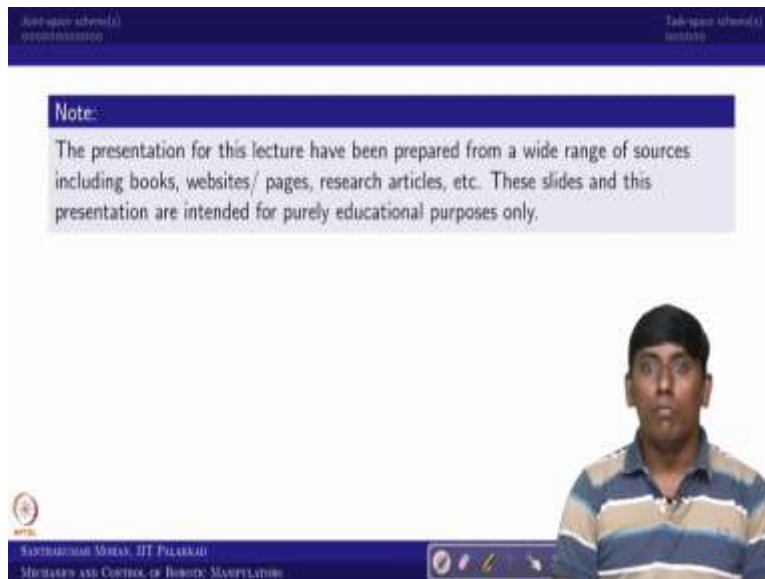


Mechanics and Control of Robotic Manipulators
Professor Santhakumar Mohan
Department of Mechanical Engineering
Indian Institute of Technology, Palakkad
Lecture No 34
Trajectory generation for serial manipulators using MATLAB

Hi, welcome back to Mechanics and Control of Robotic Manipulator. The last two classes we have seen how to generate a trajectory for a general case. So, this particular lecture we are going to see a very specific case. In the sense how to employ those schemes into a robotic manipulator trajectory generation. So, for that again we are going to take a same example which is 2R serial manipulator, and we will see how the joint space scheme and task space scheme can be visualized with the MATLAB.

(Refer Slide Time: 0:43)





The screenshot shows a presentation slide with a dark blue header and footer. The header contains the text "Joint space scheme(s)" on the left and "Task space scheme(s)" on the right. The main content area is white with a blue box containing the word "Note:". Below the note box, the text reads: "The presentation for this lecture have been prepared from a wide range of sources including books, websites/ pages, research articles, etc. These slides and this presentation are intended for purely educational purposes only." In the bottom right corner, there is a video inset showing Professor Santhakumar Mohan, who is wearing a blue and white striped shirt and glasses. The footer of the slide includes the IIT Palakkad logo, the text "SANTHAKUMAR MOHAN, IIT PALAKKAD", "MECHANICS AND CONTROL OF ROBOTIC MANIPULATORS", and a set of navigation icons.

Joint-space scheme(s) Task-space scheme(s)

TRAJECTORY GENERATION FOR ROBOTIC MANIPULATORS USING MATLAB

- 1 Joint-space scheme(s)
- 2 Task-space scheme(s)



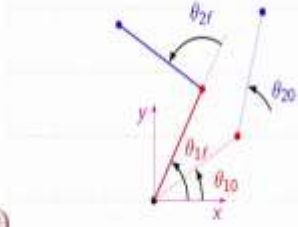


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So, for that we are taking you can say two such schemes. So, you know even the joint space scheme can be make it as a synchronous and series or you call it a simultaneous and synchronous. So, the other one is task space scheme we can do the straight-line interpolation, or we can take independent trajectory generation.

(Refer Slide Time: 1:03)

Joint-space scheme(s) Task-space scheme(s)

Example: A planar RR serial manipulator

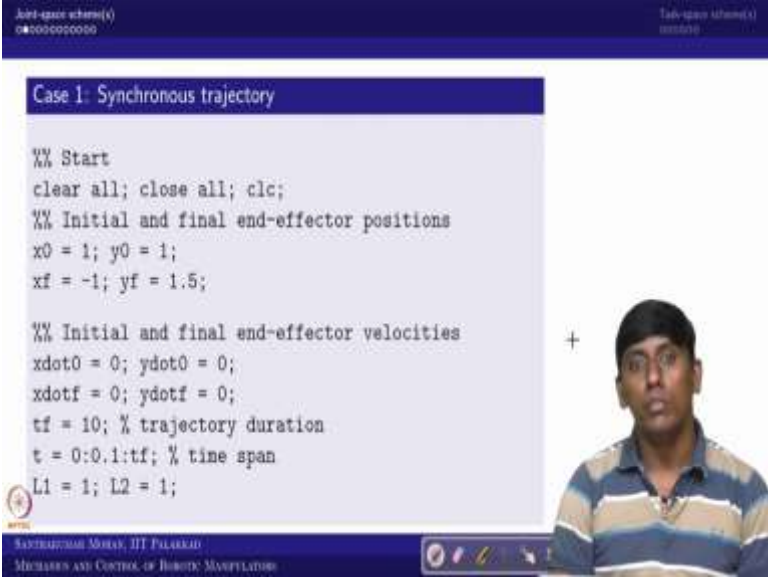

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So, in that sense we are trying to see first what is the example which is given. So, now, in this case you can see that θ_{10} and θ_{1f} and θ_{20} and θ_{2f} are given. If even the x_0 and y_0 and x_f and y_f are given, we can always calculate these 4 variables through the help of

inverse kinematics. So, in my MATLAB code I have written based on the inverse kinematics however, it is not going to restrict that way.

So, in that sense, so, we will first try to see how the joint space scheme can be generated in the sense this is the initial position and you want to go this final position. But through the help of joint space trajectory in the sense $\theta_1 0$ supposed to go to $\theta_1 f$ and $\theta_2 0$ supposed to go to $\theta_2 f$ that is what we are trying to do even you have already seen in the lecture. So, this can be done in synchronous or probably in series. So, we will try to see both in you can say this particular lecture.

(Refer Slide Time: 2:03)



```
Joint-space scheme(s)
#0000000000

Title-space scheme(s)
#0000000000

Case 1: Synchronous trajectory

%% Start
clear all; close all; clc;
%% Initial and final end-effector positions
x0 = 1; y0 = 1;
xf = -1; yf = 1.5;

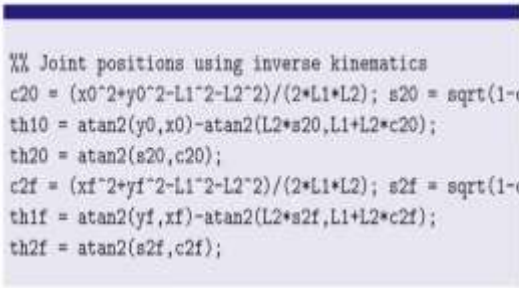
%% Initial and final end-effector velocities
xdot0 = 0; ydot0 = 0;
xdotf = 0; ydotf = 0;
tf = 10; % trajectory duration
t = 0:0.1:tf; % time span
L1 = 1; L2 = 1;
```

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So, first one is synchronous. So, where we are taking the recall initial position and you can say final position of the end effector. So, from there what we can do? We can use the inverse kinematics. So, then we can calculate θ_1 naught and $\theta_1 f$ all those things. So, in that condition, so, even the initial and final end effector velocity also can be given. Because that is what the cubic polynomial all about.

If that is the case, we can use the Jacobian in the sense the differential kinematics we can use where the inverse differential kinematics we can choose and then try to do it. So, since it is a 2R serial manipulator the system parameter or the geometry parameter or the physical parameter we need to use which is we call l_1 and l_2 in this case so, that also we have considered.

(Refer Slide Time: 2:55)



```
%% Joint positions using inverse kinematics
c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2); s20 = sqrt(1-c20^2);
th10 = atan2(y0,x0)-atan2(L2*s20,L1+L2*c20);
th20 = atan2(s20,c20);
c2f = (xf^2+yf^2-L1^2-L2^2)/(2*L1*L2); s2f = sqrt(1-c2f^2);
th1f = atan2(yf,xf)-atan2(L2*s2f,L1+L2*c2f);
th2f = atan2(s2f,c2f);
```



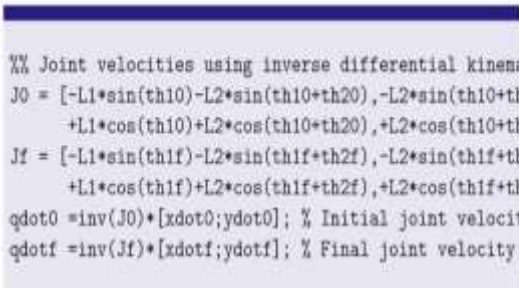
Joint-space scheme(s)
0000000000

Task-space scheme(s)
initial


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So, now, the first case we are trying to find the inverse kinematics. So, we know already inverse kinematics based on the, you can say the closed form equation. So, we have used that equation which we derived in the regular lecture. So, from there we can find theta 1 naught and theta 2 naught. So, even I would have written as a you can say sub function. However, if I write straight away sub function in the MATLAB some of you may not be familiar. So, that is why I have written explicitly here theta 1 naught theta 2 naught then theta 1 f and theta 2 f. So, now what we know? So, we know initial and final position of both joints.

(Refer Slide Time: 3:35)



```
%% Joint velocities using inverse differential kinematics
J0 = [-L1*sin(th10)-L2*sin(th10+th20), -L2*sin(th10+th20);
      +L1*cos(th10)+L2*cos(th10+th20), +L2*cos(th10+th20)];
Jf = [-L1*sin(th1f)-L2*sin(th1f+th2f), -L2*sin(th1f+th2f);
      +L1*cos(th1f)+L2*cos(th1f+th2f), +L2*cos(th1f+th2f)];
qdot0 = inv(J0)*[xdot0;ydot0]; % Initial joint velocity
qdotf = inv(Jf)*[xdotf;ydotf]; % Final joint velocity
```



Joint-space scheme(s)
0000000000

Task-space scheme(s)
initial

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So, now, we need to find out the initial and final velocity of each joint. So, for that we are taking the Jacobian which we derived in the regular lecture. So, now, that would be substituted on the initial and final location. So, we got J_0 and J_f . So, based on that you know already end effector velocities of the initial and final we can find the; you called $\theta_1 \dot{\theta}$ and $\theta_2 \dot{\theta}$ for initial and final which we have written as $q \dot{0}$ and $q \dot{f}$.

So, once these all known, then what we can do? We can go back to whatever profile generation you want to use it. So, since it is the one of the demonstrations I have taken as a cubic polynomial.

(Refer Slide Time: 4:15)

```

Joint-space scheme(s)
000000000000

Joint-space scheme(s)
000000000000

A = [1,0,0,0;
      0,1,0,0;
      1,tf,tf^2,tf^3;
      0,1,2*tf,3*tf^2];

b1 = [th10;qdot0(1);th1f;qdotf(1)];
b2 = [th20;qdot0(2);th2f;qdotf(2)];
tc1 = inv(A)*b1;
tc2 = inv(A)*b2;

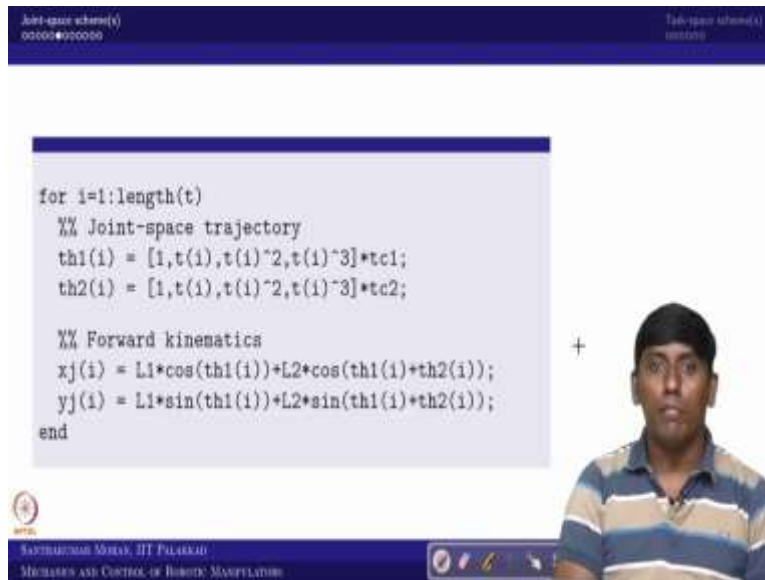
+

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```

So, for that we should know what is, we call coefficient matrix A and we should know the you can save known input as a vector. So, here the θ_1 initial $\theta_1 \dot{\theta}$ initial θ_1 final and $\theta_1 \dot{\theta}$ final need to be known for finding the first segment joint space trajectory. Similarly, second joints you can say trajectory also can be generated. So, based on that tc_1 and tc_2 for the joint 1 and joint 2 coefficients which is we call you can say a_{01} to a_{31} . Similarly, a_{02} to a_{32} that we can calculate.

(Refer Slide Time: 5:01)



Joint-space scheme(s)
000000000000

for i=1:length(t)
%% Joint-space trajectory
th1(i) = [1,t(i),t(i)^2,t(i)^3]*tc1;
th2(i) = [1,t(i),t(i)^2,t(i)^3]*tc2;

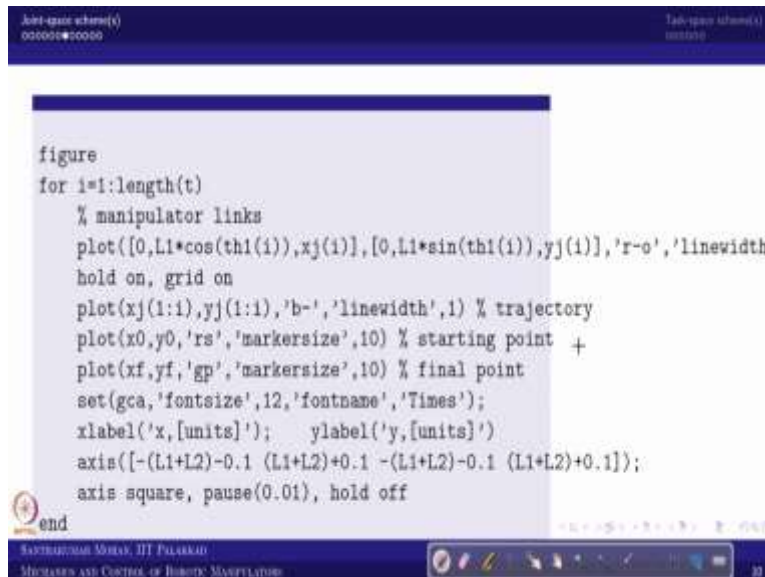
%% Forward kinematics
xj(i) = L1*cos(th1(i))+L2*cos(th1(i)+th2(i));
yj(i) = L1*sin(th1(i))+L2*sin(th1(i)+th2(i));
end

+

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So, once you calculate then we can go to the loop where you are trying to run based on t_0 to t_f profile. So, the θ_1 and θ_2 can be generated once you know since this is synchronous. So, both would be synchronously moving. So, in the sense it starts from t_0 to t_f completely both joints are rotating. So, in that sense, we can write θ_1 and θ_2 just to plot as the animated way. So, we are trying to find the forward kinematic model so, that your x end effector and the y end effector can be visualized and then you can show in an animated way.

(Refer Slide Time: 5:39)



Joint-space scheme(s)
000000000000

```
figure
for i=1:length(t)
% manipulator links
plot([0,L1*cos(th1(i)),xj(i)],[0,L1*sin(th1(i)),yj(i)],'r-o','linewidth'
hold on, grid on
plot(xj(1:i),yj(1:i),'b-', 'linewidth',1) % trajectory
plot(x0,y0,'rs','markersize',10) % starting point
plot(xf,yf,'gp','markersize',10) % final point
set(gca,'fontsize',12,'fontname','Times');
xlabel('x,[units]'); ylabel('y,[units]')
axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1]);
axis square, pause(0.01), hold off
end
```

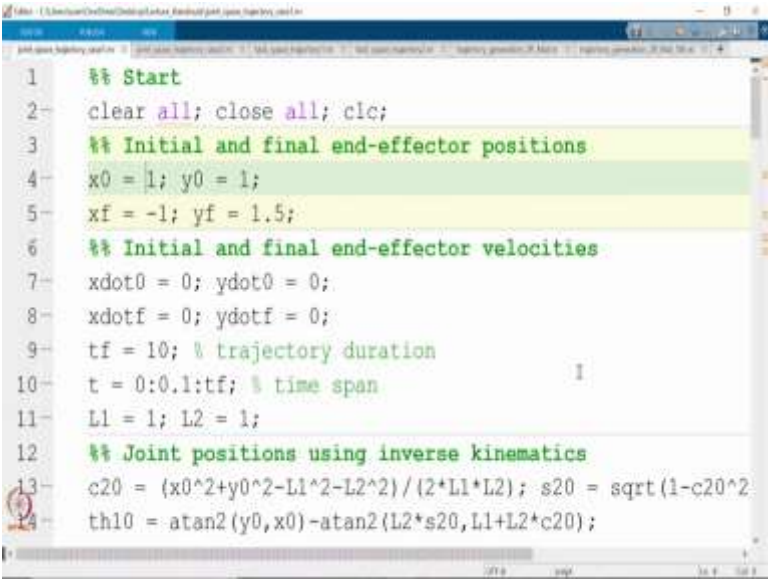
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MECHANICS AND CONTROL OF ROBOTS MANIPULATORS

For doing the animation we have tried to take this particular you can say MATLAB code. So, we are trying to plot 0 as the origin then x_1 and x_2 x_1 we can write as $l_1 \cos \theta_1$ and x_2 is nothing but you are a final end effector point. So, that is corresponding to the x similarly y we start from 0 origin then $l_1 \sin \theta_1$ is the y_1 and y_2 would be your end effector position. So, based on that we can draw, and I want to just show the continuous profile how it is generated.

So, I just added that x_j and y_j from 1 to i . So, when the loop is iterating it will show the continuous plot. And I want to show what is the initial and final point. So, I just added two additional plotting where r_s is the red square will come in the starting point. And the g_p is nothing, but you can say g pentagon will or you can say star will come at the green color at the final point.


So, and we have run everything and just to show the animated I pause and hold off. So, in the sense every time it would be super impose one image to another image but hold off command is there. So, it is look like its continuous motion.

(Refer Slide Time: 6:58)




```
1 %% Start
2 clear all; close all; clc;
3 %% Initial and final end-effector positions
4 x0 = 1; y0 = 1;
5 xf = -1; yf = 1.5;
6 %% Initial and final end-effector velocities
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tf = 10; % trajectory duration
10 t = 0:0.1:tf; % time span
11 L1 = 1; L2 = 1;
12 %% Joint positions using inverse kinematics
13 c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2); s20 = sqrt(1-c20^2);
14 th10 = atan2(y0,x0)-atan2(L2*s20,L1+L2*c20);
```

```
49 plot([0,L1*cos(th1(i)),xj(i)], [0,L1*sin(th1(i)),yj(i)])
50 hold on
51 plot(xj(1:i),yj(1:i),'b-','linewidth',1) % trajectory
52 plot(x0,y0,'rs','markersize',10) % starting point
53 plot(xf,yf,'gp','markersize',10) % final point
54 set(gca,'fontsize',12,'fontname','Times');
55 xlabel('x,[units]');
56 ylabel('y,[units]');
57 axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1]);
58 grid on
59 axis square
60 pause(0.01)
61 hold off
62 end
```



```
4 x0 = 1; y0 = 1;
5 xf = -1; yf = 1.5;
6 %% Initial and final end-effector velocities
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tf = 10; % trajectory duration
10 t = 0:0.1:tf; % time span
11 L1 = 1; L2 = 1;
12 %% Joint positions using inverse kinematics
13 c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2); s20 = sqrt(1-c20^2);
14 th10 = atan2(y0,x0)-atan2(L2*s20,L1+c20*L2);
15 th20 = atan2(s20,c20);
16 c2f = (xf^2+yf^2-L1^2-L2^2)/(2*L1*L2); s2f = sqrt(1-c2f^2);
17 th1f = atan2(yf,xf)-atan2(L2*s2f,L1+c2f*L2);
```




```
13- c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2); s20 = sqrt(1-c20^2);
14- th10 = atan2(y0,x0)-atan2(L2*s20,L1+L2*c20);
15- th20 = atan2(s20,c20);
16- c2f = (xf^2+yf^2-L1^2-L2^2)/(2*L1*L2); s2f = sqrt(1-c2f^2);
17- th1f = atan2(yf,xf)-atan2(L2*s2f,L1+L2*c2f);
18- th2f = atan2(s2f,c2f);
19- J0 = [-L1*sin(th10)-L2*sin(th10+th20),-L2*sin(th10+th20);
20-       +L1*cos(th10)+L2*cos(th10+th20),+L2*cos(th10+th20)];
21- Jf = [-L1*sin(th1f)-L2*sin(th1f+th2f),-L2*sin(th1f+th2f);
22-       +L1*cos(th1f)+L2*cos(th1f+th2f),+L2*cos(th1f+th2f)];
23- qdot0 = inv(J0)*[xdot0;ydot0]; % Initial joint velocity
24- qdotf = inv(Jf)*[xdotf;ydotf]; % Final joint velocity
25-
26- A = [1,0,0,0;
```

```
49- plot([0,L1*cos(th1(i)),xj(i)],[0,L1*sin(th1(i)),yj(i)])
50- hold on
51- plot(xj(1:i),yj(1:i),'b-','linewidth',1) % trajectory
52- plot(x0,y0,'rs','markersize',10) % starting point
53- plot(xf,yf,'gp','markersize',10) % final point
54- set(gca,'fontsize',12,'fontname','Times');
55- xlabel('x,[units]');
56- ylabel('y,[units]')
57- axis([-L1-L2)-0.1 (L1+L2)+0.1 -(L1+L2)-0.1 (L1+L2)+0.1]);
58- grid on
59- axis square
60- pause(0.01)
61- hold off
62- end
```

The top image shows a MATLAB environment with a code editor on the left and a plot window in the center. The code editor contains the following MATLAB code:

```

49- plot({0
50- hold on
51- plot(x)
52- plot(x0
53- plot(xf
54- set(gca
55- xlabel(
56- ylabel(
57- axis([-
58- grid on
59- axis sq
60- pause(0.01)
61- hold off
62- end

```

The plot window displays a 2D coordinate system with x and y axes ranging from -2 to 2. A red trajectory is plotted, starting at the origin (0,0) and ending at (1,1). The trajectory is a smooth curve that starts at the origin and ends at (1,1). The plot window also shows a toolbar with various icons.

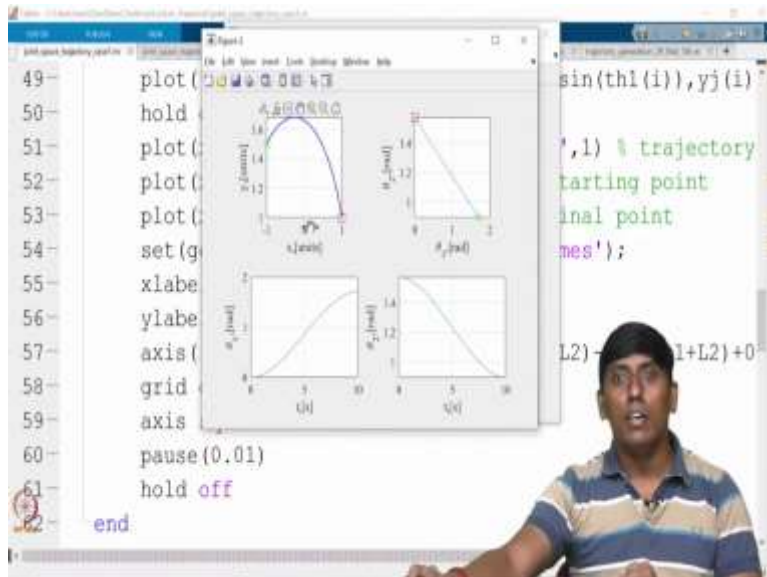
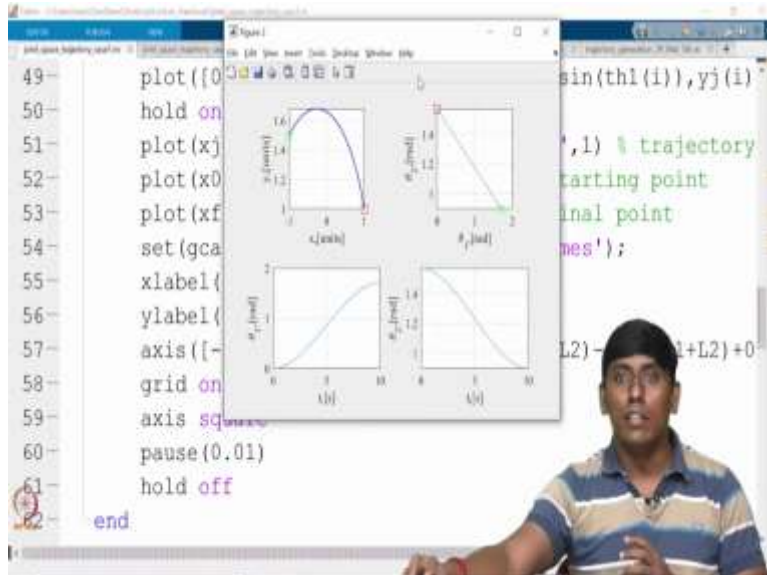
The bottom image shows the same MATLAB environment as the top image, but with a different plot. The code editor contains the same MATLAB code as in the top image:

```

49- plot({0
50- hold on
51- plot(x)
52- plot(x0
53- plot(xf
54- set(gca
55- xlabel(
56- ylabel(
57- axis([-
58- grid on
59- axis sq
60- pause(0.01)
61- hold off
62- end

```

The plot window displays a 2D coordinate system with x and y axes ranging from -2 to 2. A red trajectory is plotted, starting at the origin (0,0) and ending at (1,1). The trajectory is a smooth curve that starts at the origin and ends at (1,1). A blue arc is also plotted above the red trajectory, starting at (0,1) and ending at (1,1). The plot window also shows a toolbar with various icons.



So, just to show this I just go to the MATLAB window, and this is the code which we were discussing. So, you can see this is the code. So, now if I run this, so, what did start. So, the x initial is 1 meter and y initial also like 1 meter it is somewhere like this and the final also like this. So now we have not given θ_1 0 and θ_1 f directly. So, what we can do, we can do the inverse kinematics.

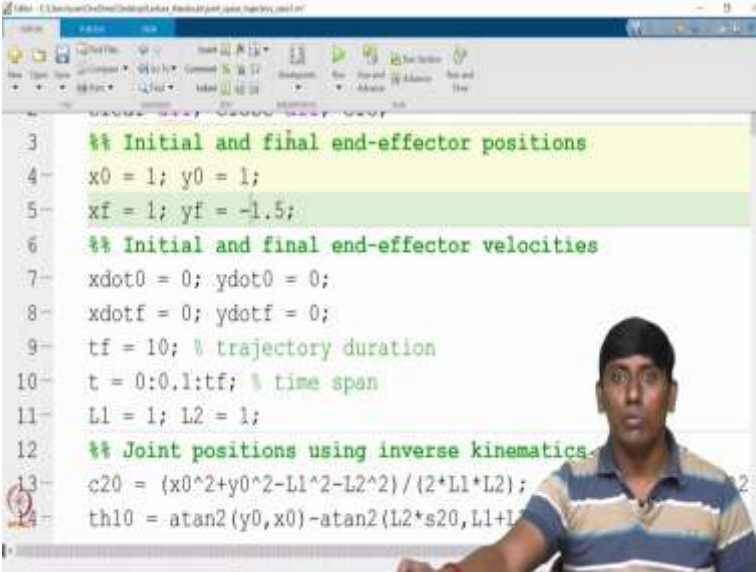
So now the inverse kinematics will have two solutions. So, we will take one solution, which is the positive solution of s^2 . So, now based on this we can run this code. So, then you can feel it what I want to explain, so, you can see like now it is starting from here and it is ending. So, you

can see like it is more like the robot is moving right. So, when I ran this so you can see like the physically the robot is moving that realization you can feel it in MATLAB animation.

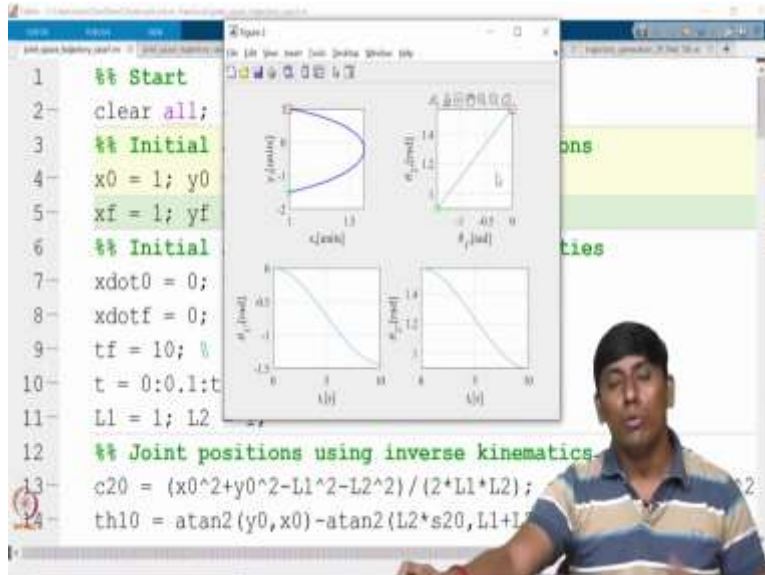
So, I did not make any video but still it looked like a video, right? Because I have made it a super impose. So, the next figure I want to show it. So, what is the; you can save profile or the trajectory generated in task space and as well as the joint space. The joint space theta 1 to theta 2 you can see it is like a straight line.

So, that is what we call it like joint space trajectory consider as a straight line the profile starts from 0 to you can say this is theta 1 0 to theta 1 f it is a smooth profile and I did not plot the velocity when you want you can do the velocity profile. So, now, this is the theta 2 f this is theta 2 0. So, like that we can generate, so, even you want to realize further.

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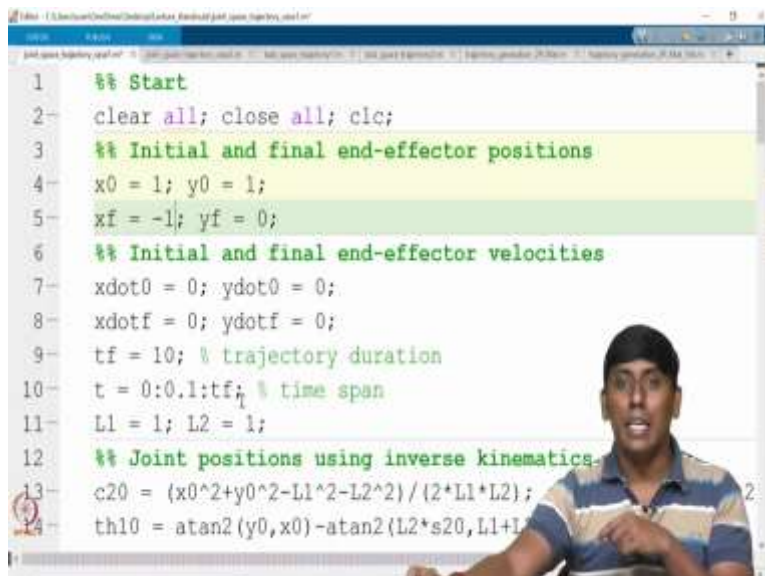


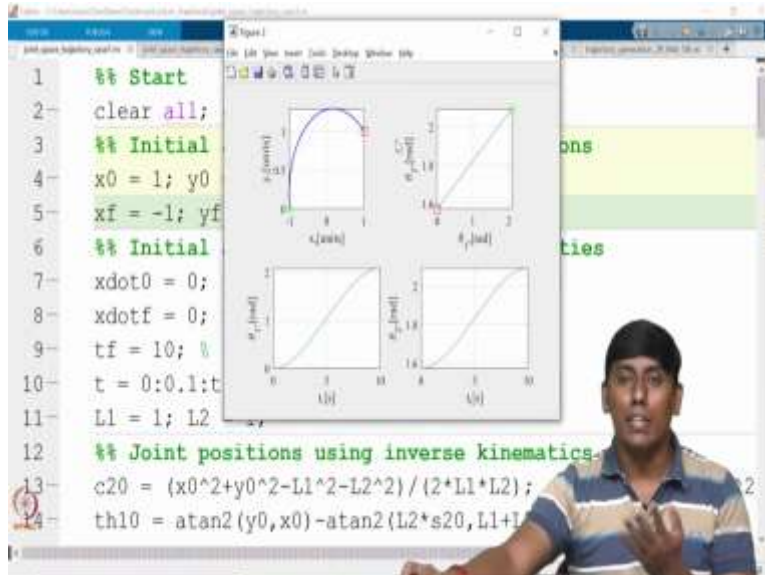
```
3 % Initial and final end-effector positions
4 x0 = 1; y0 = 1;
5 xf = 1; yf = -1.5;
6 % Initial and final end-effector velocities
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tf = 10; % trajectory duration
10 t = 0:0.1:tf; % time span
11 L1 = 1; L2 = 1;
12 % Joint positions using inverse kinematics
13 c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2);
14 th10 = atan2(y0,x0)-atan2(L2*s20,L1+L2*c20);
```



So, the only condition is this initial input what we are giving that should be in the case of what you call in the workspace. So, we can see, I have given interchange so, the initial point is same, but the final point is slightly modified. So, now, you can see that this is the initial point and the final point, you can see it is moving it. So, this is what the profile and this is what happening, right? So, now, this is more and more very clear.

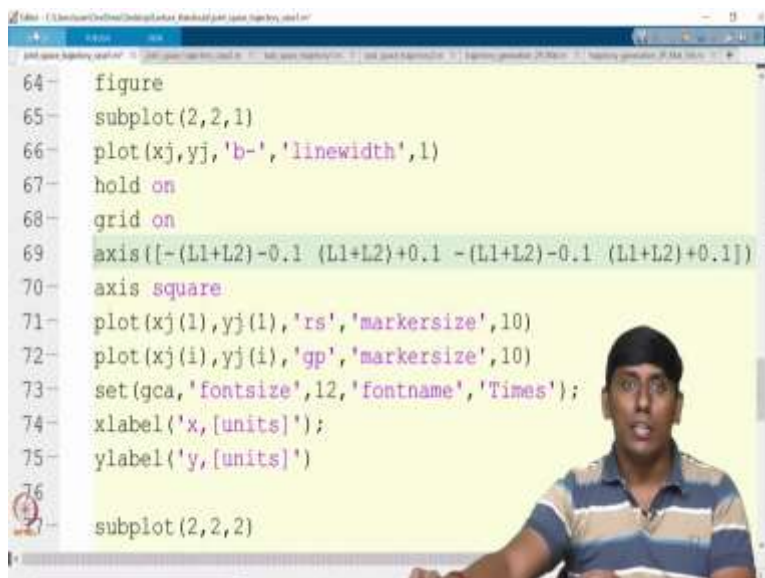
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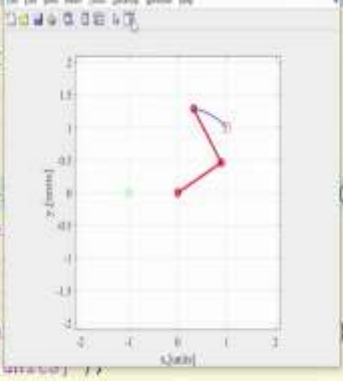


So, now, we will actually go back so go back to the code. So now I am looking at this, so I am just giving it 0. And this is it like I am putting it so minus 1. So, now, I am trying to give because you are 1 1 and 1 2 is 1 and 1. So maximum it is on the circle of radius of 2 meter. So, that is what the range within that you can choose anything. So, anyhow next lecture, we would be seeing the generation. So, you can feel it. It is more you can say beneficial. So, you want to see this in a proper shape. So even we can make the boundary.

(Refer Slide Time: 9:56)



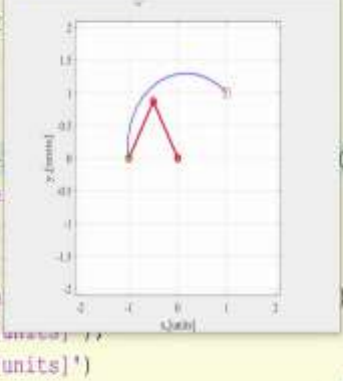
```
64 figure
65 subplot(2,2,1)
66 plot(xj,yj,
67 hold on
68 grid on
69 axis([-L1+
70 axis square
71 plot(xj(1),
72 plot(xj(i),
73 set(gca,'fo
74 xlabel('x, [
75 ylabel('y, (units)')
76
77 subplot(2,2,2)
```



0.1 (L1+L2)+0.1]]

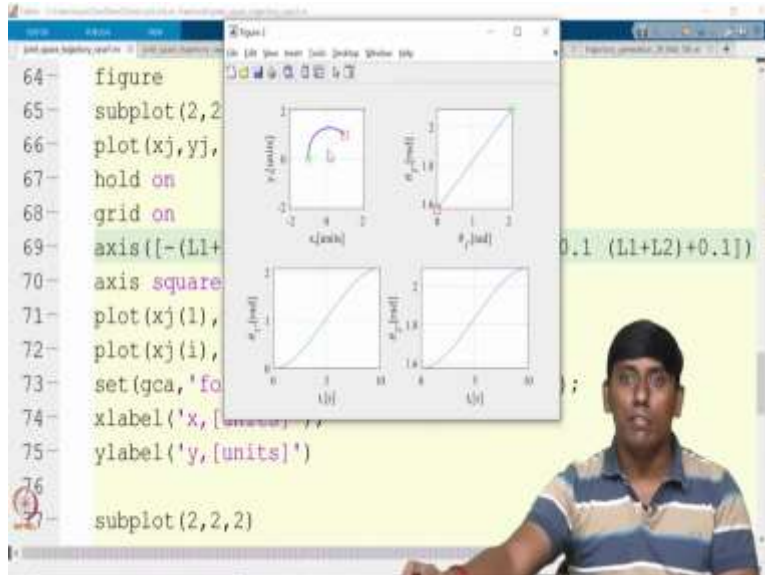
Figure

```
64 figure
65 subplot(2,2,1)
66 plot(xj,yj,
67 hold on
68 grid on
69 axis([-L1+
70 axis square
71 plot(xj(1),
72 plot(xj(i),
73 set(gca,'fo
74 xlabel('x, [
75 ylabel('y, (units)')
76
77 subplot(2,2,2)
```



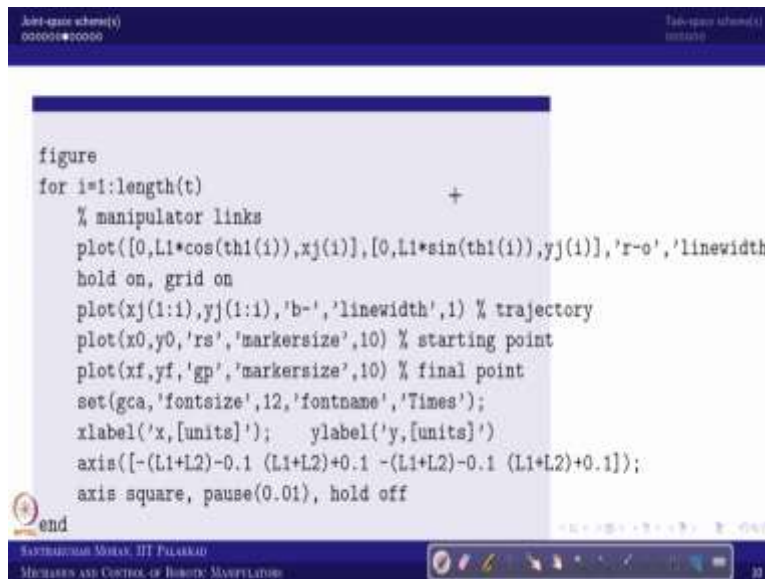
0.1 (L1+L2)+0.1]]

Figure



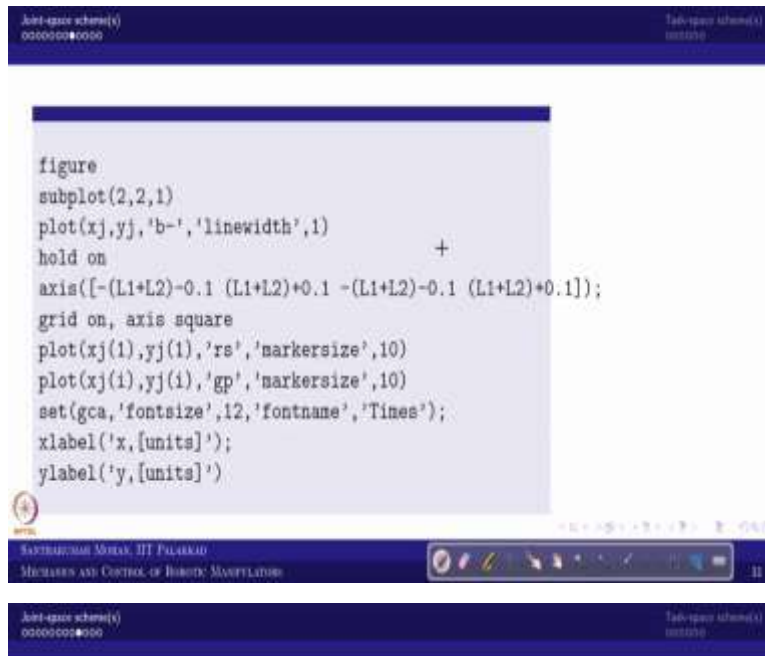
So, the boundary we can right now I make it this as commented. So, now if I make it. It would be giving a real size like how it is happening the same size it is giving this is the real size. So, now these all what you call the joint space trajectory with a case 1 where the synchronous.

(Refer Slide Time: 10:20)



We will go back to the slide where the non-synchronous we call in series of motion.

(Refer Slide Time: 10:25)



Joint-space scheme(s)
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Tab-space scheme(s)
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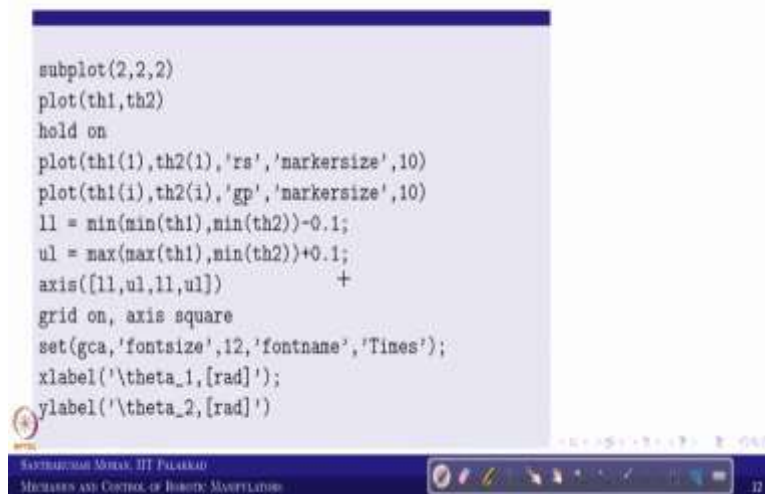
```
figure
subplot(2,2,1)
plot(xj,yj,'b-','linewidth',1)
hold on
axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1]);
grid on, axis square
plot(xj(1),yj(1),'rs','markersize',10)
plot(xj(i),yj(i),'gp','markersize',10)
set(gca,'fontsize',12,'fontname','Times');
xlabel('x,[units]');
ylabel('y,[units]')
```

Navigation icons: back, forward, search, etc.

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Joint-space scheme(s)
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Tab-space scheme(s)
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Joint-space scheme(s)
000000000000

Tab-space scheme(s)
000000

```
subplot(2,2,2)
plot(th1,th2)
hold on
plot(th1(1),th2(1),'rs','markersize',10)
plot(th1(i),th2(i),'gp','markersize',10)
l1 = min(min(th1),min(th2))-0.1;
u1 = max(max(th1),min(th2))+0.1;
axis([l1,u1,l1,u1])
grid on, axis square
set(gca,'fontsize',12,'fontname','Times');
xlabel('\theta_1,[rad]');
ylabel('\theta_2,[rad]')
```

Navigation icons: back, forward, search, etc.

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Joint-space scheme(s)
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Tab-space scheme(s)
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```

Joint-space scheme(s)
00000000000000000000

Figure
for i=1:length(t)
    % manipulator links
    plot([0,L1*cos(th1(i)),xj(i)],[0,L1*sin(th1(i)),yj(i)],'r-o','linewidth'
    hold on, grid on
    plot(xj(1:i),yj(1:i),'b-','linewidth',1) % trajectory
    plot(x0,y0,'rs','markersize',10) % starting point
    plot(xf,yf,'gp','markersize',10) % final point
    set(gca,'fontsize',12,'fontname','Times');
    xlabel('x,[units]'); ylabel('y,[units]')
    axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1]);
    axis square, pause(0.01), hold off
end

```

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So, for that what we need to do it. So, these are all the other plots. So, I will just see the non-synchronous. So, the non-synchronous you have to do it a slightly trickier way. So, these all the plot which we have already done.

(Refer Slide Time: 10:42)

```

Joint-space scheme(s)
00000000000000000000

Case 2: Sequence trajectory

tt = 10; % trajectory duration
t = 0:0.1:tt; % time span
tf = tt/2; % total duration break into number of axes

```

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So, the non-synchronous way we call sequence trajectory. So, here we call the tt is that total trajectory duration. But now I need to divide that into tf into number of segments. Because each joint would be rotating independently for example, now 10 second I have 2 joints, so, I break into 5 second each. So, first 5 second first joint will rotate the next 5 second the second joint will rotate. So, that is what we have seen as a sequence trajectory.

instead of t_f I have taken t_t and the t_t and t_f why I have made it because I make it the t_f always the segment time. The segment time here is you can say t_t divided by 2. So, based on that you will get the inverse kinematics and all.

(Refer Slide Time: 12:17)

```
37- for i=1:length(t)
38-     %% Joint-space trajectory
39-     if t(i)<=tf
40-         th1(i) = [1,t(i),t(i)^2,t(i)^3]*tc1;
41-         th2(i) = th20;
42-     else
43-         th1(i) = th1f;
44-         th2(i) = [1,(t(i)-tf),(t(i)-tf)^2,(t(i)-tf)^3]*tc2;
45-     end
46-     %% Forward kinematics
47-     xj(i) = L1*cos(th1(i))+L2*cos(th1(i)+th2(i));
48-     yj(i) = L1*sin(th1(i))+L2*sin(th1(i)+th2(i));
49- end
```

```
52-
53- for i=1:length(t)
54-
55-     % manipulator links
56-     plot([0,L1*cos(th1(i)),xj(i)],[0,L1*sin(th1(i)),yj(i)])
57-     hold on
58-     plot(xj(1:i),yj(1:i),'b-','linewidth',1) % trajectory
59-     plot(x0,y0,'rs','markersize',10) % starting point
60-     plot(xf,yf,'gp','markersize',10) % final
61-     set(gca,'fontsize',12,'fontname','Times')
62-     xlabel('x,[units]');
63-     ylabel('y,[units]')
64-     axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)+0.1 (L1+L2)+0.1])
65-     grid on
```

The top image shows a MATLAB code editor with the following code:

```

52
53 for i=1:len
54
55 % manip
56 plot([0
57 hold on
58 plot(xj
59 plot(x0
60 plot(xf
61 set(gca
62 xlabel('x [m]')
63 ylabel('y, [units]')
64 axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+
65 grid on

```

The plot window shows a 2D coordinate system with x and y axes. A red line represents the trajectory of a pendulum, starting from a green dot at the origin (0,0) and ending at a blue dot. A blue arc indicates the path of the pendulum bob. The x-axis is labeled 'x [m]' and the y-axis is labeled 'y, [units]'. The plot title is 'A100000'.

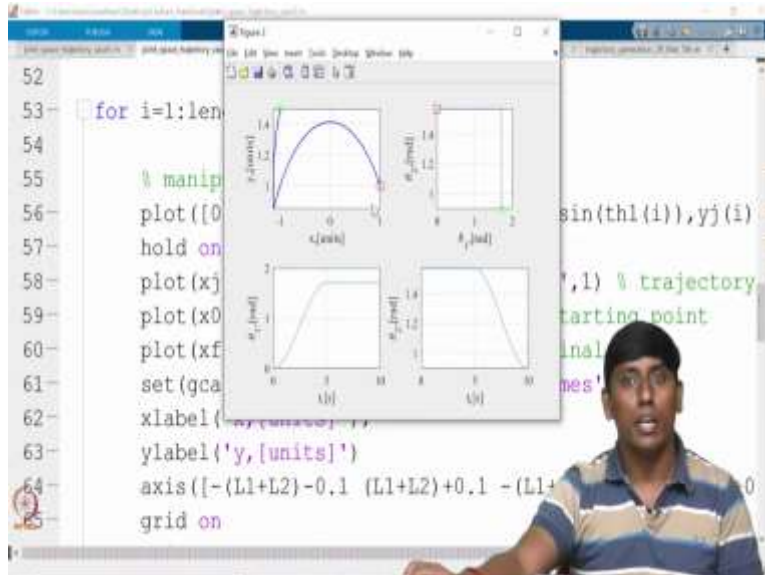
The bottom image shows a MATLAB code editor with the following code:

```

52
53 for i=1:len
54
55 % manip
56 plot([0
57 hold on
58 plot(xj
59 plot(x0
60 plot(xf
61 set(gca
62 xlabel('x [m]')
63 ylabel('y, [units]')
64 axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+
65 grid on

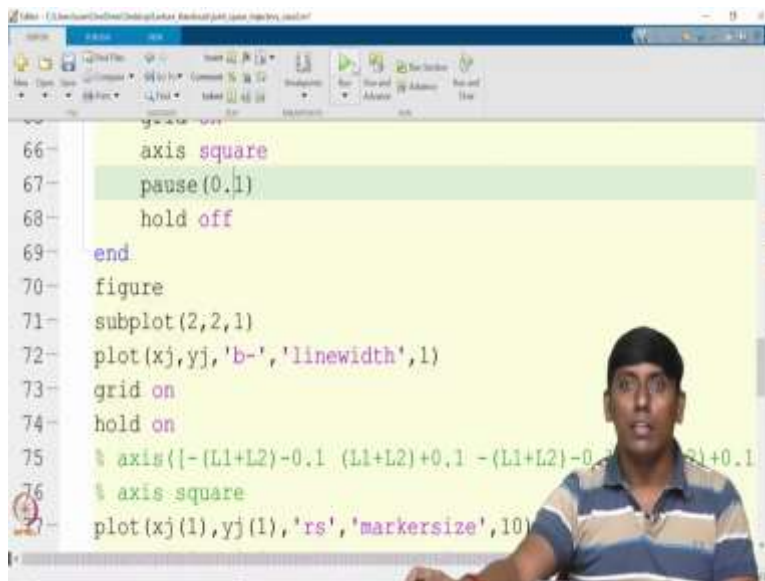
```

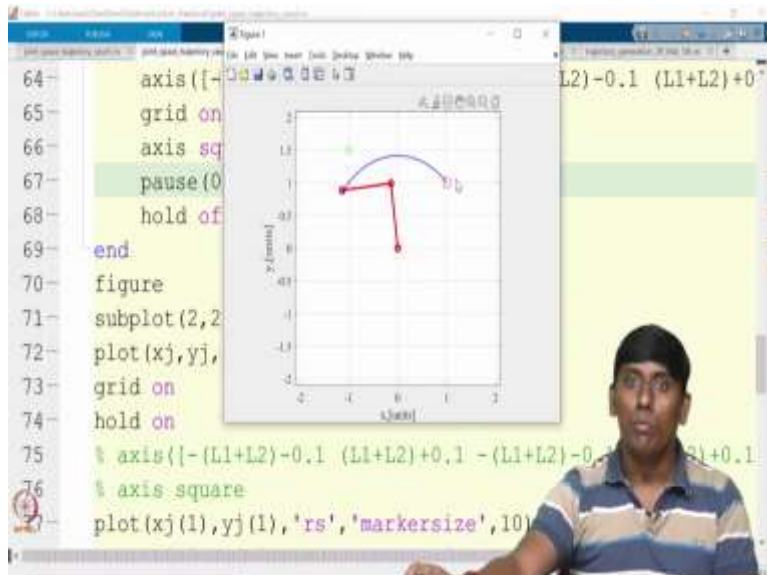
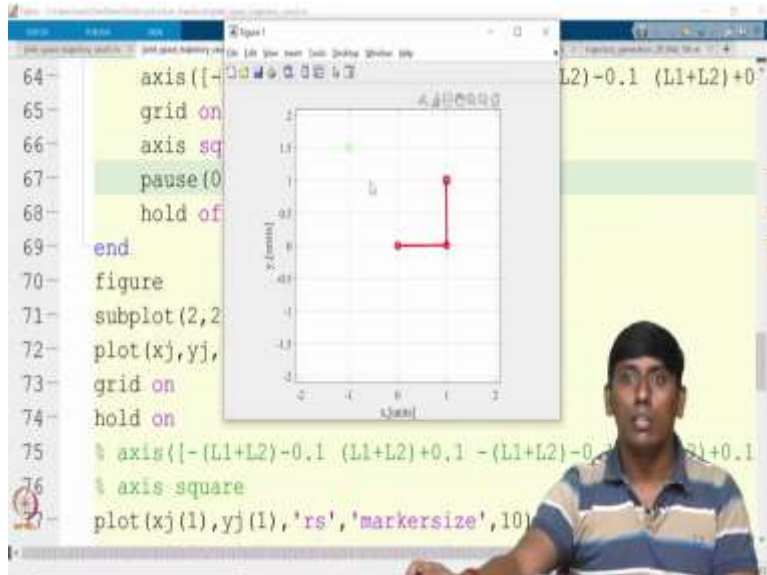
The plot window shows a 2D coordinate system with x and y axes. A red line represents the trajectory of a pendulum, starting from a green dot at the origin (0,0) and ending at a blue dot. A blue arc indicates the path of the pendulum bob. The x-axis is labeled 'x [m]' and the y-axis is labeled 'y, [units]'. The plot title is 'A100000'.

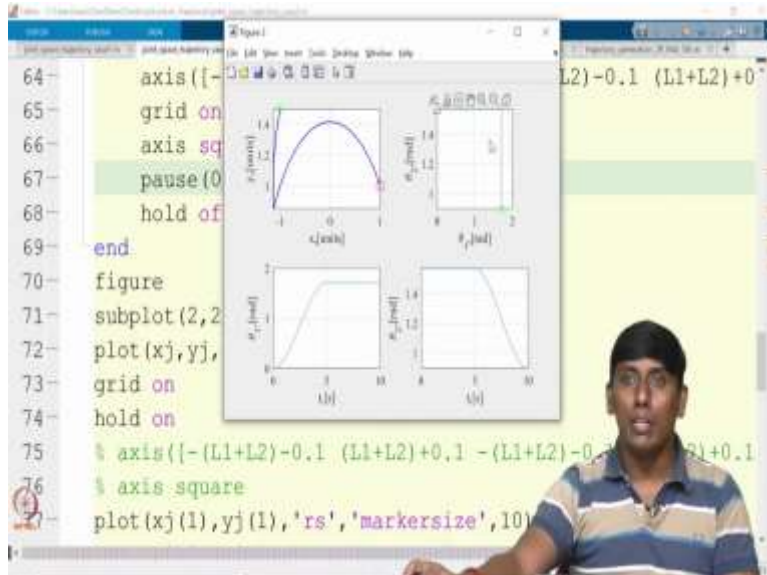


So, based on that you will get the joint space trajectory in two ways. So, if I run you will be very clear to this. So, I will just run this then you will feel it. You can see like the first joint is rotated the second joint is rotating. So, it is little faster.

(Refer Slide Time: 12:33)

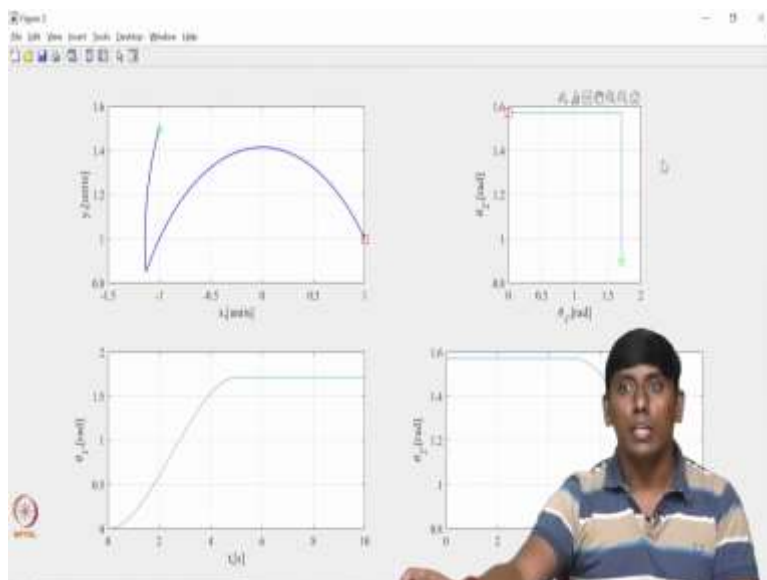






So, I will just make it the pause command probably 100 milliseconds rather than the you can say 10 milliseconds. So, now you can see the first joint only rotate the second joint is as whatever this theta 2 0 is maintained. The first joint rotated once it is reached that final then it is the second joint rotate. So, while second joint rotate you can see the first joint remain same. So, you can look at it.

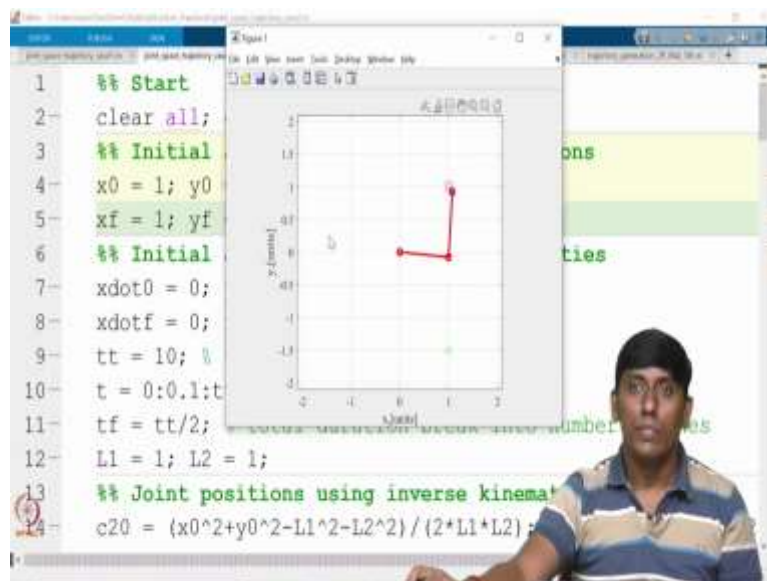
(Refer Slide Time: 13:04)

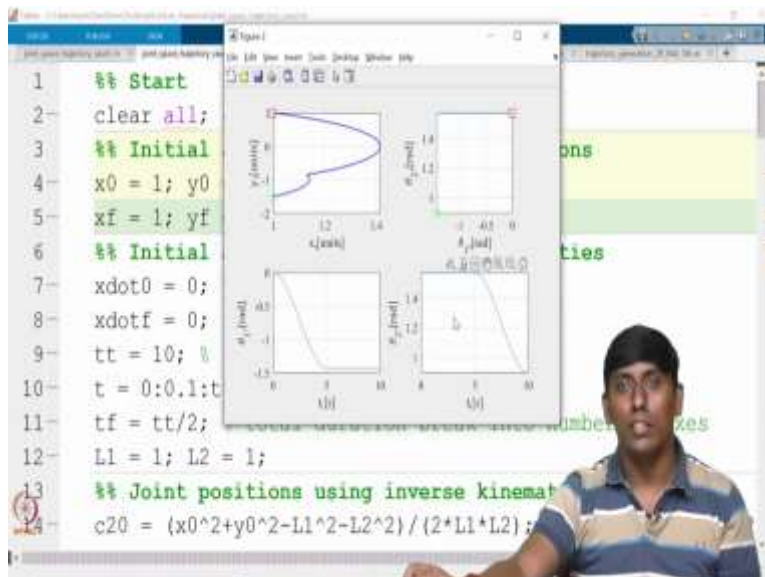
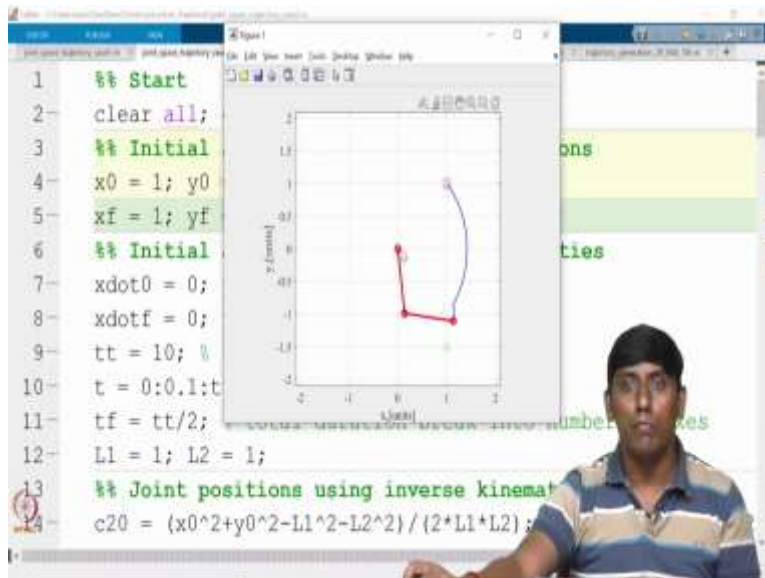


So, from here. So, you can see like the first joint rotated. So, that time the theta 1 vary from theta initial to final. So, the second segment, So, theta final theta 1 final remains same, but the correspondingly theta 2 starts from 1 to final. So, that is what the realization.

(Refer Slide Time: 13:26)

```
1 %% Start
2 clear all; close all; clc;
3 %% Initial and final end-effector positions
4 x0 = 1; y0 = 1;
5 xf = -1; yf = -1.5;
6 %% Initial and final end-effector velocities
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tt = 10; % trajectory duration
10 t = 0:0.1:tt; % time span
11 tf = tt/2; % total duration break into number of axes
12 L1 = 1; L2 = 1;
13 %% Joint positions using inverse kinematics
14 c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2);
```

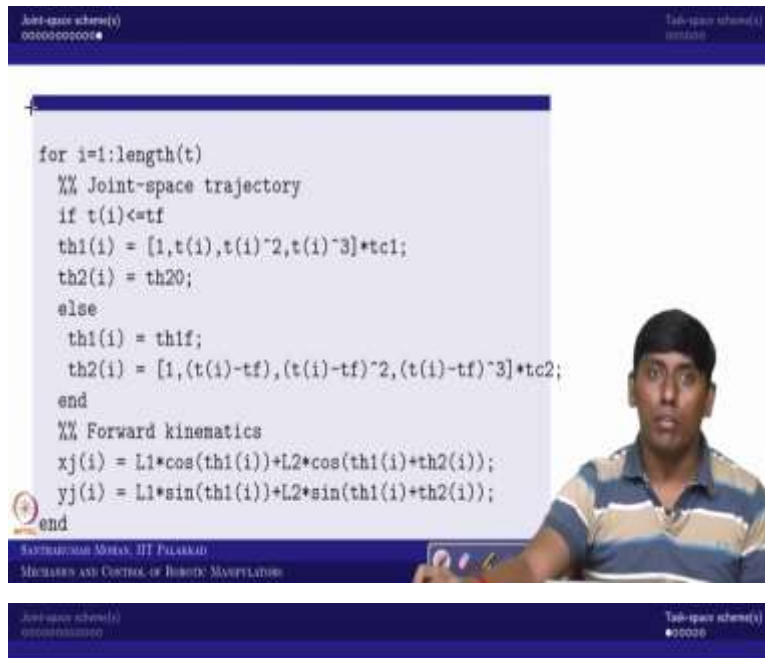




So, now, you can want to more clear, so, I will just give you random theta 1f. So, in the sense, so, I just give it probably a different point. So, I just put it this minus. So, I just show it this is vertical. So, then you can see it sorry. So, you can see it while running it you can see the first joint only rotating the second joint remains same constant. So, once it reached a particular location, you can see the second joint rotate that time the first joint remains same.

So, it is very clearly seen that the explicit trajectory generation. But this is not preferred at all. However, so, most of the you can say kinematics solvers, and all done with this way individual joint would be move in series. So especially if you go to ROS, we call robot operating system. So, I already told in the lecture these things are very clearly visible there.

(Refer Slide Time: 14:35)

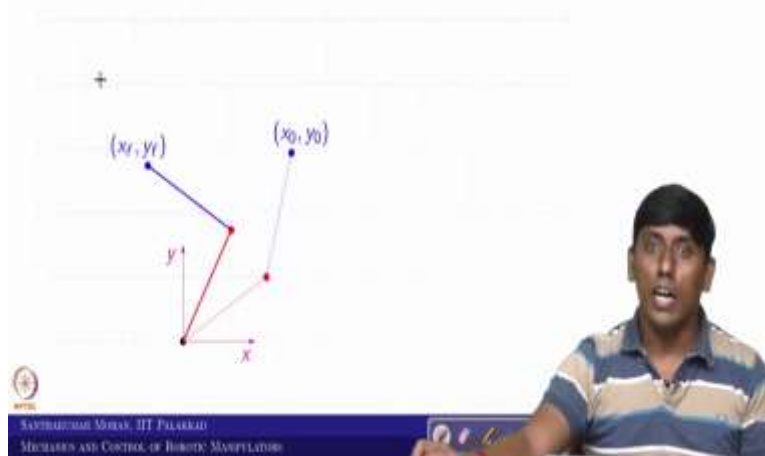


```
Joint-space scheme(t)
00000000000000000000

Task-space scheme(t)
00000000000000000000

for i=1:length(t)
    %% Joint-space trajectory
    if t(i)<=tf
        th1(i) = [1,t(i),t(i)^2,t(i)^3]*tc1;
        th2(i) = th20;
    else
        th1(i) = th1f;
        th2(i) = [1,(t(i)-tf),(t(i)-tf)^2,(t(i)-tf)^3]*tc2;
    end
    %% Forward kinematics
    xj(i) = L1*cos(th1(i))+L2*cos(th1(i)+th2(i));
    yj(i) = L1*sin(th1(i))+L2*sin(th1(i)+th2(i));
end
```

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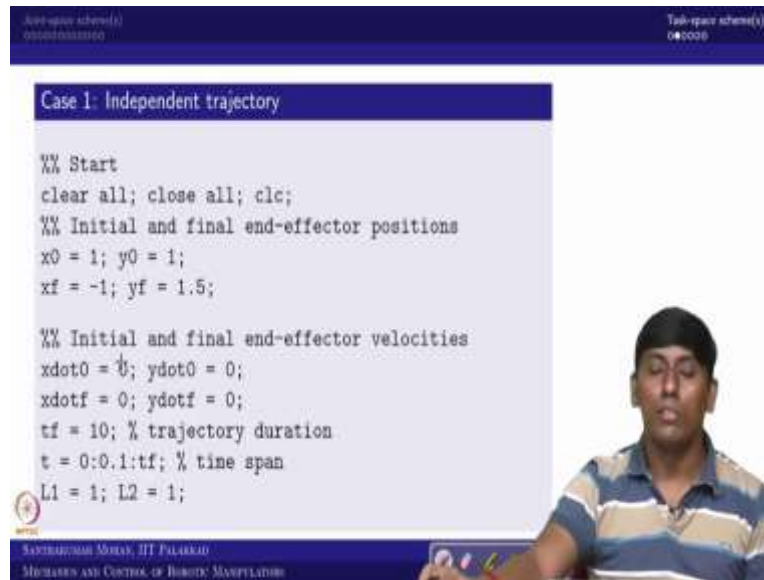


The diagram shows a 2-link planar robot arm in a 2D Cartesian coordinate system with x and y axes. The base of the arm is at the origin (0,0). The first link is shown in red, extending to a joint at position (x₁, y₁). The second link is shown in blue, extending from the first joint to the end effector at position (x₂, y₂). The end effector is marked with a blue dot.

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So, now going back to the slide. So, we will see this whatever we have seen is the joint space trajectory can we see in a task space even the task space we can see in that case. So, x initial and final would be given. Similarly, y initial and final would be given. So, this is the initial position, and this is the final position. We want to follow this in this sense it is supposed to be followed as a straight line. So, the straight line is not assured because the x dot of 0 and x dot f if it is nonzero then it will not maintain.

(Refer Slide Time: 15:05)



Case 1: Independent trajectory

```
%% Start
clear all; close all; clc;
%% Initial and final end-effector positions
x0 = 1; y0 = 1;
xf = -1; yf = 1.5;

%% Initial and final end-effector velocities
xdot0 = 0; ydot0 = 0;
xdotf = 0; ydotf = 0;
tf = 10; % trajectory duration
t = 0:0.1:tf; % time span
L1 = 1; L2 = 1;
```

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So, we will see that how it comes. So, in the sense here you can see so, we are trying to do the first case independent trajectory in the sense x trajectory and y trajectory we are trying to do it in independent cubic polynomial, we are not doing it a synchronous me in the sense once you do the x based on the x, y can be generated no not like that. So, in that sense we are trying to do this. So, here you can see initial and final velocity are given. And we have taken the same time which is 5 you can say second and l 1 and l 2 are same.

(Refer Slide Time: 15:41)

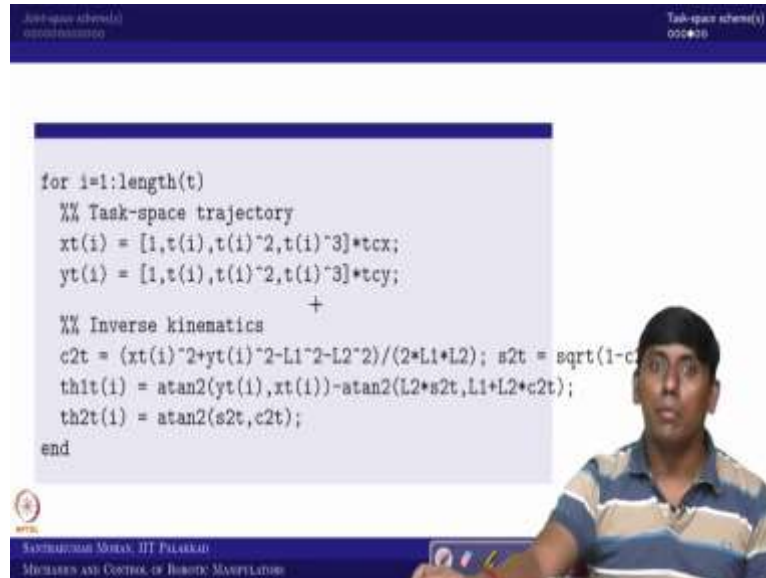


```
A = [1,0,0,0;
      0,1,0,0;
      1,tf,tf^2,tf^3;
      0,1,2*tf,3*tf^2];
+
bx = [x0;xdot0;xf;xdotf];
by = [y0;ydot0;yf;ydotf];
tcx = inv(A)*bx;
tcy = inv(A)*by;
```

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So, now, we have taken the same A matrix, but bx and by are the two other vector based on that the coefficient what we calculate t_{cx} and t_{cy}.

(Refer Slide Time: 15:51)



```
for i=1:length(t)
    %% Task-space trajectory
    xt(i) = [1,t(i),t(i)^2,t(i)^3]*tcx;
    yt(i) = [1,t(i),t(i)^2,t(i)^3]*tcy;

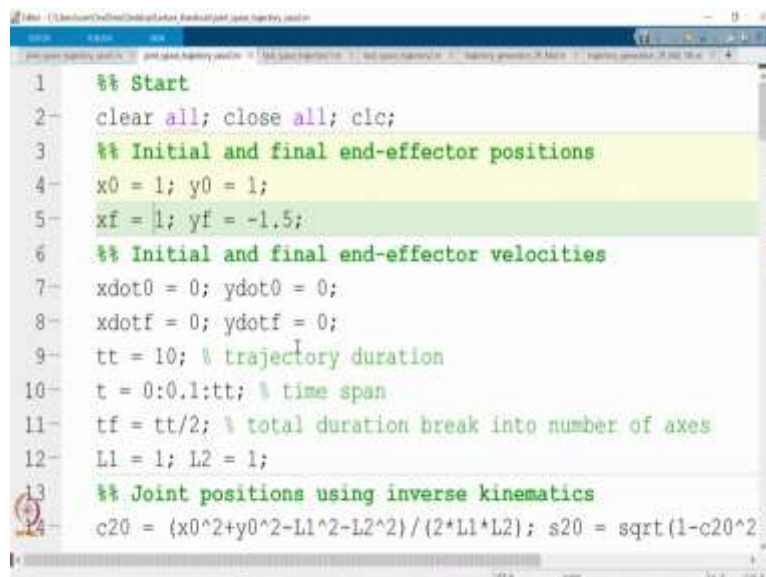
    %% Inverse kinematics
    c2t = (xt(i)^2+yt(i)^2-L1^2-L2^2)/(2*L1*L2); s2t = sqrt(1-c2t);
    th1t(i) = atan2(yt(i),xt(i))-atan2(L2*s2t,L1+L2*c2t);
    th2t(i) = atan2(s2t,c2t);
end
```

Task-space scheme(s)
000*10

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So, now in that case so, xt and yt which is written as t_{cx} t_{cy}. So, in the sense the x trajectory and y trajectory independently derived. So, in order to check in order to compare so, we have done the inverse kinematics just to see how the theta profile will go. So, we have done the inverse kinematics. So, now, we have used the same plotting command.

(Refer Slide Time: 16:18)



```
1 %% Start
2 clear all; close all; clc;
3 %% Initial and final end-effector positions
4 x0 = 1; y0 = 1;
5 xf = 1; yf = -1.5;
6 %% Initial and final end-effector velocities
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tt = 10; % trajectory duration
10 t = 0:0.1:tt; % time span
11 tf = tt/2; % total duration break into number of axes
12 L1 = 1; L2 = 1;
13 %% Joint positions using inverse kinematics
14 c20 = (x0^2+y0^2-L1^2-L2^2)/(2*L1*L2); s20 = sqrt(1-c20^2)
```

```
1 %% Start
2 clear all; close all; clc;
3 %% Initial and final end-effector positions
4 x0 = 1; y0 = 1;
5 xf = -1; yf = 1.5;
6 %% Initial and final end-effector velocities
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tf = 10; % trajectory duration
10 t = 0:0.1:tf; % time span
11 L1 = 1; L2 = 1;
12 A = [1,0,0,0;
13      0,1,0,0;
14      1,tf,tf^2,tf^3;
```

```
7 xdot0 = 0; ydot0 = 0;
8 xdotf = 0; ydotf = 0;
9 tf = 10; % trajectory duration
10 t = 0:0.1:tf; % time span
11 L1 = 1; L2 = 1;
12 A = [1,0,0,0;
13      0,1,0,0;
14      1,tf,tf^2,tf^3;
15      0,1,2*tf,3*tf^2];
16 bx = [x0;xdot0;xf;xdotf];
17 by = [y0;ydot0;yf;ydotf];
18 tcx = inv(A)*bx;
19 tcy = inv(A)*by;
```

```

10- t = 0:0.1:tf; % time span
11- L1 = 1; L2 = 1;
12- A = [1,0,0,0;
13-      0,1,0,0;
14-      1,tf,tf^2,tf^3;
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- by = [y0;ydot0;yf;ydotf];
18- tcx = inv(A)*bx;
19- tcy = inv(A)*by;
20
21- for i=1:length(t)
22-     %% Task-space trajectory
23-     xt(i) = [1,t(i),t(i)^2,t(i)^3]*tcx;

```

```

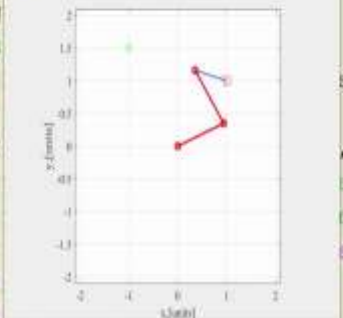
19- tcy = inv(A)*by;
20
21- for i=1:length(t)
22-     %% Task-space trajectory
23-     xt(i) = [1,t(i),t(i)^2,t(i)^3]*tcx;
24-     yt(i) = [1,t(i),t(i)^2,t(i)^3]*tcy;
25-     %% Inverse kinematics
26-     c2t = (xt(i)^2+yt(i)^2-L1^2-L2^2)/(2*L1*L2); s2t = sqrt(
27-         1-c2t^2); th1t(i) = atan2(yt(i),xt(i))-atan2(L2*s2t,L1*c2t);
28-     th2t(i) = atan2(s2t,c2t);
29- end
30
31- figure
32- for i=1:length(t)

```

So, we will go to this the task-based space trajectory one. So, you can see that, so, these are the cases which we have shown in the MATLAB. Where initial and final you can see initial and final initial and final of y and the initial velocity of x and y and final velocity of x and y and total duration is 10 second and you can say this all we have tried. And the A matrix bx by are given and based on the tcx and tcy can be calculated. And you got the task space trajectory. Then the inverse kinematics just for our reference we calculate so that we can do the plotting option.

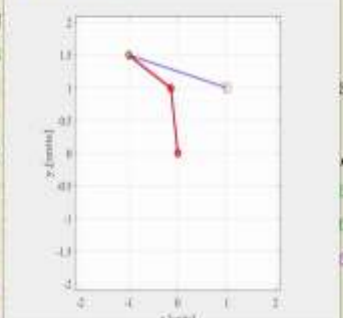
(Refer Slide Time: 16:58)

```
31- figure
32- for i=1:leng
33-     % manipu
34-     plot([0,
35-         hold on
36-         plot(xt(
37-         plot(x0,
38-         plot(xf,
39-         set(gca,
40-         xlabel('
41-         ylabel('
42-         axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1)
43-         grid on
44-         axis square
```



The plot shows a 2D coordinate system with x and y axes ranging from -2 to 2. A red line starts at the origin (0,0) and ends at (1,1). A blue line starts at (0,0) and ends at (1,1). The plot is titled 'Figure' and has a grid.

```
31- figure
32- for i=1:leng
33-     % manipu
34-     plot([0,
35-         hold on
36-     plot(xt(
37-     plot(x0,
38-     plot(xf,
39-     set(gca,
40-     xlabel('
41-     ylabel('
42-     axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1)
43-     grid on
44-     axis square
```



The plot shows a 2D coordinate system with x and y axes ranging from -2 to 2. A red line starts at the origin (0,0) and ends at (1,1). A blue line starts at (0,0) and ends at (1,1). The plot is titled 'Figure' and has a grid.


```

31 figure
32 for i=1:length
33     % manipu
34     plot([0,
35         hold on
36         plot(xt(
37         plot(x0,
38         plot(xf,
39         set(gca,
40         xlabel('
41         ylabel('
42         axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1
43         grid on
44         axis square

```

So, the plotting option we have used the same thing. So, now if I run so, what you have expected earlier it was curved. Now, in this case it is a straight line which is followed but the straight line is not assured if the initial and final velocity are nonzero. So, now you can see it.

(Refer Slide Time: 17:20)

```

37 plot(x0,
38 plot(xf,
39 set(gca,
40 xlabel('
41 ylabel('
42 axis([- (L1+L2)-0.1 (L1+L2)+0.1 - (L1+L2)-0.1 (L1+L2)+0.1
43 grid on
44 axis squ
45 pause(0.
46 hold off
47 end
48
49 figure
50 subplot(2,2,1)

```

```

37- plot(x0,
38- plot(xf,
39- set(gca,
40- xlabel('
41- ylabel('
42- axis([-1
43- grid on
44- axis squ
45- pause(0.
46- hold off
47- end
48-
49- figure
50- subplot(2,2,1)

```

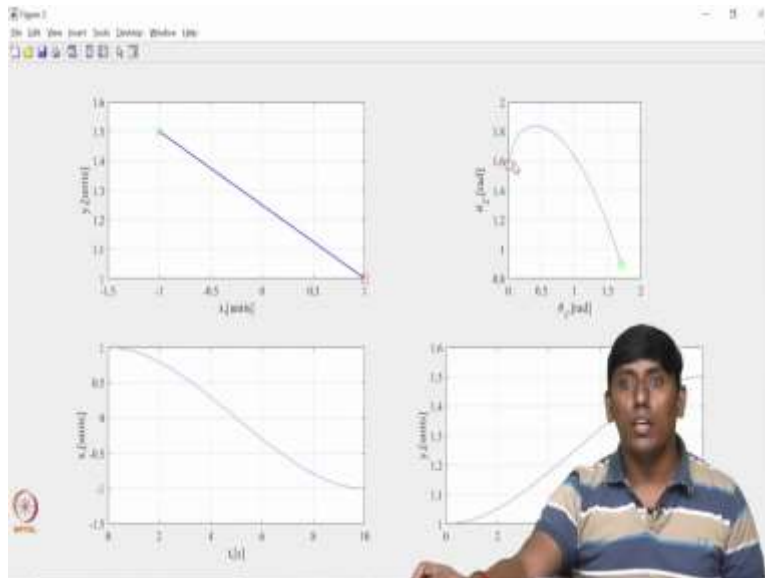
Starting point
Final point
es');
 $(2) - 0.1 (L1 + L2) + 0.$

```

37- plot(x0,
38- plot(xf,
39- set(gca,
40- xlabel('
41- ylabel('
42- axis([-1
43- grid on
44- axis squ
45- pause(0.
46- hold off
47- end
48-
49- figure
50- subplot(2,2,1)

```

Starting point
Final point
es');
 $(2) - 0.1 (L1 + L2) + 0.$



So, I will just make it the time duration is higher. So, now in that case the delay time is actually higher. So, you can see like it starts from this point and it is supposed to go here it is trying to follow a straight line because the initial and final velocity are 0. If the initial and final velocity are nonzero what will happen you can see it by changing that.

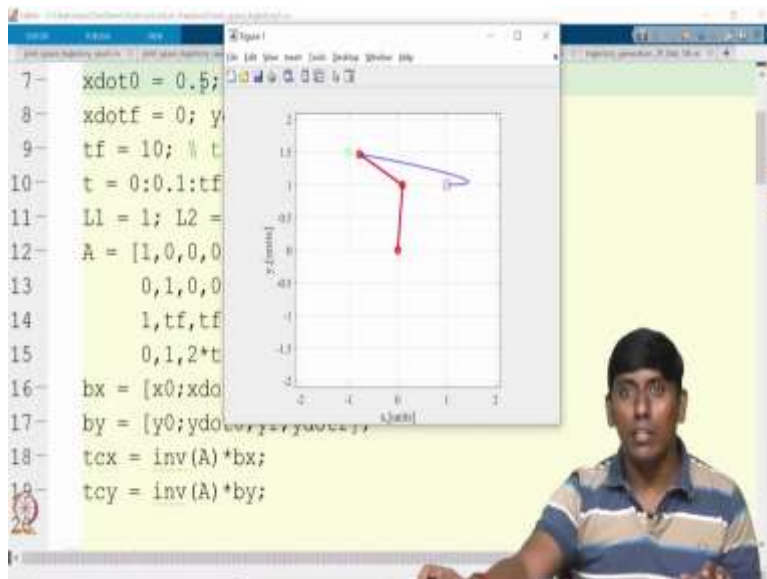
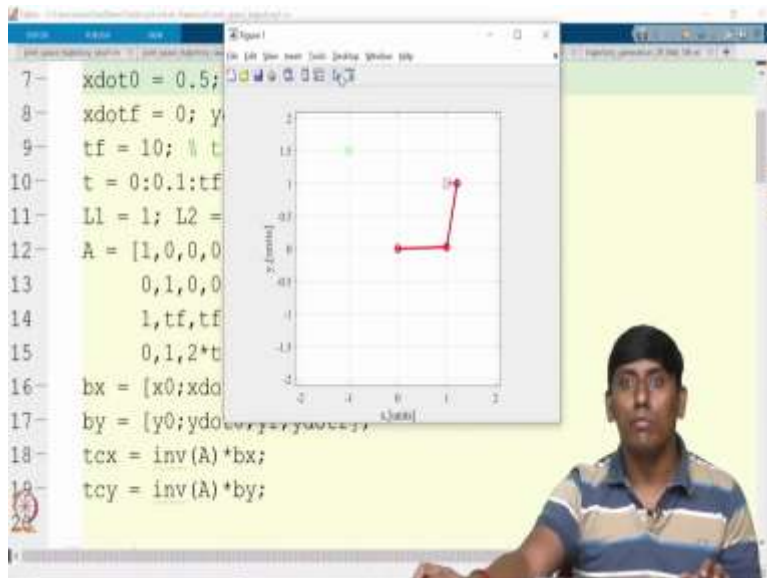
So, now in that case, you can see the joint space you can see like it is not following as a straight-line earlier case it was followed a straight-line earlier case the task space trajectory is followed as curve. But in this case is the straight line. So, the same way x trajectory is smooth y trajectory is again smooth these all we can realize it.

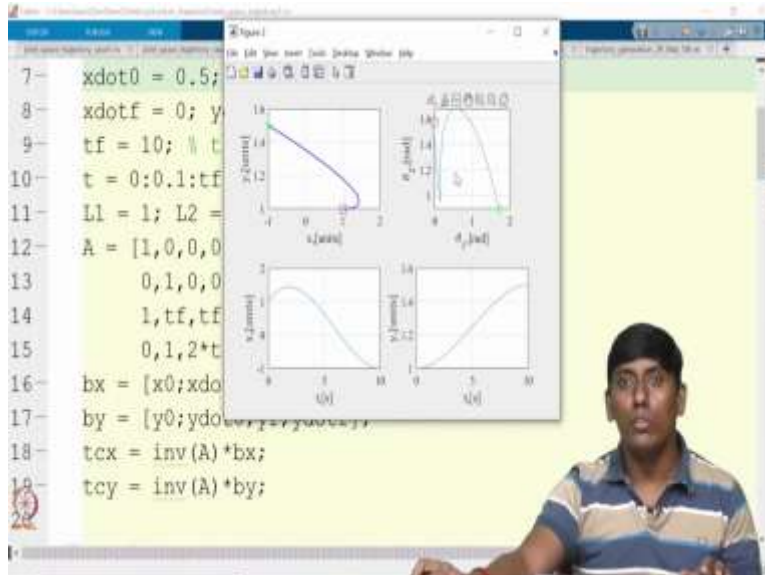
(Refer Slide Time: 18:03)

```

7- xdot0 = [ ydot0 = 0;
8- xdotf = 0; ydotf = 0;
9- tf = 10; % trajectory duration
10- t = 0:0.1:tf; % time span
11- L1 = 1; L2 = 1;
12- A = [1,0,0,0;
13-      0,1,0,0;
14-      1,tf,tf^2,tf^3;
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- by = [y0;ydot0;yf;ydotf];
18- tcx = inv(A)*bx;
19- tcy = inv(A)*by;

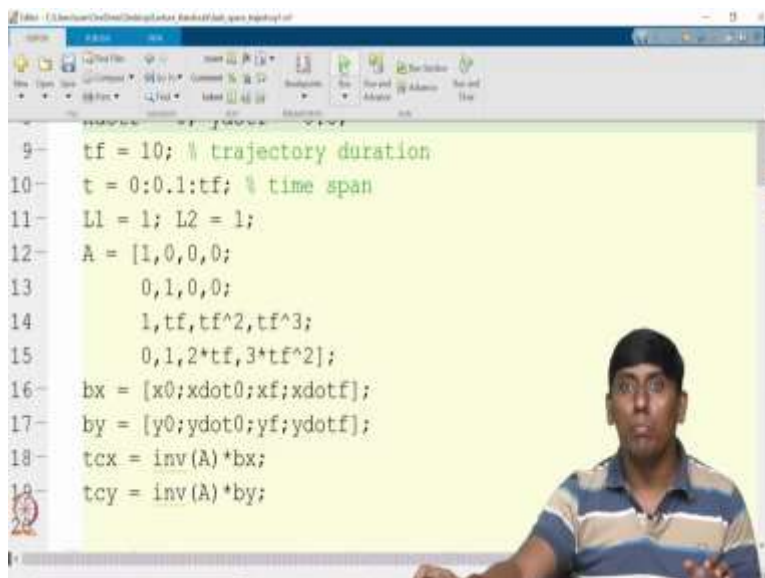
```



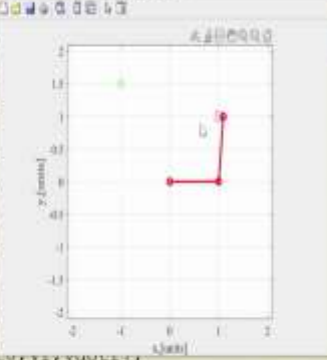


So just to show that how the nonzero initial velocity will involve. So, I am just saying that only. So, the nonzero x is there, which is 0.5 meters per second. So, you can see that it will not be straight line it will be giving a small curve because it is already having some nonzero initial velocity you can see that it is trying to follow. However, it is going to at rest, it will not get any residue there. But here it is starting from nonzero you can see that the profile is inherently modified. So, that is what you can look at it.

(Refer Slide Time: 18:42)

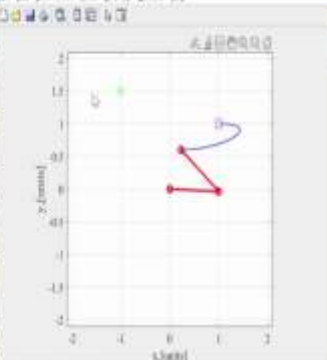


```
7- xdot0 = 0.5;
8- xdotf = 0; y
9- tf = 10; % t
10- t = 0:0.1:tf
11- L1 = 1; L2 =
12- A = [1,0,0,0
13-       0,1,0,0
14-       1,tf,tf
15-       0,1,2*t
16- bx = [x0; xdo
17- by = [y0; ydo
18- tcx = inv(A)*bx;
19- tcy = inv(A)*by;
```

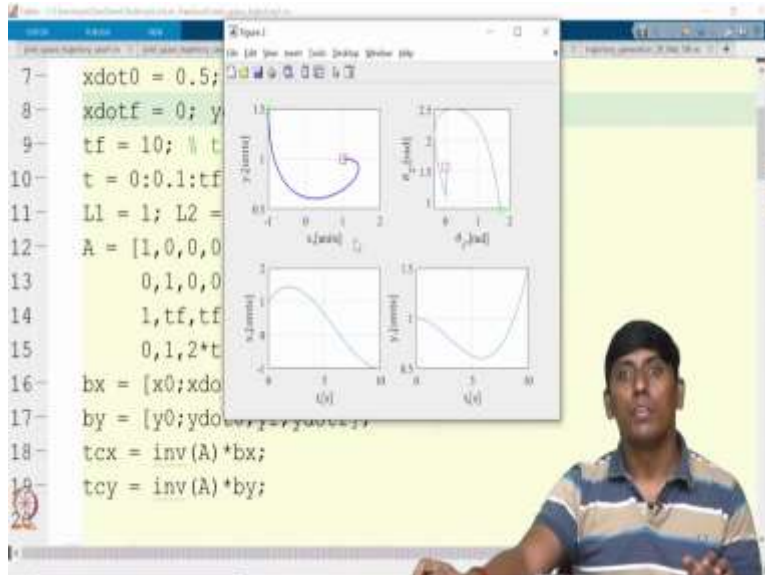


A man in a striped polo shirt is speaking in front of a yellow background.

```
7- xdot0 = 0.5;
8- xdotf = 0; y
9- tf = 10; % t
10- t = 0:0.1:tf
11- L1 = 1; L2 =
12- A = [1,0,0,0
13-       0,1,0,0
14-       1,tf,tf
15-       0,1,2*t
16- bx = [x0; xdo
17- by = [y0; ydo
18- tcx = inv(A)*bx;
19- tcy = inv(A)*by;
```

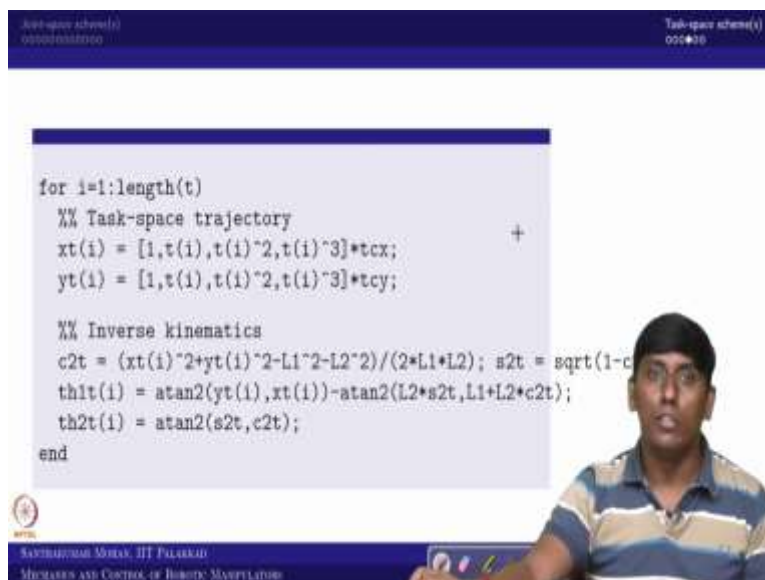


A man in a striped polo shirt is speaking in front of a yellow background.



So even if I give something like the final velocity of this I am just giving randomly. So, the final velocity is 0.5 meters per second. So, you can see like, it is for the x axis it is there. So that is why it is trying to go and now you can see it is modified because the y final velocity is supposed to be somewhere 0.5 meter per second. So, in the sense vertically up. So that is why it is start from horizontal and the end with vertical that is what it is showing. So, now you are I hope you are getting the clarity. Now, we will move to the second case.


(Refer Slide Time: 19:21)



(Refer Slide Time: 20:07)

```
7- xdot0 = 0.5; ydot0 = 0;
8- xdotf = 0; ydotf = 0.5;
9- tf = 10; % trajectory duration
10- t = 0:0.1:tf; % time span
11- l1 = 1; l2 = 1;
12- A = [1,0,0,0;
13-      0,1,0,0;
14-      1,tf,tf^2,tf^3;
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- by = [y0;ydot0;yf;ydotf];
18- tcx = inv(A)*bx;
19- tcy = inv(A)*by;
```

```
1 %% Start
2 clear all; close all; clc;
3 %% Initial and final end-effector positions
4 x0 = 1; y0 = 1;
5 xf = -1; yf = 1.5;
6 %% Initial and final end-effector velocities
7 xdot0 = 0;
8 xdotf = 0;
9 tf = 10; % trajectory duration
10 t = 0:0.1:tf; % time span
11 l1 = 1; l2 = 1;
12 A = [1,0,0,0;
13      0,1,0,0;
14      1,tf,tf^2,tf^3;
```



```
16- bx = [x0;xdot0;xf;xdotf];
17- tcx = inv(A)*bx;
18- for i=1:length(t)
19-     %% Task-space trajectory
20-     xt(i) = [1,t(i),t(i)^2,t(i)^3]*tcx;
21-     yt(i) = (yf-y0)/(xf-x0)*(xt(i)-xf)+yf;
22-     %% Inverse kinematics
23-     c2t = (xt(i)^2+yt(i)^2-L1^2-L2^2)/(2*L1*L2); s2t = sqrt(
24-     th1t(i) = atan2(yt(i),xt(i))-atan2(L2*s2t,L1*c2t);
25-     th2t(i) = atan2(s2t,c2t);
26- end
27-
28- figure
29- for i=1:length(t)
```



```
16- bx = [x0;xdot0;xf;xdotf];
17- tcx = inv(A)*bx;
18- for i=1:length(t)
19-     %% Task-space trajectory
20-     xt(i) = [1,t(i),t(i)^2,t(i)^3]*tcx;
21-     yt(i) = (yf-y0)/(xf-x0)*(xt(i)-xf)+yf;
22-     %% Inverse kinematics
23-     c2t = (xt(i)^2+yt(i)^2-L1^2-L2^2)/(2*L1*L2); s2t = sqrt(
24-     th1t(i) = atan2(yt(i),xt(i))-atan2(L2*s2t,L1*c2t);
25-     th2t(i) = atan2(s2t,c2t);
26- end
27-
28- figure
29- for i=1:length(t)
```



```

16- bx = [x0; xdo
17- tcx = inv(A)
18- for i=1:leng
19-     %% Task-sp
20-     xt(i) = [l
21-     yt(i) = (y
22-     %% Inverse
23-     c2t = (xt(
24-     th1t(i) =
25-     th2t(i) =
26- end
27-
28- figure
29- for i=1:length(t)

```

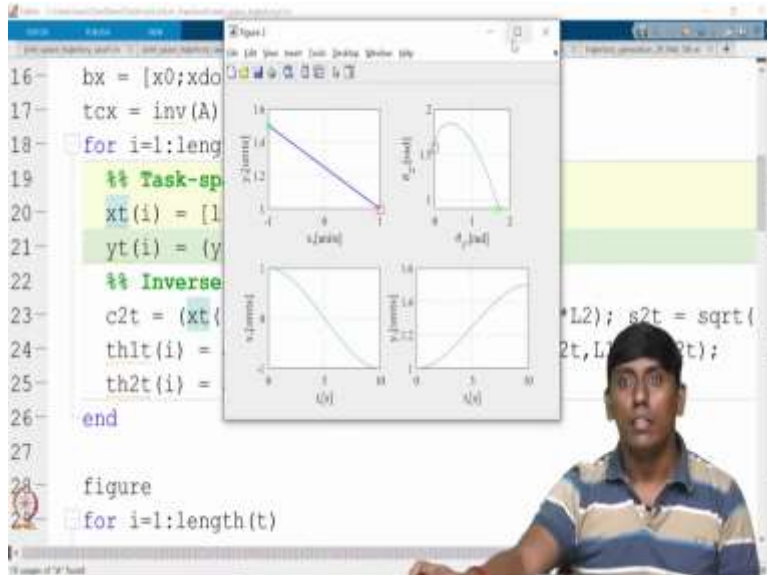
Figure 1: A MATLAB plot showing a red line segment in the x-y plane. The x-axis is labeled 'x (mm)' and the y-axis is labeled 'y (mm)'. The red line starts at approximately (0, 0) and ends at (1, 1). The plot is titled 'Figure 1' and has a toolbar with various icons.

```

16- bx = [x0; xdo
17- tcx = inv(A)
18- for i=1:leng
19-     %% Task-sp
20-     xt(i) = [l
21-     yt(i) = (y
22-     %% Inverse
23-     c2t = (xt(
24-     th1t(i) =
25-     th2t(i) =
26- end
27-
28- figure
29- for i=1:length(t)

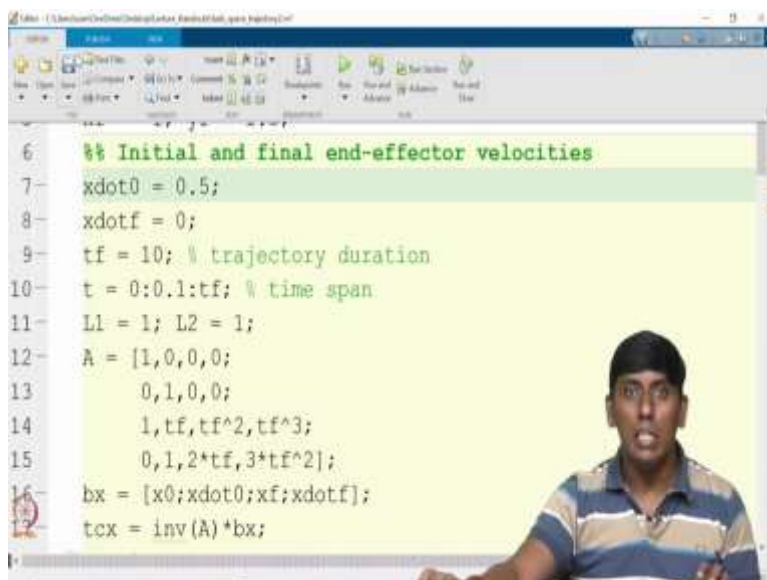
```

Figure 1: A MATLAB plot showing a red line segment in the x-y plane. The x-axis is labeled 'x (mm)' and the y-axis is labeled 'y (mm)'. The red line starts at approximately (0, 0) and ends at (1, 1). The plot is titled 'Figure 1' and has a toolbar with various icons.



So, the task space trajectory 2 you can look at it. So, this is what the given condition. So, the same condition we have taken and the \dot{x} and \dot{y} like \dot{x} initial and final only we are taking for the trajectory generation. So, then we are deriving the y of t or y . So, based on the x So, that is what you can see explicitly. So, then we are moving to the same thing. So, now, you can look at the profile which is going to be very smooth because the y would be given.

(Refer Slide Time: 20:49)



```

4- x0 = 1; y0 =
5- xf = -1; yf =
6- %% Initial a
7- xdot0 = 0.5;
8- xdotf = 0;
9- tf = 10; % t
10- t = 0:0.1:tf
11- L1 = 1; L2 =
12- A = [1,0,0,0
13-      0,1,0,0
14-      1,tf,tf
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- tcx = inv(A)*bx;

```

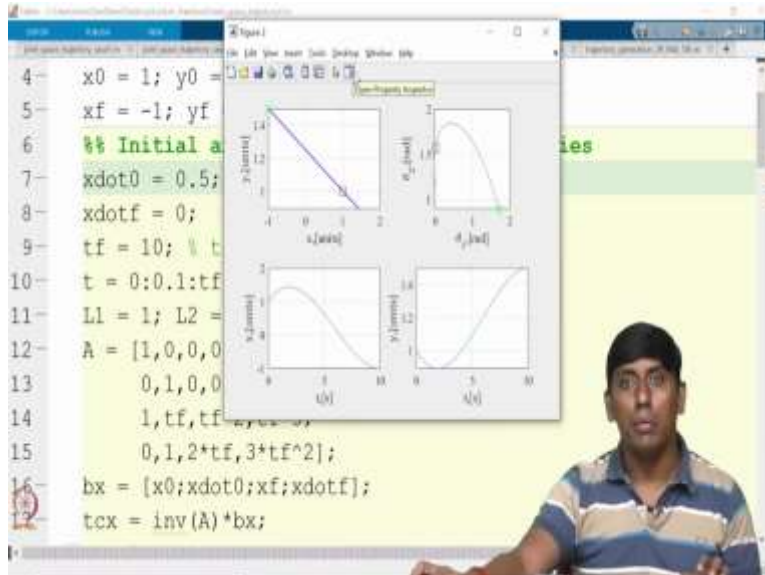
The plot shows a red trajectory in the x-y plane. The x-axis ranges from -2 to 2, and the y-axis ranges from -2 to 2. The trajectory starts at (1, 0) and ends at (-1, 1). It consists of a horizontal segment from x=1 to x=-1 at y=0, and a vertical segment from y=0 to y=1 at x=-1.

```

4- x0 = 1; y0 =
5- xf = -1; yf =
6- %% Initial a
7- xdot0 = 0.5;
8- xdotf = 0;
9- tf = 10; % t
10- t = 0:0.1:tf
11- L1 = 1; L2 =
12- A = [1,0,0,0
13-      0,1,0,0
14-      1,tf,tf
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- tcx = inv(A)*bx;

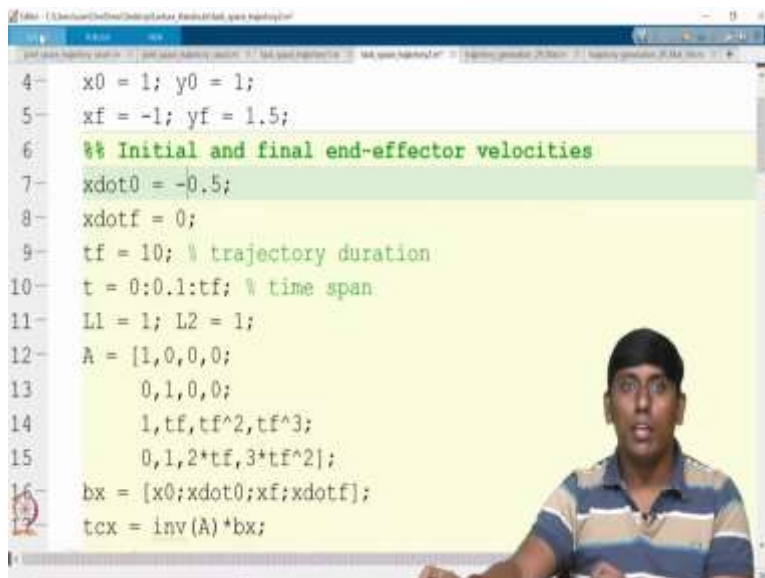
```

The plot shows a red trajectory in the x-y plane. The x-axis ranges from -2 to 2, and the y-axis ranges from -2 to 2. The trajectory starts at (1, 0) and ends at (-1, 1). It consists of a horizontal segment from x=1 to x=-1 at y=0, and a vertical segment from y=0 to y=1 at x=-1. A blue line is also shown, starting at (1, 0) and ending at (-1, 1).



So, now, you can feel it. So, now, even if I give some nonzero initial velocities. So, for example, I am just taking it this is 0.5. So, now, the profile would be you can say goes like that however, you can see the y is not coming. So, that is what you can feel it. So, it is going across the line and goes because you do not have any you can say case. Because we assume that this is followed on the same line.

(Refer Slide Time: 21:19)



```

4- x0 = 1; y0 =
5- xf = -1; yf =
6- %% Initial a
7- xdot0 = -0.5
8- xdotf = 0;
9- tf = 10; % t
10- t = 0:0.1:tf
11- L1 = 1; L2 =
12- A = [1,0,0,0
13-      0,1,0,0
14-      1,tf,tf
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- tcx = inv(A)*bx;

```

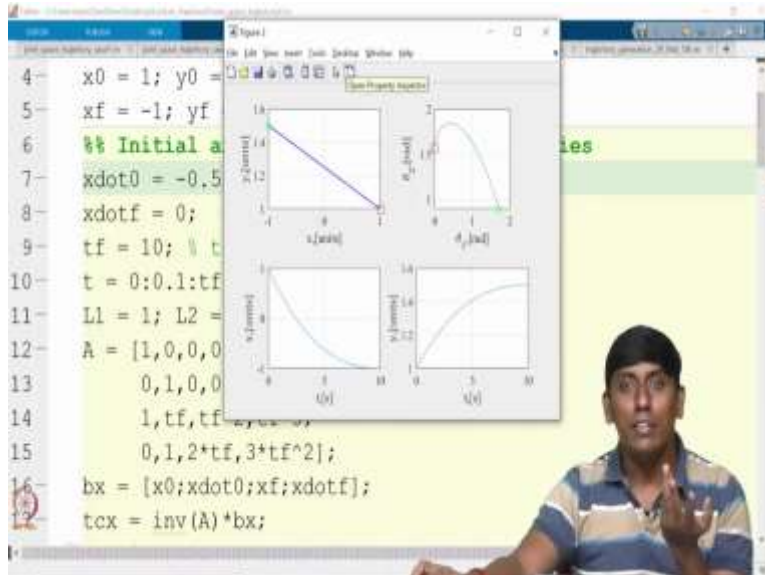
The plot shows a trajectory in the x-y plane. The x-axis ranges from -2 to 2, and the y-axis ranges from -2 to 2. A red line starts at (1, 1) and ends at (-1, 1). A blue line starts at (1, 1) and ends at (0, 0). A green line starts at (0, 0) and ends at (0, -1).

```

4- x0 = 1; y0 =
5- xf = -1; yf =
6- %% Initial a
7- xdot0 = -0.5
8- xdotf = 0;
9- tf = 10; % t
10- t = 0:0.1:tf
11- L1 = 1; L2 =
12- A = [1,0,0,0
13-      0,1,0,0
14-      1,tf,tf
15-      0,1,2*tf,3*tf^2];
16- bx = [x0;xdot0;xf;xdotf];
17- tcx = inv(A)*bx;

```

The plot shows a trajectory in the x-y plane. The x-axis ranges from -2 to 2, and the y-axis ranges from -2 to 2. A red line starts at (1, 1) and ends at (-1, 1). A blue line starts at (1, 1) and ends at (0, 0). A green line starts at (0, 0) and ends at (0, -1).



For example, if I take minus 5. So, you can feel it that would be come on the same line and go. So, there is no change in your velocity, only thing this profile is interchanged that is what you would have felt. So, this is what the; you call straight line interpolation even the straight-line interpolation you want to do it even little more, then you are to bring the; you can say \dot{x} and \dot{y} dot term these terms I do not want to bring it not to make a much much you can say complex.

This is sufficient for you can say run the code. So, even you want to more smoother. So, what we can do instead of, you can say cubic polynomial for the x even we can use fifth order polynomial or cycloidal. In that case the y consequently will come smooth enough. So, that way we can do it. So, I hope you are clear to this. So, with that, we are ending this particular lecture.

The next lecture I will be showing how to generate a workspace with the MATLAB in that we will see like how the task space trajectory will be having some you can see difficulty and all. So, in the sense we will take a simple MATLAB code, single MATLAB code, including workspace and you can say the task space trajectory and you can say joint space trajectory generation and we can see the comparison in a simple single short video. So, with that, I am saying thank you here and see you in next lecture until then, bye, take care.