Mechanics and Control of Robotic Manipulators
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Lecture No. 24
Newton-Euler Method

Hi, I am really happy to welcome you again for Mechanics and Control of Robotic Manipulator. So, so far, we have seen up to motion dynamics. So, in specific we have seen one such method called Lagrangian Euler. So, this particular lecture we are going to talk about what you call Newton Euler Method. So, although this method is very popular in rigid body or even vehicle dynamics and all. We will see a slightly different version here because it is iterative one here. So, how we can employ that into a serial manipulator, that is what the focus of this particular lecture.

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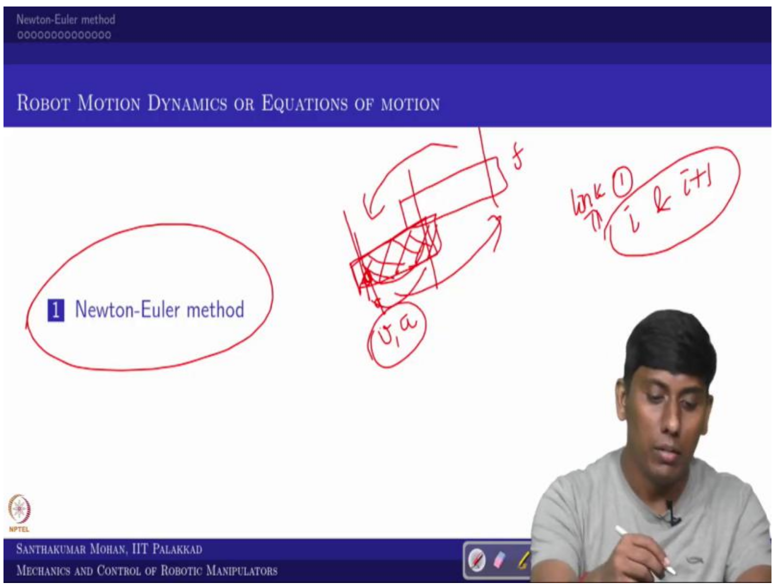

So, let us see that how we are doing it. So, in this sense straight away we are going to talk about Newton Euler method, this Newton Euler method is a slightly a tricky one. So, what that means, so, you have multiple bodies, so we will be talking about joints. So, in the sense we will go your velocity acceleration all from the base to end and then we will talk about the forces from the you can see end to base and while going you can see that the mass is part of link.

So, the link i is having you can say i axis and i plus 1 axis. So, in the sense we are not talking about the inertia on a joint. So, please be clear, the inertial forces are due to the link or the mass which is pertaining. So, that we will be clear, clarifying and then we will go the velocity propagation and acceleration propagation from one joint to another joint. So, in the sense we will be doing three stage process. So, that three stage process is what, Newton Euler method call iterative one or you can say iterative Newton Euler method.

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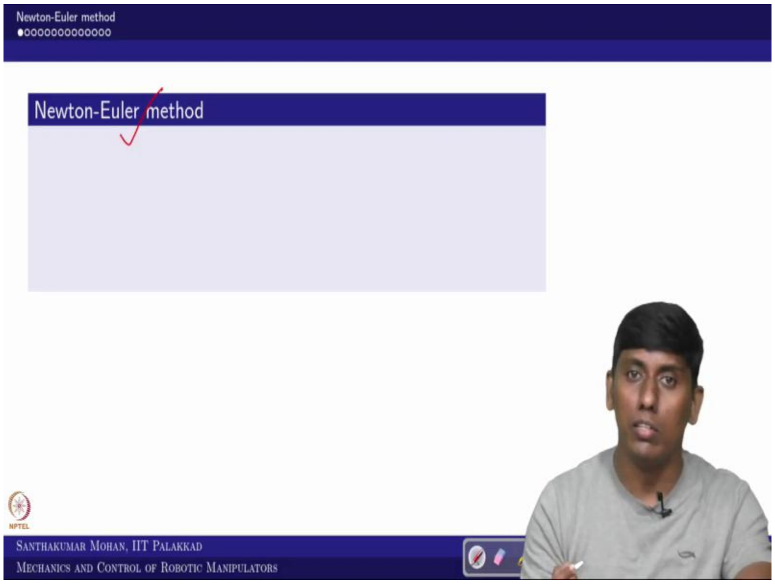

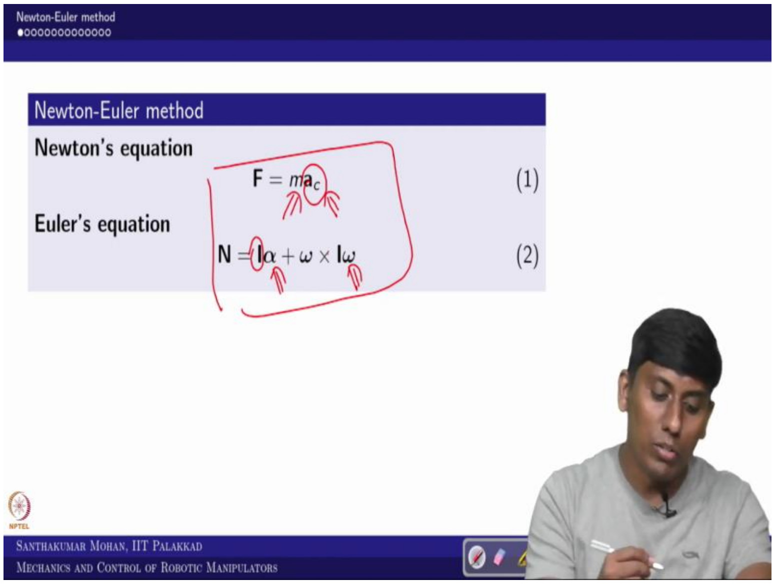

So, now we will go the generalized Newton Euler, so where this is based on the linear momentum and angular momentum. The angular momentum would brought from the Euler equation and the linear momentum brought by the Newton's equation. So, now the Newton equation and the you can say Euler equation, the simplified Euler equation we are going to use. So, where ma c, so here you can see this a c is the linear acceleration of the mass at centroid. So, where I is inertial tensor and omega is angular velocity and alpha is you can say angular acceleration and mass is the link mass.

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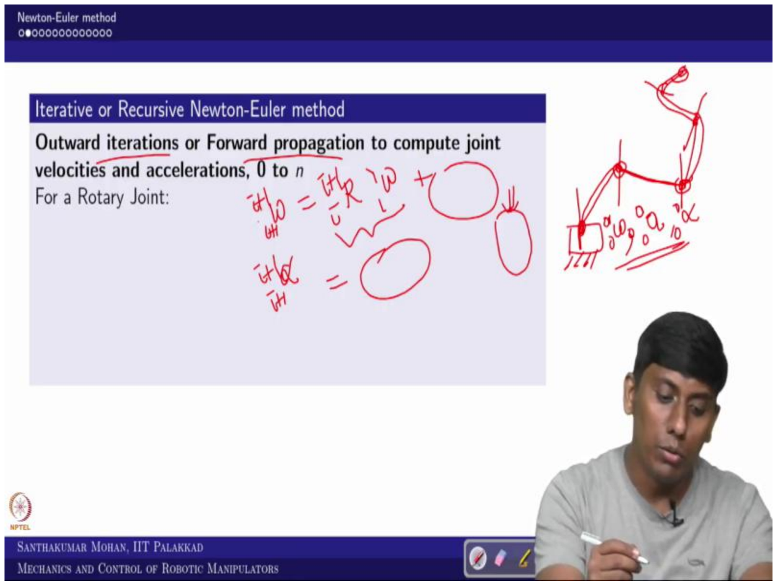

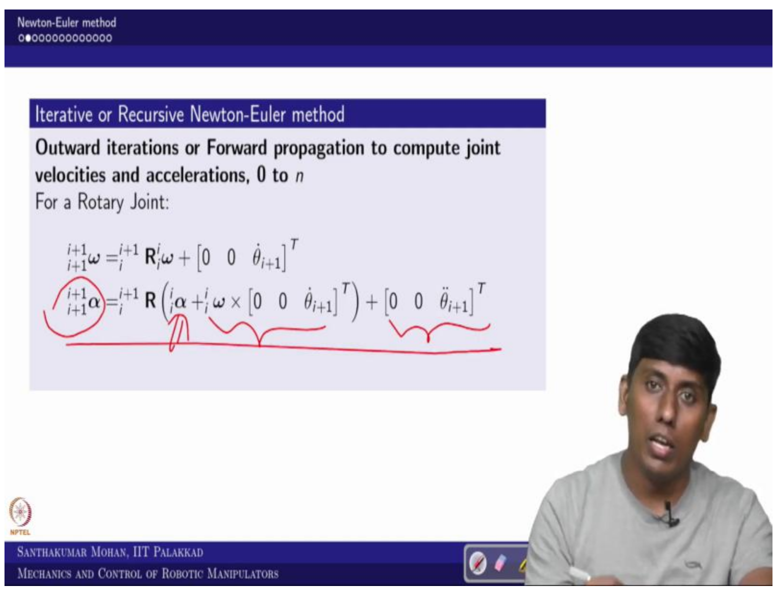

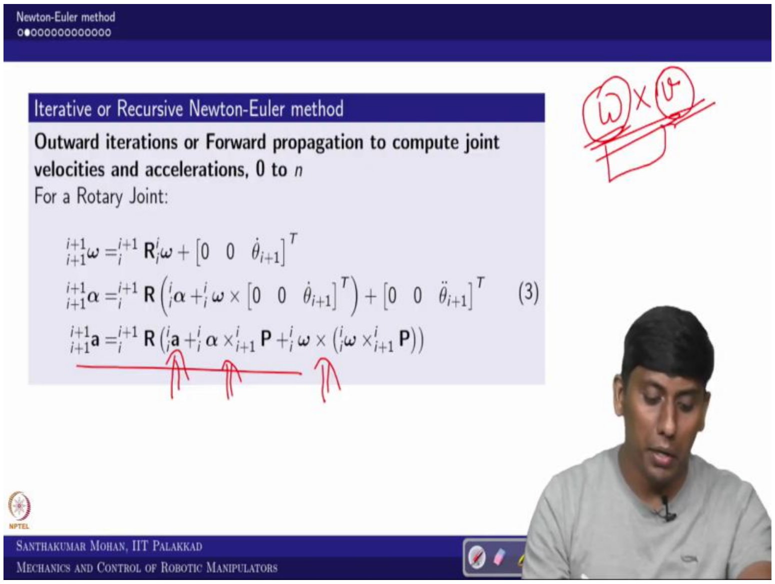

So now, based on this equation what one can try to see, we can make iterative one. So, here iterative is different. So, what that means, so we will start, this is the base. So, I am trying to show that these are number of bodies. So, we can actually see that the base velocities, so angular velocity or you can say acceleration all alpha also 0, all are zeros. So, these all 0, so in the sense from here, I can go first frame second frame, third frame, and fourth frame, fifth frame to n frame in the sense what I can do, I can do outward iteration or forward propagation to compute joint velocities.

So, corresponding joint velocities I can calculate with the help of whatever the equation which we derived. In the sense omega i plus 1 2 i plus 1 we know like already it is R i plus 1 2 i omega i if it is a rotary joint, it would be added if it is a prismatic joint not. So, similarly alpha also you get it.

So, here what addition will be there, this is angular velocity the similar way angular acceleration component of previous joint would be there. Further, the current angular velocity or in the sense current active joint angular velocity which will be interact with the previous joint angular velocity because it is the same part of link. So, that would give a gyroscopic effect. So, that would be added.

So, in the sense what we are trying to do, we are trying to compute the; you can say angular velocity and as well as angular acceleration. So, you can see the angular acceleration previous joint angular acceleration further, you can see the gyroscopic effort, in addition to that the active you can say joint acceleration. So, these three are coming into a picture. If it is linear, the linear acceleration which we are interested is this. So, which would be you can see one with the come from the you can say slip acceleration of previous, then you can say tangential, then the radial which we call centripetal.

Some people even uses the Coriolis, the Coriolis will happen only if you have an interaction this. So, in the sense you have angular velocity and the linear velocity active in the sense this is having and there is an active linear joint, then this Coriolis will come otherwise there is no Coriolis component.

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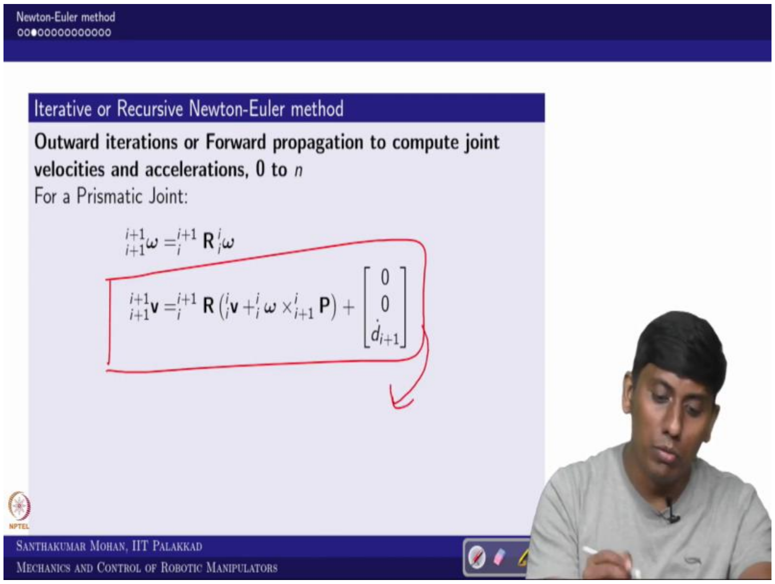

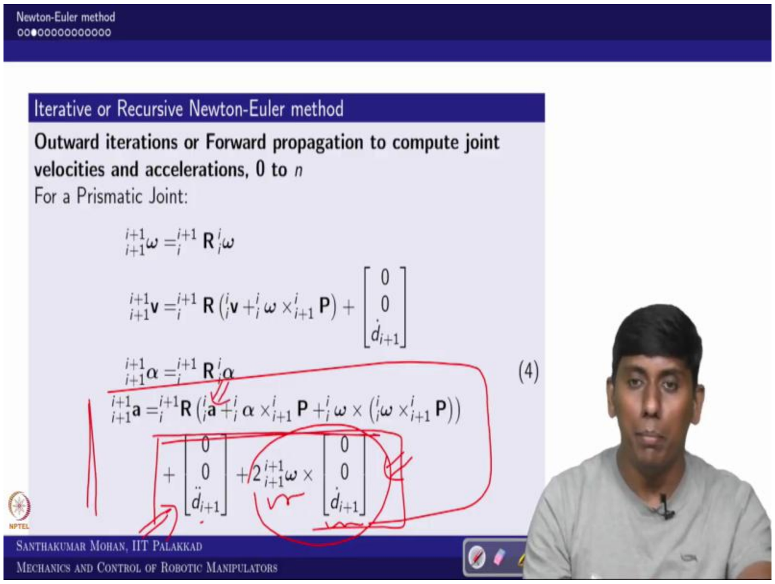

So, that is what we are trying to show the same thing I am writing in the prismatic joint. You can say prismatic joint the angular velocity is very simple. Similarly, angular acceleration also like would be simple, but the velocity component you can recall because this would be useful further on. So, this is the angular acceleration part, and this is the linear acceleration part.

The linear acceleration part you can see the slip acceleration added which is slip on the joint and you can see the Coriolis acceleration. The Coriolis acceleration is omega of i plus 1 to i plus 1 and d dot i plus 1 but this is d double dot. So, you please make it this in your mind. So, if it is a linear joint there would be two additional components will come further to what you can say the previous joint accelerations all play in a picture.

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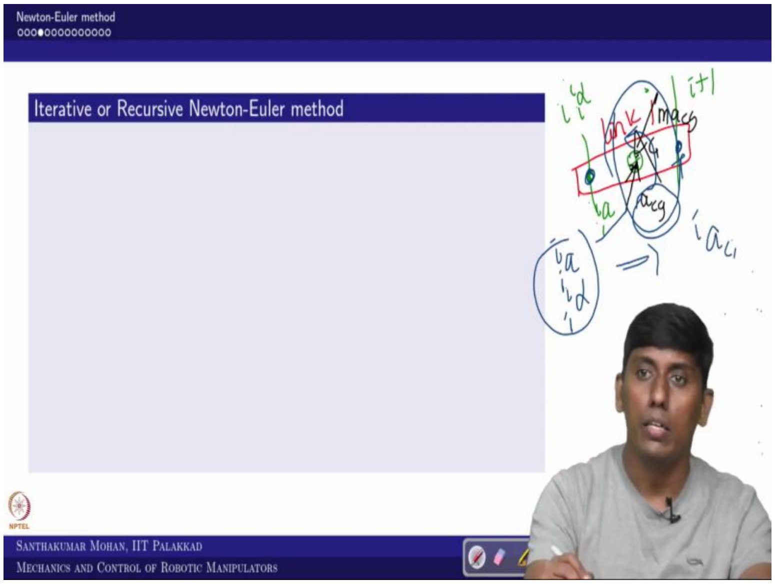

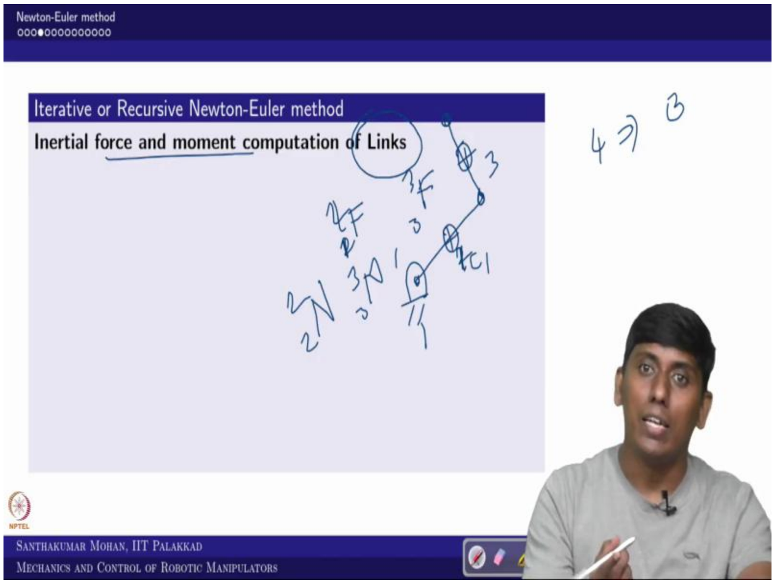

So now, in that sense, what one can do, for example now, I take a link i. So, link i would be having you can say ith access and i plus 1 access, so I can calculate the angular acceleration and linear acceleration, but the mass would be concentrated somewhere. So, I assume that this is mass concentrated as a CG. So, here only the inertia force would act. So, if you see the acceleration is happening here like this.

So, what would be the force acting here the force would be acting, so just opposite to this. So, m a cg would be acting. So, similarly alpha so that you can say the torque would be in this way. So, this is what the inertial torque, so these two we need to calculate. So now, what we are doing, we are not doing with the joint we are doing work link. So, in the sense what you need to know, you need to know the link inertial force and moments for that you need to know the link centroidal acceleration.

So, for that what one can do you can go from any one of the joint the prefered is forward propagation. So, once you know a i to i alpha i to i, so you can go to a c i to i. So, that is what we are trying to do. So, you can see inertial force and moment computation of link. So far what we have calculated of joints, please be clear. So, this is of link. So, if you have 4 joints, so how many links would be there only 3 links would be there because including ground floor. So, in the middle there would be 3 links.

So, for example, you take a 2 R serial manipulator. So, how many you can say link, so the ground, so 2. But in the sense, there are 2 moving links, the center centroid would be playing these links, in the sense they link inertial force and moments would be 2 components. So, F you have you can say in this case, F 2 2 and F 3 3 and N 2 2 and N 3 3 that is all will come, but how you are denoting that is different. So, we denote this one. So, this is one c. So, in the sense this is 1 that way you can make it.

Thisn is we will come the representation or how you are approaching but otherwise 1 supposed to know we are calculating inertia forces and moments offer link, where the acceleration we have calculated till the previous slide of you can say joints. So, this is joint velocity and acceleration, because several of time the students always confused. Sir, why you are calculating additionally 1 more velocity or 1 more acceleration? This is just because you are having a link which is consists of 2 joints, that is what the idea.

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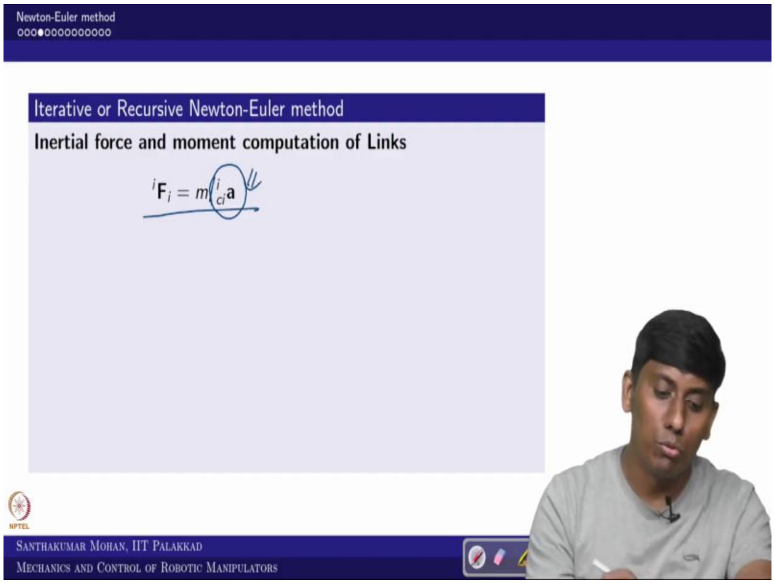

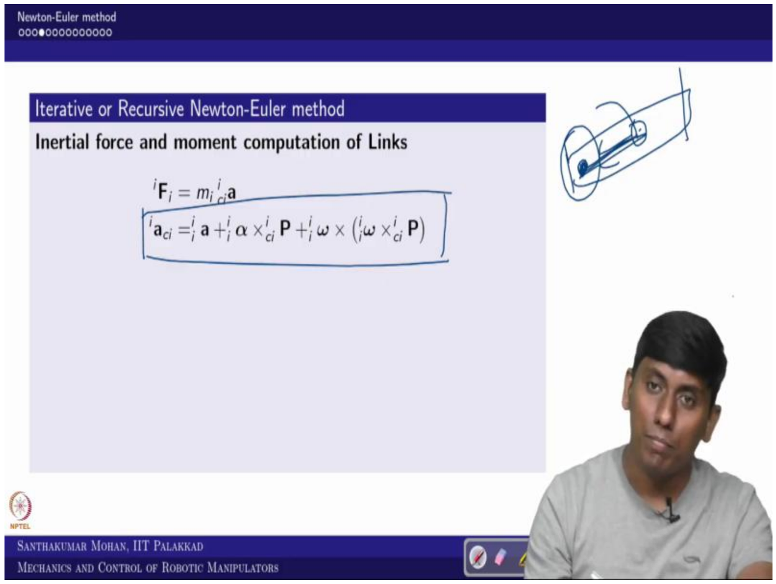
Newton-Euler method
Iterative or Recursive Newton-Euler method
Inertial force and moment computation of Links
${}^{i}\mathbf{F}_i = m_i\, {}^{i}_{ci}\mathbf{a}$
${}^{i}\mathbf{a}_{ci} = {}^{i}_{i}\mathbf{a} + {}^{i}_{i}\alpha \times {}^{i}_{ci}\mathbf{P} + {}^{i}_{i}\omega \times \left({}^{i}_{i}\omega \times {}^{i}_{ci}\mathbf{P}\right)$
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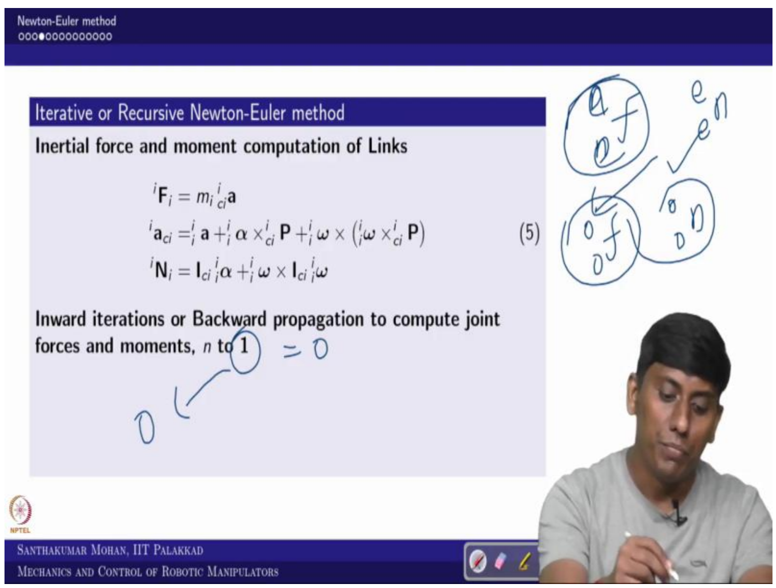
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${}^{i}\mathbf{N}_i = \mathbf{I}_{ci}\, {}^{i}_{i}\alpha + {}^{i}_{i}\omega \times \mathbf{I}_{ci}\, {}^{i}_{i}\omega$
Inward iterations or Backward propagation to compute joint forces and moments, n to 1
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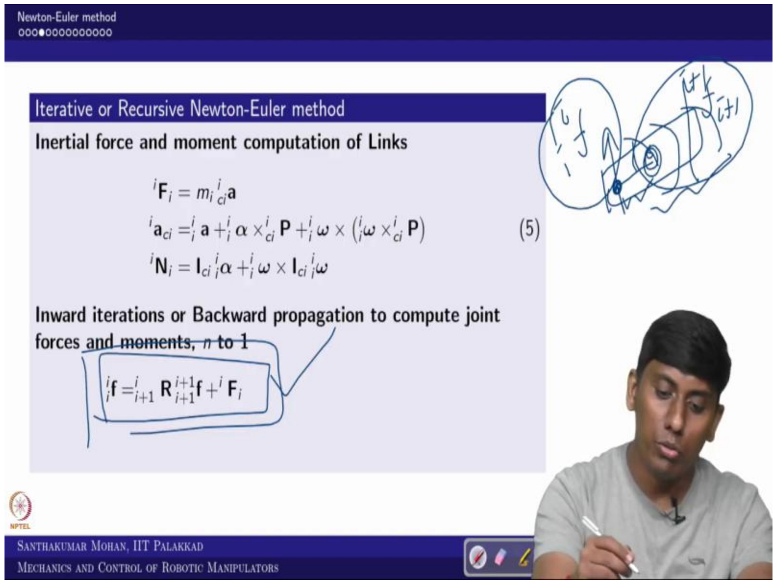

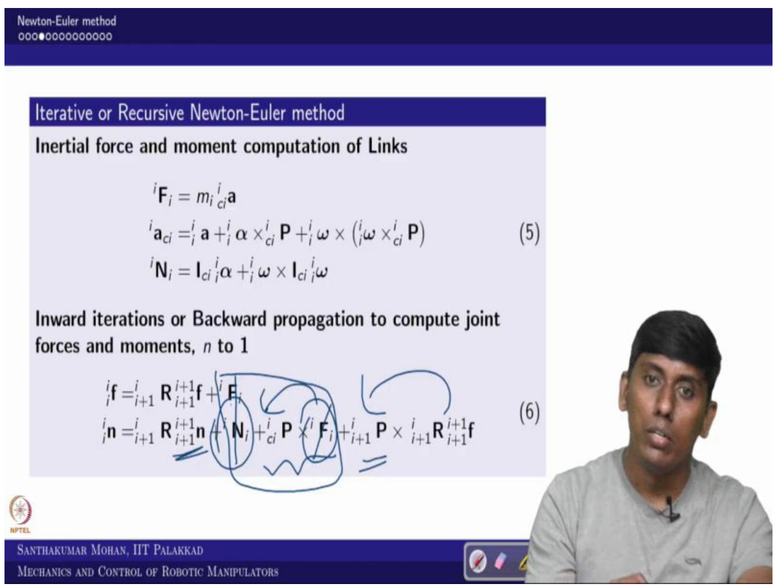

We will see an example, which is very much clear to you. So now you can see the inertial force, which we need to have the you can say centroidal acceleration, linear acceleration you should know. Similarly, you can calculate what we did the previous because now it is again already you calculated your acceleration here. So, you are going here. So, then you can see there would be a tangential acceleration, there would be a radial and there would be a same acceleration would go across.

Since these two are in the same frame, so you no need to do any transformation that is why you can see there is no you can say rotation matrix on here. So, the next one is you need to calculate the; what you call angular moment at the inertial location. So, you can see the inertia tensor at centroid you calculate, then you can see that angular velocity and angular acceleration, you will calculate these two forces and moments.

So, once you do this, so what we can do, you can come back from n because the end effector the forces and the end effector you can say I put e the end effector moments are known to us. So, then we can come backward up to 1 because 1 is the active joint up to 1 only active joints would be there. If you come to 0, that would be the base you can just support force and moment in addition to that, what this base force we simply call shaking forces. So, whatever is transmitted to the base, whatever transmitted as a moment to the base, that would be shaking moments.

So, if you actually go to 0 that, so f 00 and the n 00 that would be shaking force and moment. Right now, we are bothering about only motion dynamics will do it. One simple advantage I want to tell here. So, the comparison between Lagrangian Euler and Newton Euler, the Newton Euler would do all 3 you can say access force component and moment component. So, in that sense, when you want to do the structural dynamics or you want to do a design, so these equations would be beneficial.

So, that is why we are always encouraged to use Newton Euler rather than the Lagrangian Euler. But we will be seeing in the next lecture, what is the comparison between these two, but any how you can get this idea. So, now, this is what the equation, which is we have, because when you talk about this is the link. So, this link would be having your fi because ith the frame, this is f i plus 1 to i plus 1, but this having inertial force. Now when you see this point, if you assume that this is a dynamic equilibrium or equilibrium, then these two forces sum supposed to be equal to this. So, that is what we are writing here.

The same way if you talk about moment, so there is a moment here. So, that moment would be having you can say couple happening here, that also will come. So, that is what you can see. So, this is from the you can say forth going joint or you can say the joint which is in the front, and this is a centroidal force is how much couple it will generate. Similarly, they end effector force, how much couple it can produce, and this is the inertial moment.

So now, if you do not have inertial efforts. So, if you do not have inertial effort, it is a static equilibrium. Now, if you consider that would be the dynamic case. Now, you are clear. So, the inertial forces if you include that become dynamic case, if you ignore that would be a static case, it is very straightforward.

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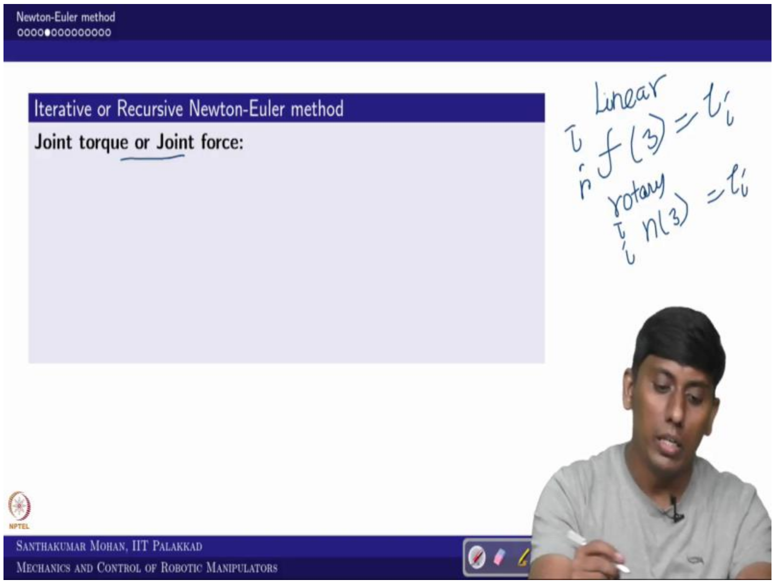

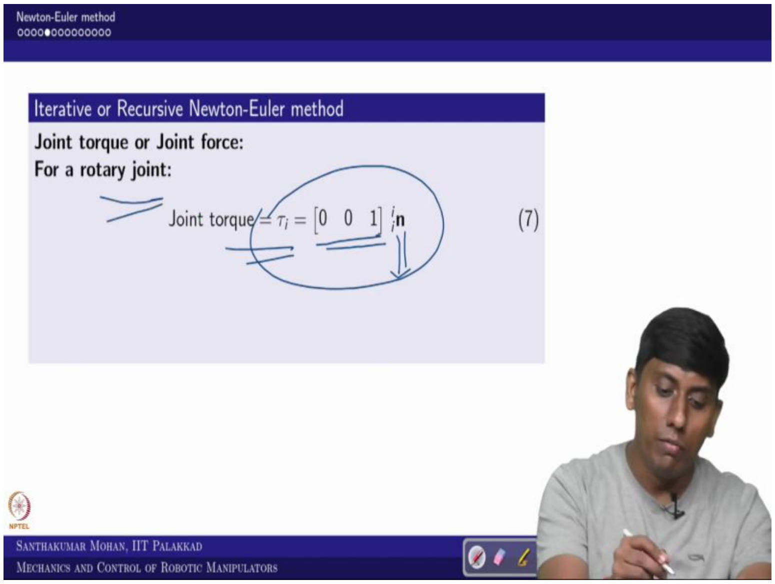

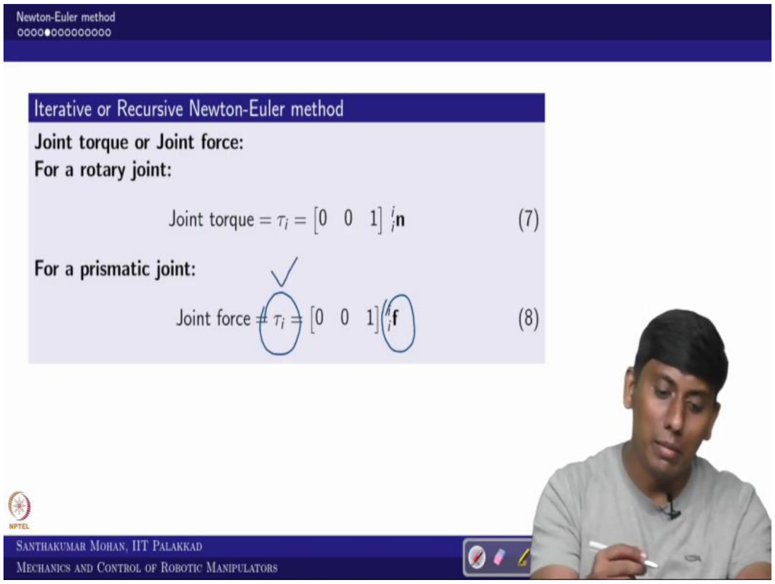

So, now, we will try to calculate, so what will be the joint torque or joint force, if it is a linear joint, so then the f corresponding to you can say that ith joint, the third quantity in the sense z aces component would be your tau i, if it is a rotary so then the n of 3 the third component of i would be your tau i. That is what I am writing in, you can say proper vector form. So, you can see it this.

So now, this is you can say a row vector, and this is actually a column vector. So, now if you multiply that would be coming the third component which is given us a torque, which is a rotary joint. If it is a prismatic joint, so that would be the force vector. So, then this would be a joint force, the third axis or the third component of you can say force vector would be your tau.

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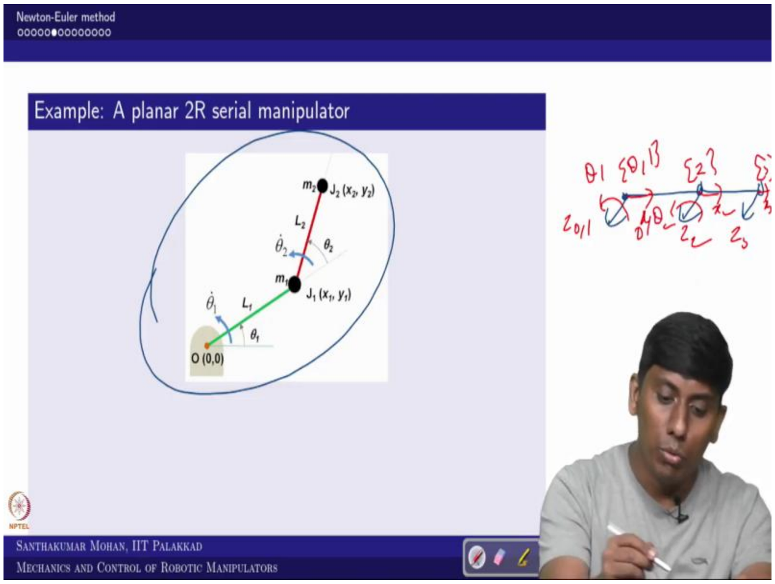

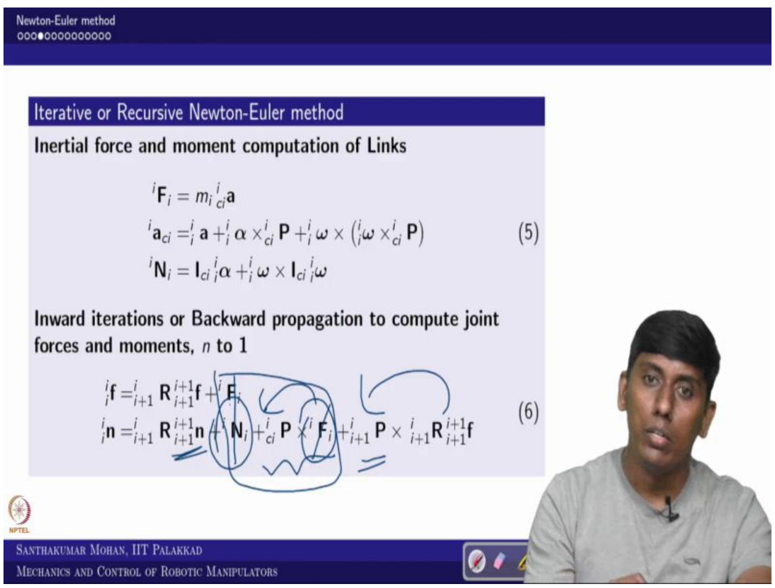

So now, we will see one simple example. Then we will close this particular lecture. For that we have taken the same picture which we have shown in the previous lecture as Newton, you can say Lagrangian Euler, the same thing we are using for Newton Euler also, only thing this is, you are to fix the frame. So, I assume that this is the frame arrangement. So, I am keeping it. This is z 0, 1. So, this is theta 1 and this is theta 2, z2 and this is z3, this is 2 and this is 3 and this is 0 and 1.

So, now what one can see where the x axis goes, this is x1 and 0 and this is the x2 and this is x3. So, in that sense, so in the sense of what one can find, you can find the, what you call the DH parameter, so, the DH parameter I just to show it here for your purpose because last time I have drawn which is gone out. So, this is z 0, 1 and this is z2 and this is z3. So, I am just showing it how this x 0, 1 goes this is x2 and this is x3. So now, this is theta 1 and this is theta 2. Right now, we assume theta 1 and theta 2 is 0, but that is active joint.

So, in the sense you can see 0 and 1 are same point in the sense both the distance So, both distance 0 and both are parallel, the angle is 0, but x 0 and 1 are parallel, but there is active to joint that is why theta 1 is coming, in the sense 0 plus theta 1. Second similarly, there is no angle between z1 and z2, but there is a distance along x1 that is the L1 and that is the active joint theta 2 and there is no distance along the z axis. So, these are 0. So, now you can see this is passive joints, so only L2 comes.

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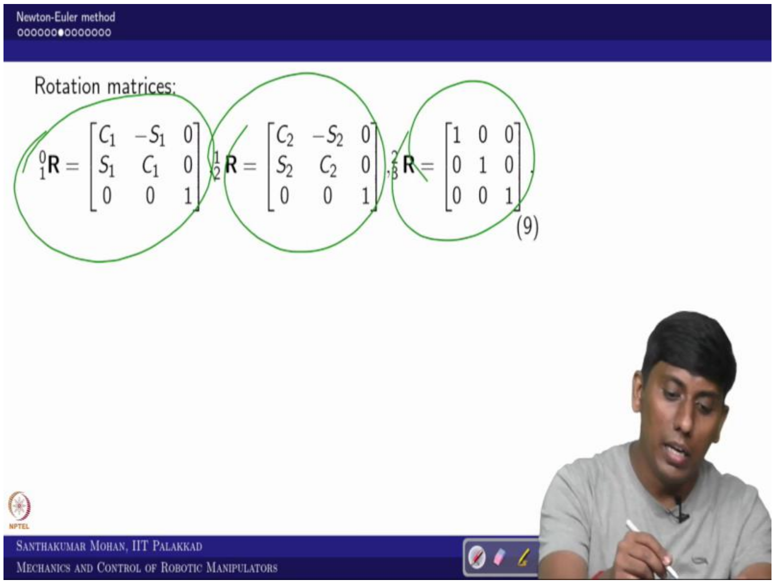


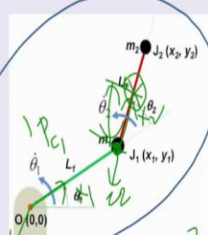
Rotation matrices:

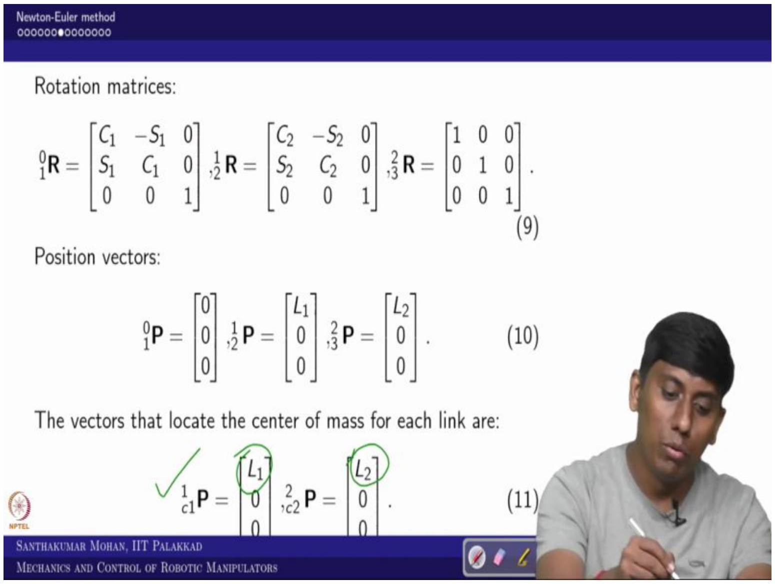
$$ {}^0_1\mathbf{R} = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^1_2\mathbf{R} = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, {}^2_3\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (9) $$

Position vectors:

$$ {}^0_1\mathbf{P} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, {}^1_2\mathbf{P} = \begin{bmatrix} L_1 \\ 0 \\ 0 \end{bmatrix}, {}^2_3\mathbf{P} = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}. \quad (10) $$

Example: A planar 2R serial manipulator





So now based on this what one can do, we can get the individual transformation matrix from there you can extract the rotation matrices. And you can extract position vectors. In addition to that, you need one more, what one more so you need to extract the centroidal location because you are trying to find out vci and aci. So, because you need to calculate the inertial you can say forces Fi. So, this aci you need to calculate for that you need to know the Pci. In this case, what would be the Pci.

So, you can see that this is along x. So, this is a mass concentrated, I will show it here. So, what is Pc 1 with respect to 1; 1 is here z1. So, this is the fastest centroidal location. So that is along x 1. So, in this and the Pc 1 with respect to 1 would be L1 0 0. So, Pc 2 with respect to 2, this is 2 z2. So, what is it this is x2, so then it is L2 0 0. So, please be clear. So now, for example, the centroid is coming here. So, then this is Lc 2. So, now, you have a triangular link like this, this is z 2 and you are centroid somewhere coming here.

So, then you can see like this is x axis and this is, this is your y axis. So, this is down. So, what would be the Pc in this case, so 2 2, so this would be x and minus y the value will say and 0. So, you be clear, so this is actually with respect to link. So now, that also we are actually trying to find out, so in this case it is Pc 1 would be L1 0 0, Pc 2 would be L2 0 0. In this case only two links, so two link centroidal we have calculated.

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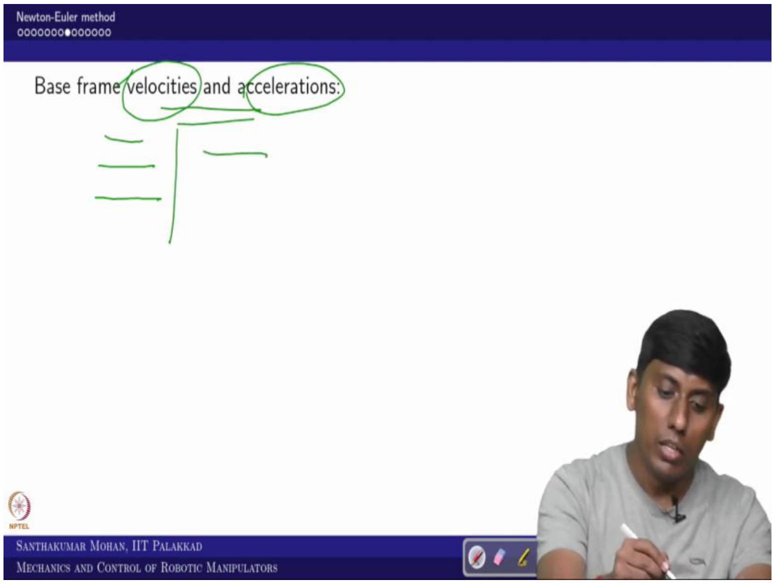

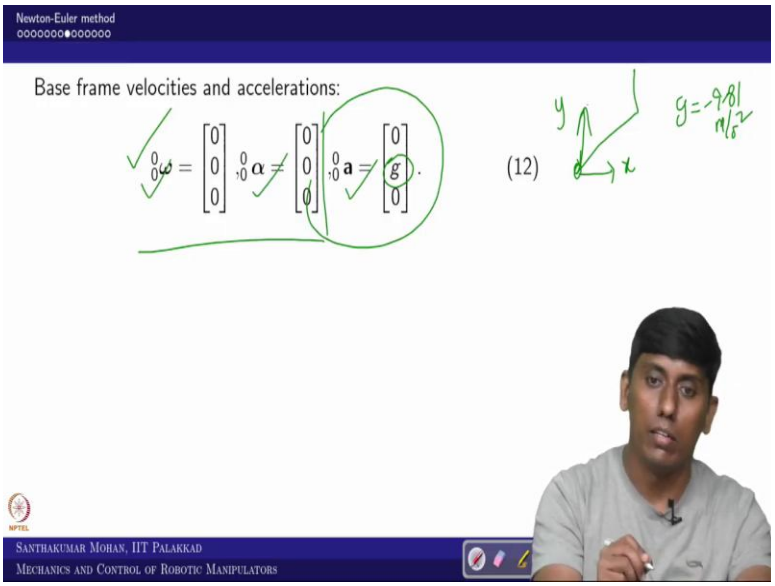

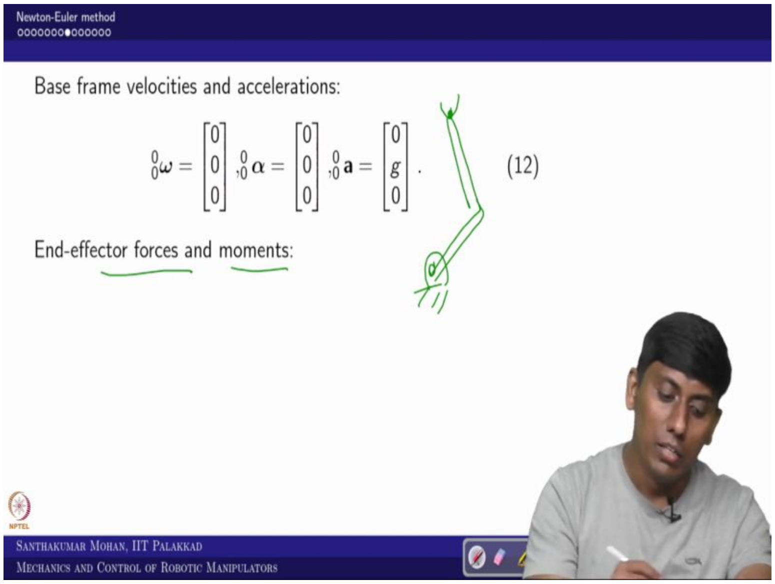

Once you calculate it, then we can go forward for the forward propagation, we calculate the angular velocity linear velocity and acceleration. In this case linear velocity is not important. So, we will calculate angular velocity, angular acceleration and linear acceleration further we will end with the linear acceleration at centroidal location. So, that is what we are trying to do.

So, far that we assume that the first case the linear and angular, angular velocity and angular acceleration 0 and the linear acceleration we have consider with gravity. So, in the sense this is your serial manipulator 2 R. The gravity is on this axis. So, which is y in addition to that what the g is minus 9.81 we are going to substitute meter per second squared. So, you are clear.

So, now this is the initial you can say frame, angular velocity angular acceleration and linear acceleration. Now, we will go for propagation. Further in order to make it simplification the end effector forces we are assume 0 and end effector moments also 0. In the sense it is a free end there is no body is considered, so only these 2 are there. So, it is a free end. So, there is no end effector force and moments acting.

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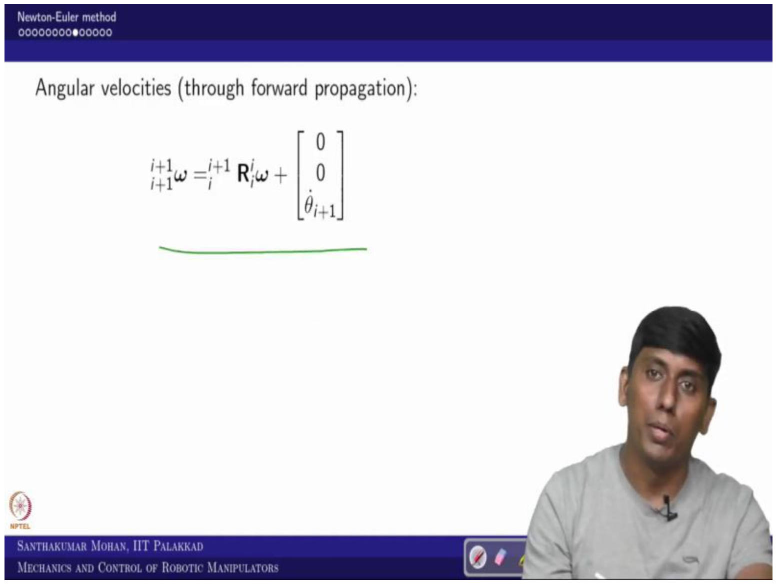

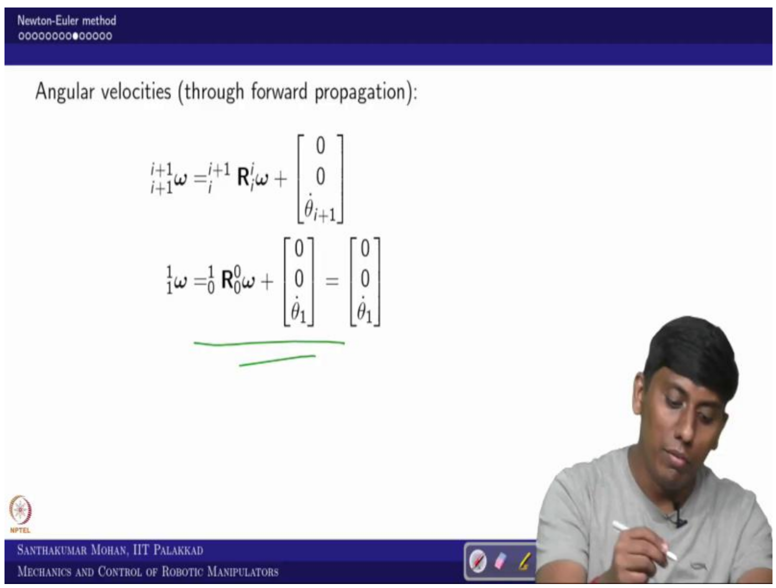

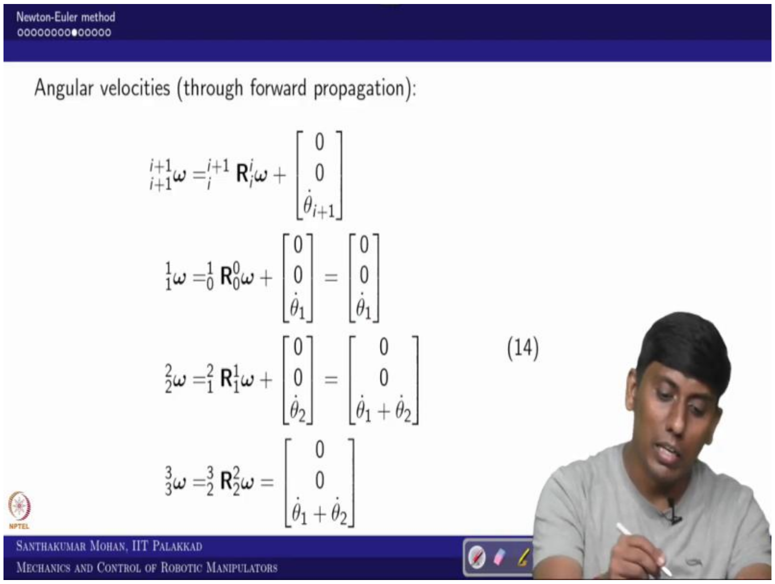

So now, in that sense we will go to the angular velocity propagation, which is we have done in the last two last class. So, this is what we obtained, and we will actually go to the second this is also we obtained and the third quantity we can calculate.

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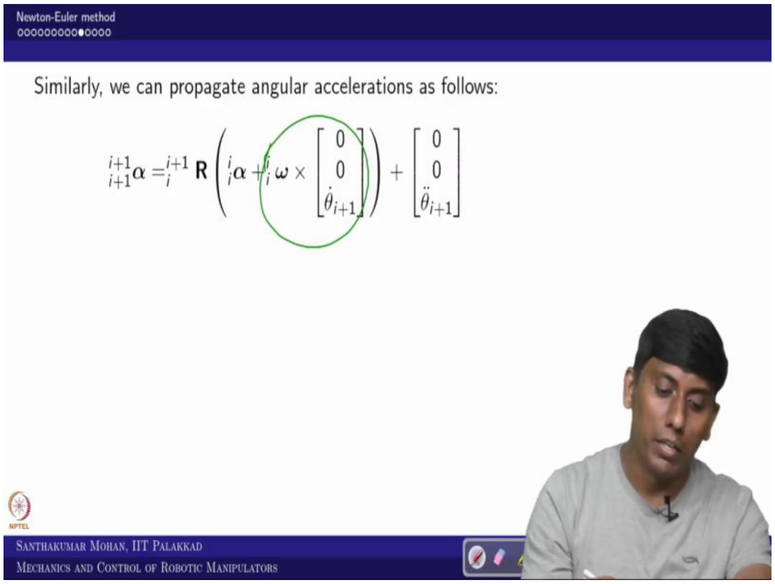

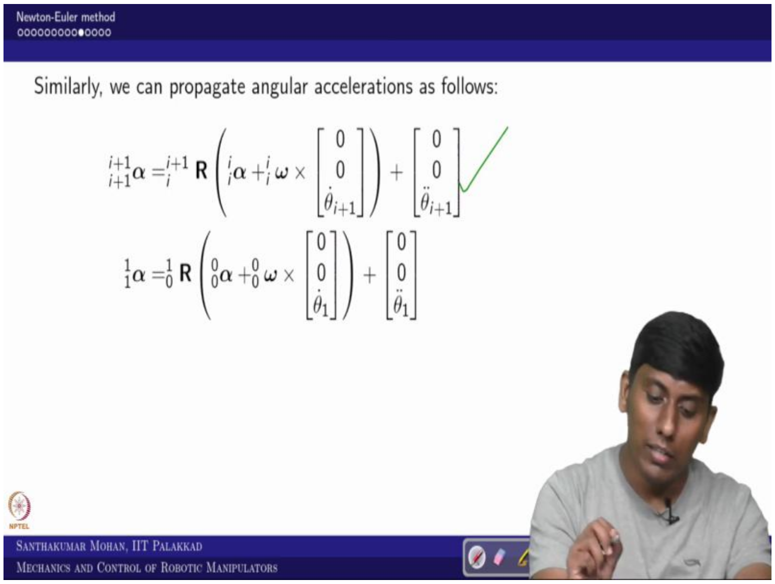

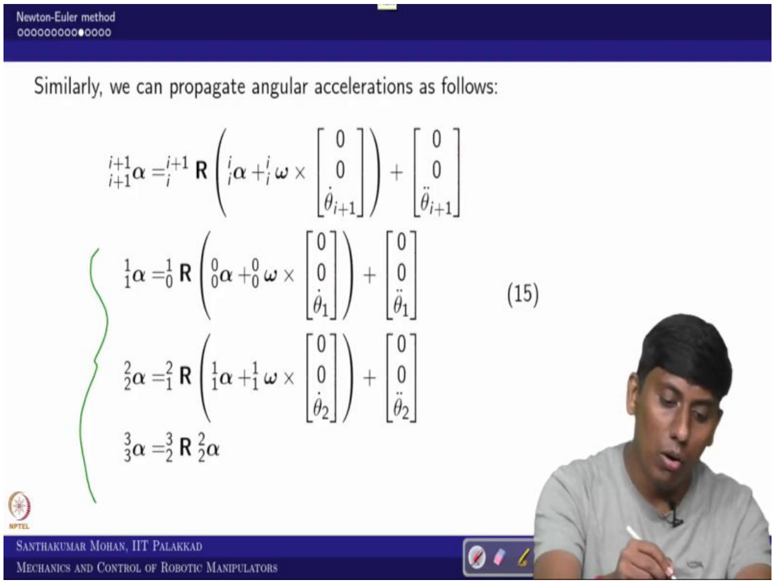

Similarly, we can calculate the angular acceleration where the gyroscopic effect is coming. So, then you can see the slip, or you can say that corresponding angular acceleration component is coming. So, you can calculate, so all three.

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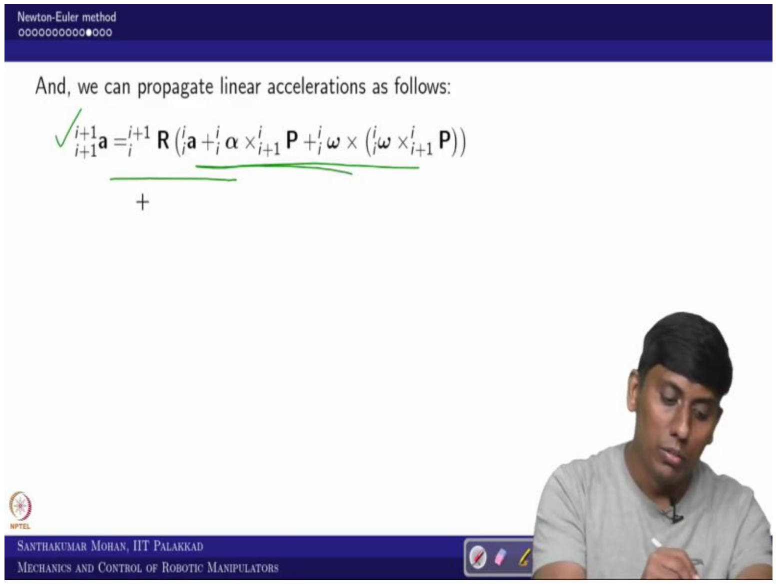

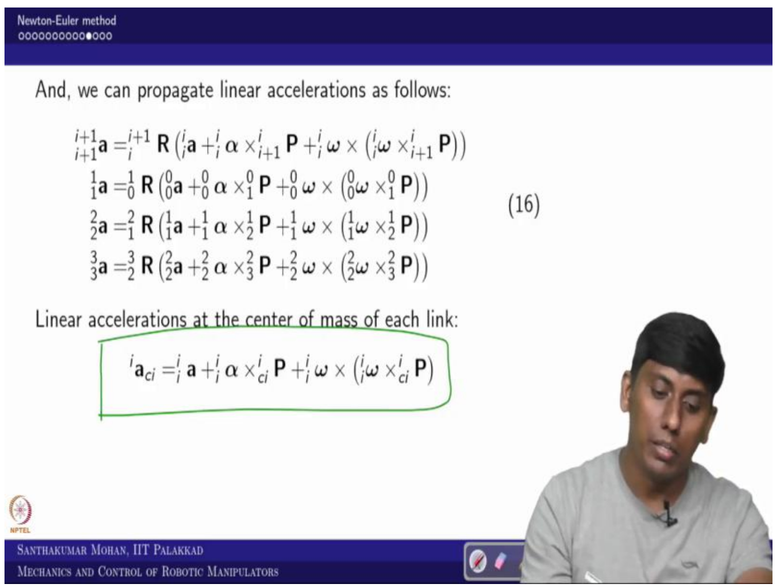

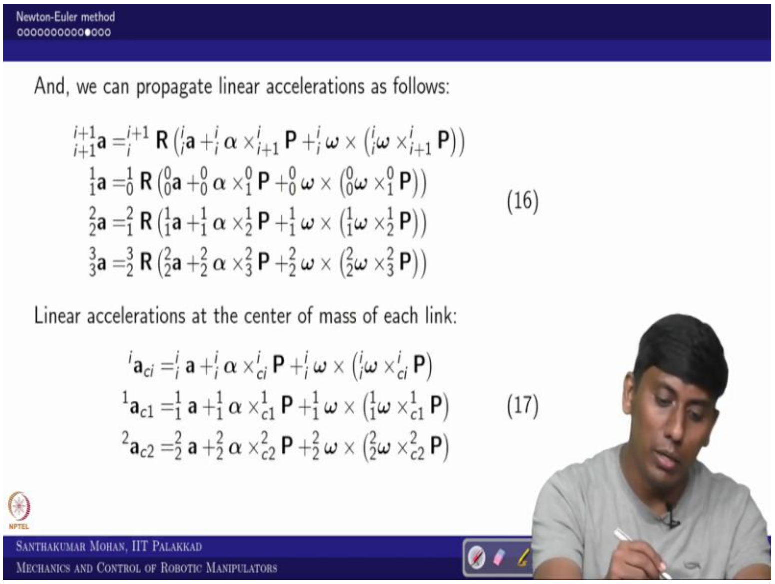

So, once you calculate all three, so we can go to the linear acceleration, the linear acceleration would be in this case only rotary joint there is no Coriolis and slip. So, these three would be from the previous joint. So, we can calculate that. So, we can start from 0 to 1, 1 to 2 3. So, that we can do it. So, once you have obtained then you can actually go to centroid. So, centroidal we can calculate based on the corresponding frame. So, if you talk about link 1, so the joint 1 you take and then do it. So, that we can do it. Similarly, ac 1 and ac 2.

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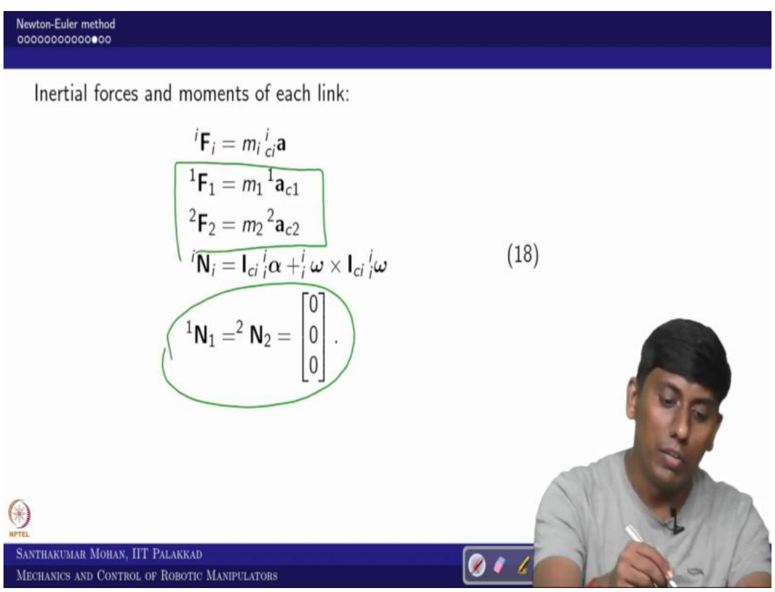

Then you can calculate the inertial forces and moment. In this case they assume it is a point mass. So, in the sense the moment would be 0, inertial moment would be 0. So, that is what we can see. So, these 2 we can calculate,

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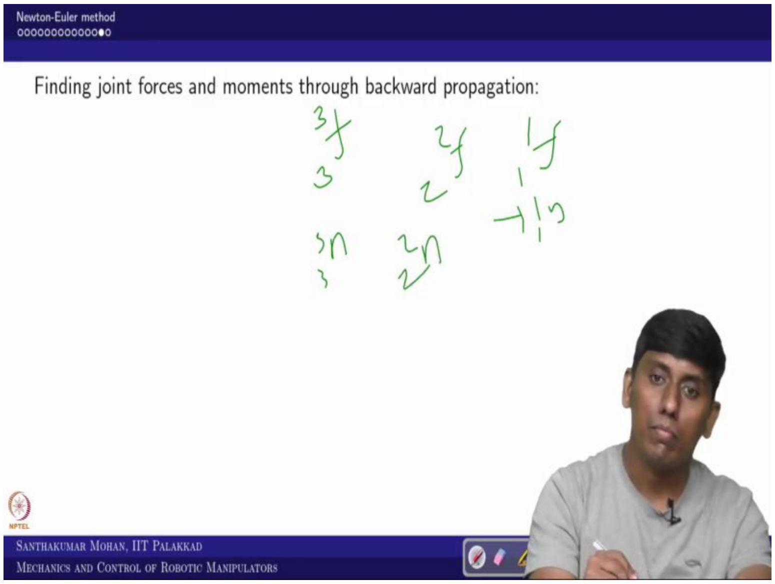

So, then we can actually go for the backward propagation. So, where we start from the f3 3 then we come back to f2 2, f1 1 to 1 we stop. Similarly, n3 3 to so n 1 1 we will come back via n 2 2.

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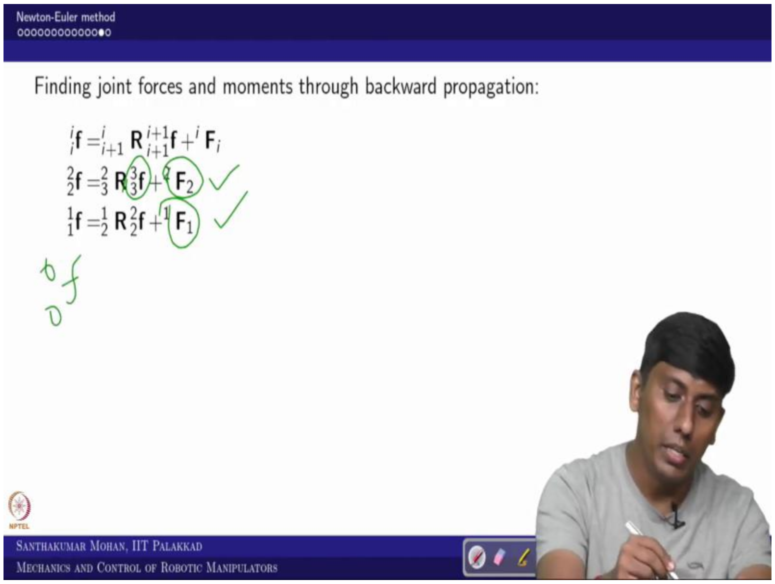

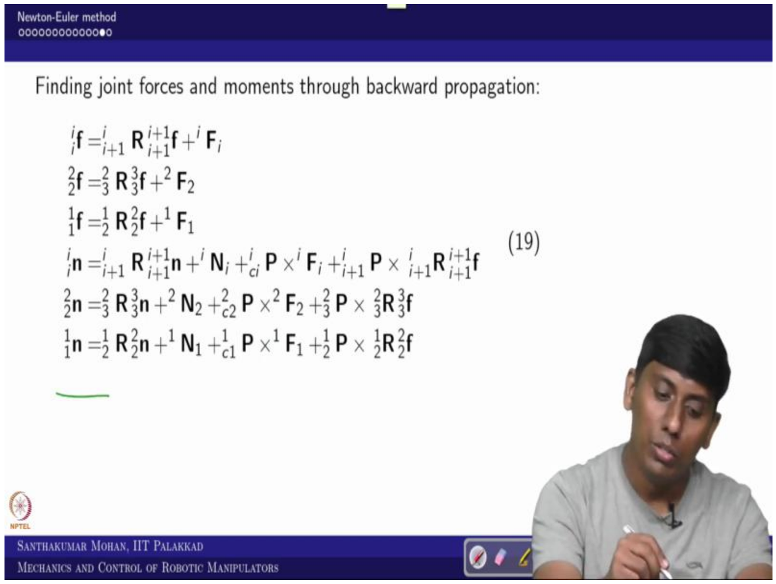

So, that is what we are trying to do. So, these are 0, but this is their inertial, inertial forces. So, then you can calculate if you want shaking forces you can go till 0. So, now similarly moment you can calculate backward, and you can calculate up to 1 if you want to know the shaking moments you go up to 0.

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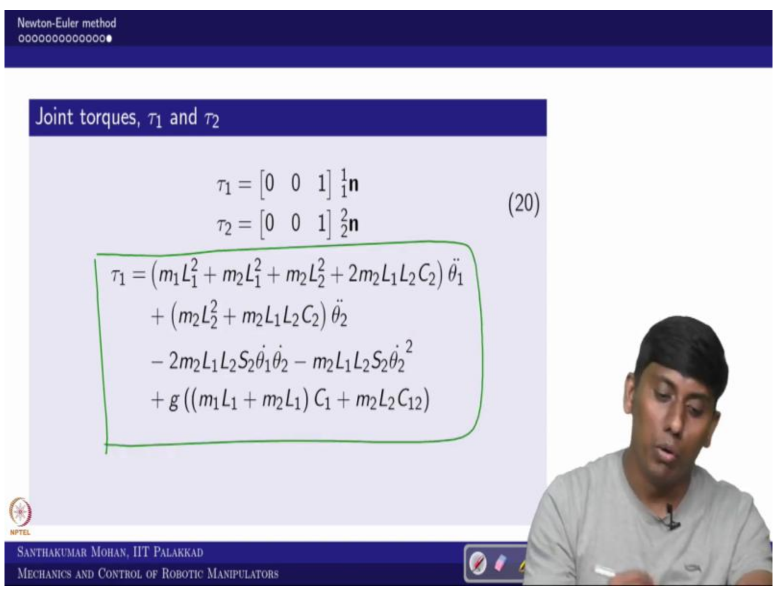

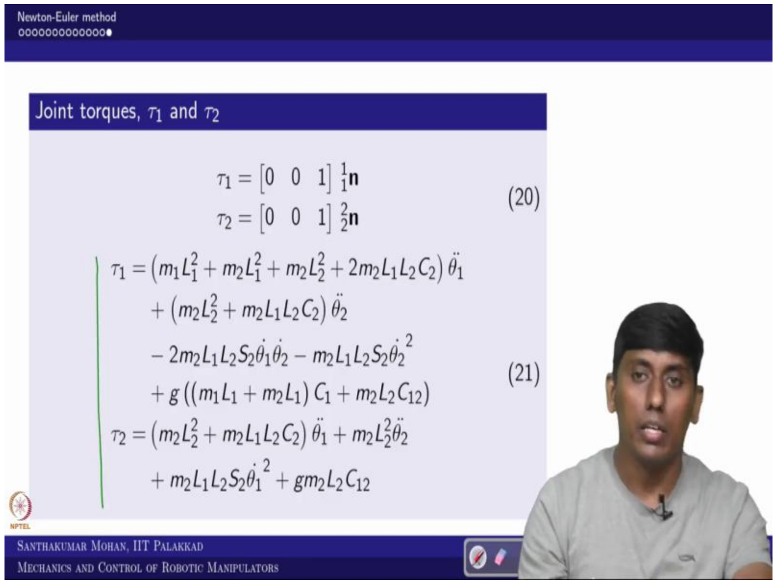

So, now we can calculate this is what the tau. So, now, if you do manually by yourself, so finally, you can see that the equation will come as similar to what you obtained in the previous lecture. So, this is what the case. So, with that, you can see that the Newton Euler method also we have demonstrated, so the demonstration if we do it in MATLAB, that would be very much beneficial.

The next lecture, you will be seeing the Newton Euler method or Lagrangian Euler method, do it in MATLAB and we can actually see whether the equations are matching with what we have done and we can show that any complex serial manipulator we can do these two method, then you can see that what is the difficulty or the complexity coming one to another method that we can see it in upcoming lectures. So, with that I am ending this particular lecture. So, until then see you. Bye. Take care.