Mechanics and Control of Robotic Manipulators
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Lecture No. 25
Equation of Motion in State-Space Form

Welcome back to mechanics and control of robotic manipulator, last few classes we have seen basically robot dynamics mainly we call manipulator dynamics. So, you know like manipulator dynamics again divided into two, one is we call motion dynamics the other one is what you call structural dynamics. So, we have seen more focused on motion dynamics. So, we have seen what way we can formulate this.

So, there are two formulation methods, or you can say derivation methods we have seen. So, in this particular class once you derive the equation, we can write it in certain form. That is what we are calling equation of motion in state space form.

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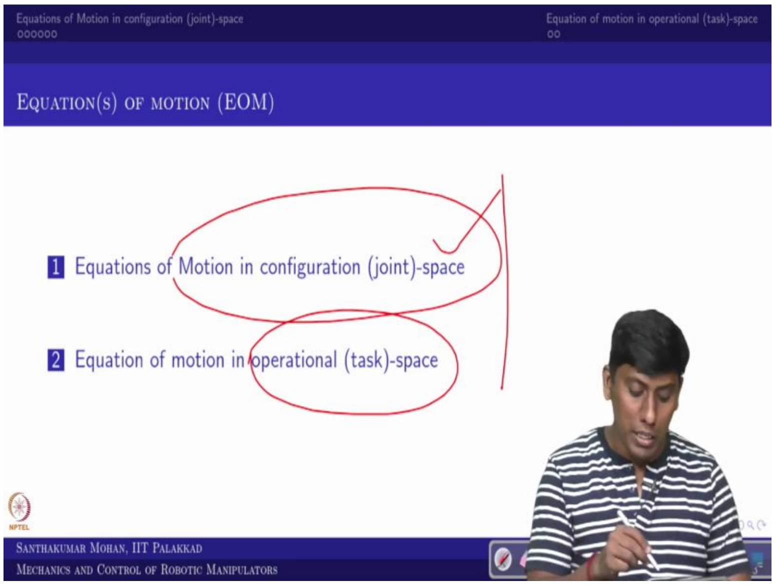

So, that is what we are trying to see. In this case, if you look at it the equation of motion can be written in the general space, we call configuration space which is what the manipulator space or we can write at the tool or you can say end of vector space which we call operational space or task space. So, in the sense the equation of motion can be written in state space form either in joint space or task space. In this particular lecture, we are going to see both in detail.

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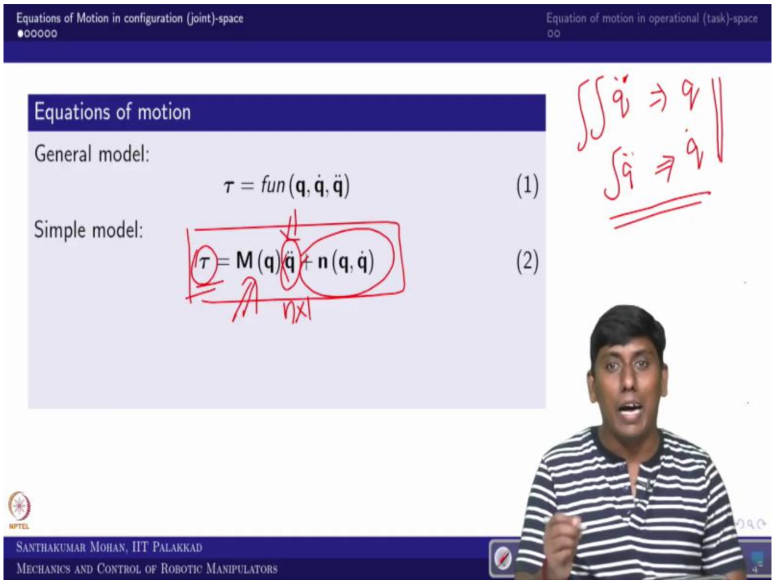

So, in that sense what we are rewriting this equation what we derived in the last few classes where tau is the actual like a cause of or you can say causes. And where this q q q dot and q double dot are the motion. So, we have a relation between you can say input and you can say output, here output is motion and input is actually like you are input forces or you can say joint forces and torque.

So, in that sense the general model can be returned in a nonlinear form as a simple function. But this can be made it for you can say numerical integration point of view or in the other way around you can say for a forward dynamics point of view we can rewrite this equation in this manner. The sense for us you can say simplification or for the integration part you know q double dot then you can integrate double times what you will get you will get the q.

If you integrate single time so, you will get what you call q dot. So, in the sense these are the two states variable this you can obtain by integrating in that. So, in that sense you need to have a separation of q double dot. So, that separation will come. So, in that sense you can see one side without having any acceleration term. So, other side you have acceleration term alone but in that acceleration term alone we are taking n cross 1 as a vector.

Now, the coefficient of this n cross 1 vector in this tau equation would give a coefficient matrix. This coefficient matters what we are going to call as an inertia matrix. So, we will see in detail what are the property of this inertia matrix. But this is the simplification, or you can say simple model for numerical integration. In that sense what you have you have input, and you have output in acceleration and other form.

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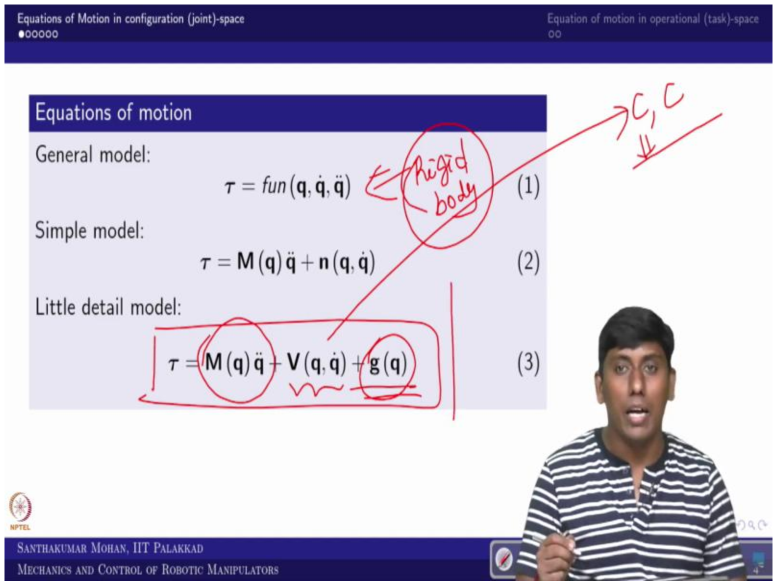

So, even some people even further you can say make a detail, where you can see that the gravity they separated. So, we have separated already the initial term. So, now, we can separate the gravity term, then you can see whatever left that we can call as other effect. So, that other effect I put it in a capital V as the vector. So, now, in that sense you can see this is the little detail model even you know like this other vector would be having further component as a centripetal and Coriolis component and even you bring the friction.

But right now, we are not bothering about friction because if you read the if you re you can say remember the derivation which we derived which is based on rigid body. So, based on rigid body we have seen that there are only kinetic and potential energy we have taken in Lagrangian Euler. And even in the; what you call Newton Euler also we have seen acceleration then inertial forces only right.

So, in this sense the dissipative or you can say conservative forces we have not seen. So, in that sense the frictional term directly will not be coming. So, we will see how the other terms can be divided.

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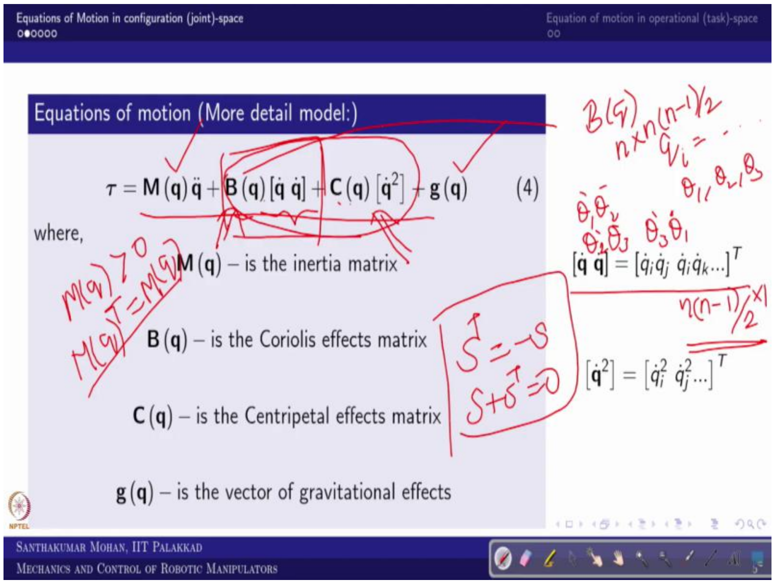

So, in that sense, what one can see the equation of motion can be further you can see detail it. Where you can see that the B of q comma q dot further divided into two different terms. So, one is we call Coriolis effects. So, the other one is centripetal effect. Where this would be come with you can say vector. So, now this B of q is a rectangular matrix, but the C of q is you can see it is a square matrix.

So, what is the dimension of q dot that would be for example you have so q you can say i to something for example, I say theta 1 theta 2 theta 3 are the three variables. So, now theta 1 dot theta 2 dot then theta 2 dot theta 3 dot and theta 3 dot and theta 1 dot are the three combination it can come which will generate the; what you call the Coriolis effect. In that sense, this is the; you call a vector this would be having n into n minus 1 divided by 2 by a cross 1.

This is the; you can say size of this particular vector in that sense this would be B of q would be n cross n into n minus 1, whole divided by 2. So, this would be the size. So, now, you got idea what is the Coriolis Effect. Coriolis effect would be happening due to a combination of linear and angular velocity so of that particular link that way we can generate.

And similarly, you can see the; you call radial or centripetal effects that would give you can say something specific matrix the C of q would be giving something call as skew symmetricity matrix in the sense S transpose equal to minus S or S plus S transpose equal to 0. So, this way it will come we will see these all detail in later. Similarly, M of q which is the inertial term. Inertial term cannot be negative.

And in that sense what you can see M of q can be positive definite matrix further, this M of q will give one additional aspect that what we call symmetric matrix. In the sense M transpose of q is a this. So, this all we would be seeing in detail in upcoming you can say slides. So, now, this is the detail model it is a more detailed model where gravity terms separated inertial term separated further the V of q comma q dot we have further divided into two sub terms.

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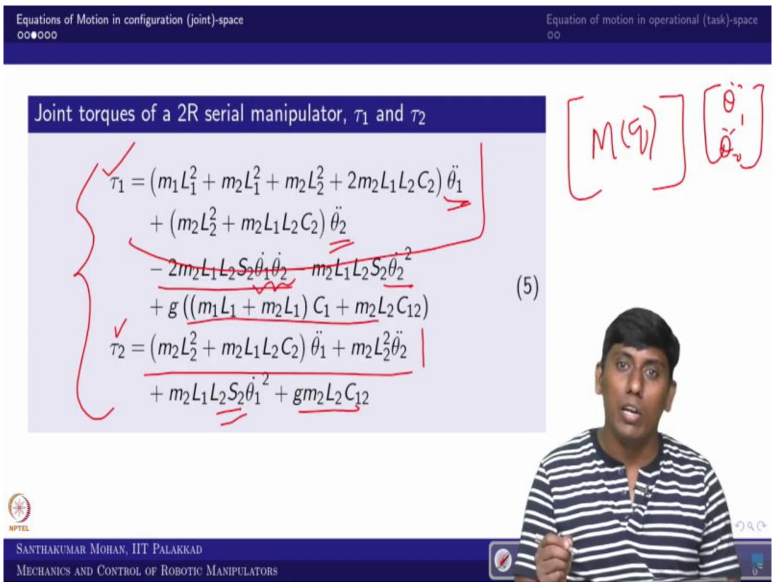

Further if you want you can make it friction, we can bring it. But right now, we will rewrite the equation which we derived in both you can say Newton Euler and Lagrangian Euler method. So, these are the two equations which we derived for a 2R serial manipulator which we call tau 1 and tau 2. So, now, you can see that the theta 1 double dot theta 2 double dot you separate it.

Similarly, in the second equation also you separate it. So, what that will come? So, you are interested this way. So, you are rewriting this equation, then you will get a coefficient. This coefficient matters what you call M of q which is inertia matrix. And further you can see I already told once you derived you have to match the unit. So, you can see like this is giving theta 1 dot and theta 2 dot this will give you the Coriolis component.

And you can see theta 2 dot squared this is giving this centripetal component. The same way you can see here. So, here you can see the centripetal term alone is there, because the second body is not getting the Coriolis effect that is very clear because the second body is freely rotating. And you can see the other term is the gravity terms. So, now we will rewrite in the; what you call this form more detail form we can rewrite that.

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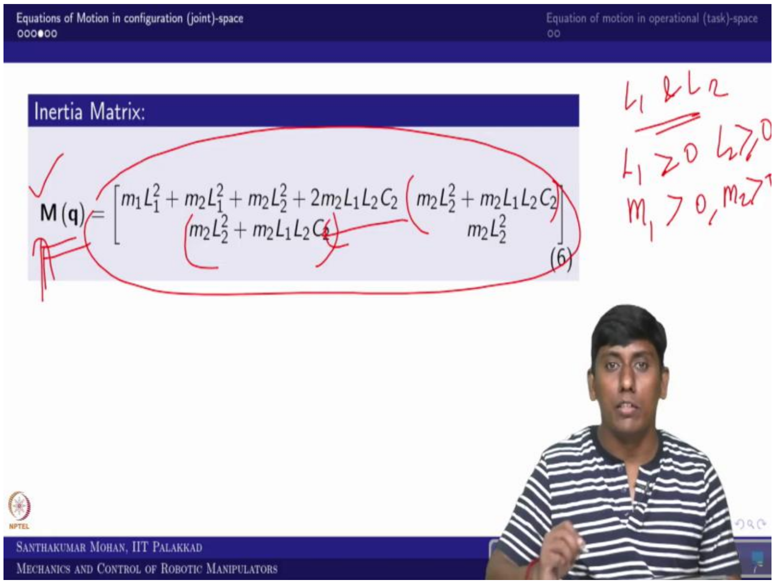

So, in that sense you can see that M of q will come in this. So, now, after seeing this matrix so, Ll and L2 are geometrical variable. And as per the criteria L1 is positive or equal to 0. So, L2 also positive or equal to 0 and mass cannot be 0. If you have a link that cannot be 0 in the sense m1 greater than 0 and m2 also greater than 0. And you substitute these all so, what one can see this matrix is definitely going to be a positive definite that we will see in the numerical simulation side.

And further you can see this term is matching with this term in the sense even you take a transpose is giving the same matrix. So, that is what we have seen M transpose of q equal to M of q in the sense it is a symmetric matrix. Further this is going to give a positive definiteness. So, that is what we have seen as an inertial matrix.

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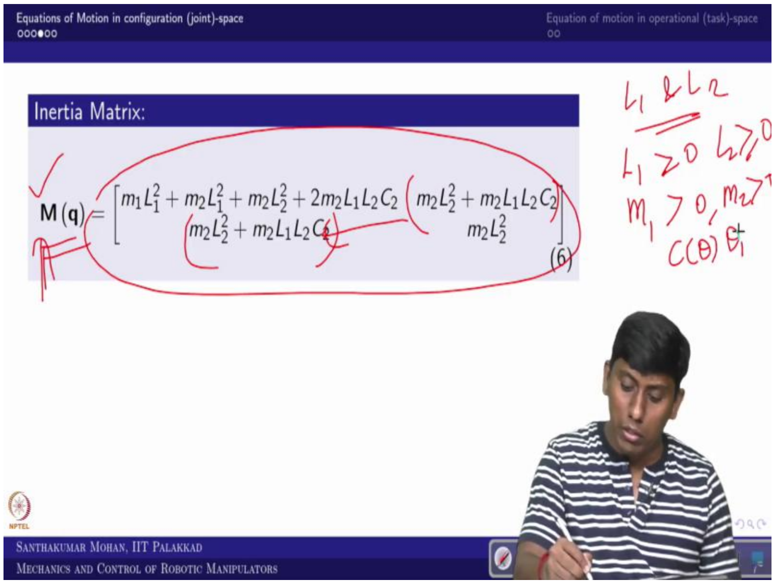

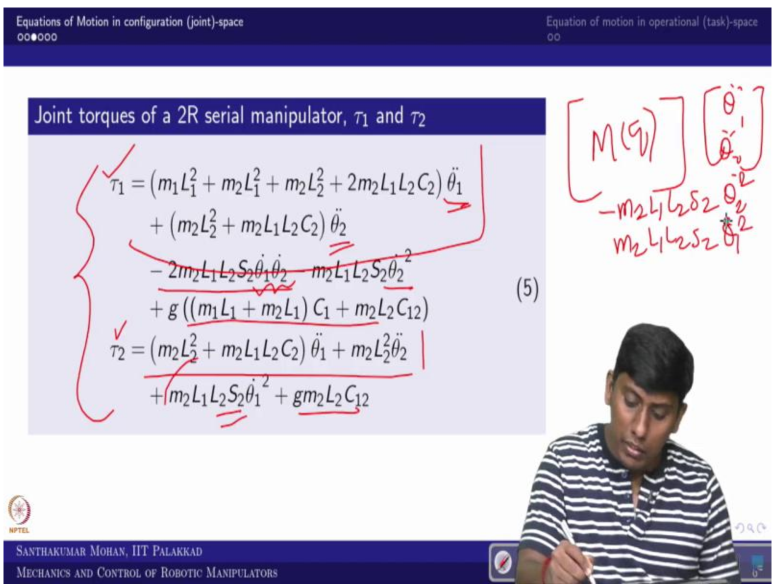

Let us move to the second term, we call it like centripetal effects. So, if you recall from this, the interpreted term is here is so minus m2 L1 L2 S2 theta 2 dot squared. So, in the second term, it is so m2 Ll L2 S2 so theta 1 dot square. So, these are the two terms. So, now if we rewrite that in a matter form where you can write C of you can write the theta into theta 1 dot squared and theta 2 dot squared. So, what that matrix looks like?

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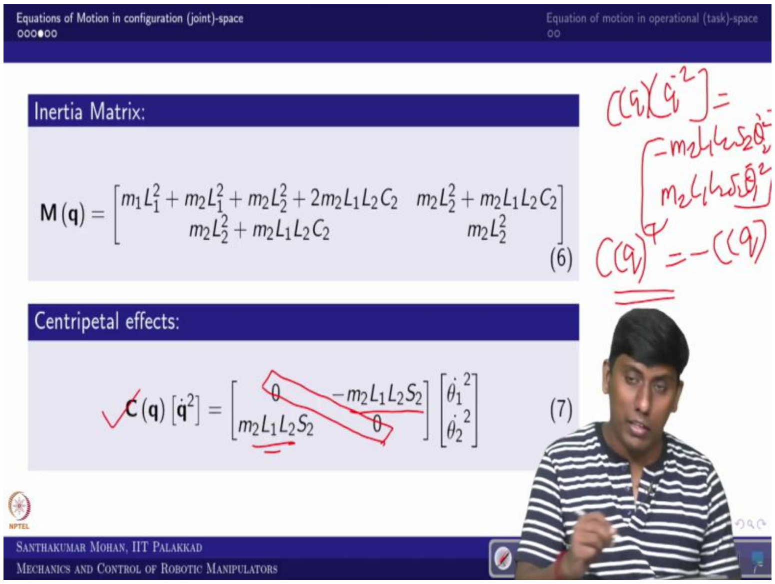

This look like this. You just multiply you can see that related to tau 1. So, the C of q comma q dot squared is giving the terms so, minus m2 L1 L2 S2 so, theta 2 dot squared. This is m2 L1 L2 S2 theta 1 dot square. So, this is matching with your three previous derivations. But by looking this matrix what one can see the diagonal is 0. This is matching with a skew symmetric the first criteria and off diagonal matrix is just you can say transpose of that you can see our conjugate term.

So, you can see like this is minus m2 L1 L2 S2 but here you can see. So, in the sense if you take a transpose of C of q. So, this would be minus of C of q that is what matching in the sense this is skew symmetric matrix you have seen that.

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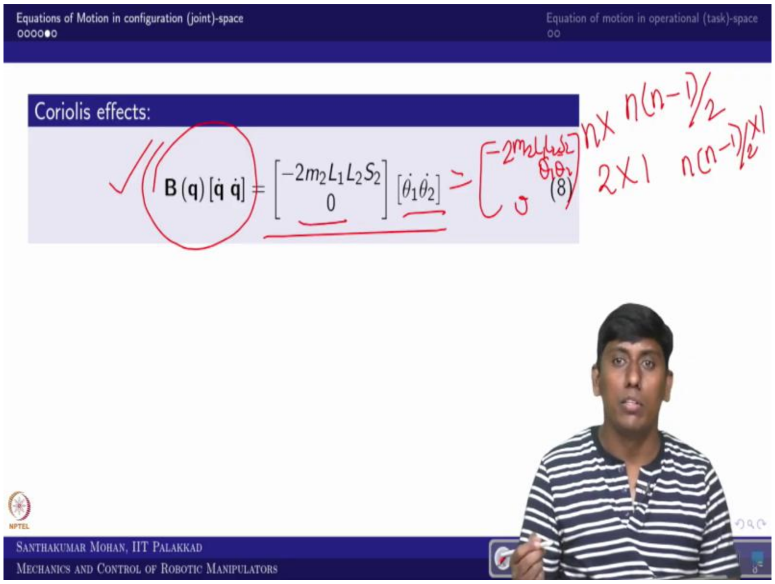

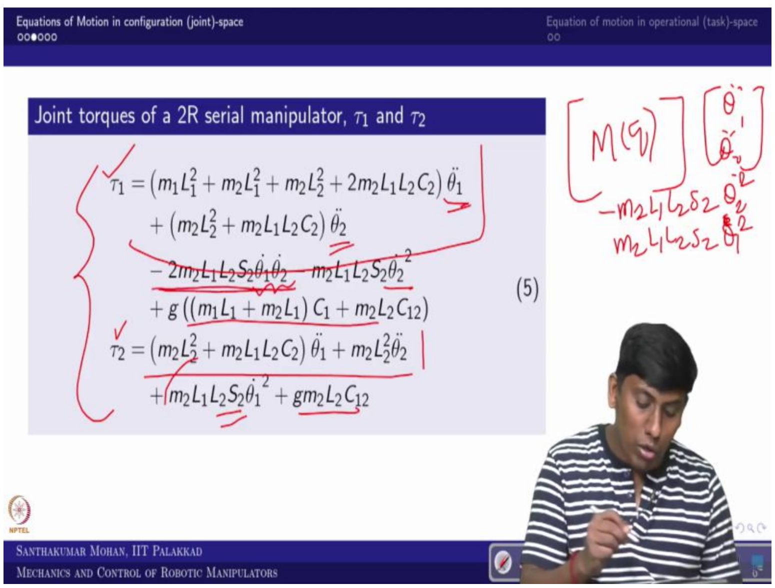

So, now, we will go the next one. So, this is not having any specific property but what you can see that the Coriolis effect can be written in this form. So, as I already said n into n minus 1 whole divided by 2 is the size of the vector. So, now, this matrix is the n into n of n minus 1 whole divide by 2 here n is 2 so, 2 cross so, 2 into 1. So, in the sense this is 2 cross 1. This is 2 row and 1 column. And similarly, the other one is n to n minus 1 by 2 cross 1. So, in the sense it is 1 cross 1. So, this is what we have seen as the Coriolis effect.

So, if you a take a product so, you can see that the first term would be minus 2 m2 L1 L2 S2 theta 1 dot theta 2 dot. The second term is 0 that is what coming into this also you can see. So, only the tau 1 is having a Coriolis effect and tau 2 is having 0. So, that is what we are also getting in this you can say effect right. So, now coming to the last one. What is the last one the gravity effect?

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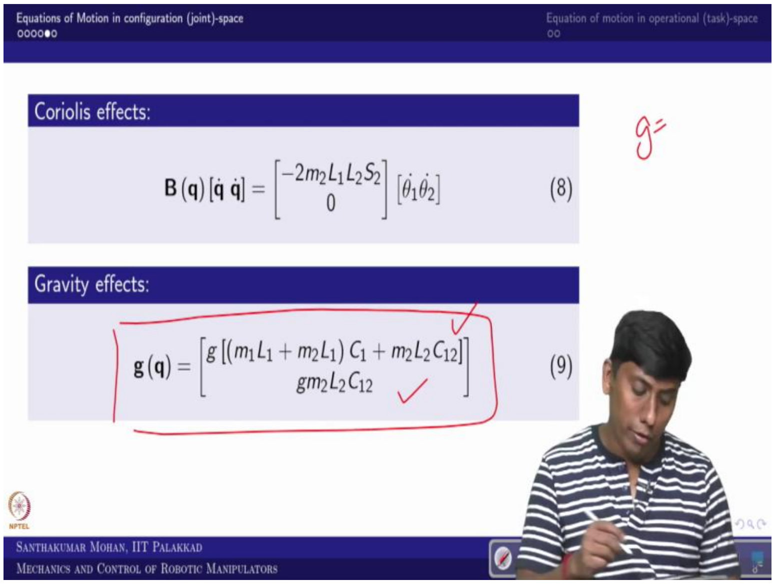

The gravity effect you want g can be brought in out or you can keep a g, but this is the two terms which we obtained in tau 1 and tau 2. So, now, this is the gravity vector, and you can substitute g is minus 9.81 or plus 9.81 based on what direction you will derive your equation of motion. So, this is the basic idea about what you call the equation of motion in configuration space. So, let us move further.

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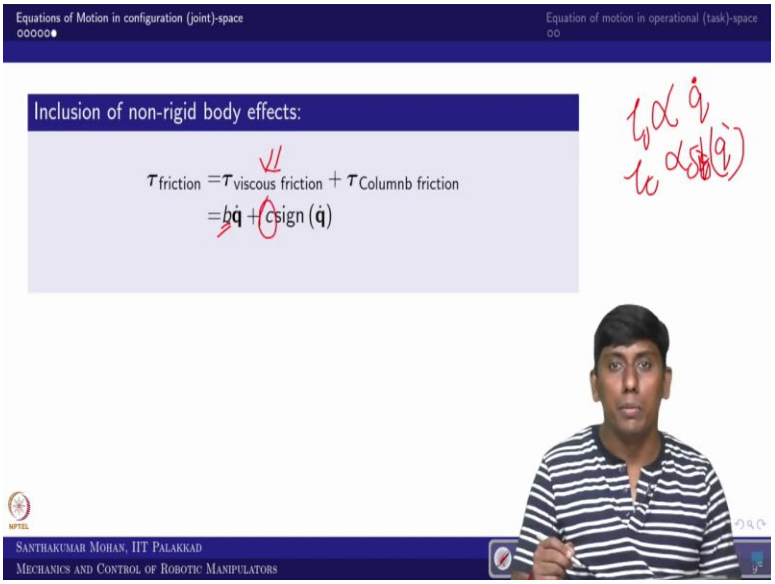

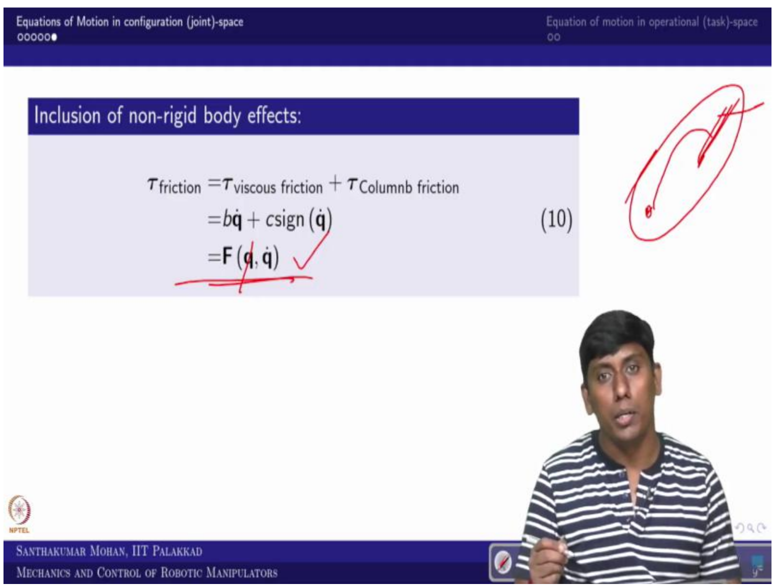

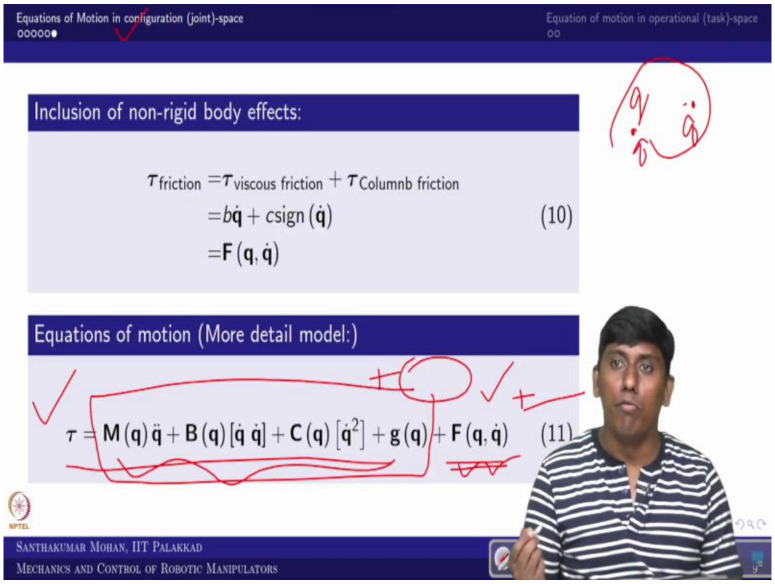

So, you can see if you want to include the non rigid body effect. So, here what we are calling non rigid body effect is the friction, the friction would be usually having three frictions. So, we usually consider viscous, Coulomb, and static friction but we assume that the static or you can say kinematic friction is not important. So, then the static only Coulomb is coming and viscous is a dynamic is coming.

In the sense you can write this tau friction term as additional to other terms even these two additional terms can brought into a coefficient and further quality. In the sense so, you can say stage space vector. So, this viscous friction is what? So, the tau viscous friction directly proportional to the velocity. So, in the sense if the proportionality constant you bring it that proportionality constant I brought as a B.

So, the Coulomb friction is similar the tau this is called a viscous the coulomb friction directly proportional to the sign of the q dot. So, then you can see this is what coming so, now the Coulomb friction coefficient also we brought in. So, now, this is the frictional term, if you rewrite that in a matrix or vector form, I am writing as a capital F as the frictional effect. So, that would be written as q comma q dot.

So, in fact this q is not really necessary but some time you may look the friction is happening somewhere else for example, you have the joint is here but there is a friction is happening because of some other term. Then you had to bring that q so, that is why we have generalized the frictional effect F of q comma q dot. So, now, based on these all if you rewrite the more detailed model, the frictional term is added.

Now, these are all our rigid body effect, and this is non rigid body effect. So, now if you assume that this is at like moving in a space or it is an aerial case and all then even you can add the non-rigid body effect as you can say aerodynamic effect then you can add that. If this manipulator is moving inside the you can see submersible water or some other fluid, then you will get a buoyancy force that you can add. But right now, it is ground based normal serial manipulator, we can assume these all rigid body effect and this is the non-rigid body effect.

So, now, this is what we have seen that the equation of motion in joint space. Why we say joint space? So, q we call joint space state vector so, q dot is the velocity of joint space vector and q double dot is acceleration of joint space vector. So, in the sense this equation of motion is written in the form of you can say joint space or you can say configuration space that is what you can say the equation of motion here. So, now, we are talking about equation of motion in operational space.

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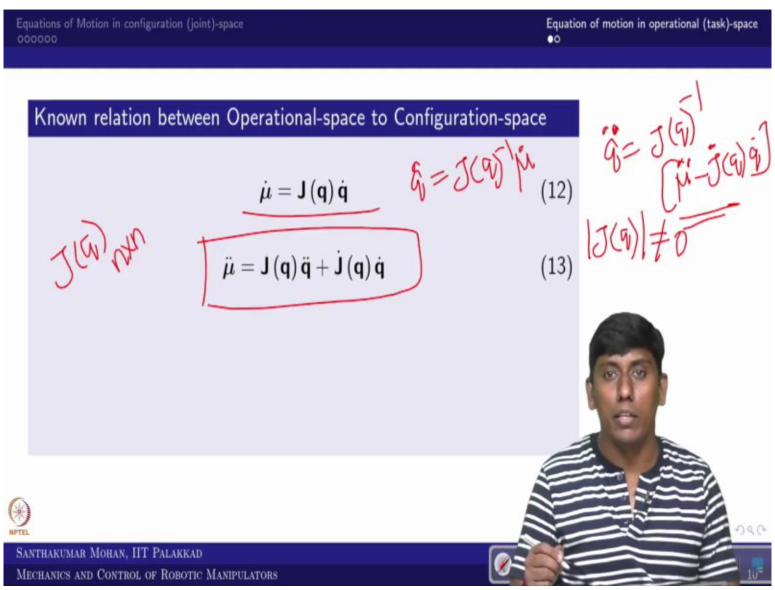

So, for that what we can see. So, what are the relations we know? We know mu dot as J of q into q dot. So that equation I can take. Further what I know the tau I can write as J of q transpose into f. So, this we have derived. So, these two equations we are going to use it. So, this is the differential kinematic equation that to like forward differential kinematic equation. Then by differentiate further I will get mu double dot in this form.

So, from there I can take so, q double dot as you can write J of q inverse into mu double dot. So, minus J dot q into q dot is coming as q double dot. So, what we derived in the previous equation? So, in the previous equation we derived a q double dot we can substitute that into mu double dot term. Further what we know so, the q dot I can again write as so, J of q inverse into mu dot. So, now only one consideration is there that mod, or you can say determinant of this should be nonzero.

Further, if it is J of q is a square matrix, we are fine if it is non square then you are to take J you can say pseudo inverse. So, that we have already seen in the differential kinematics. So, now this equation we have.

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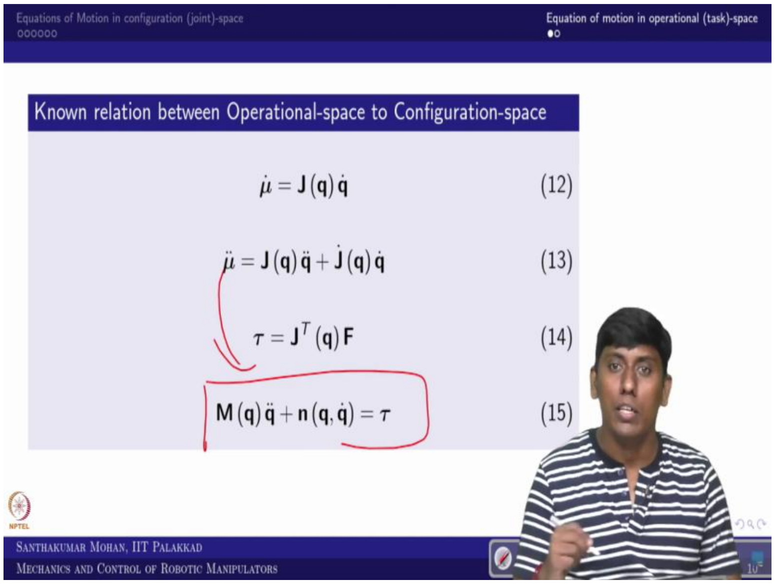

And this equation we already know from the statics so, now we can use this again so, tau I can substitute this and the q double dot I can substitute in terms of mu double dot and mu dot. So, that is what we are actually taking this equation and going to substitute in this form. I am not going to take more detail equation I am taking a very simple equation which is easy for me to derive.

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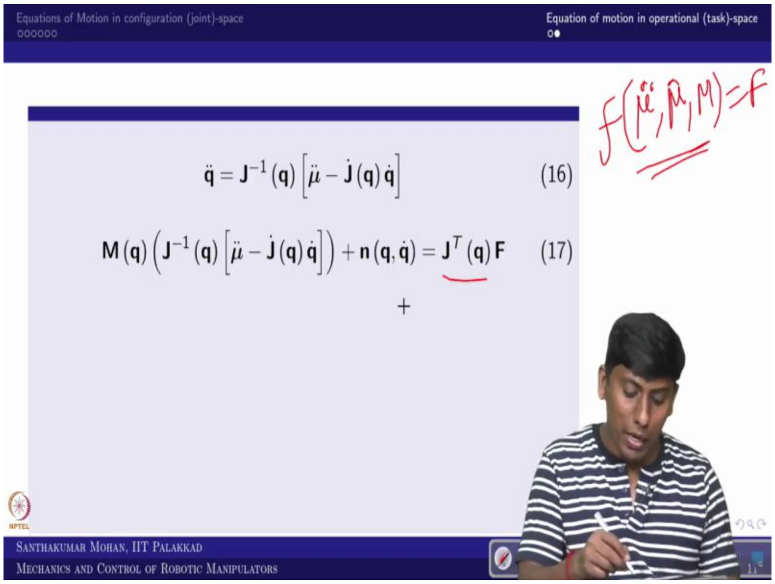

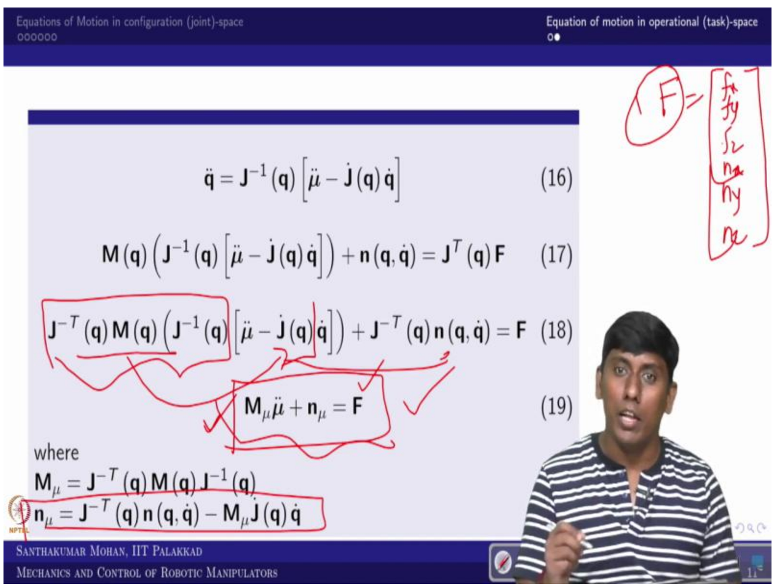

So, in the sense I have returned the q double dot in the form of mu double dot even here I bring it q dot as J of q inverse into mu dot. Then you can see that the q double dot is in the form of mu double dot and mu dot. So, right now I do not want but I want to write it in mu double dot form. So, in that sense, I can write this equation in this. So, and I know this but what I wanted I wanted mu double dot function of this as F.

If I want to write this so, I can multiply and bring all the terms. So then, you can see like this is what all coming into a picture. So, now I make it separation. So, my m of mu is these three terms together. And n of mu is this term multiply with this and this term all together. You can see that is what we have derived. So, now you can see like the equation of motion we can write it even in the task space.

Now, this F would be the end effector forces and moment. So, we call fx fy fz and nx ny and nz. So, this is what you call the F vector. So, in the sense at the end what is the effect that I know I can actually propagate the model and then do it. So, this is the whole idea. So, now, what we have come across in this particular lecture. We have seen so what is the equation of motion that equation of motion can be written in two different spaces.

One is the operational space that is what we have derived here but more convenient is actually configurational or you call joint space. So, that is what we derived in the equation of motion. Then we can rewrite in more detail form. So, in that sense, so, this particular lecture is giving a initial push. So, how we can actually go for in real cases. So, based on this you can give your F or tau and then you can see your system simulation. And then you can actually design your entire system.

So, before going to see that in detail, let me show how to derive the equation of motion either in Newton Euler or Lagrangian Euler through the help of MATLAB. Then we will move to this form and make it as a numerical simulation in after this you can say demonstration in MATLAB. So, the next class we would be talking about so, how to derive the equation of motion for a given example in MATLAB that is what we are going to see. So, I hope this is useful we will see in next class with MATLAB simulation. Thank you. See you then bye.