Mechanics and Control of Robotic Manipulators
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Lecture No. 29
Introduction to Trajectory Generation

Welcome back to mechanics and control of robotic manipulator. Last few classes we have seen kinematics and dynamics. So, this particular class we are going little ahead, where we are going to touch upon the control. For doing the control we should actually like have some principle called the trajectory planning or you call trajectory generation. So, why the trajectory is coming and what is the difference between trajectory and path and how we can generate the trajectory these all we would be covering in today's lecture and a few other lectures upcoming.

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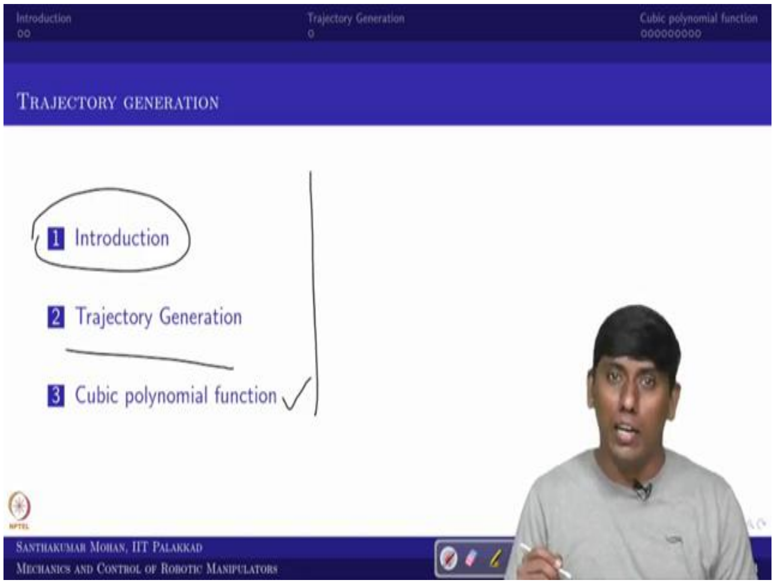

So, in that sense of what we are trying to cover here basically like why we need trajectory generation and like what is trajectory generation, and you will be seeing one trajectory generation principle called cubic polynomial function. So, these are the things which we are bounded to cover in this particular lecture. So, let us actually like move a little ahead.

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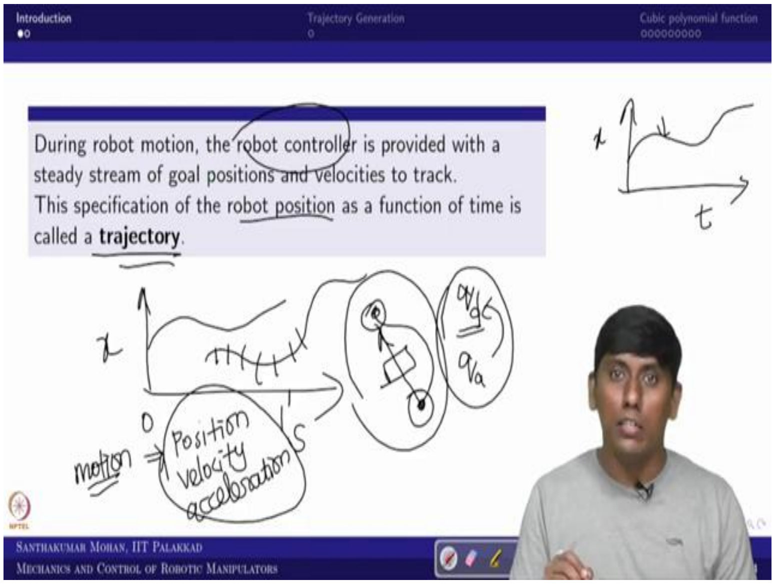

So, what is trajectory? So, trajectory is nothing but a path with the function of time for example, now, I say that, so, this is what you call time versus x. So, I am saying that this is a such a like having this, so, now, this is a such a like time history. So, this is going to give a trajectory, but right now, I say that this is actually like not time dependent, I set some parameter called S. So, this is actually like very 0 to 1 and I am actually like writing the same thing as the x. So, now, this is actually like not dependent on time. So, in that sense, what it is called actually like a kind of path.

But why this is required, because the robot controller actually like need to follow certain things. So, for example, now, I am saying that the manipulator supposed to grab this particular point from this point, so, it needs to actually like come and do this. So, for coming this you need to actually like identify some kind of predefined path or you call trajectory. So, why this is required, because sometimes there will be obstacles.

So, it needs to go are probably in order to make the energy optimal or time optimal. So, you need to have some predefined manner of you can say following one point to another point. So, this is what the prerequisites and when you talk about controllers, so, you have all the time actually like some desired and you have actual. So, you are actually trying to compare and trying to maintain that q actual become to decide. So, this can be actually like just a constant for example, you take an air conditioner, you keep 20, 22, 24 or 16. So, these are actually like set.

So, then the air conditioner supposed to follow this in a, you can say piecewise manner, whereas you assume that you are actually like taking a properly a milling machine or you are actually like trying to draw a diagram. So, then you are touching like follows certain manner. So, then this is a actually like each and every time would be actually like to defined, so, these are all we are trying to see. So, that is point we are actually like bringing it here.

So, now, we are talking about the motion. So, the motion is actually like what we bother, we bother about the actual like the position and its derivative which is actually like velocity. So, these two things are very important, even if you are going for a dynamic level, even the desired acceleration also important, so then you can see like, so, acceleration is the third part which we are bringing in.

So, in the sense we are actually like trying to see these all. So, for that we are actually going to specify the robot position as a function of time that is what we are going to call as a trajectory. So, now, you can actually like understand the trajectory can be given one way of definition it is nothing but specification of a robot position as a function of time that is what we are calling trajectory.

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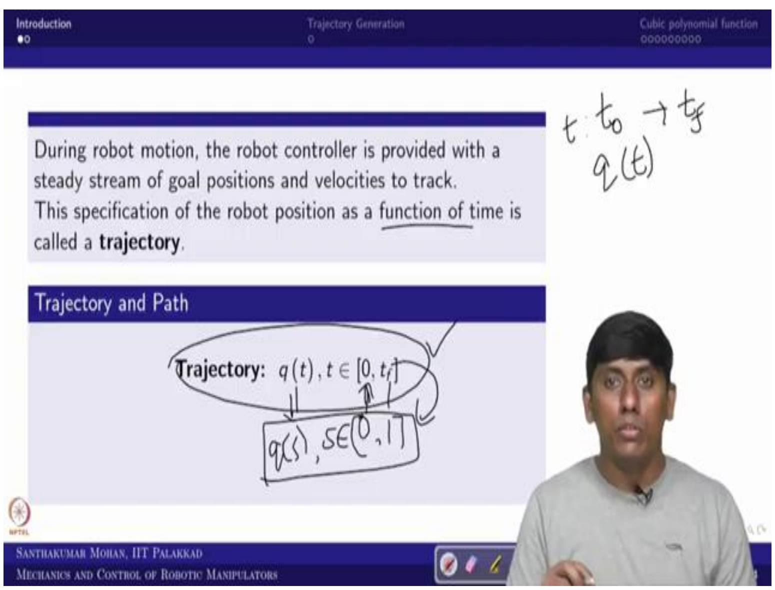

So, then we need to actually like differentiate what is path and trajectory. So, for that, we are actually like taking the trajectory means, so, we take a Q is the; you call the position, so, the q of t is actually like even where it is starting from 0, it may not be 0 all the time. So, I said that starting point is t0, and it is actually like ending as tf. So, where I define this 2 as actually like t so, the t vary from t0 to t f, so, this is what we call trajectory.

So, in the sense is time dependent in the sense you can see it is a function of time. Now, this tf when t0 I actually like converted into 0 and 1 and this I actually like call q of s. So, where there is only you call subset of S, that S actually like very you can say 0 to 1. So, if I make this you see there a time dependency is gone and you no need to actually like bother even it can be easily scaled up to any given time range. So, this is what we call actual like path. It need not to be a function of time.

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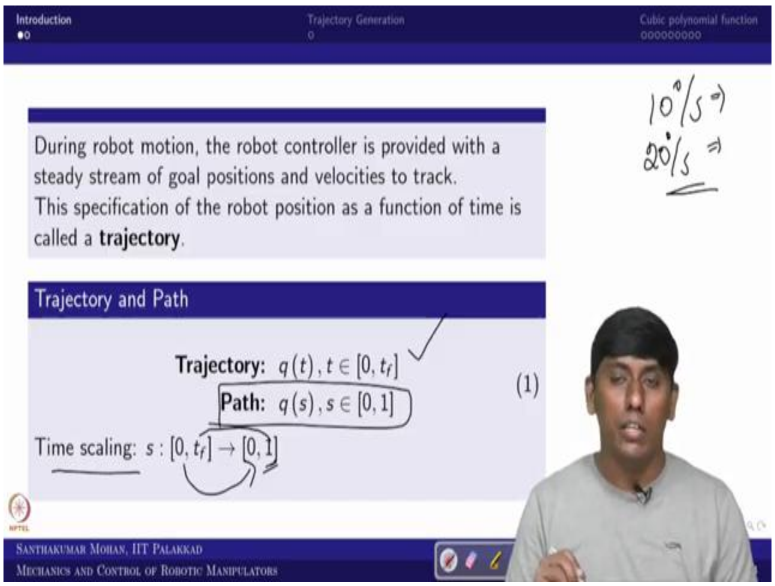

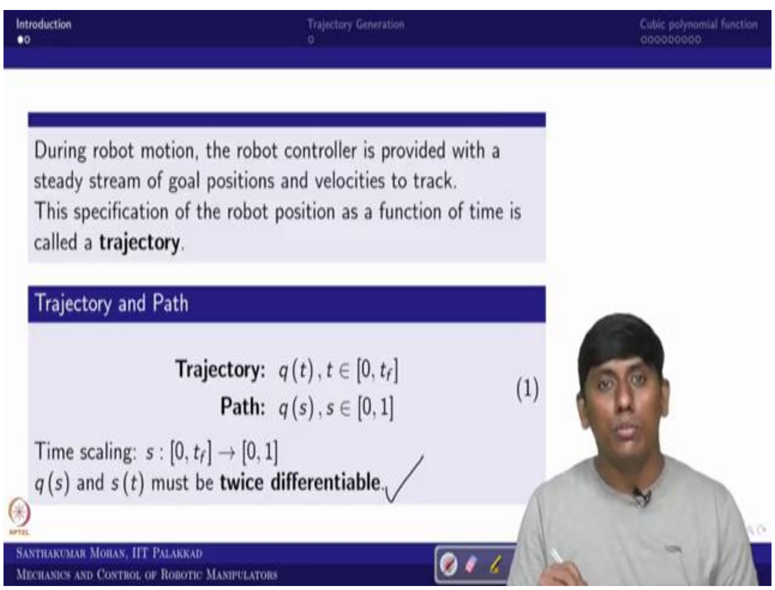

So, that is what we are calling like path we can actually like re-written as another set of function the q of s where S actually like belongs to or you can say S varies 0 to 1. So, how we convert the t0 be equal to 0 and the tf or you can say tf minus t0, so, that we are actually like equating to 1. So, now, this is what we call time scaling. So, most of the cases we do the time scaling. So, because the path easily we can generate with respect to 0 comma 1 or the S is varying 0 to 1 that way we can generate, and we can actually like scaled up whatever we want.

For example, now, my end effector supposed to move in 10 degrees per second as the; you can say speed. So, then I can see what would be the time required, I want to actually like increase the speed of that system, then I can actually like change the time step like that I can make it. So, that is what the whole idea, here we are insisting the; you can say robot position as a function of time that is what we are focusing as a trajectory here.

So, in that sense, so, either you call q of t or q of s that should be at least twice differentiable in the sense the acceleration is definite the sense so, acceleration is actually like what you call finite value or it is function of time it there. So, in that sense, you our actuator will start in a proper way. So, that is what we are actually like putting it, so, in that sense, we are actually trying to see a little more in detail.

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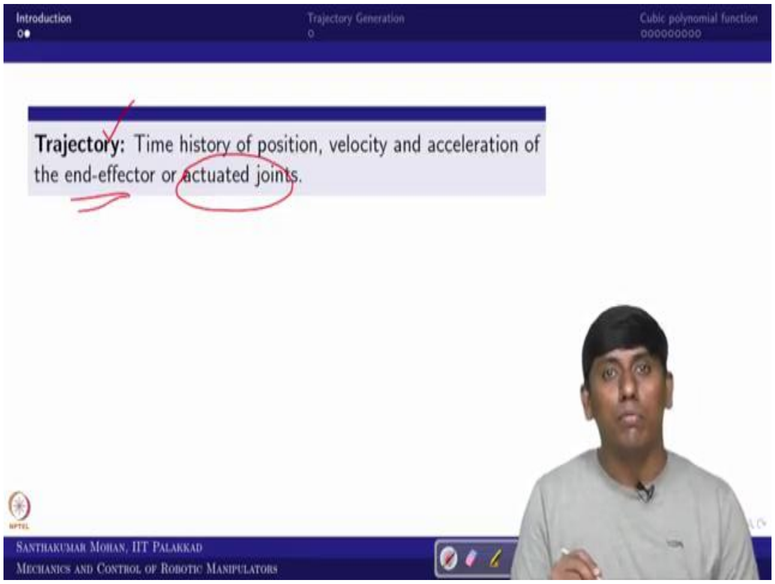

So, what that means, so, we are again bringing back the different definition. So, you should actually like should be familiar, so, the trajectory is nothing but time history of position velocity and acceleration of the end effector or sometimes it would be actuated joint it depends on what space you are actually like working on. So, for example, you are working on a configuration space then you can say time history of you can say actuated joints, in the sense joint position, joint velocity and joint acceleration be write in time history, that is what you call trajectory.

Or you are actually like bothering about the operational space where the end effector point you can say the position velocity and acceleration need to be generated then that is actual like end effector trajectory. So, that is what we are seeing it.

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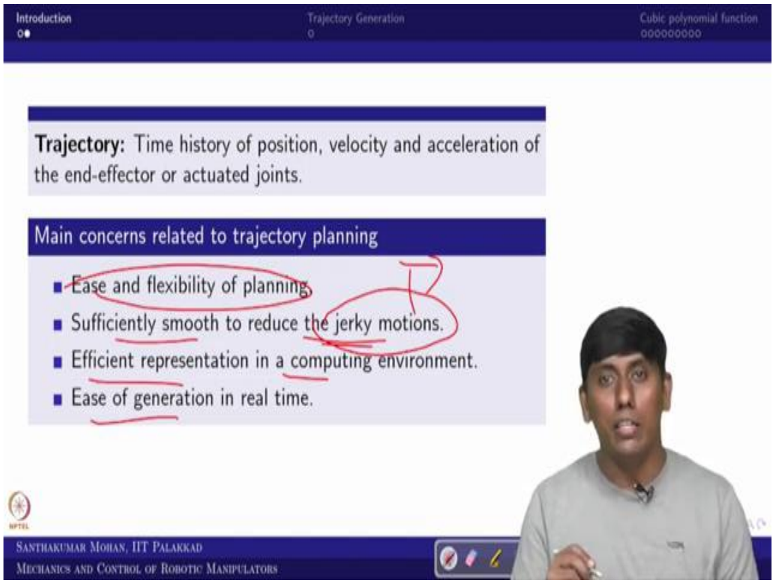

So, in that sense, what would be the main concern? So, already is I said it is supposed to be twice differentiable, but what if you are thinking about in terms of real time implementation, so, one is ease, so, it is supposed to be self-generating, and it should have a flexibility. So, that is one important point we need to think about further. So, when you generate any trajectory that trajectory should be very smooth.

So, in such a way that the robot manipulator or even robotic system that should actually like take away from the jerk. So, for example, now, you take a car, and you start and you actually like release the clutch arbitrarily, so what happened there would be a small jerk. So, however, if they experienced driver, you can see that there would not be any you call jerk you can experience when the car is starting, when start moving.

So, that is what you call this smooth trajectory. So, in the sense your trajectory should be smooth so that you can avoid the jerky motion in invariantly it will actually like reduce the cost of you can say overall system and long life of the actuator will come. So, that is what the whole idea So, further in talk in terms of actually like real time you can say computational point of view, so, it should be efficient, so, the representation of the algorithm or the trajectory planner should be efficient.

And as well as it should be actually like fast enough, this is the processing time for computing should be actually like good. So, that is what we said ease of generation in real time, so that you can easily implement it any point of time that is what the whole idea.

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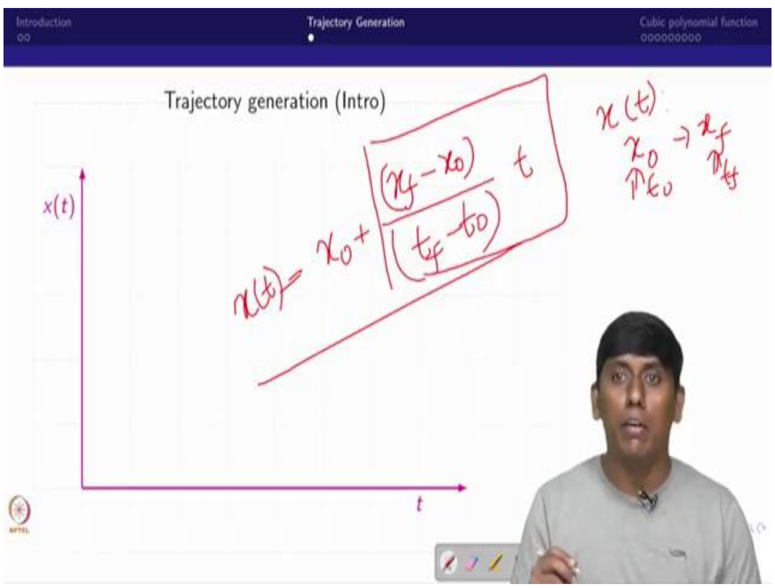

So, just to take that, so, what would be the easiest one, so, I have x of t, so, that is actually like varying x naught to x f, I am just saying, so, this is corresponding to t0, and this is corresponding to tf. So, what would be the easiest one, so, we will say that what is xf minus x0. So, this is actually like tf minus t0. So, this I call as a slope in the sense linear interpolation.

So, then I will actually like take it as a function of time. So, this would be x naught plus this, this is what X of t. So, now, in that case what it is giving it is giving a linear interpolation, but this linear interpolation is giving what? So, it is actually giving a constant velocity generation, if you think about constant velocity, what would be the acceleration?

So, it would be infinite in both acceleration and deceleration at the initial and at the end also infinite deceleration and acceleration right in the sense. So, you are acceleration at initial and endpoint should be infinite, which is not possible you do not have a smooth curve. So, for that only we are actually trying to see, so, whatever I have written in it like analytically, so, that we can see in symbolic, or you can say graphical manner.

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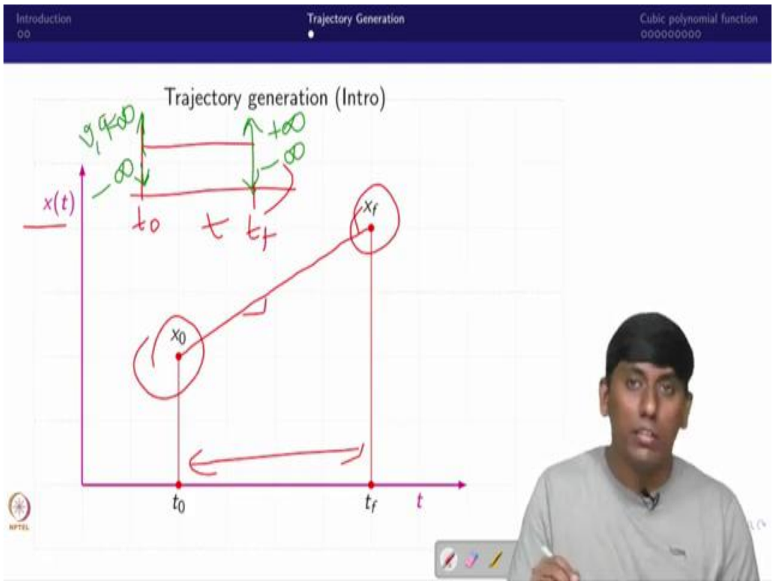

So, this is what I said the initial point, and this is the final point of X of t and this is their duration which I want to actually like see. So, what is the one of the easiest one I connect as a straight line which is actually like really a better one, but what happened the velocity would be a constant when I actually like draw time versus start t0 to tf so the velocity would be constant. So, if the velocity is constant, what would be the acceleration?

So, the acceleration would be so, infinite acceleration and deceleration. So, similarly here it would be so, that is actually like a little concern for us. So, this is actually like velocity and acceleration.

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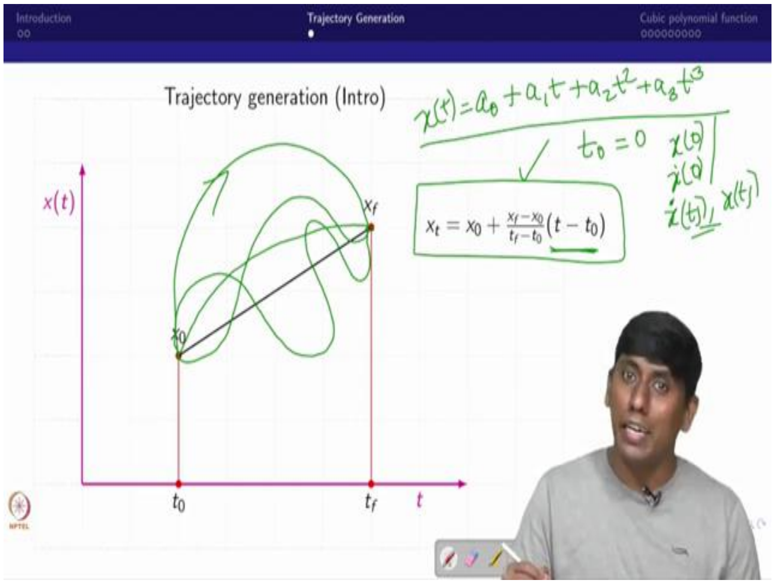

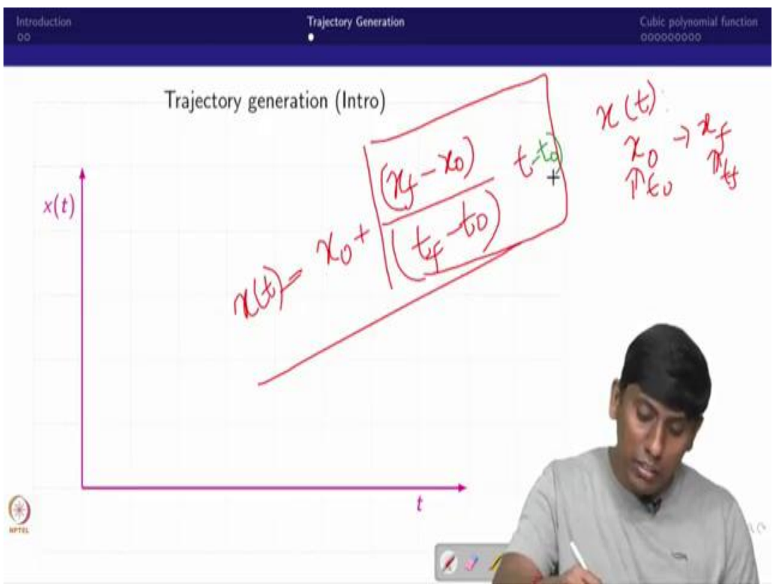

So, in order to avoid that what people are actually like thinking about this is what the linear interpolation So, in order to actually like get this So, the t naught we always assume 0. So, but if it is t naught is there then you are actually like substitute that so, naught simple t. So, I will just rewrite this equation. So, this is actually like minus t naught. So that you can actually like get it clear.

So, now, in that case, so, what we are actually like writing this is linear interpolation this is not smooth, it is actually like a jerky motion. So, in order to avoid what one can see, so, there are actually like several possible you can see these two points can be connected like this. So, like this and there are several possibilities to reach from this. So, we are trying to see what would be the efficient.

So, this is actually like first order the second order is possible, which is what you call the quadratic equation, but quadratic equation would be having. So, the number of you can see equation and number of unknowns are actually like not properly matching. So, we will go with you can say cubic polynomial where the cubic polynomial the equation I can write, so, a naught plus a1 t plus a2 t square plus a3 t cube where t naught I assume as 0.

So, which is I called this so, in the sense of what you can see there are four unknowns. So, these four unknowns what I can do x of 0, x dot of 0 I can equate and similarly, x you can say x dot of t f and x of t f I can actually like substitute you can see, so, 4 boundary condition and 4 unknowns I can get it. So, that is what we are trying to see.

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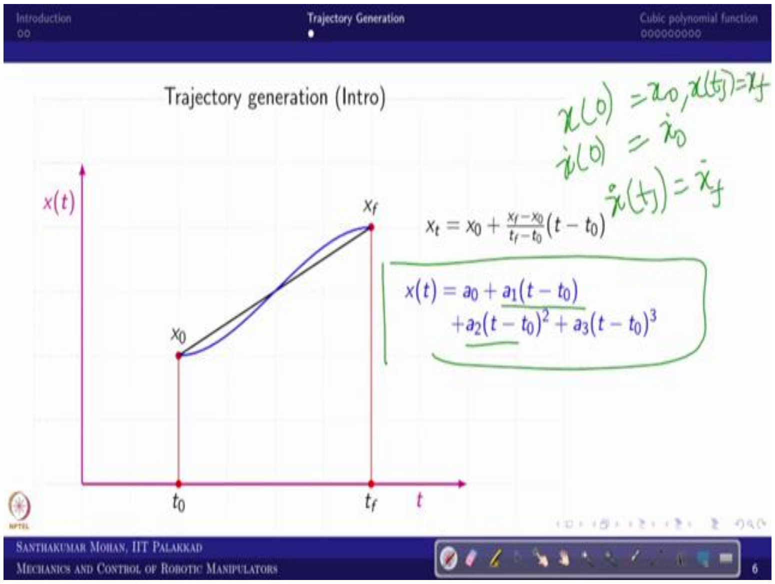

So, in that sense the cubic polynomial which I have shown it is very smooth. So, which is actually like very close to what you call the straight line, but it is actually very smooth, it is actually like going to give at least twice differentiable in the sense the acceleration would be finite. So, that is what we are trying to see here. So, in the sense of what one can see, this is what the equation which we generated.

So, in that sense, you can actually like see, so, these four unknowns, we can actually like calculate by bringing the, you can say four conditions. So, I call this as x naught and this is x f and x, this is x dot naught and this is tf. So, then x tf is called x, so, in the sense of four unknowns and we can actually like get you can say four equation and then we can solve.

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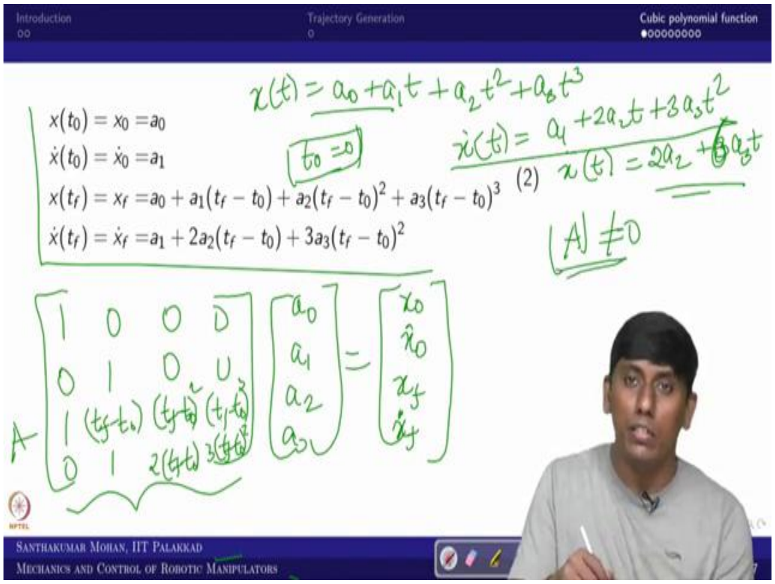

So, that is what we are trying to write it here you can see further you differentiate, so, what will come. So, further if you differentiate, so, it would be actually like further if you differentiate so the equation x of t is actual like, so a naught plus a1 t plus, I am just assuming that t naught is 0, this is explicit assumption here.

Otherwise, I have to write t minus of t naught. So, in order to avoid that I am writing, so what would be x dot of t, that would be a1 plus 2 a2 t3 a3 t squared, what is the x double dot? So, that would be so 2 a2 plus you can say 3 a3 t. So, this will be sorry this is 6. So, this will be coming, but you need actually like only four equations. So, we will actually like take this and this and we can actually like substitute, and you can do it.

So, now for solving this there are several ways, so, one of the easiest way what we can do it we can rewrite this in a state space form. So, what we can do so, we can write the unknown so, a1 a naught, a1 a2, a3 or I can write it to one side and the other side actually like x naught x dot naught and xf, xf dot I can write and you can see the first equation that is 1 0 0 0; the second equation 0 1 0 0 and the third equation 1 then you can see tf minus t naught and tf minus t naught squared and the third one is tf minus t naught cubed.

And the third one 0 1 and 2, so, tf minus t naught and tf minus t naught squared. So, now, this is actually like linearly independent you call rows containing, so, in the sense this I call A. So, the determinant of A is actually like non-zero. So, in that sense of what one can see, so, if I take inverse of this and multiplied with this So, that will give a naught a1 a2 a3. So, that is what we are actually like doing it.

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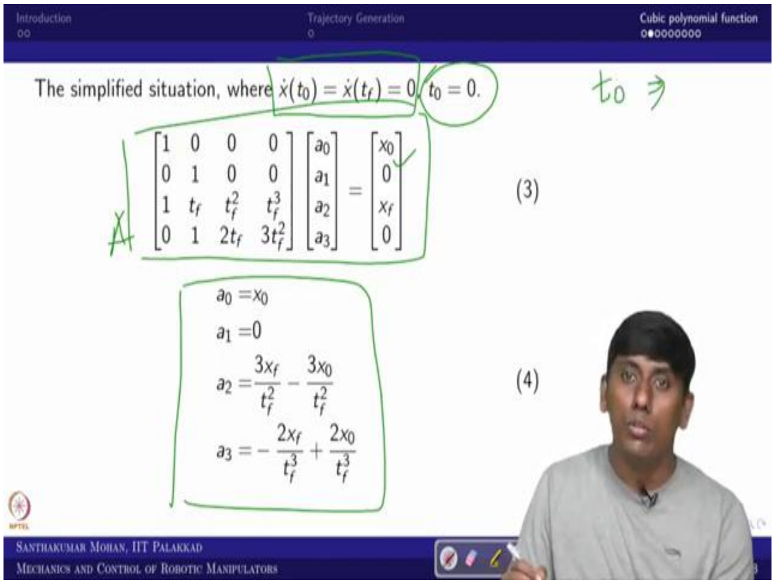

So, you can see I take this I take this A inverse. So, this A inverse I multiply with this, and I can do. For making simplification, I assume t naught is 0 and the initial velocity and final velocity are 0. So, in that sense what happened x dot of 0 and the x dot of f both are 0 so, which gives the A naught a1 a2 a3 in a simplified manner. So, the same way you can actually like even you put it non-zero value, but t naught we can make it always 0 because we can actually like bring it t naught at the end of wherever it is required.

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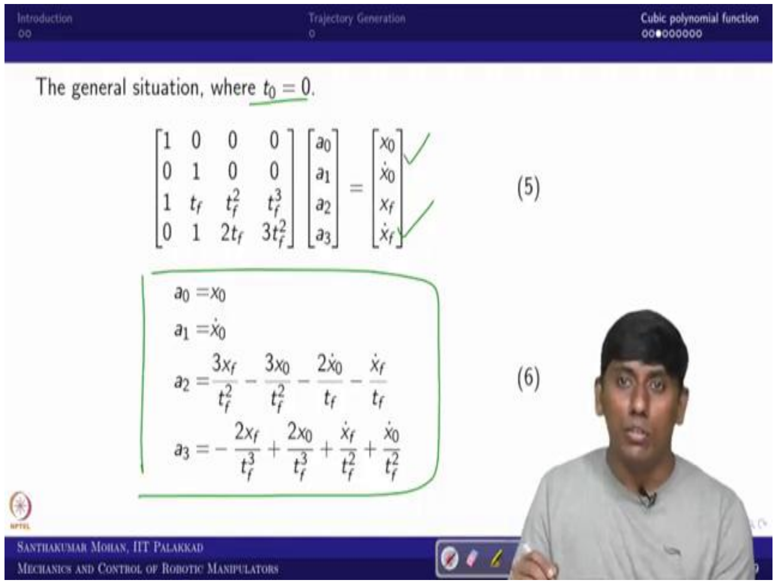

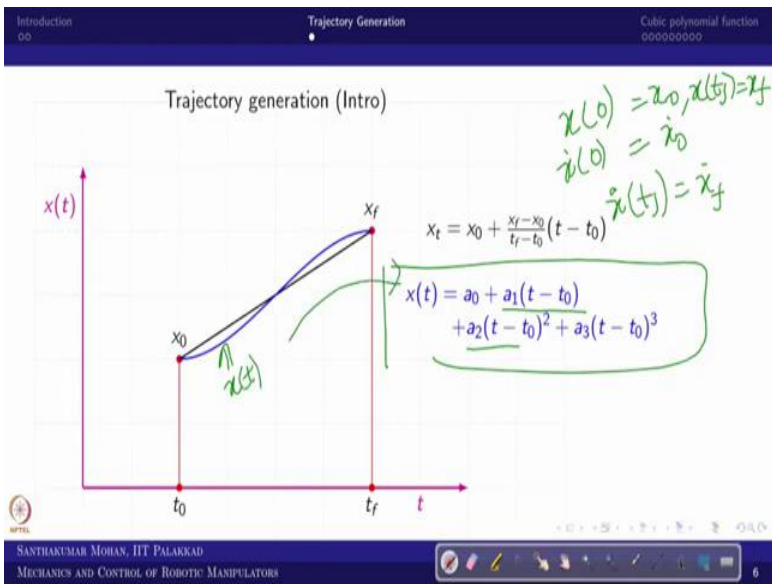

So, in that sense, you can actually like assume x dot of 0 and x dot of f exists so, then the a naught a1 a2 a3 are attainable in this way. So, now, you know all the coefficients once you know the boundary condition, then what you can do you can actually like generate the trajectory. So, that is what we have done in the; here. So, this is what you call x of t as a cubic polynomial. So, now, this equation actually like array.

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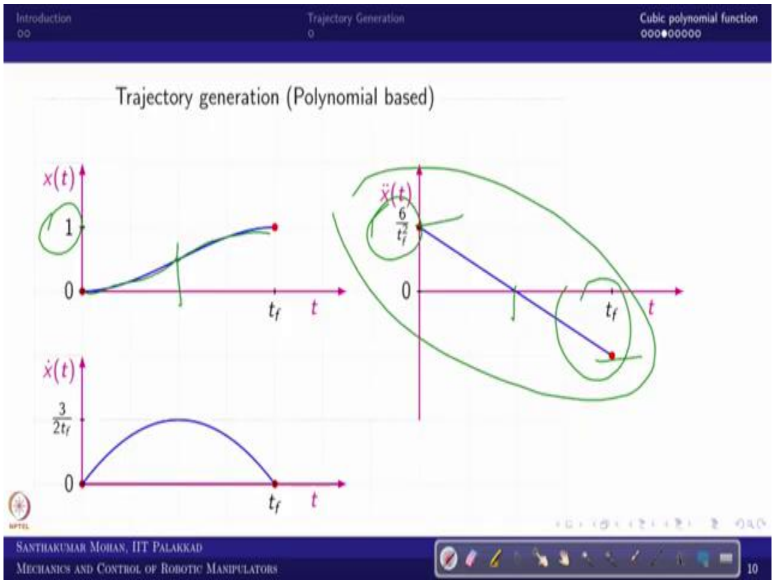

So, now, we will actually like go back individual component. So, in the sense how the velocity how the acceleration will come, because the position profile we have seen. So, now, I assume that it has scaled down to 1. So, in that sense at t0 I assume 0 and tf. So, this is the duration and the S displacement, or you can say x of t as the you can say displacement as 1 so, then you can see the profile is smooth.

So, now, if you rewrite this into a velocity form you differentiate you can see the velocity would be maximum at the middle of the duration. So, that value would be equivalent to 3 into 2 tf. And even you bring the non-zero initial condition all those things will come theta f minus theta 0 would be added. So, now, the same way you go to acceleration you will feel a little strange because the escalation will start from non-zero value. And similarly end with a non-zero value.

So, now, we assume that this is actually like one. So, based on that you can actually like see that the acceleration is actually like non-zero at initial phase and non-zero at the final and zero acceleration at middle point. So, here actually like you can see the middle point it is actually like this curve is actually like making another parabolic.

So, it is actually like positive parabolic, and this is negative parabolic kind of thing it is generated. So, now, this is actually like proper what you call cubic polynomial, this is giving a smooth profile, and it is actually like you can say twice differentiable in that sense, this is a one of the easiest ways to apply.

So, most of the industrial manipulator use the cubic polynomial that is why we straight away started talking about this but the problem here is we do not have any control over the initial and final acceleration. So, if you want to do that, so we need to go little, you can see improvised algorithm that we will see in the next lecture. But now we will actually go further.

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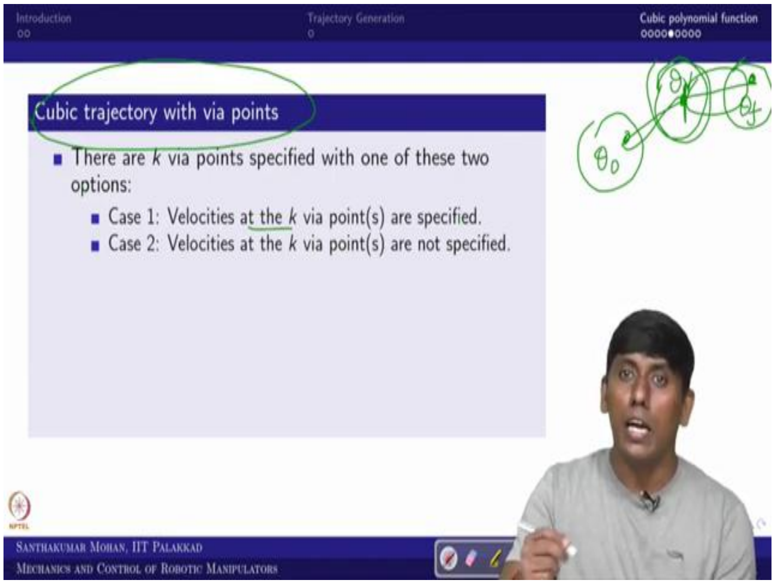

The cubic trajectory can come with via point for example, I said that, so this is theta 0 or x0 whatever. So, I said this is actually like supposed to reach here theta F, but I have given a via point as theta v. So, then how I can actually like do so, even the cubic trajectory can be generated in two modes; one is this what you call via point velocity also specified then that would give actual like you can say 8 equations and 8 unknowns and you can actually like solve it.

And that would give a slightly trickier way that we will be seeing in the next slide, but the other one is actually like the velocity of the you call the via point is not specified, then we can see that the velocity is what generated from here that we can continue as that is the velocity in the sense the initial acceleration and velocity of the second segment would be the final acceleration and velocity of the fastest segment that way we can equate. So, that that would give further add-on that make life simple. So, that is what we are trying to see.

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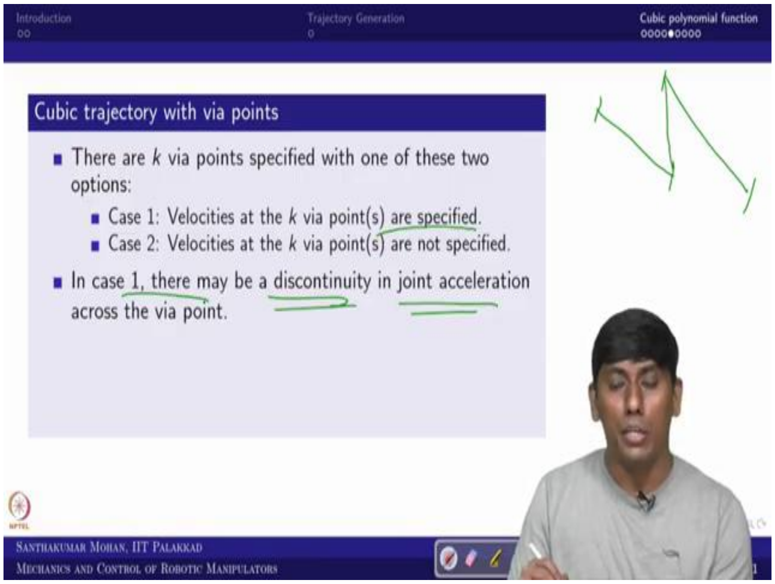

So, in that sense you can see that in the case-1 there may be a discontinuity because you are specifying the, what you call the via point velocity that make the joint acceleration is actually like jerky. So, in the sense so, it ends here and another one into somewhere like this. So, this is the first segment, and this the second segment may come like this, we can see that. So, this is actually going to give a discontinuity which is not encourage able. So, and this and this point is discontinuous happening.

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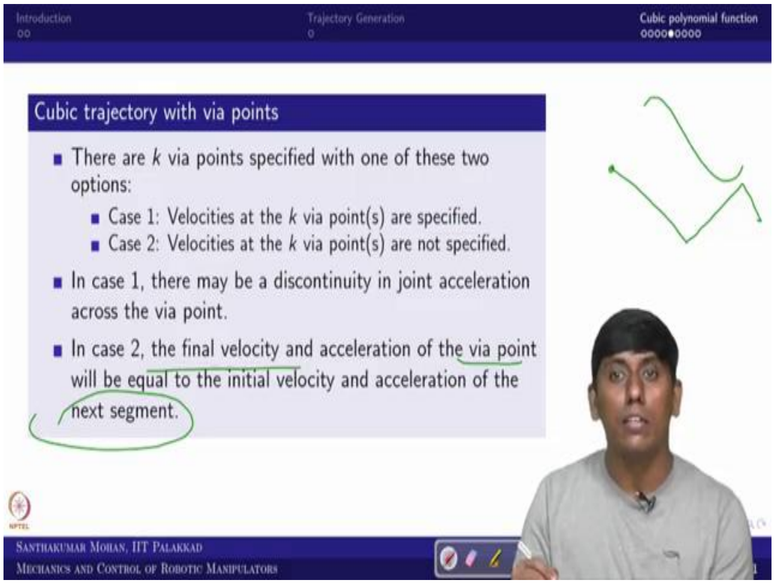

So, whereas the second one; the final velocity and final acceleration of via point would be equal to the initial velocity and acceleration of the second segment so, in the sense we can actually like use this way. So, in that sense it is actually like give the smooth transition and as well as the acceleration would be for example, this way so, it comes like this it goes. There would not be any discontinuous happened you can see that how it actually like happening it.

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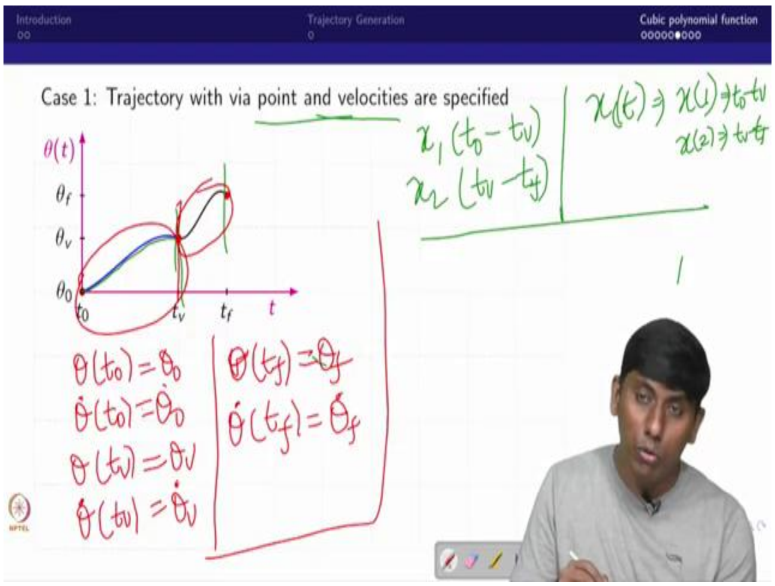

So, in that sense, we take the via points segment, so, you can see like, we take first one with velocity specified, so, then you can see like, this is actually like solved as an independent cubic polynomial, then this is actually like solved another independent cubic polynomial, and you can actually like use it, so, in the sense, so, you have first set of equations, so, t0 to tv. The second segment is actually like tv to, so, tf, so, that way we will make it.

So, in the sense x of, you can say t would be constraint actually like x of 1, for actually like the first duration, so, t0 to tv and x of you can say 2 that would be, so, tv to tf. So, like that we can actually like use it. So, in the sense, you have actually like via point velocity. So, in the sense, I said that x1 of t0 is actual like x of 0, and the x1 dot of t0 is so x dot of 0.

So, x1 of tv is xv. So, in this case, it is actually like, I will write it consistent. So, I will just use the consistent in the sense and write so theta, I will write this again. So, theta, you can say t0 is actual like theta 0 and theta dot t0 is actual like theta dot 0 and theta of tv is theta v. So, theta dot of tv is defined like this. So, second segment is actually like, so, you have six variables.

So, second segment is actually like a tf, which is you call theta f and so, these all 6 variables will be given, but based on this how many equations you can generate? So, you can actually like take this as the first segment. So then, you can actually like make it so then the second segment can actually like do it. So, in that sense, you will get 8 unknowns and 8 equations you can solve it. So, the profile will come like this.

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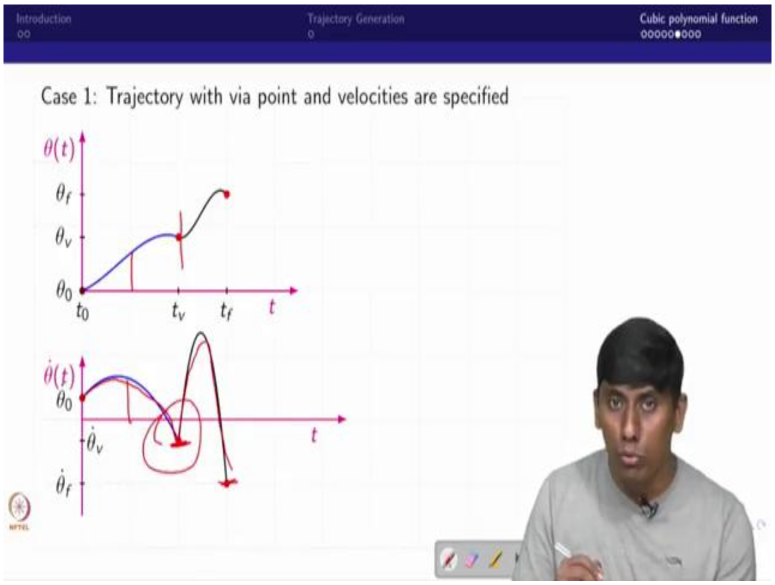

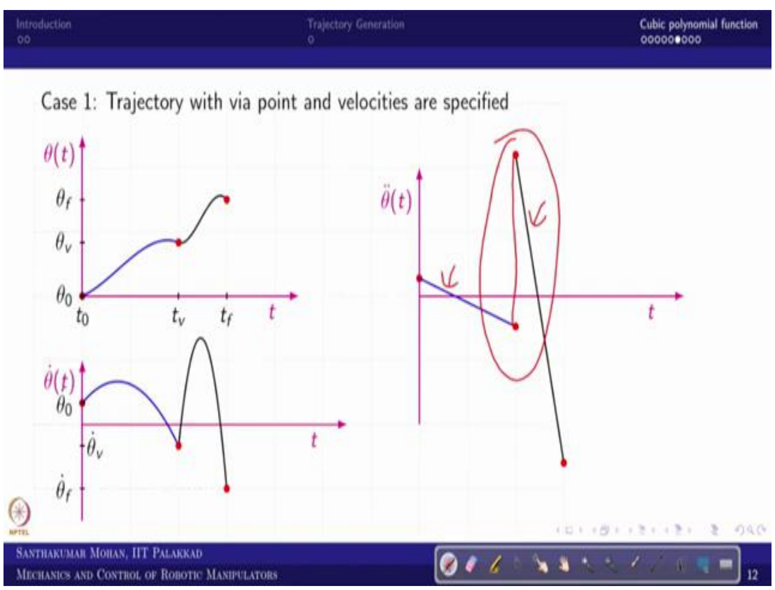

So, based on that velocity, you can see I already said, there is theta dot v exists. So, you can see like it is first curve is smooth, where maximum at the middle. So, where you can actually like see, and after that what happened that trajectory is actually like second segment, where it is start from non-zero via point velocity and end with the theta f it is actually like going again. So, here itself you can see that there is non-smooth transition.

So that is what we can actually like see it from here. You can see this is the fastest segment acceleration and there is a discontinuity happen at this point. So, this is the second segment acceleration and this the first segment acceleration, so, there is a discontinuity happened here. So, in order to avoid this what we can actually like do it.

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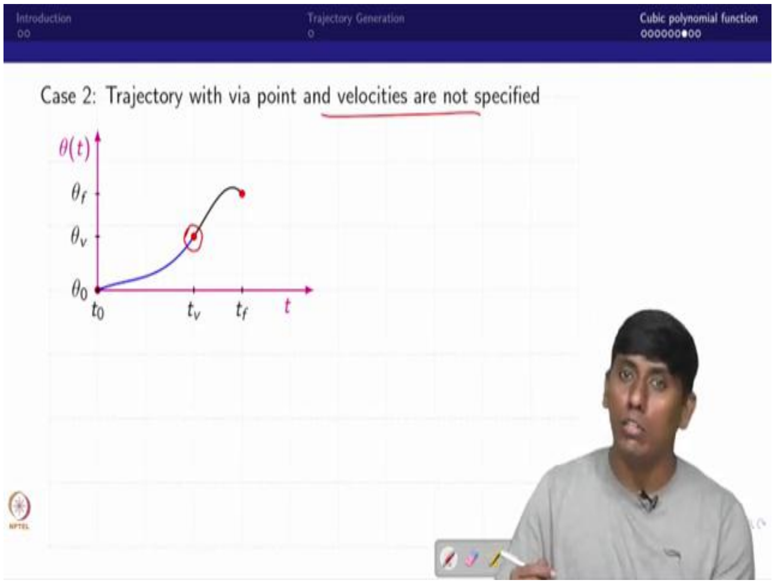

So, we can assume that the; you can see the final velocity of this segment and final acceleration of this via point would be equal to the second segment initial velocity and initial acceleration. So, in that sense you will be using given the; you can say acceleration equation and one another you can say constraint. So, then what happened you will be getting again you can say six valid equation and you can add these two additional constraints. So, then it would do eight equations and eight unknowns and then you can solve it. So, I will show you in the simulation how these variables and all generated.

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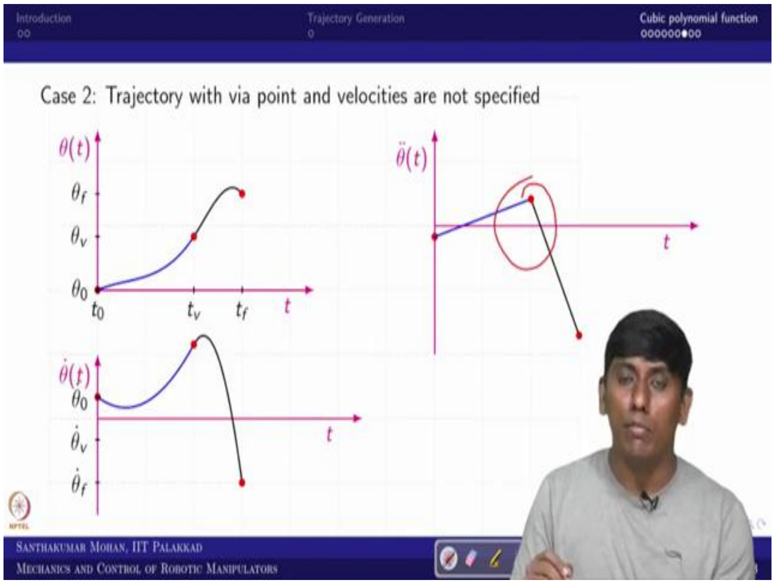

So, now, based on this you can see the velocity is actually like smooth enough. And you can see even the acceleration is actually like, so, no discontinuities are happening. So, this is what we call actually like a cubic polynomial with via point. So, we will see how the cubic polynomial used in real time. So, real time we usually use in the nested. So, before going to the nested we can see one another constraint.

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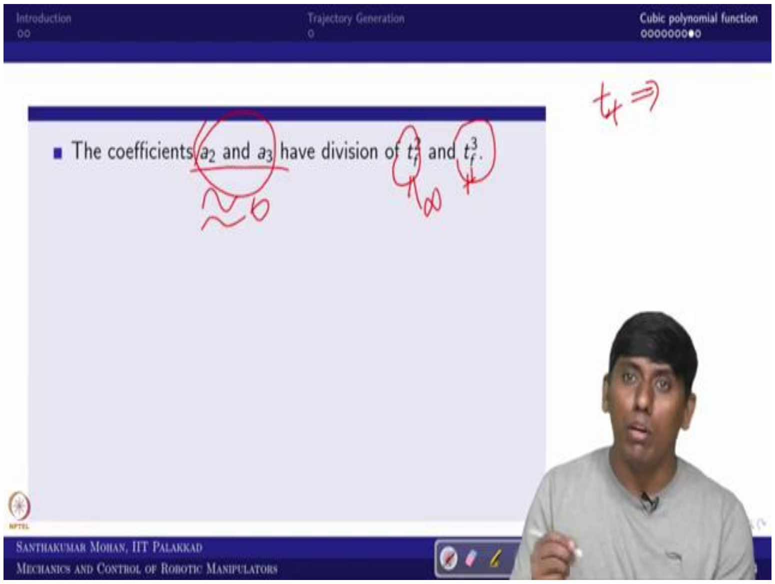

So, where a2 and a3 if you recall the variables, so, it comes actually like a tf squared in the you call denominator, similarly, t3 tf cube are in the denominator which makes. So, when tf is actually like very large what happened this becomes actually like infinity in the sense this a2 and a3 approximately to 0 which we need to avoid. So, for that what we used to do, so, we will actually like take one of the variable new variable call u.

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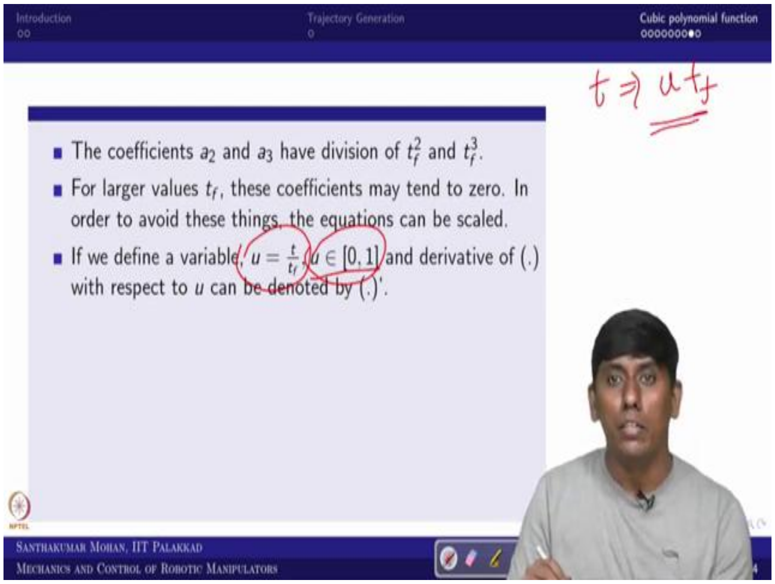

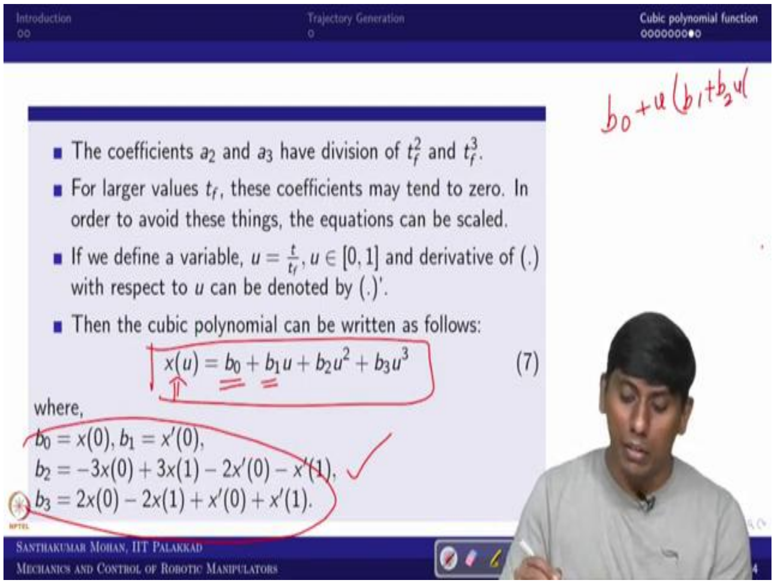

So, in order to avoid this tends to 0, so, then what we are actually like trying to take we are trying to take x as t by t f and in that case what happened this is actually like a path. So, now, you got it why we have used so, q of s, so, now in that way, so, it is 0 to 1, so, in the time no need to actually like worry. So, now simply use scaled up to tf. So, now you want t then that is u into tf. So, that way you can actually like multiply.

So, based on that what happened so, you are equation would be very simple. So, we write that x of u in the coefficient of b0, b1, b2, b3 the coefficients you can see like there is no divisions. So, in that sense, it is actually very, very simple. And even this we can rewrite as a nested form, so, I am writing b0 okay. So, plus u, you can actually like see that b1 plus so b2 u then I can actually like write plus that way, so, like that I can write as a nested form.

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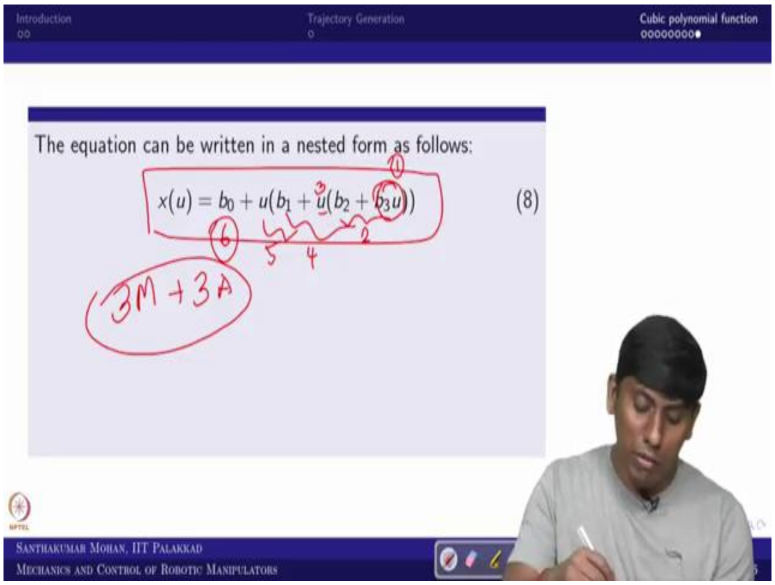

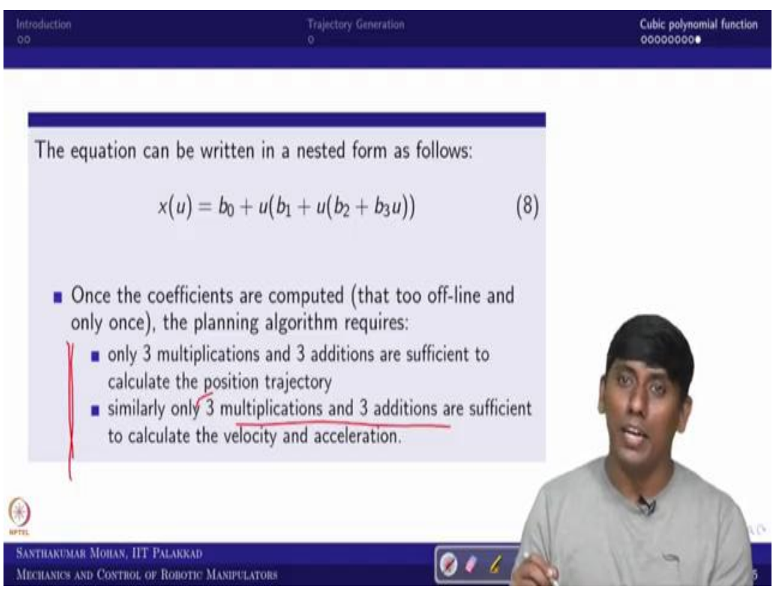

So, you can see this nested form is actually like one of the easiest one. So, in this nested form what you can see, so, first one addition then multiplication, then addition another multiplication. So, here are one multiplication, so, first you multiply this, the first operator then second, then third, the fourth and fifth and sixth. So, in the sense what you can see three multiplications plus three additions only coming, so, which is easy to derive the equations in real time the computational point of view it is very easy.

So, that is why we are using this nested even this nested is making even, you can see very, very simple because the; you call the velocity and acceleration also like contain only three additional multiplication and you can say additions only. So, in that case we can easily solve this all. So, if that is what we actually like bound, so, then what you can actually like see that this can be easily implemented.

So, now, you have actually like seen, so, how the cubic trajectory has come, but I already said the cubic trajectory is actually like a not having any control over initial and final acceleration, so, that we can think about some kind of higher order, you can see polynomials or some other polynomial that is what we are going to focus in the next lecture.

So, once that is done, then we will bring back to the manipulator and see how to generate the trajectory in joint space or you call task space. So, with that I am ending this short lecture. See you then thank you. Bye.