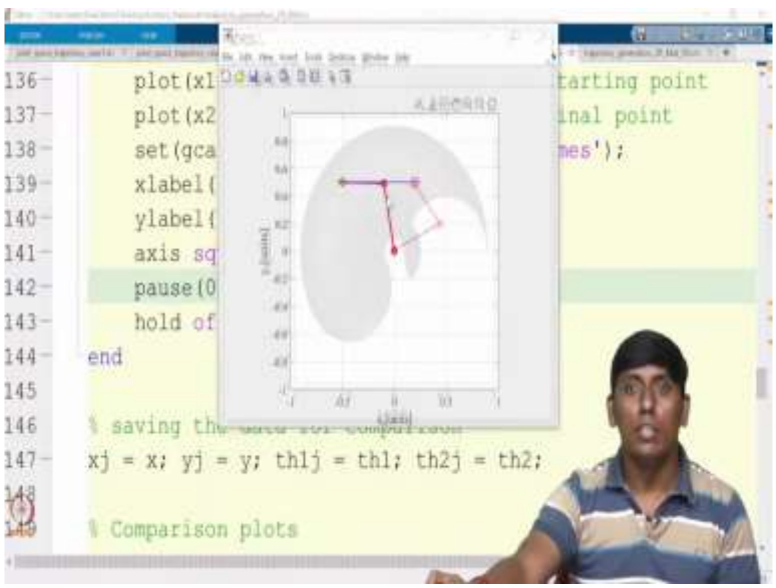
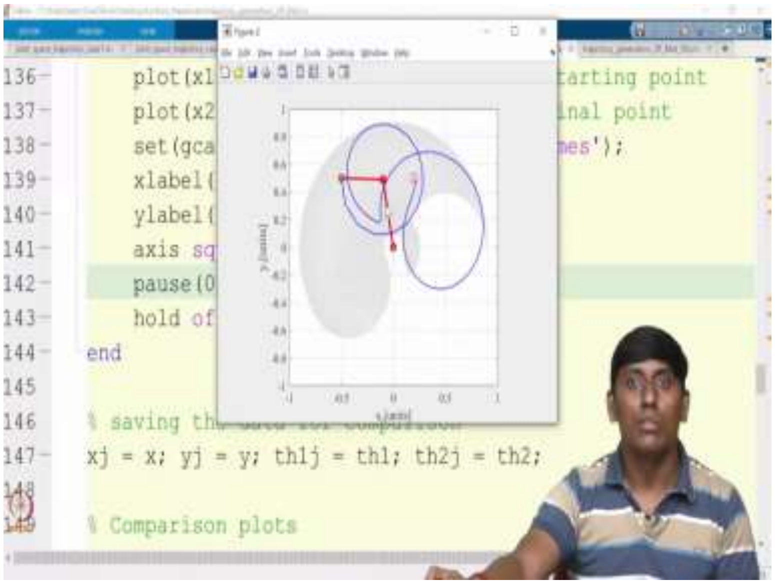



% Initial and final velocities are zero
x1dot = 0.1; x2dot = 0.1; y1dot = 0; y2dot = 0;
% Time for reaching initial to final points
ts = 10;
% time span
t = 0:0.1:ts;
%% Trajectory generation using cubic polynomial
A = [1,0,0,0;
1,ts,ts^2,ts^3;
0,1,0,0;
0,1,2*ts,3*ts^2];
%% Case 1: Trajectory generated based on
bx = [x1;x2;x1dot;x2dot];


plot(x1,y1,'rs','markersize',10) % starting point
plot(x2,y2,'gp','markersize',10) % final point
set(gca,'fontsize',12,'fontname','Times');
xlabel('x, [units]');
ylabel('y, [units]')
axis square
pause(0.1)
hold off
end
% saving the data for comparison
xj = x; yj = y; th1j = th1; th2j = th2;
% Comparison plots


plot(x1
plot(x2
set(gca
xlabel(
ylabel(
axis sq
pause(0
hold of
end
tarting point
inal point
mes');
% saving th
xj = x; yj = y; th1j = th1; th2j = th2;
% Comparison plots
Figure 1


plot(x1
plot(x2
set(gca
xlabel(
ylabel(
axis sq
pause(0
hold of
end
tarting point
inal point
mes');
% saving th
xj = x; yj = y; th1j = th1; th2j = th2;
% Comparison plots



So first I will try to show this is the joint space trajectory. This is the task space trajectory. So, tasks space trajectory is there are based on this, the initial velocity and final velocity is making it a little unrealizable. So, I will just make it both 0. And in order to make it stand by just to give idea. So, I am making it 100 milliseconds for the step for one image to another image. So, now, I am trying to show here.

So, you can see this is the workspace the patch part which is the workspace. So, the first joint is rotate 0 to 210 degree second joint also rotate maximum 210 degree starting from 0. So, if I plot then this is a workspace will come within the workspace this is the initial point and this is the final point. So, now, it is from nonzero initial velocity to you can see finite velocity. So, you can see how it is so, complex.

Because, we have taken the inverse Jacobian, which is one such complex so, that is what it is trying to do it based on so-on-so case. So, here there is a peculiar thing we need to see it.

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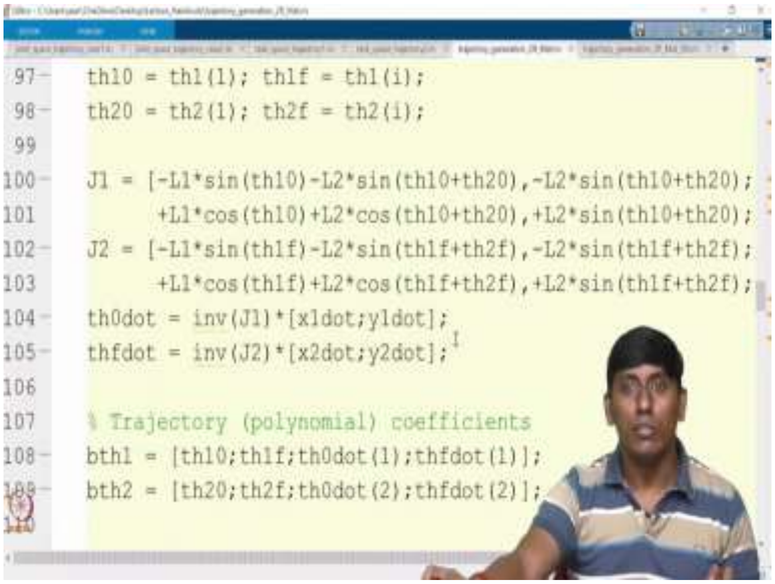
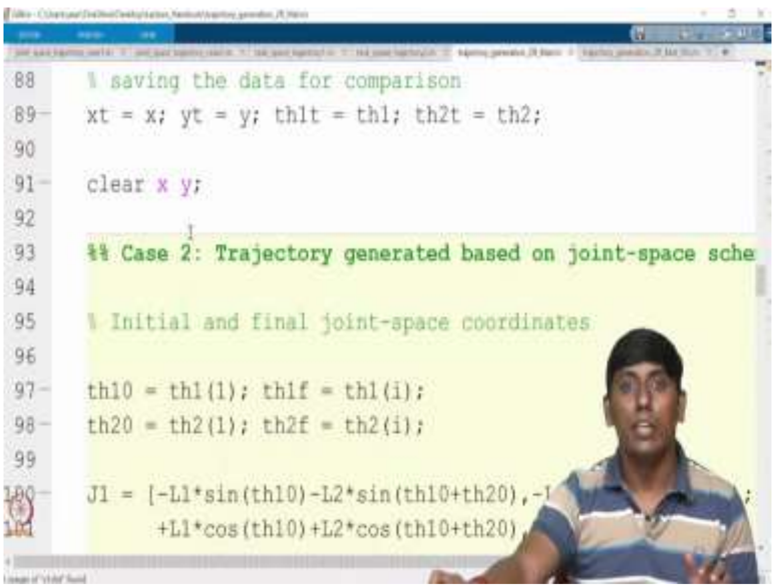
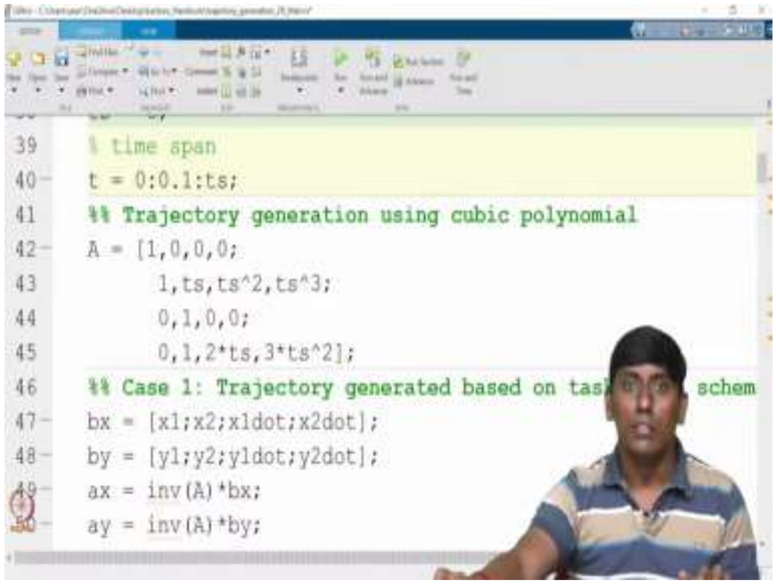
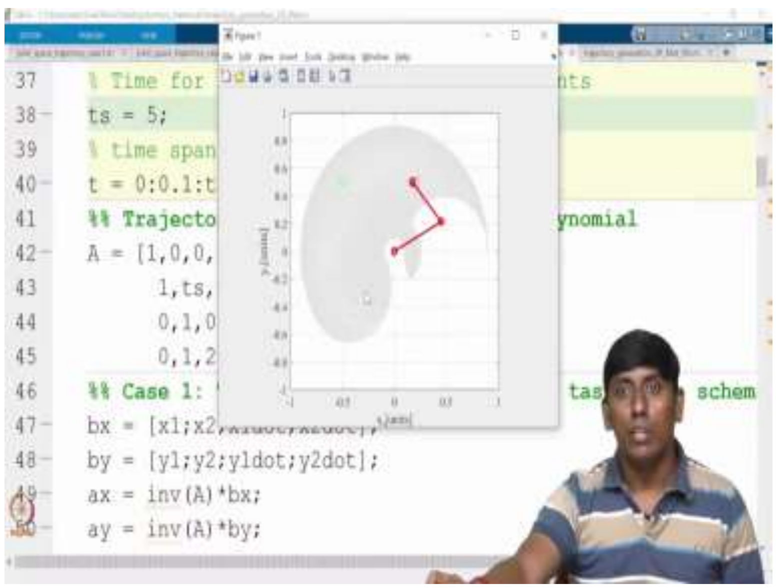
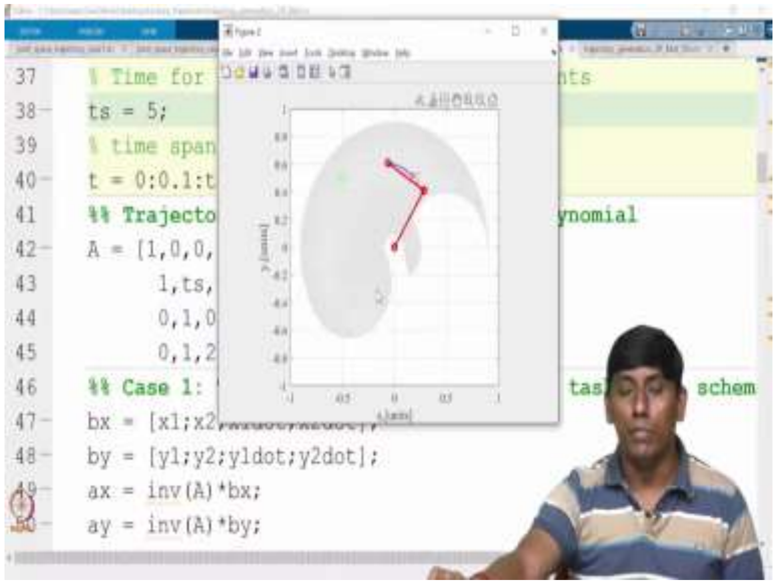
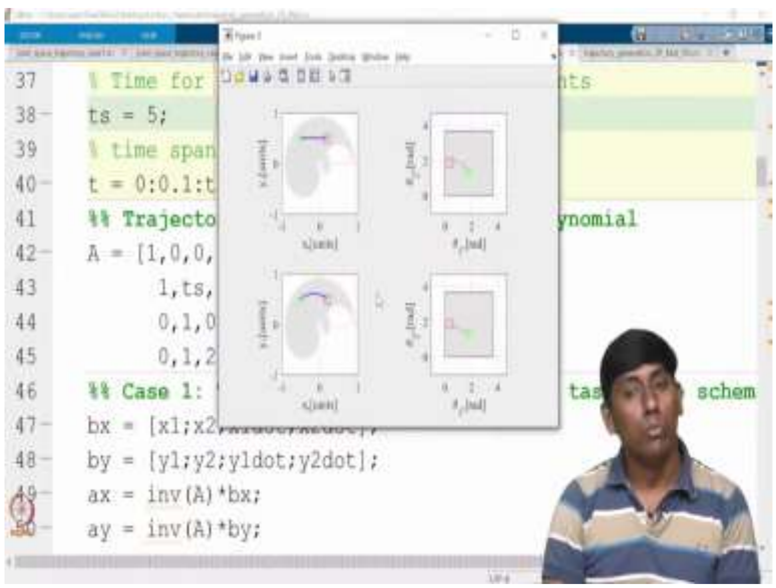





% time span
t = 0:0.1:ts;
%% Trajectory generation using cubic polynomial
A = [1,0,0,0;
1,ts,ts^2,ts^3;
0,1,0,0;
0,1,2*ts,3*ts^2];
%% Case 1: Trajectory generated based on tas
schem
bx = [x1;x2;x1dot;x2dot];
by = [y1;y2;y1dot;y2dot];
ax = inv(A)*bx;
ay = inv(A)*by;


% Time for
nts
ts = 5;
% time span
t = 0:0.1:t
%% Trajecto
ynomial
A = [1,0,0,
1,ts,
0,1,0
0,1,2
%% Case 1:
tas
schem
bx = [x1;x2
by = [y1;y2;y1dot;y2dot];
ax = inv(A)*bx;
ay = inv(A)*by;

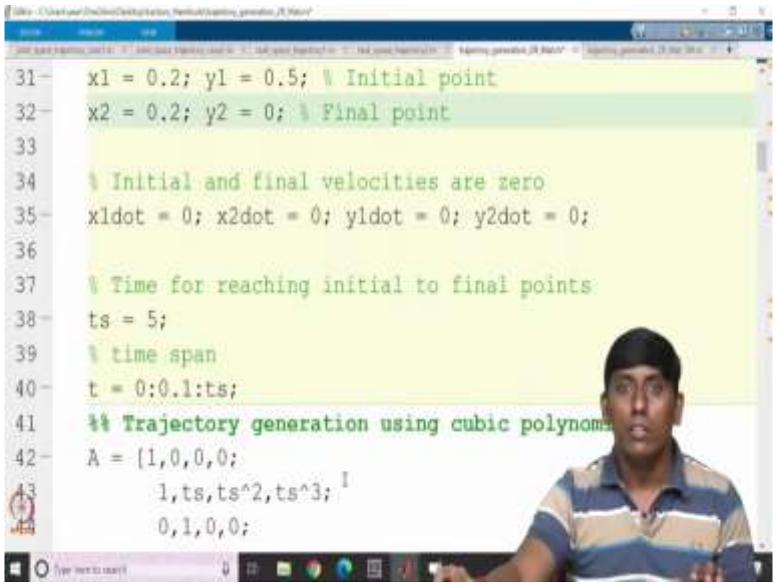


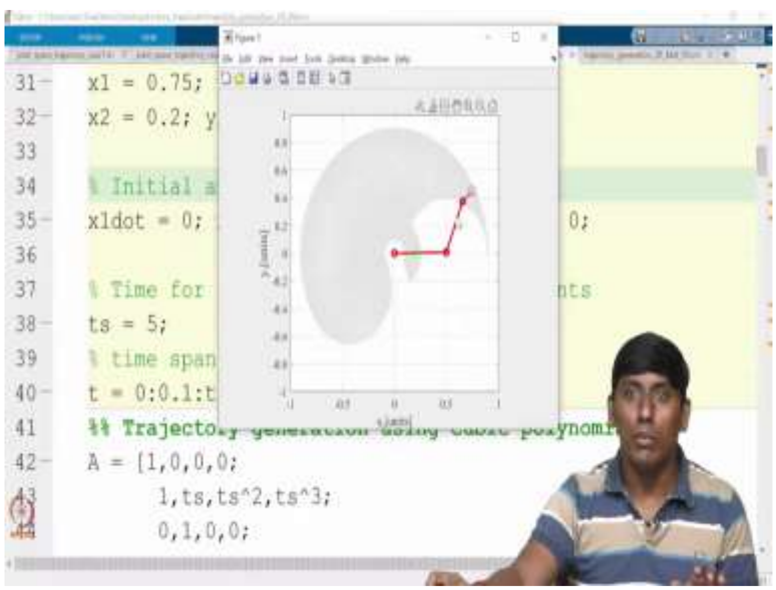


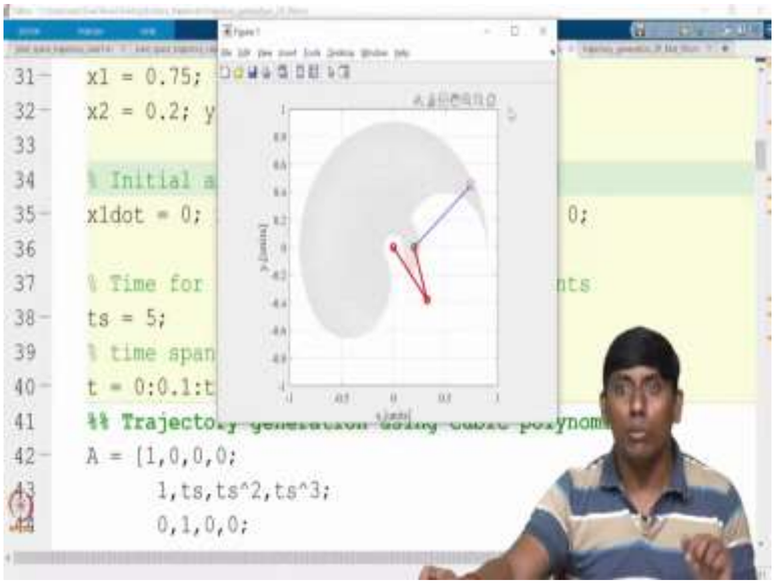
So, I like to like to try to show so, what happened here. So, here we are trying to calculate so and so this x 1 dot and x 2 dot so since we are clearing it there so, we are clearing everything clear x y. So, these initial conditions, which we have calculated, you can see like these are still exist, so I make it initial and final velocity of both x and y are 0. And I just tried to show with a very simple 5 second. So, I just want to show it here. So, now I just you can see it.

So, it takes this point to this point it is taking 5 second and it is running. So, this make it if initial and final velocities are 0 so you are getting much, much smoother profile. So now we can realistic. So, for example, I am taking one point here, so another point is somewhere here. So, what will happen in the task space trajectory it will come here. So, this point, I am taking it 0.15 in the x and probably 0 in y I just take it.

(Refer Slide Time: 12:14)







So, I will take the initial point is the same. So, the final point is 0.2 this is 0. I just so I just try to see the plot. So here I will be taking the initial point is something around probably, I will take it here. So, which is 0.75 and 0.45 I just randomly take 0.75 and 0.45 just to show that. So, the workspace since we have not restricted the joint here. So, the workspace will show, but the manipulator will show the animation says that it is going out of the workspace.

So, you can see like so you can see like it is going to go outside the workspace in the task space trajectory. This is what we have seen one of the constraint, which we were discussing one of the lecture. So, you can see it here. So, this is one of the solution we have taken. That is why it is coming this way. So now the solution, I am taking it the other solution.

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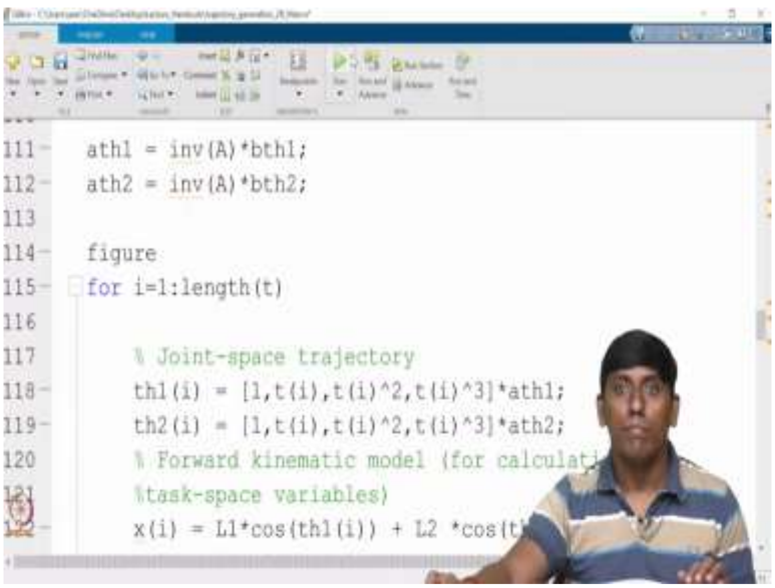
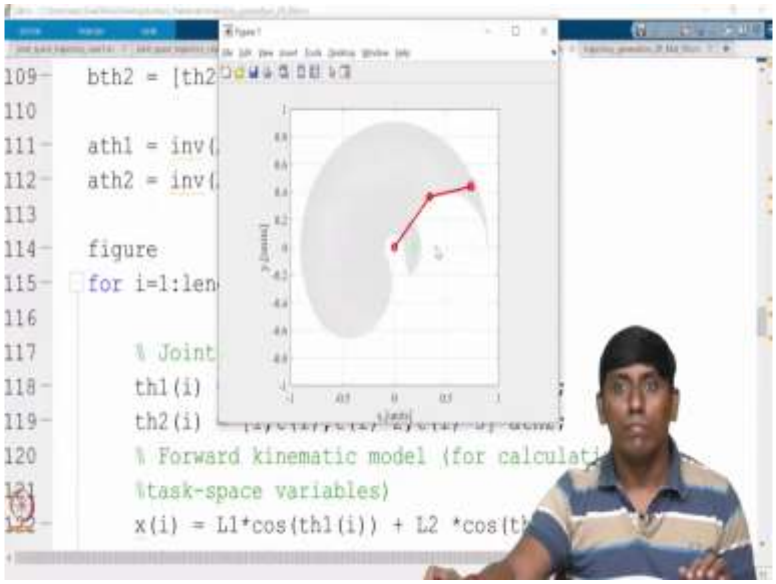
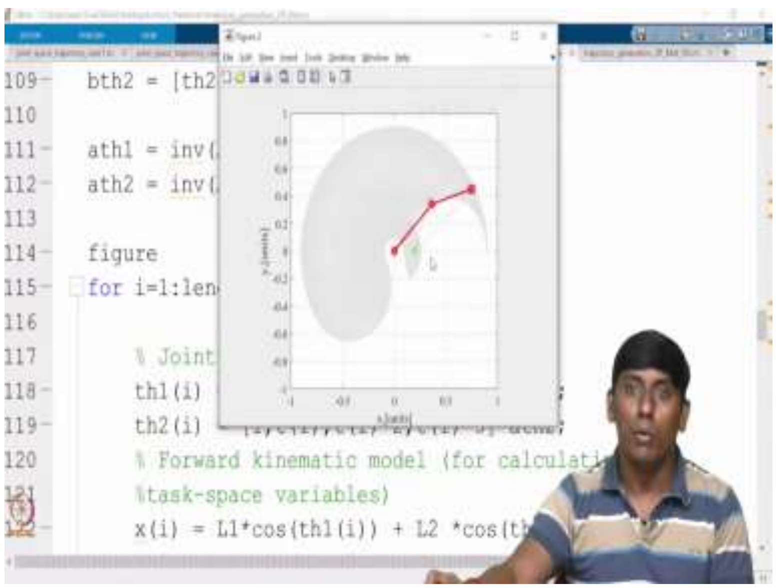
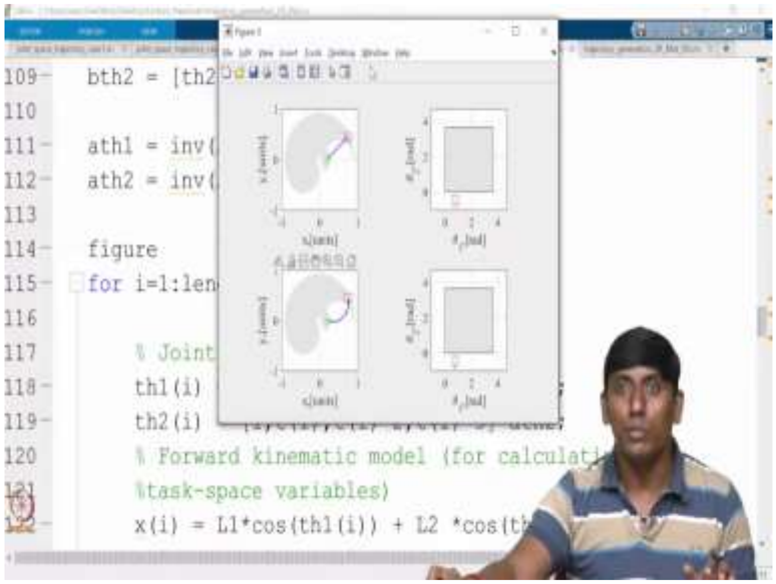


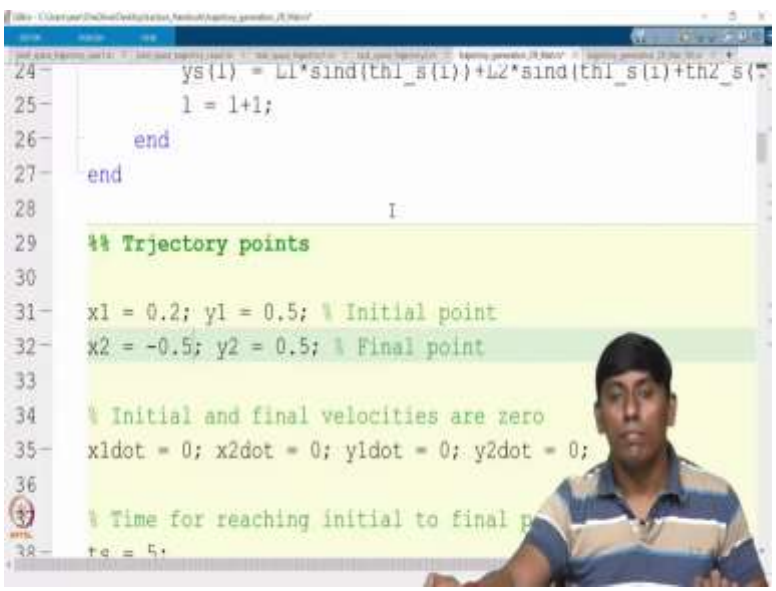

bth2 = [th2
ath1 = inv(
ath2 = inv(
figure
for i=1:len
% Joint
th1(i)
th2(i)
% Forward kinematic model (for calculati
%task-space variables)
x(i) = L1*cos(th1(i)) + L2 *cos(t

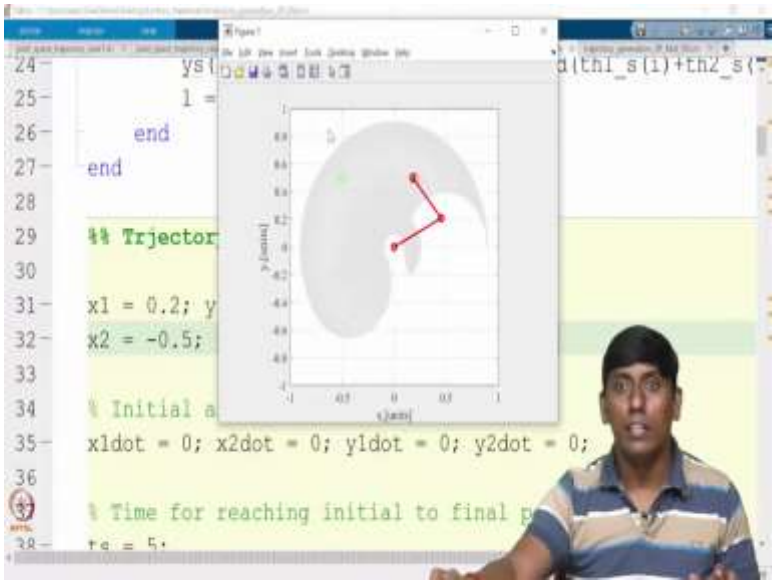

bth2 = [th2
ath1 = inv(
ath2 = inv(
figure
for i=1:len
% Joint
th1(i)
th2(i)
% Forward kinematic model (for calculati
%task-space variables)
x(i) = L1*cos(th1(i)) + L2 *cos(t

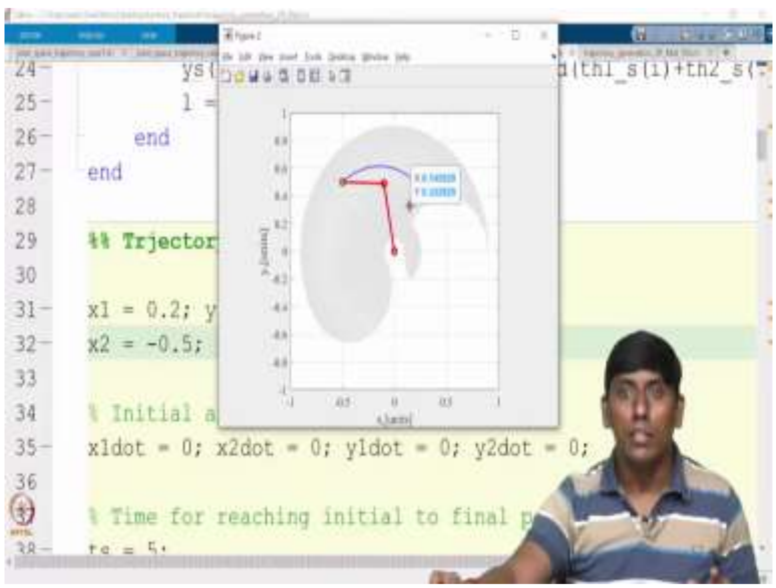


We can see that other solution. So, what is the other solution? So, I am taking it this is s2 is minus sign. So, in that case so you can find. So, this would be still outside workspace. at least one solution is possible. This is also like going out. So probably I have taken some point from here to here. that would be clearly visible because both are outside the workspace even the input is outside the workspace in this case. So that is why it is not doing it.

(Refer Slide Time: 14:18)







So, I will take it back. So, I will just run this. And we will modify that. So, I will take one point here and one point here. Then that would be within the workspace of both we can see at least. So probably I take 0.75 and 0.45 and here somewhere around 0.25 or something. So, I will just see that. So, in fact, I would have taken a grade as the point. So, this is the point 0.15 and 0.35 I take.

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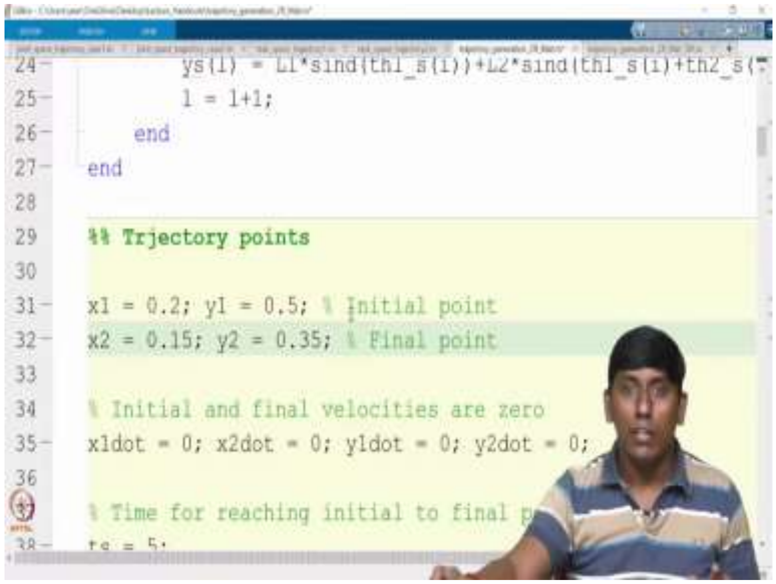
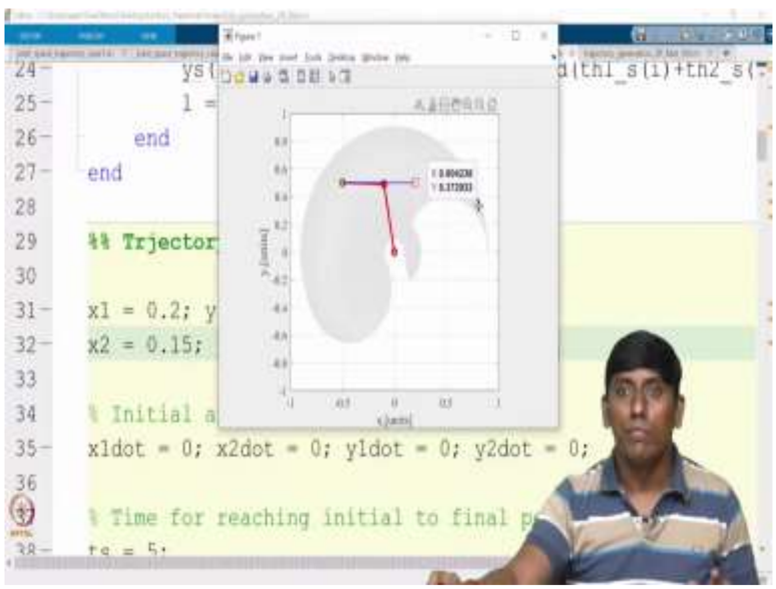
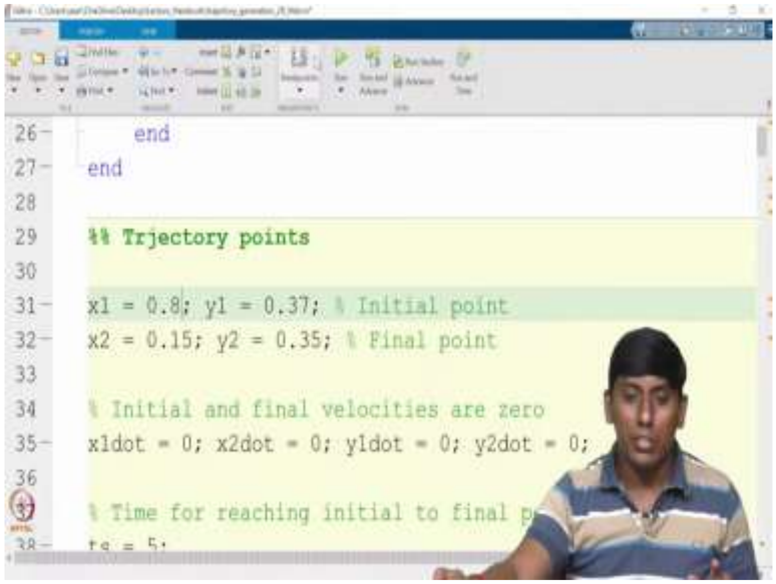
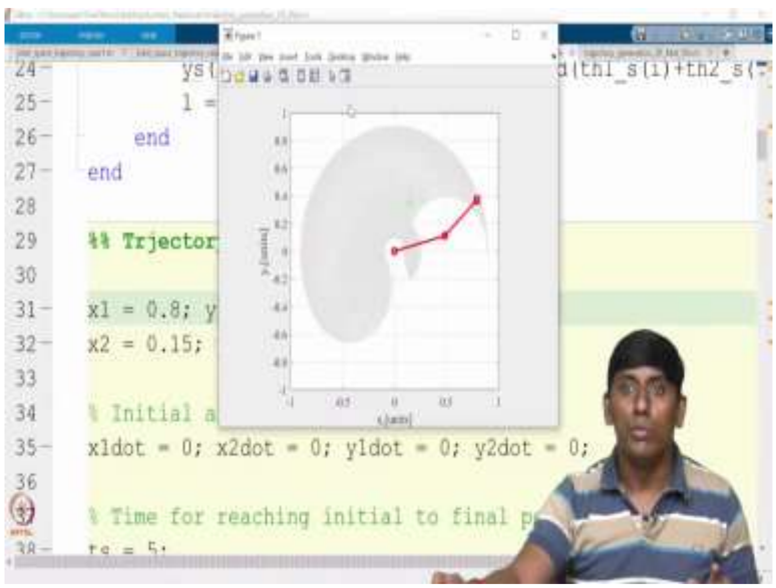


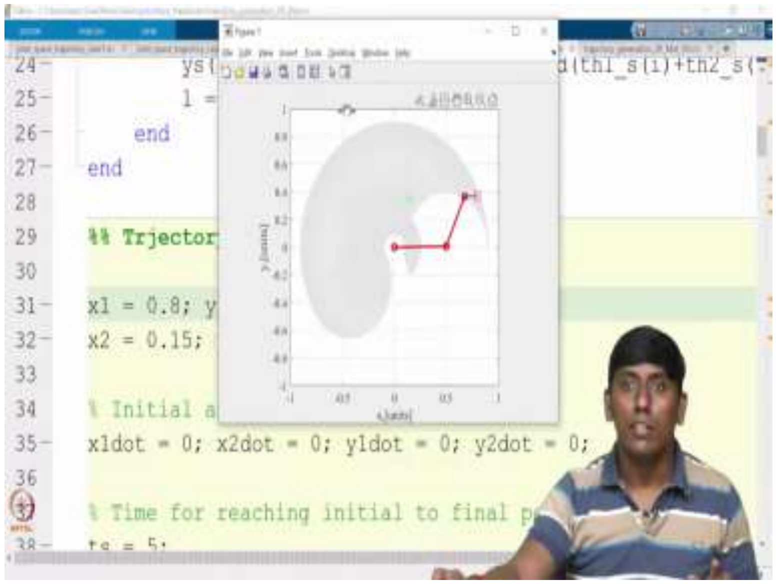


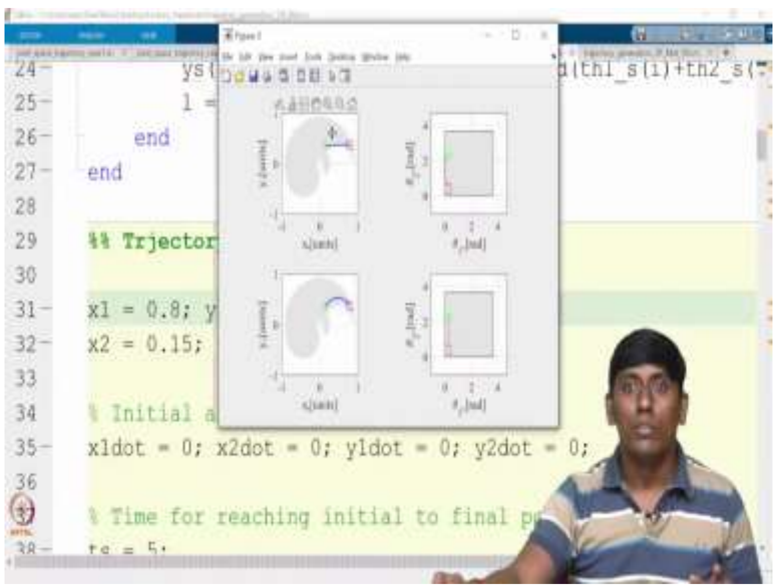


end
end
%% Trjectory points
x1 = 0.8; y1 = 0.37; % Initial point
x2 = 0.15; y2 = 0.35; % Final point
% Initial and final velocities are zero
x1dot = 0; x2dot = 0; y1dot = 0; y2dot = 0;
% Time for reaching initial to final p
ts = 5;



end
end
%% Trjector
x1 = 0.8; y
x2 = 0.15;
% Initial a
x1dot = 0; x2dot = 0; y1dot = 0; y2dot = 0;
% Time for reaching initial to final p
ts = 5;





So, this is so this is 0.15 and so 0.35 and other point I am trying to take the input point somewhere here. So, 0.8 and 0.3 so 0.37 and 0.8 and just to taking a random point. So, I am just trying to demonstrate So, you can see it. So, it is going to make it outside but provided the joint space scheme is still following that profile. This is what we have seen as one simple case. So even I will take the other one.

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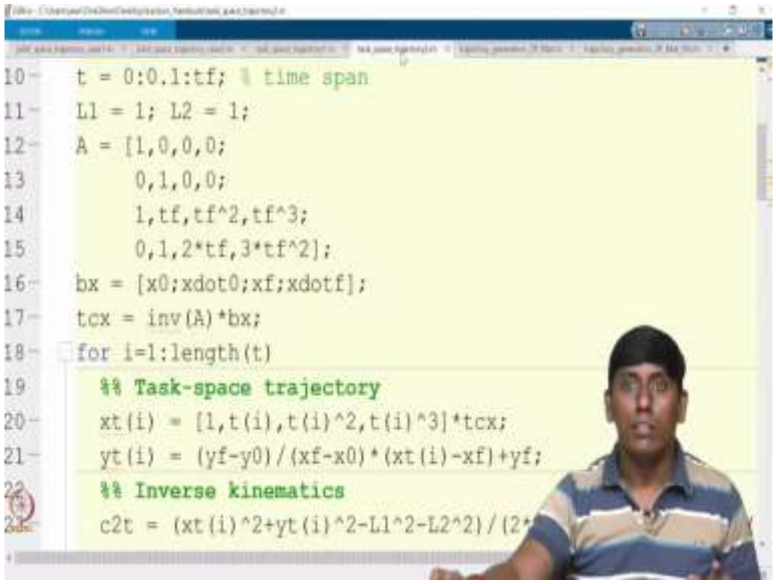
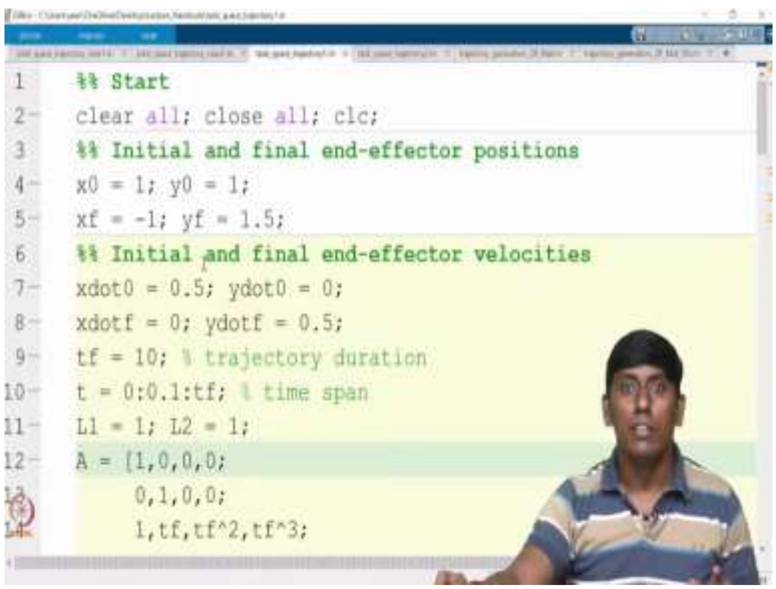
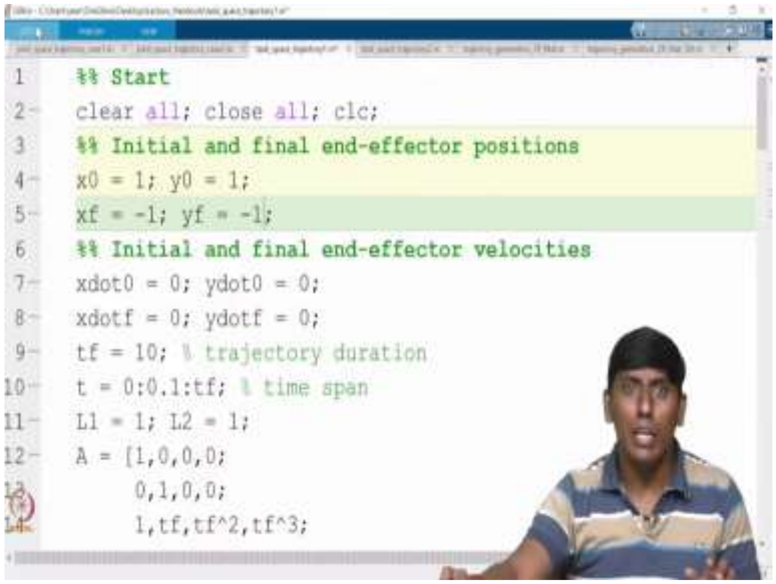
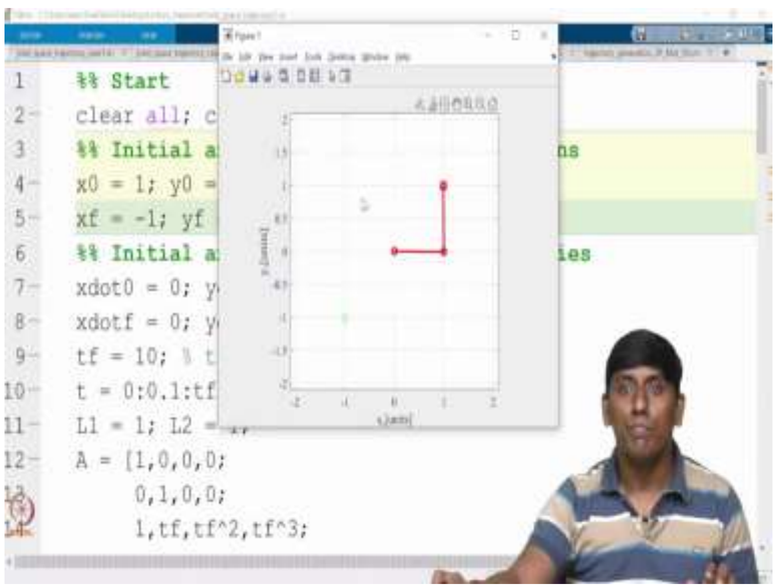
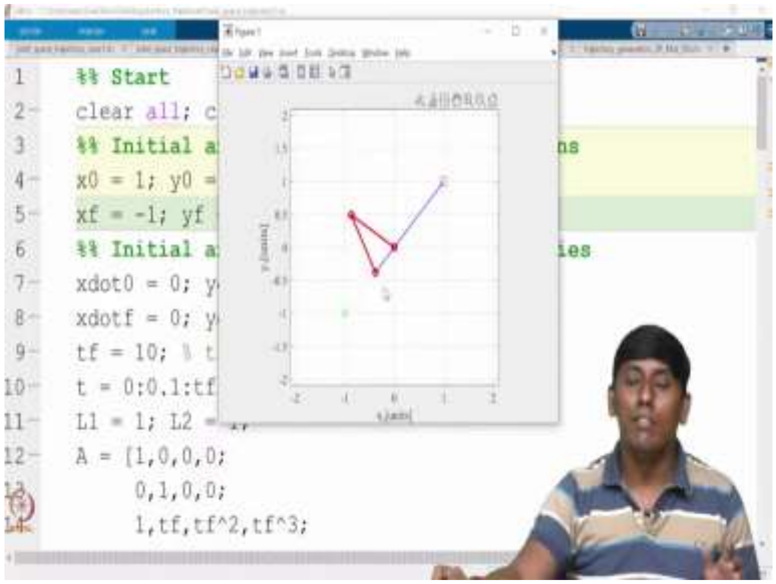
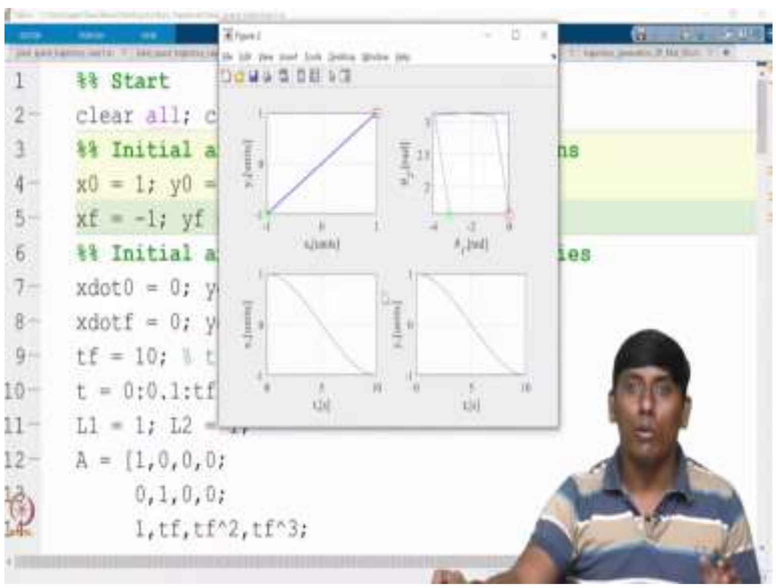





%% Start
clear all; close all; clc;
%% Initial and final end-effector positions
x0 = 1; y0 = 1;
xf = -1; yf = -1;
%% Initial and final end-effector velocities
xdot0 = 0; ydot0 = 0;
xdotf = 0; ydotf = 0;
tf = 10; % trajectory duration
t = 0:0.1:tf; % time span
L1 = 1; L2 = 1;
A = [1,0,0,0;
0,1,0,0;
1,tf,tf^2,tf^3;


%% Start
clear all; c
%% Initial a
x0 = 1; y0 =
xf = -1; yf
%% Initial a
xdot0 = 0; y
xdotf = 0; y
tf = 10; % t
t = 0:0.1:tf
L1 = 1; L2 =
A = [1,0,0,0;
0,1,0,0;
1,tf,tf^2,tf^3;





So, the task space trajectory itself. So, I will take it here. So, just the mirror image I will take. So, this I will take it 0. So, just to show that where it failed so it mirror image in the sense it is minus 1 you can feel it in a single simulation itself. You can feel it how this is going to be very tough; you can see it is going to fall in here single in the sense that it is going to fall on a singular point.

And you can see that it is giving the other solution. The task space which is giving the other solution. So, you can follow it. So, this is what I was saying if the solution of the same point, which is having multiple solution, the scheme can let go it in the end of this. So why it is it is it is just a mirror image of this. So that is why it is giving the other solution.

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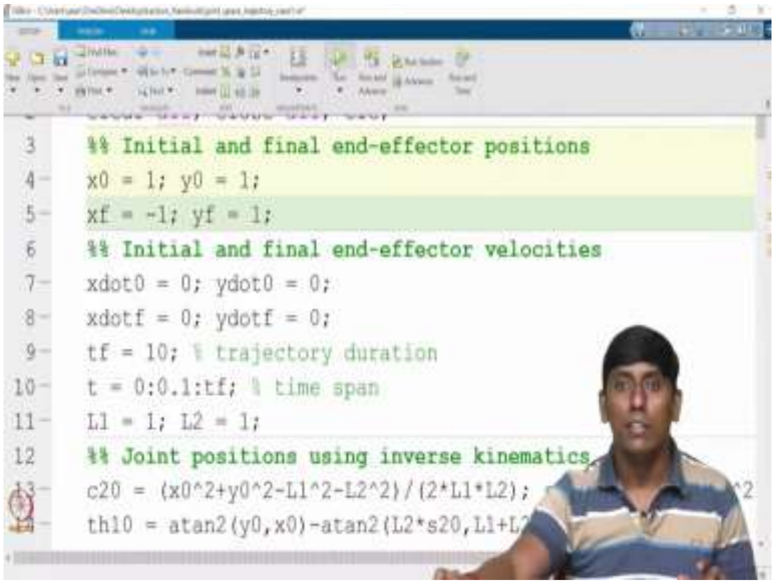

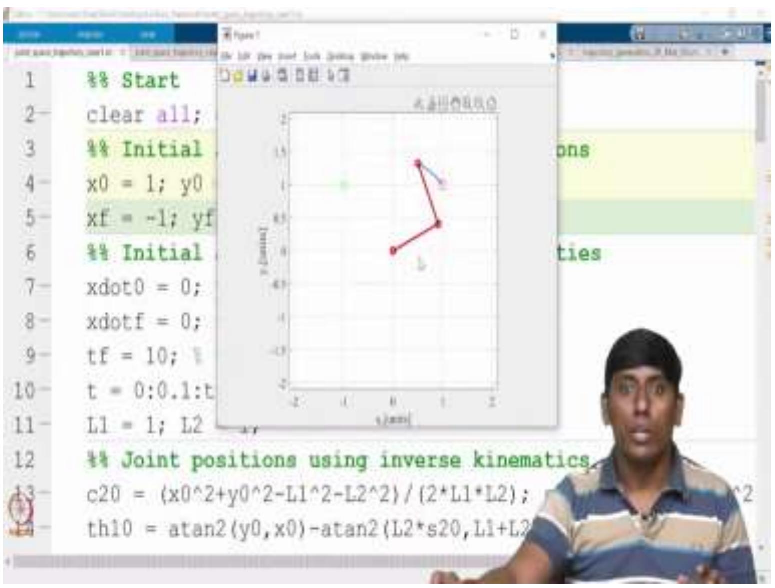

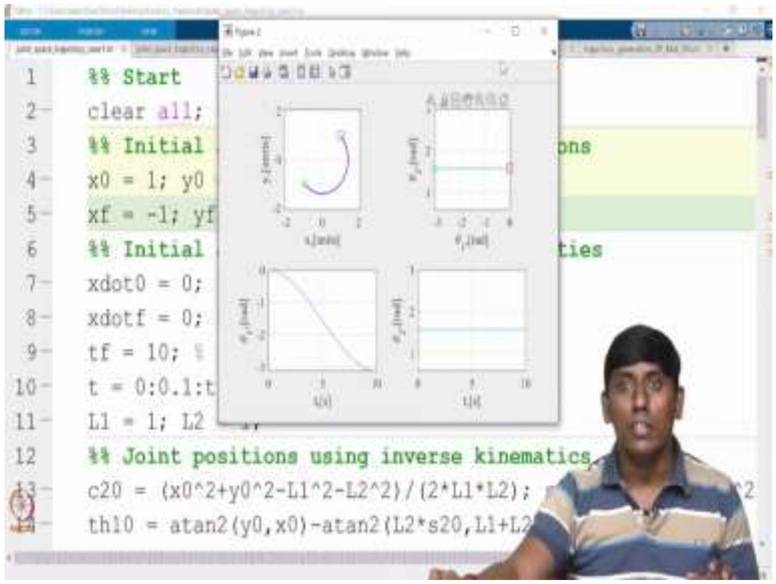

So now the same situation if you do it in joint space, we will try to see whether that is matching at least or not. So, with that, I like to like end it. So, this one and this one and we are trying to see. This is minus 1 I have to take. So, this is minus 1. So, I will try to see whether that consideration of near singular and other thing is restricted you can see it is overcome. So, this way as long the workspace is available even that solution is exists.

So, that is what we wanted to see. So, you can see like this point to this point in the task space is going with what you call overlap with a singular point. But the other case it is not happening it. So, these all we wanted to check.

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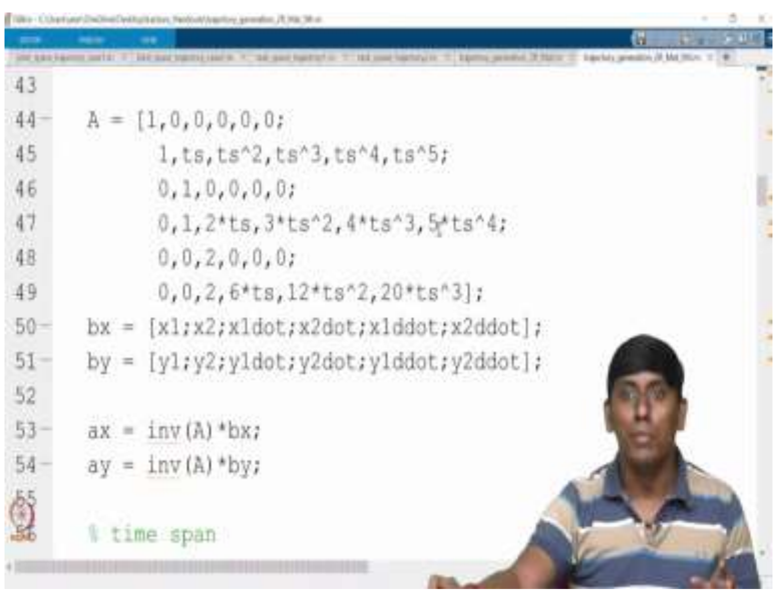
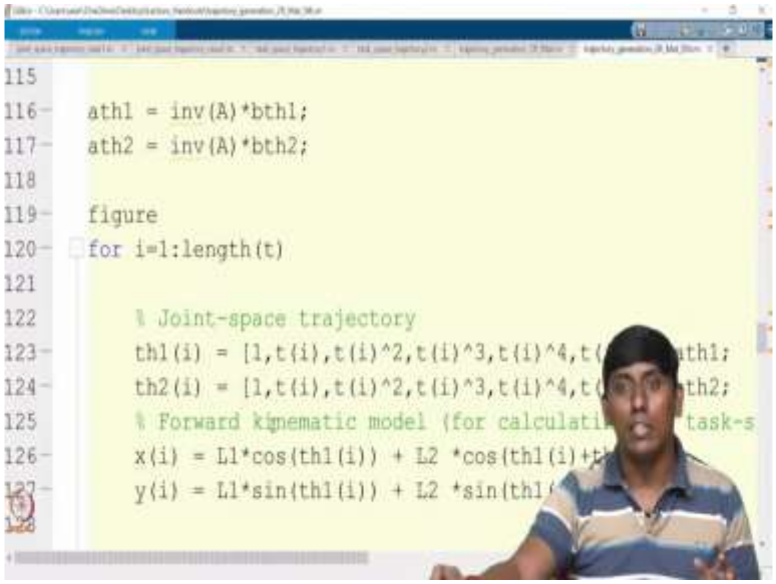
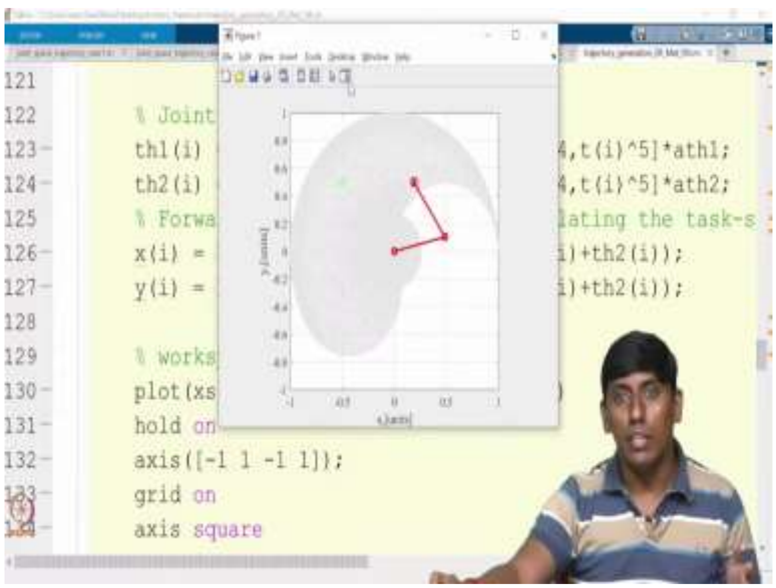
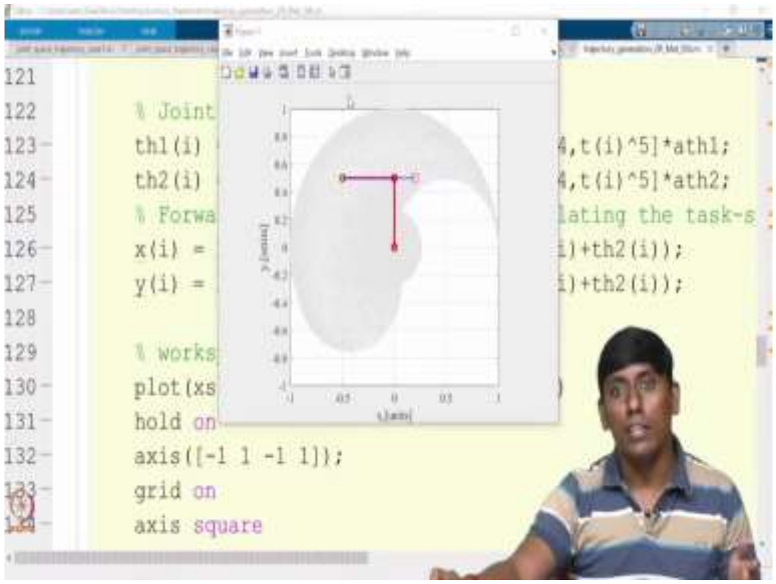
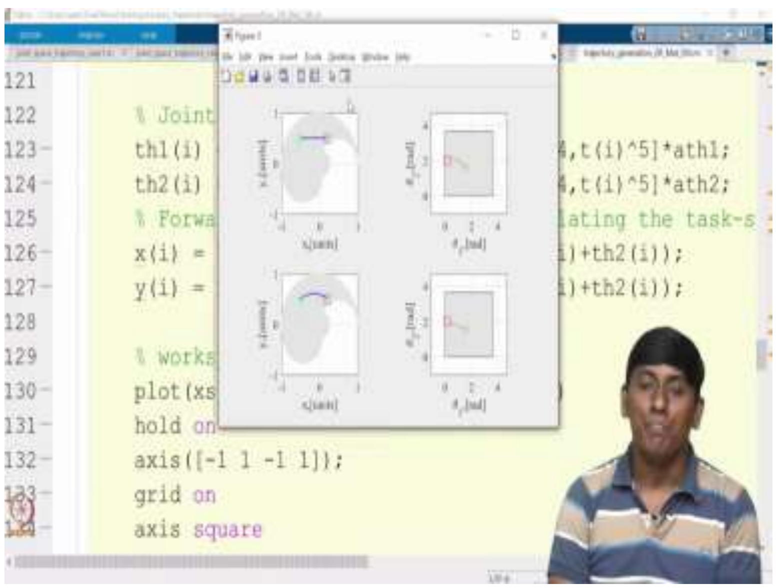
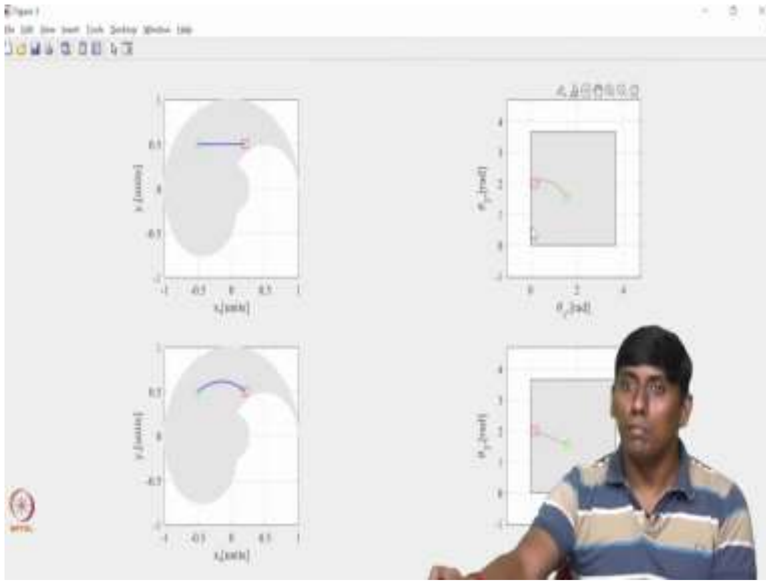



ath1 = inv(A)*bth1;
ath2 = inv(A)*bth2;
figure
for i=1:length(t)
% Joint-space trajectory
th1(i) = [1,t(i),t(i)^2,t(i)^3,t(i)^4,t(
th2(i) = [1,t(i),t(i)^2,t(i)^3,t(i)^4,t(
% Forward kinematic model (for calculati
x(i) = L1*cos(th1(i)) + L2 *cos(th1(i)+t
y(i) = L1*sin(th1(i)) + L2 *sin(th1(


% Joint
th1(i)
th2(i)
% Forwa
x(i) =
y(i) =
% works
plot(xs
hold on
axis([-1 1 -1 1]);
grid on
axis square
4,t(i)^5]*ath1;
4,t(i)^5]*ath2;
lating the task-s
i)+th2(i));
i)+th2(i));


% Joint
th1(i)
4,t(i)^5]*ath1;
th2(i)
4,t(i)^5]*ath2;
% Forwa
lating the task-s
x(i) =
i)+th2(i));
y(i) =
i)+th2(i));
% works
plot(xs
hold on
axis([-1 1 -1 1]);
grid on
axis square
x_i [units]


% Joint
th1(i)
4,t(i)^5]*ath1;
th2(i)
4,t(i)^5]*ath2;
% Forwa
lating the task-s
x(i) =
i)+th2(i));
y(i) =
i)+th2(i));
% works
plot(xs
hold on
axis([-1 1 -1 1]);
grid on
axis square



So, the same thing we can even do for a fifth order. So, the fifth order you can see why only 1 change will come the A matrix and the coefficients would get changed. So, because of that, this you can say task space trajectory or joint space trajectory so that you can say values will change remaining all are same. So, for example, I just if you I run. So, the same thing we can realize it and it is going to happen.

So, now, I hope you are clear to this what you call how to find the workspace and how to get the task space trajectory and joint space trajectory and one if it is failing can we use the second one or which is beneficial and what instant it is beneficial all those things you can look at it. However, the task space trajectory is always better. However, like you can avoid certain constraint points because that will give more realistic.

Because that is going to give in real time. Sometime workspace means we always see only in the task space, but the workspace even we can realize it in the joint space. So here the workspace is minimum to maximum of each joint. So here are only 2 joints it is in a plane, if it is a 3 joint it would come as a volume. So similarly, if it is xyz that would come in a volume. So even we include orientation that would come in the higher order dimensional which is not directly visible to us.

So, with that I am closing this particular lecture. I hope this is giving some clarity on so what is trajectory and how to generate a trajectory? How we can imply or how we can employ these games in the you can say serial manipulator and serial manipulator what is the difference between joint space and task space these all you realize it. So, with that, I am ending this particular lecture.

And the next lecture we will be talking about control we will initially start with open loop, then we will go to feedback which we call closed loop. Then we will go the nonlinear scheme, then we can like close the subject with the advanced topic. With that see you then, thank you bye, take care.