Mechanics and Control of Robotic Manipulators
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Lecture No. 30
Trajectory Generation using smooth functions

Hi, welcome back to mechanics and control of robotic manipulator. The last lecture we have seen what is trajectory and how to generate a trajectory using cubic polynomial. In the end of the lecture itself I gave us small limitations of what you call cubic polynomial. So, which has no control over what you call the initial and final acceleration or straightaway you can say no control over the acceleration. So, in that sense of what we are trying to see, so, how to overcome that with what you call higher order polynomial.

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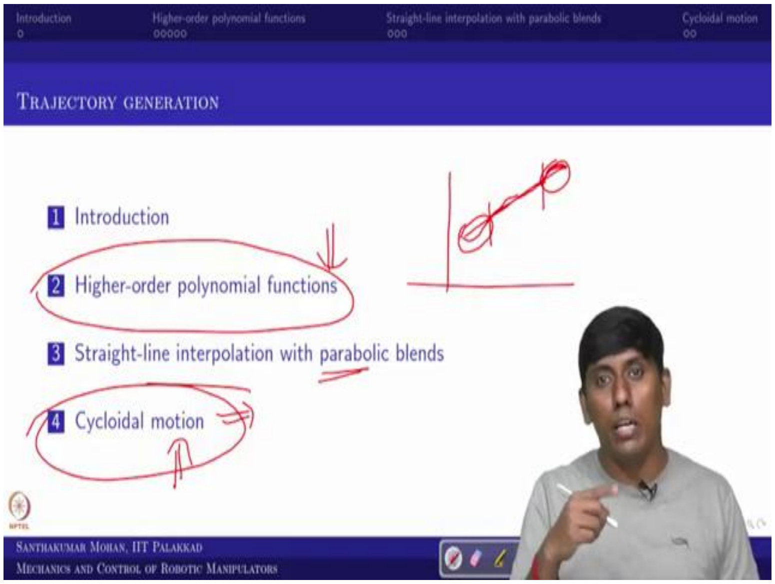

So, in that sense we are going to talk about higher order polynomial. So, in that sense we are going to restrict to only one polynomial which is fifth order, and I will be giving why it is so, then even the straight-line interpolation can be modified with the some kind of parabolic blend in the sense.

So, you have a straight line. So, this straight line I break into certain part, and I actually like make it a smooth and then curved kind of thing. So, it is like that, so, we can actually like make it so, these two segments would be actually like a parabolic. So, that is what actually like we are trying to do it as a parabolic blend and finally, we will see one interesting phenomenon called cycloidal motion.

So, how the cycloidal motion can be used as you can say trajectory generation, which is very common in most of the; what you call legged robot. So, in fact, the legged robot the cycloidal motion both x and y we will be using, but in this particular robotic manipulator, we would be using that so, how the x displacement happening so, that we would be trying to use as the trajectory.

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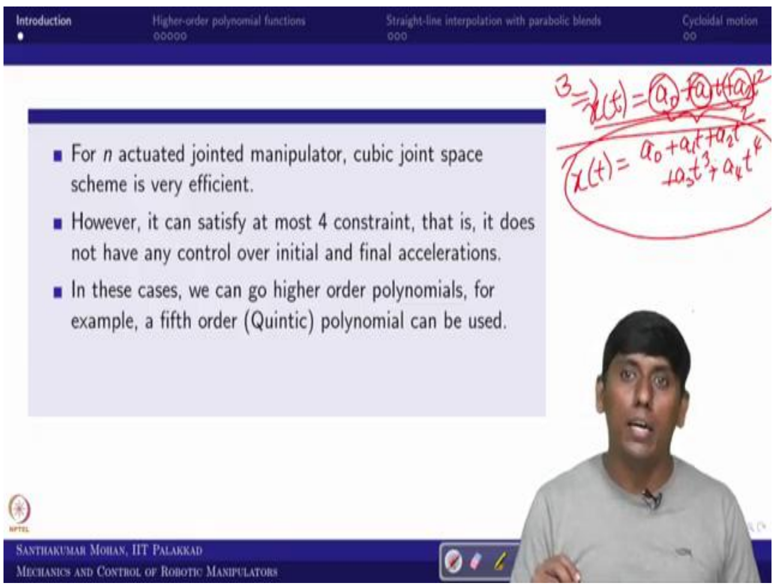

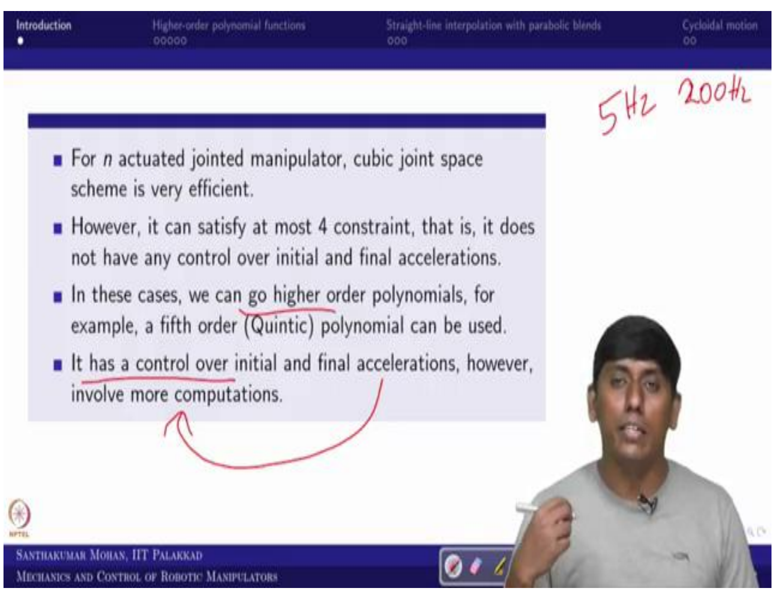

So, in that sense, we will be trying to come back what is we have done in a cubic polynomial, if it is n actuated joint manipulator, obviously, cubic joint space scheme is very-very efficient, but what happened it is satisfied only 4 constraint we already seen. So, initial and final positions and similarly, initial and final velocity is only going to you can say work you can say make a constraint, but it has no control over anything called initial and final acceleration which is one of the limitations.

So, based on that what people think about. People think about going to a higher order polynomial. So, for example, people use the Quintic polynomial which is nothing but a fifth order polynomial. So, how we call the third order polynomial cubic. So, similarly, the Quintic is the fifth order polynomial. So, sometimes people may ask why we are not using fourth order or sixth order like that.

So, I already said even the quadratic equation we are not using because you can see that the X of t can be written as a naught plus a1 t plus a2 t squared. So, similarly, if I write the fourth order polynomial, so, a naught plus a1 t plus a2 t squared plus a3 t cube plus a4. So, t power 4. So, now, what you can see it is giving the even number, but equation, but here actually what you can see odd number of unknowns are there.

So, in this case it is 3 unknowns, but the equation which you can make it usually you can say multiples of 2. So, in this sense you can take initial and final position, then one variable will not be coming. So, then we have to bring initial and final velocity, then it comes four equations, then you had to forced to fit into you can say 4 equations for 3 unknowns the other way around if you are actually like using fourth order, so, then again the same issue will come.

So, you have 5 unknowns, but you can make either 6 equation or 4 equations, which actually like make some kind of handicap. So, sometime actually like people try to use this what do you call the quadratic equation with final velocity equal to 0 initial velocity they will not bother usually initial loss also starting from 0, but the fourth order or sixth order is not straight forward.

Further the coefficients are actually prone to make the what you call the value get into unstable, but if you use actually like odd number of you can say order of polynomial that make you can say number of unknowns and number of equation equal in the sense even number. And make the equation is stable that is what the whole idea that is why we are using higher order polynomial also like fifth or seventh like that.

So, third five seven like, so in that sense, what you can see there is a control over initial and final acceleration, but it comes with you can say some additional burden which we call additional computation. But nowadays we no need to bother because the modern computer is actually like having high capability. So, in that sense, these computations are actually like in fraction of second milliseconds it can be calculated.

So, usually the robotic system will work on 5 Hertz to 200 Hertz in the other way around you can see it come around 5 millisecond or probably 2 milliseconds to probably 10 to 50 milliseconds as the fastness. So, in that sense our existing processor can easily do this.

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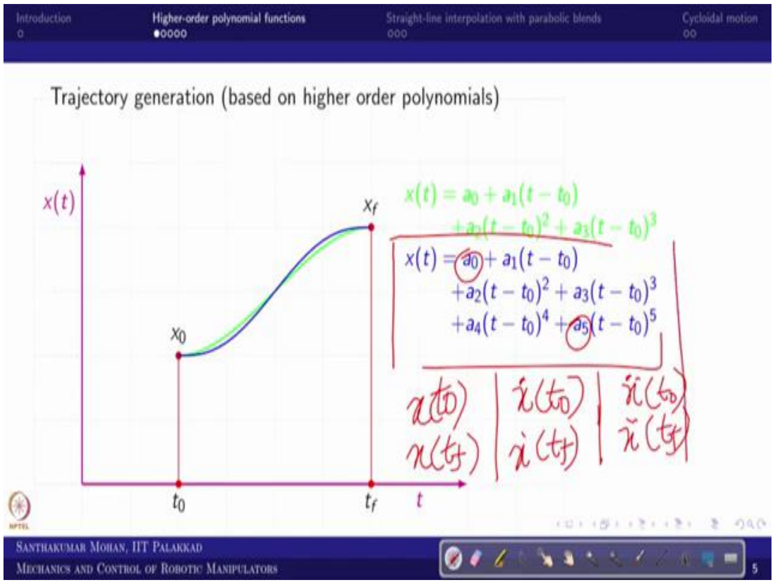

So, let us come what is the higher order polynomial how it is actually beneficial. So, you have seen this is the third order polynomial which we have seen which is smooth but fifth order polynomial if you write so, how it looks like so, in this is what you can see is slightly improved, it is a little more smooth, so, which gives you the acceleration also like smoother. In fact, we can actually like start even the initial and final acceleration is 0, even initial and final velocity also 0 that will give very smooth curve.

The coefficient also like very, very simple, but right now, we will see what would be the equation equal. So, the equation would be having 6 unknowns, so, where it starts from a naught to a5 and you can write 6 equations, so, where x of 0, x of tf. So, I will write this. So, similarly x dot of t0 and x dot of tf and you can write x double dot of t0 and the x double dot of tf. So, in that sense you have 6 equations and 6 unknowns, you can do it.

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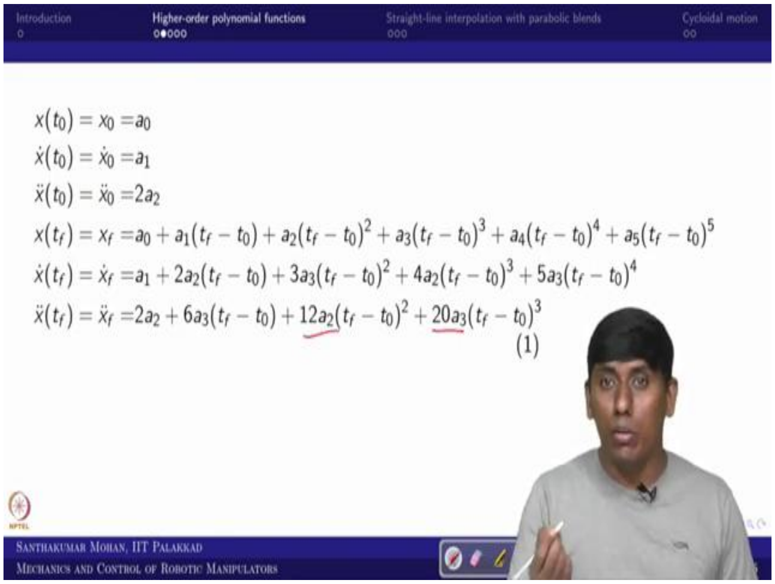

So, whatever we did in the earlier case, we can first write the 6 equations which are actually like possible. So, in that sense, you can actually like see, these are the 6 equations which were equal in that you can see the coefficients. So, and you can actually like take it out.

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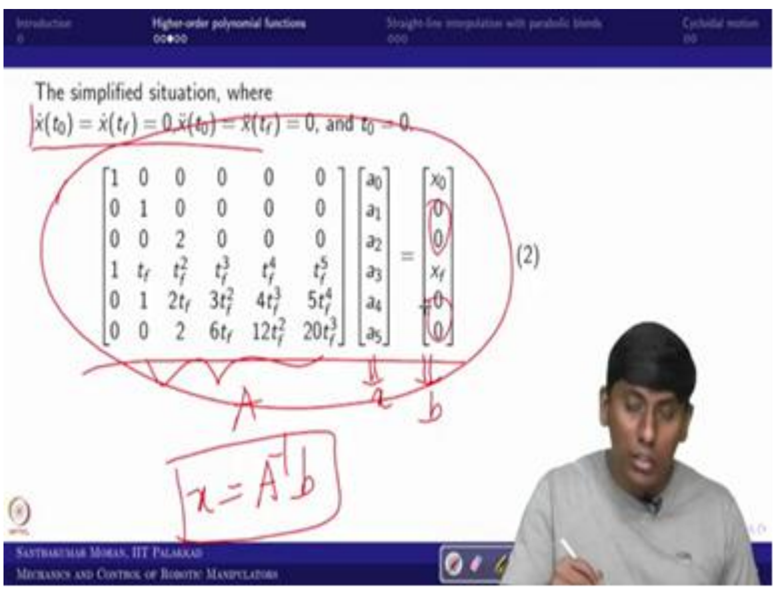

So, for example, this is the way we can write in a state space form, or you can say simple Ax equal to b form. So, where this A is actually like invertible. So, then now, I can call this as capital A and this is small x and this is a small b, So, I can write so, A inverse of b is actual like x vector. So, I can do this. So, then I can actually like do further and all.

So, these computations I can do it in MATLAB. So, I do not want to show that here. But, we will come back in the next to next lecture, I will be showing everything in MATLAB, then you can see how easy it is and you do not need to even write much more complex you can write it as it is and then you can solve it.

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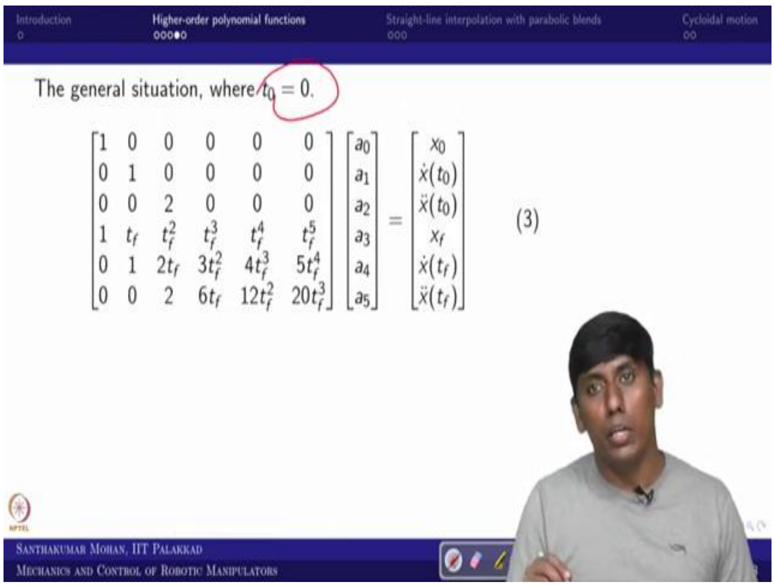

So, let us actually like go what we can see if t0 is actually like 0, then you will actually like see this equation which actually like simplified. Here what we have taken the tf you can see initial velocity final velocity initial acceleration final acceleration all are 0, but this is actually like a generalized especially when you are using in MATLAB you no need to actually like even make this kind of simplification directly you can write this and you can solve it.

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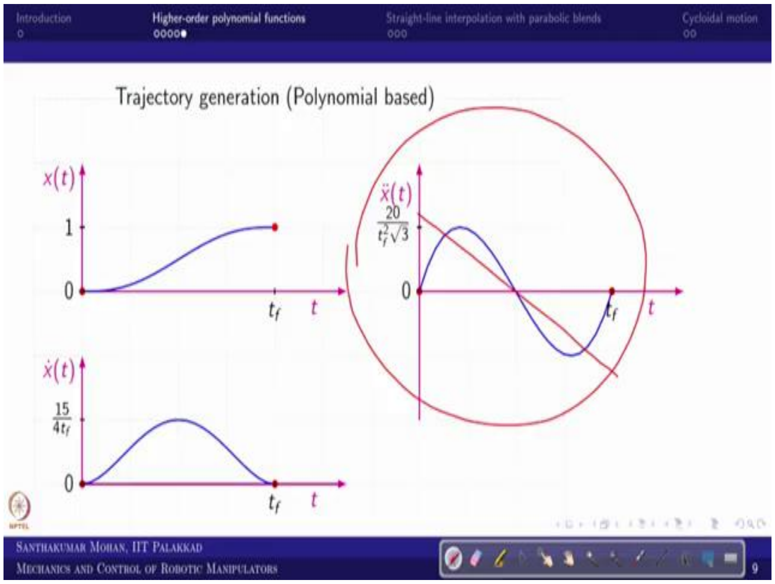

So, let us move how individual velocity and acceleration will go across again we are trying to take for one unit reading the profile is actually like a little smoother than the cubic polynomial definitely that you can all agree because the earlier one is third order polynomial there is a fifth order, but how the velocity look like. So, velocity we have seen that maximum at you can say tf by 2 or you can say t f minus t naught whole divide by 2 in the sense midpoint.

So, there the maximum velocity in cubic polynomial, but here you can see, so, here also the same. So, the maximum velocity at the middle point, so, but what one can expect from the acceleration constraint. So, if I assume that 0 initial and final acceleration, final velocity also 0 the same way I can do even initial and final acceleration is 0 or I can keep whatever I wanted.

Right now, I assume that 0 initial and final acceleration in that sense you can see it is simple you can say sinusoidal curve, it starts from 0 and end also from 0. So, even you start from some finite so then also you will get only sinusoidal acceleration. So, in that way this is actually like much-much smoother. So, there is no you call linear acceleration.

So, in the sense you would have seen in cubic polynomial that was actually like linear acceleration like this, but here it is actually like sinusoidal it is much smoother. So, this is what the advantage of using what you call higher order polynomial. So, now, we will see how the straight-line interpolation can be brought into parabolic blend.

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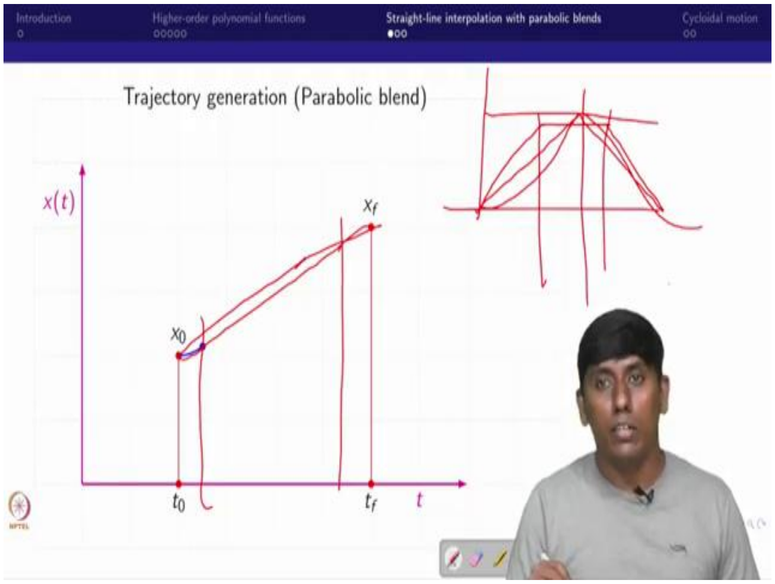

For that, we have actually like taken the straight line, the straight line I am breaking into, you can say 3 segments. The first segment I am actually like assuming it as a parabolic then I do it and then again, a parabolic. So, this way I am trying to do. So far that I need to make a three blend you can say three blends in the sense the simple straight line make it to parabolic along with one straight line in the other way around if you look at the velocity.

So, earlier the velocity was actually like straight line distance constant now, it will actually let go like this. So, even this can be actually like go like this also. So, then this would be you call a uniform acceleration and deceleration case, but in this case, there is a parabolic will come, so, here the parabolic there is no blend. So, it is actually like going to come in the; what you call a acceleration curve that we will see in upcoming slides.

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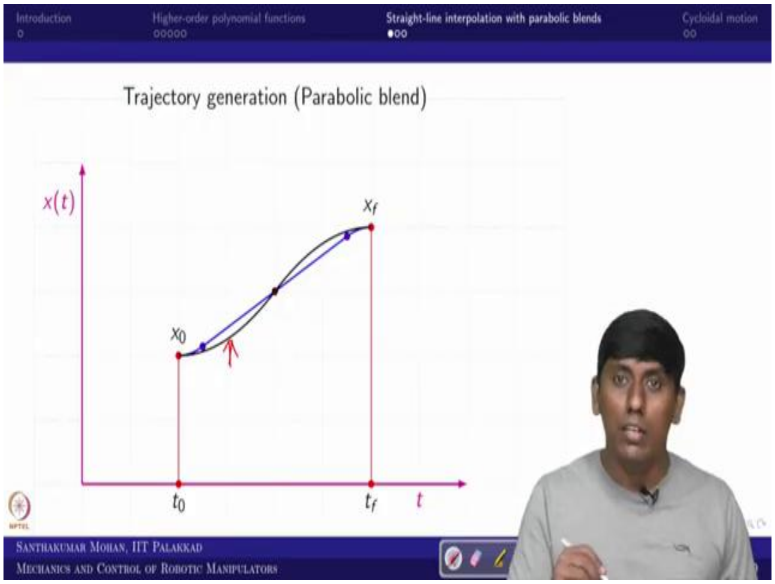

So, now, you can see this is the straight line, so this to this is the straight line after that again you make it up parabolic. Now, what you can see the starting point and the end point would be a smooth starting and ending. In other way around the middle wherever you can see it is a smooth straight line which is the shortest best or you can say the other way around it is the best path which we expected.

So, in that sense, this is actually like what you call parabolic blend, even this blend can be modified without even a straight line you can make a parabolic like this. So, that is what we call uniform acceleration and deceleration. So, where this is actually like very smooth than this, because there is no straight line involved.

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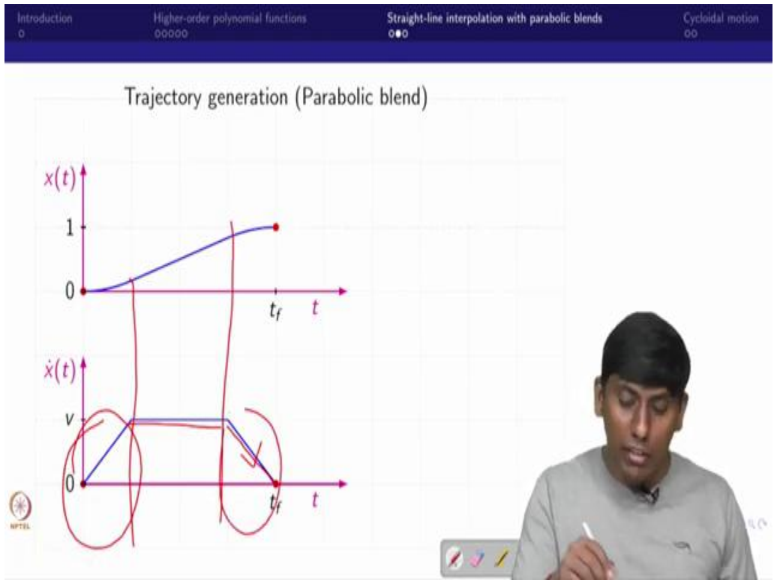

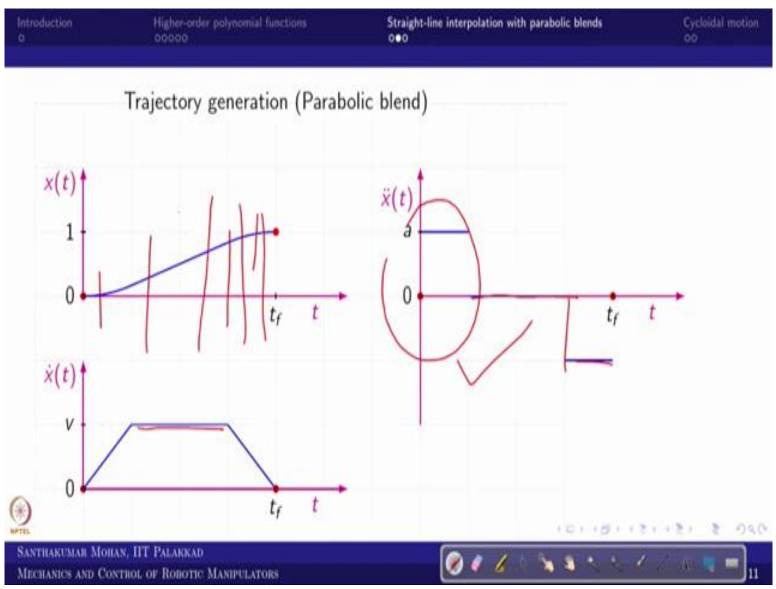

So, we will see how the individual velocity and acceleration look like. So, you can see this is the parabolic blend, so, this point this point is having blend but in the middle, it is a straight line. So, in this case, it is actually like 1 second probably. So, till this it is smooth then straight line and again a smooth curve. So, now how the velocity look like you can see, so, till this point, it is actually like uniformly increasing then constant, then uniformly decelerating or you can say decreasing.

So, in that sense what you can see you will see the acceleration you can say going up and down and then you can actually like see here also up and down. So, in the sense the acceleration reaches a certain point here, then you can see it 0 acceleration during this phase, then the (non-zero) you can say again a finite deceleration. So, this is actually like one simple step, where you can say the parabolic blend can come.

Now, this blend can actually like make it instead of this big even you can make it as small or you can actually like make it big according to your joint restrictions, for example, your motor is having so and so, speed as the restriction you can actually like make the blend accordingly.

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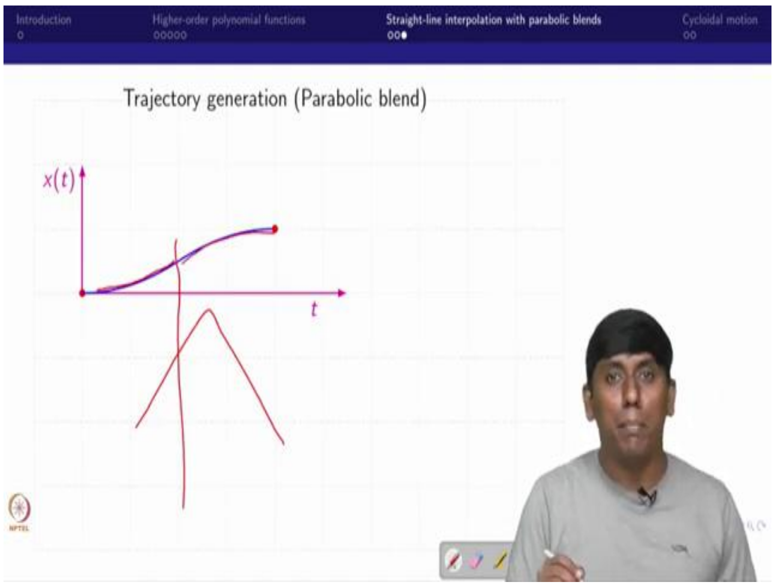

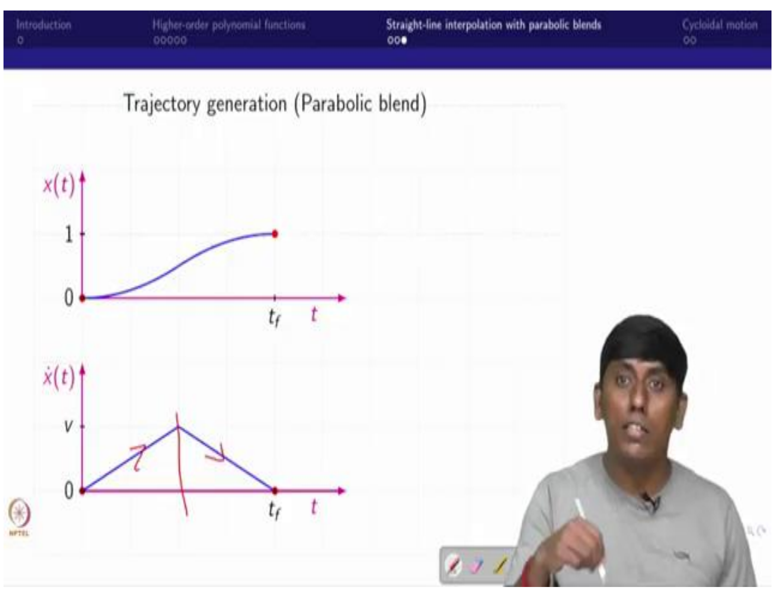

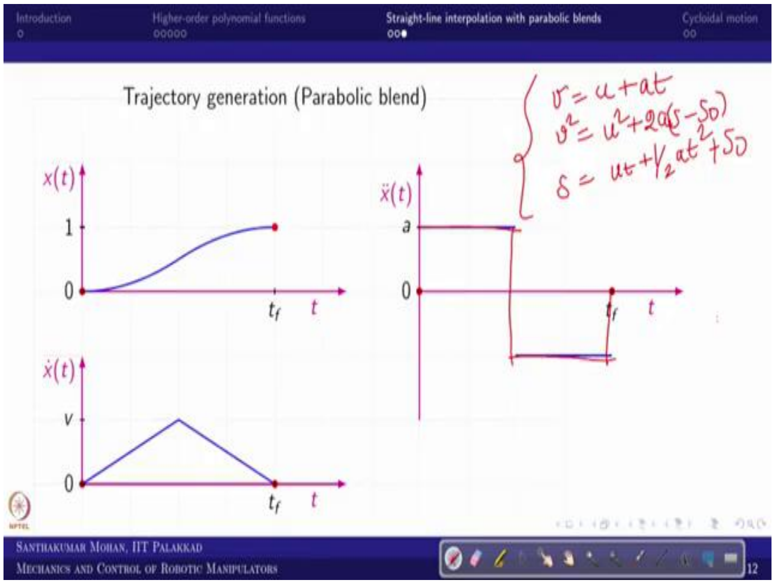

So, the same way we have seen the parabolic blend with uniform acceleration and deceleration in the sense there is no velocity at all. So, the velocity gradually increasing then suddenly it start decreasing the sense the velocity will go up and down like this. So, then you can see up to this the acceleration is happening then after that the deceleration is happening that is what we are trying to see here.

The velocity is actually like increasing, then decreasing in the sense the acceleration curve you can actually like look at it, so, the finite acceleration then finite deceleration is happening. So, now, you can see that this is what the whole idea for bringing straight line interpolation, but with parabolic blend. So, now the blend you can actually like do it, it is actually like blending into the straight line that is why it is called parabolic blend.

So, this is we can use a simple equation of motion what we know already. So, I said that v is u plus 80 then you know like v squared is actually like u squared plus 2as, then s is actually like you can write u t plus half a t squared. So, even you bring the initial then this is actually like so, plus S naught and right. So, like that you can actually like bring it. So, these are the 3 equations of motion that we can use it to get this what you call parabolic blend.

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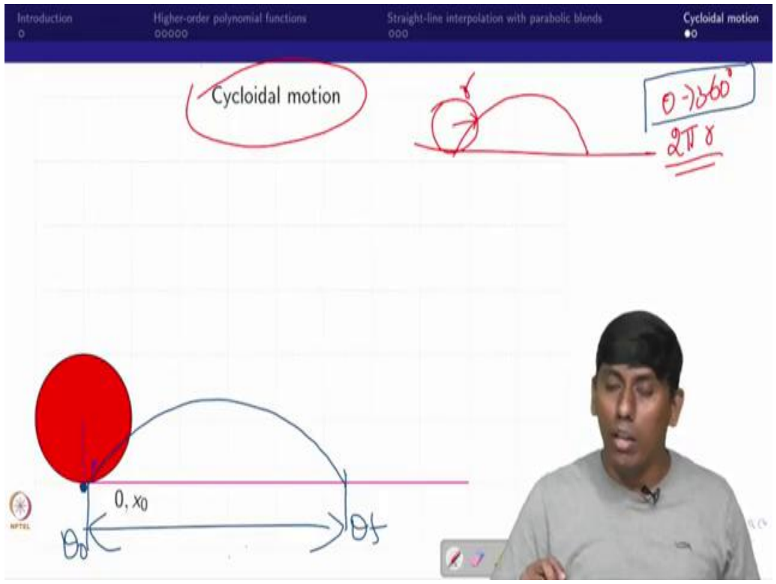

So, now, in that sense, we will go one additional trajectory generation what you call cycloidal motion. So, what is cycloidal motion, you take a flat surface and take one cylinder and roll the cylinder. We assume it is pure rolling in the sense. So, this I also assumed as R as the radius, so, if the cylinder rolled, so 0-to-360-degree rotation, so, this distance covered is actually like 2 pi r.

So, this much actually like covered like this. So, now, if I see this is the point, this point, how it is actually like followed when this is rolling. So, I see that is what going to give what do you call cycloidal motion. So, now the total distance covered I assume that this is you can say theta f and this is the theta naught. So, then I can actually like relate. So, where this 0 to 360 degrees I can relate with respect to time, then I can generate this what you call cycloidal motion. So, that is what we are trying to do.

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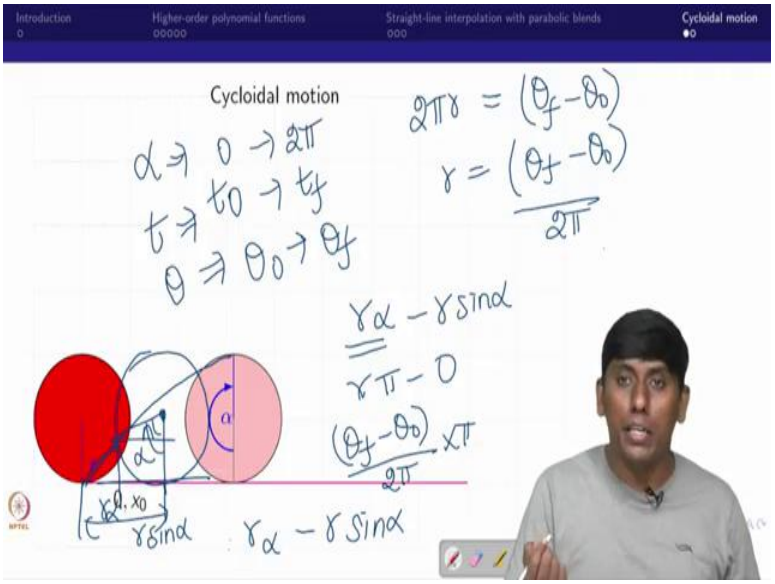

So, you can see this is the same thing is rotated 180 degrees. So, now, you can see that this is I call alpha, this alpha varies, so, 0 to 2 pi the same time the T vary 0 to, t0 to tf. So, what additionally it is covered it goes theta goes theta 02 to theta f. So, this is what we are trying to cover. So, in that sense what would be the total distance? The total distance covered is 2 pi r. So, that would be equal to theta f minus theta naught.

So, in this sense I can find what would be the radius which I can use the sense theta f minus theta naught whole divided by pi. So, then I can actually like find further. So, I want to actually like find out what would be the distance covered here. So, that distance covered is actually like I take any random point that would be covered is actually like r alpha minus r sin alpha for understanding this I am just taking one another curve so where it is actually like a traveled some alpha.

So, now, what would be this, so, this is actual like you can see, so, this is what you can actually like see, so r, so this is alpha right. So, this would be r sin alpha. And what is the distance covered here? This is r alpha, so, r alpha minus r sin alpha would be the distance which is actual like moved from here to here. So, that way we will relate.

So, now, you can see like this much, so, this is what we can see. So, now, r alpha in this case is r pi. So, r sin alpha would be 0 because it is pi right. So, then we can actually like check, so, theta f minus theta naught, so, whole divided by 2 pi into 2 pi. So, you can see that it is actually like a crossed, so, half of the distance that is what happening here also. Like that you can actually like make another.

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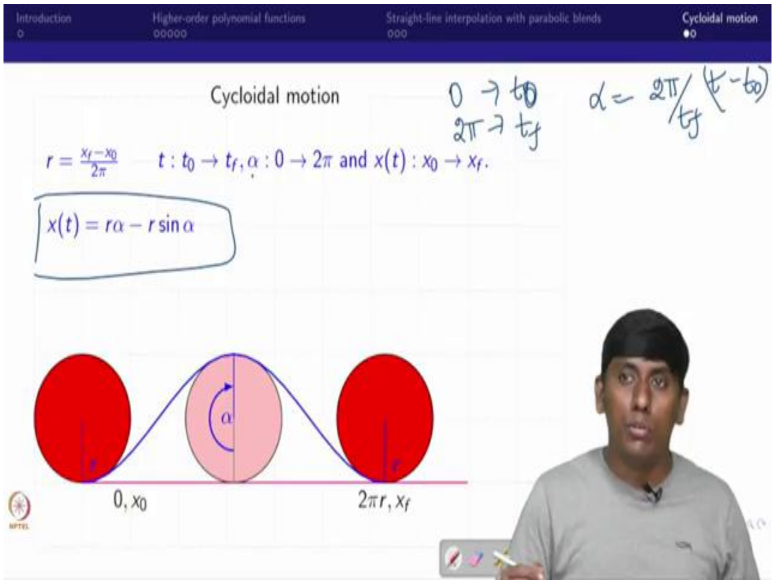

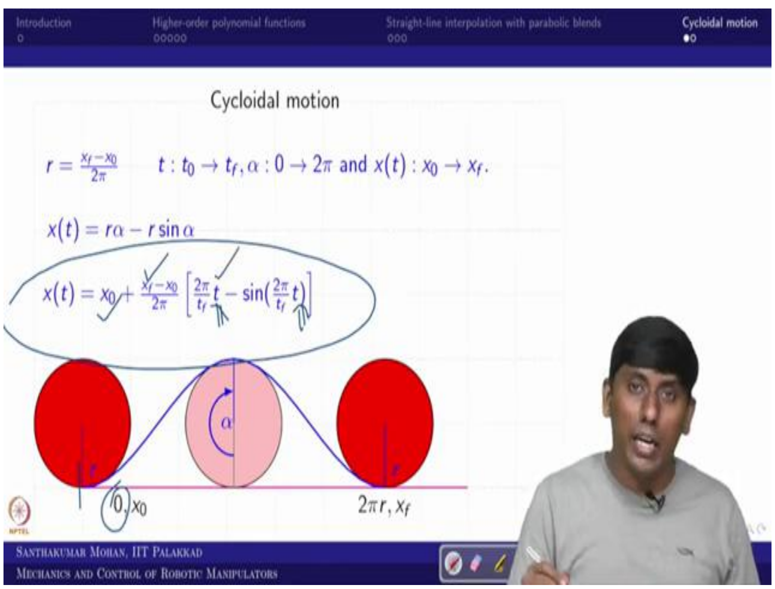

So, you can see that the profile would be smooth and as well as when it starts from zero to you can say 2 pi r distance, it is actually like travel x 0 to x f. So, that is what we have written, and you know, this is what the r equation and the t vary from t0 to t f the same time the alpha varies from zero to 2 pi. So, that is what we have written. So, in that sense x of t vary x naught to xf.

So, if that is the case, so, I already said the x of t I can write as r alpha minus r sin alpha. So, you can actually like find alpha, so, how you can find alpha in terms of t, so, 0 equal to tf, t0 and so,2 pi equal to tf, this is t 0. So, in the sense alpha can be written as 2 pi by tf into t. So, I can write this way. So, even if I write strictly, it is t minus t0, so, if I write this way, so, you can see like when t equal to tf, so, that would be actually like equal to so and so thing.

So, that way we can actually like make it so, that is what we are actually like writing it. So, we always assume that the t0 equal to 0 we start from this that is what we said. So, in that sense, I can write straight away t and you can see this is what the overall equation. So, now, if I substitute x naught is known xf is known and t is actually like variable which is varying 0 to tf and then I can actually like get this equation.

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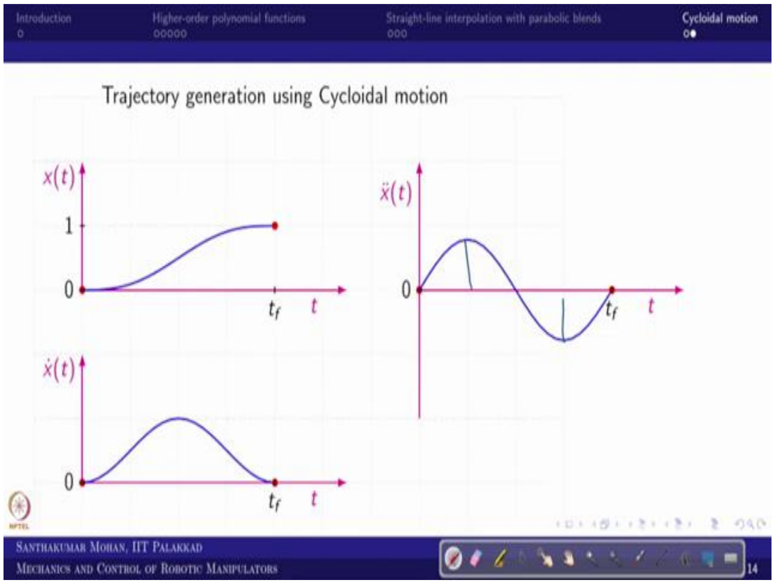

So, now, if I draw that you can see this is very smooth okay. And further what you can see the velocity earlier you have seen like this, but here it would be the same, but it is a cos function. So, our sine function whatever way you can actually like a take it. So, now, here also midpoint is the velocity is maximum and acceleration is actual like you can see maximum at the quadrant, and you get the overall idea.

So, now you can actually like see. This is very similar to what you can say higher order polynomial with 0 initial acceleration and zero final acceleration, the cycloidal we assumed that way then we have actually like derive this equation. So, that is what the whole idea, but this equation is the important one.

So, now, what we have seen we have seen is actually like how we can actually like use higher order polynomial or straight-line interpolation with parabolic blend or what do you call the cycloidal motion can be used as the trajectory. So, now, we have seen a few trajectory generation schemes which start from cubic polynomial to cycloidal motion.

The next lecture we would be seeing how the manipulator will generate, so, what would be the constraints? So, what are the schemes I already said joint space scheme and task space scheme. So, these are all we will see and then we will go to the simulation. So, in that sense the next class we will be talking about joint space and task space schemes. So, we will see in that, until then see you. Thank you. Bye.