Mechanics and Control of Robotic Manipulators
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Lecture No. 23
Introduction to Robot Dynamics and Lagrange-Euler Method

Welcome back to Mechanics and Control of Robotic Manipulator. So, so far what we have seen in this course the mechanic parts, especially the mechanics part, I can clearly say that the kinematics side, so where we started with the geometrical model we call forward and inverse kinematics, then we started talking about what you call differential kinematics.

So, where in the differential kinematics we were talking about one important matter is called Jacobian. So, after that we have slightly moved away where we have touched upon statics. So, this particular lecture is going to talk about more you can say into a real system, in the sense we are going to talk about dynamics. So, the mechanics part is going to end with the dynamics.

So, what we are trying to see in this particular lecture, we will be seeing robot motion dynamics not a structural dynamics, the structural dynamics we will see in the end. So, robot motion dynamics, in that we will be seeing what are the types and we will see how to derive the equation of motion.

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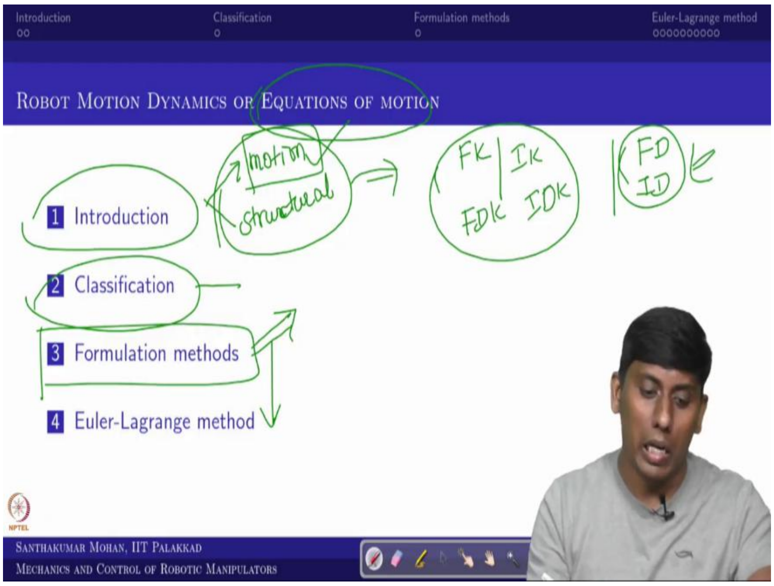

In the sense this particular lecture mainly specific to give an introduction to the robot dynamics, the robot dynamics again in the beginning we said so, we can write as a motion dynamics and structural, so it is like a structural. So, both we are going to see but, this particular lecture is going to talk about more on motion, this structural thing will come in manipulator design aspect.

So however, this particular course is mechanics and control. So, we would be seeing more on motion. So, that is what we are going to give in this introduction, then this motion dynamics can be classified, or this can be even some people call equation of motion. So, this further classified into two. So, that is what we are going to see.

So, how the forward kinematic model inverse kinematic model was there, then we have seen forward differential kinematic model and inverse differential kinematics model the same way we would be talking about forward dynamics and inverse dynamics, but the scenario is not the same. So, here it is mapping between you can say configuration space and you can say the joint or you can say operational space, but here the spaces are different, that we would be seeing.

Then, for getting this equation of motion there are several, you can say formulation methods, we are going to talk about 2 popular methods in that 1 we are going to see in this particular lecture in detail with an example. So, I hope now you are clear about the structure of this lecture. So, we will go further.

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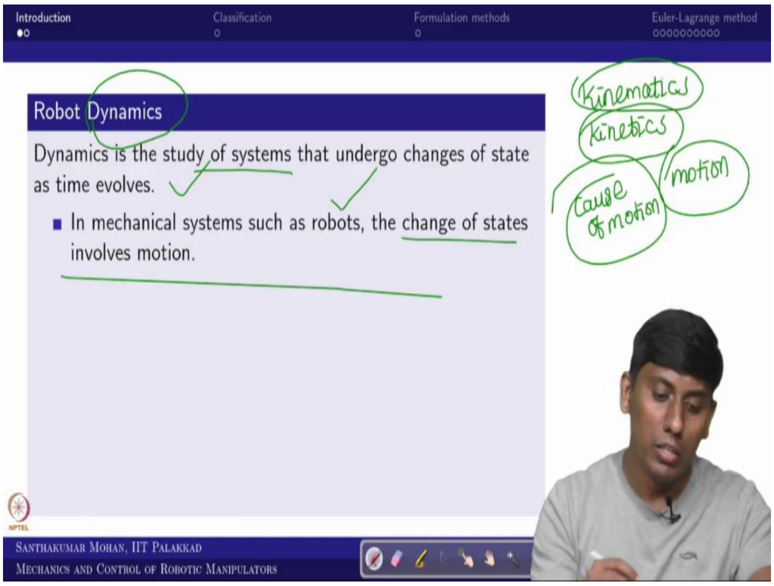

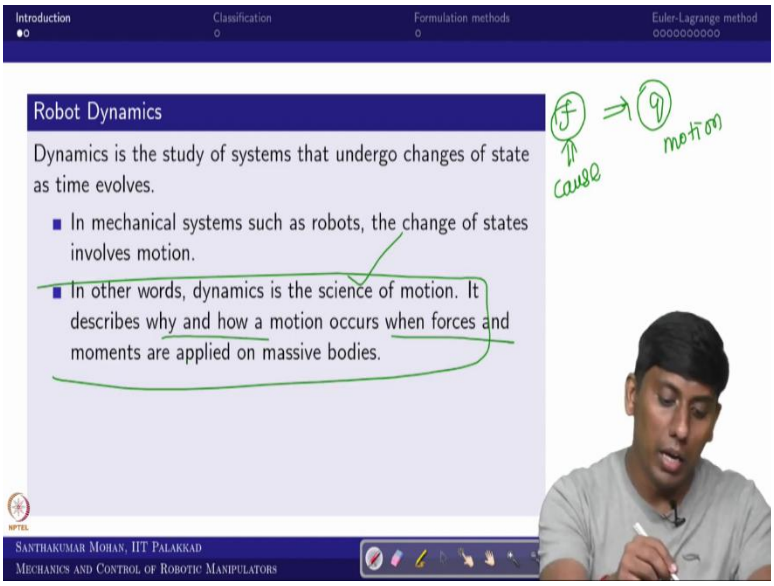

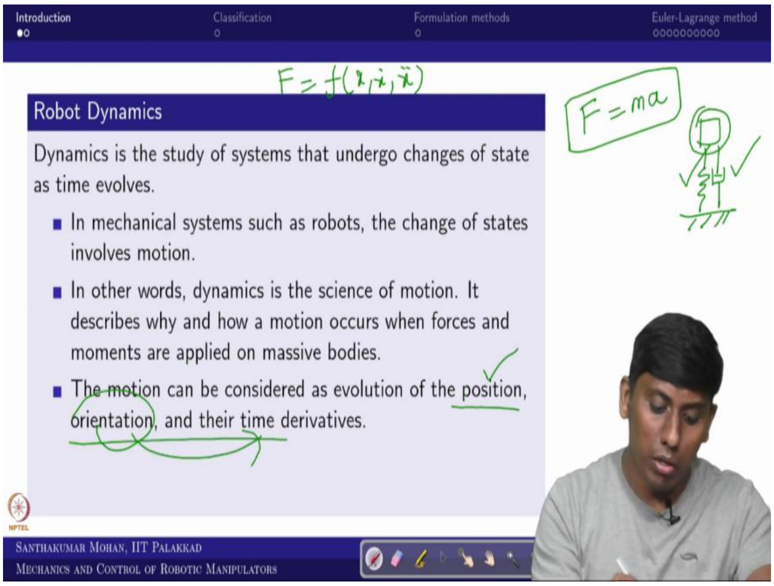

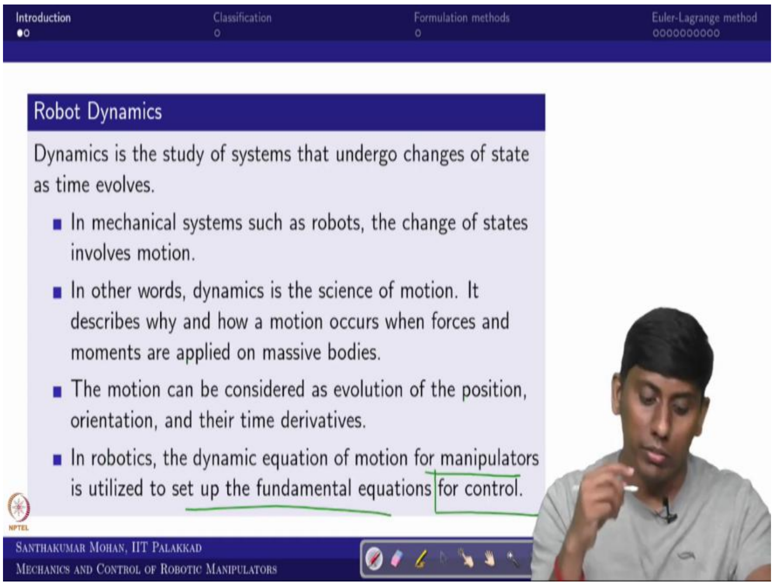

So, you know dynamics is nothing but study of systems that undergo changes of state as time evolves. So, in that sense the robot dynamics is more general it would be having you call kinematics and the kinetics. So, where one with regard forces our efforts, the other one is without you can say considering that, but here we are going to call as a general, in the sense you are going to relate the motion and cause of motion or causes.

So, this relation we are going to relate, not like simple kinematics. So, that is why we always robot dynamics, kinematics, we will not we will call robot kinematics, but robot kinetics is very rarely people put it forward. So now, in that sense, what mechanical system also such as robot the changes of states involve motion, but what else will be coming. So, it gives you science of motion, but it described why and how the motion occurred, that is what I said.

So, we are relating the input to the output. So, here the input is your efforts or causes, the causes, which is causing the motion, and we are trying to see the motion relation. So, but what it is a multi-body system? So, we are trying to see how it can be done one by one. So, in the sense when forces and moments are applied on a massive bodies, so how we can understand the system of motion, that is what the idea.

So, why this is very, very important, because this is evolving from you can say position, so orientation that derivative is making. That is what I said F equal to ma, it is Newton's second law. But if you take a very general, if you assume that there is a mass that mass is connected with a spring that is connected with another spring under dashboard.

So, then you can see this would give a frictional component, this will give a restoring and this will give an inertial component, in the sense this same equation F, you have can written as function of x x dot x double dot. So, this is what we are trying to write that is what we have written here. So, in addition to that what so, this is mainly to utilize. So, two things, so it will give you a fundamental equation, but this equation is for control, and it can be used for even what you call design the overall system.

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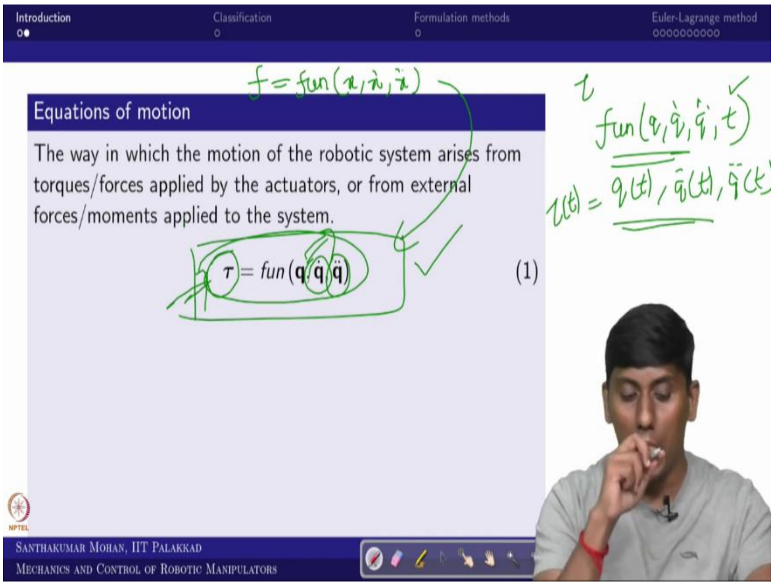

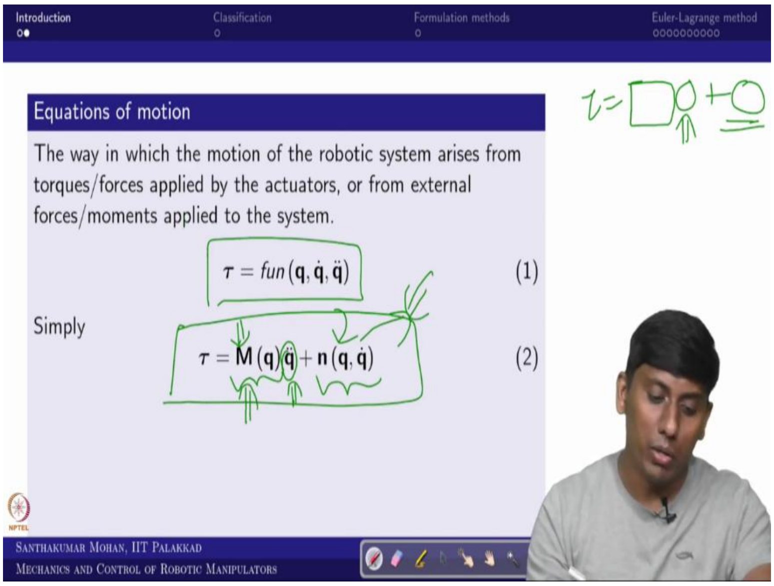

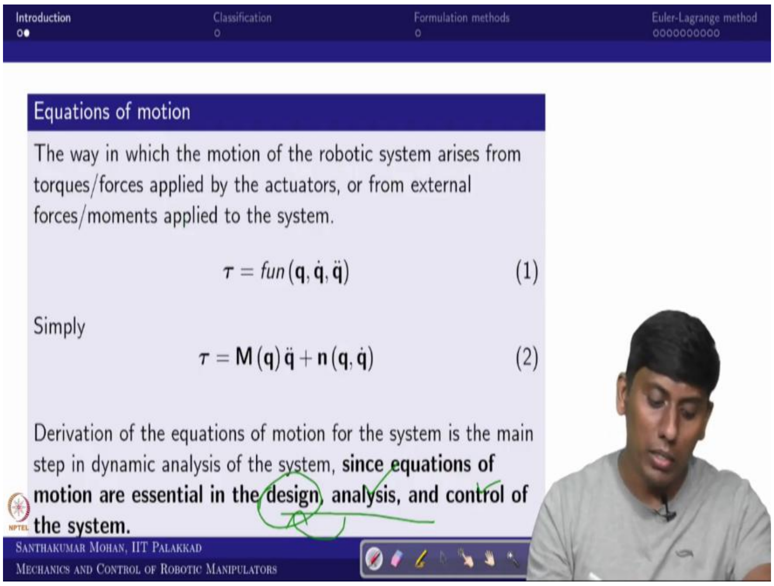

So, if you look at that aspect, so what one we expect we are writing a generalize, so earlier I said f can be written as, so function of you can say x, x dot and the x double dot, so this equation we are writing in a generalized manner with a vector. So, this is tau is the input vector and this q is you can say the joint vector of joint variable then the velocity and acceleration, in the sense we are writing a generalized form.

So, this is more important for one because this is nonlinear. So, that is why we are writing straight forward in fact, we have to write strictly. So, q, q dot and q double dot comma t, because this is a dynamic system it is function of time, but we are ignoring because q also function of t, so q dot also function of t and q double dot also function of time. So, we are ignoring similarly tau is also function of time. So, that is the whole idea. So, we will see.

Some people this bigger equation can be reduced into a small you can say state space form that to like we can write as you can say acceleration plus other terms. So, in the sense acceleration coefficient will come as a matrix and this other term as a vector and this is acceleration vector that is what we have written here, this is acceleration vector, and the coefficient of acceleration vector is what we are calling as the inertial.

So, this is the inertia matrix, these are all called other effects. Even some people even further decompose this into gravity and other effect, some people call even gravity frictional other effect, some people even further go on. So, centripetal and Coriolis separate, and you can say friction separate, gravity separate like that you can make it. But I think we, we will restrict this up to this because we are going to do the second part as the simulation side. So, I hope this would be very much beneficial.

So, why this is very important, because it is helpful for analysis and control. And similarly, this can be used as a design tool, in order to cross verify your system and further use for design aspect. So, that is why this is very, very important.

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Forward Dynamics

For a given, input vector, τ and the known states $\mathbf{q}$, $\dot{\mathbf{q}}$, find the resultant motion of the manipulator, in other words, find, $\ddot{\mathbf{q}}$.

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For a given, input vector, τ and the known states $\mathbf{q}$, $\dot{\mathbf{q}}$, find the resultant motion of the manipulator, in other words, find, $\ddot{\mathbf{q}}$.
Simply

$$\ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{q})\left(\tau - \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}})\right) \tag{3}$$

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It is the problem of **simulating or analyzing the manipulator**.

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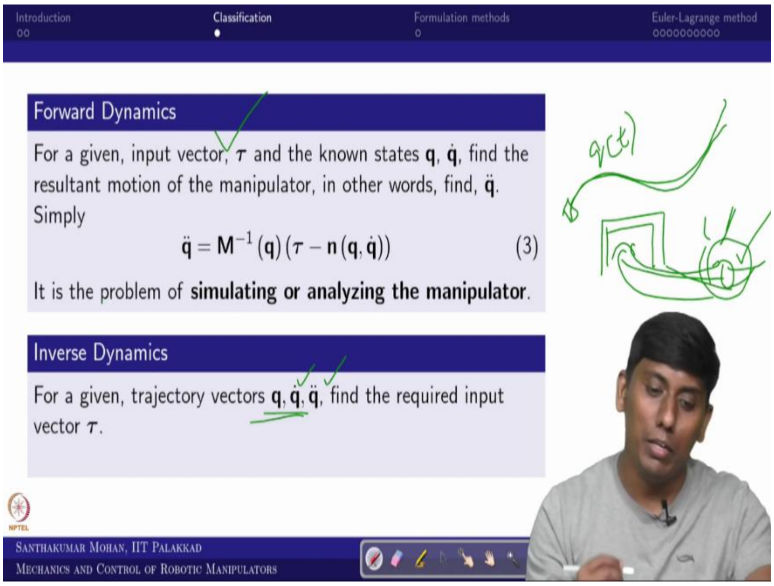

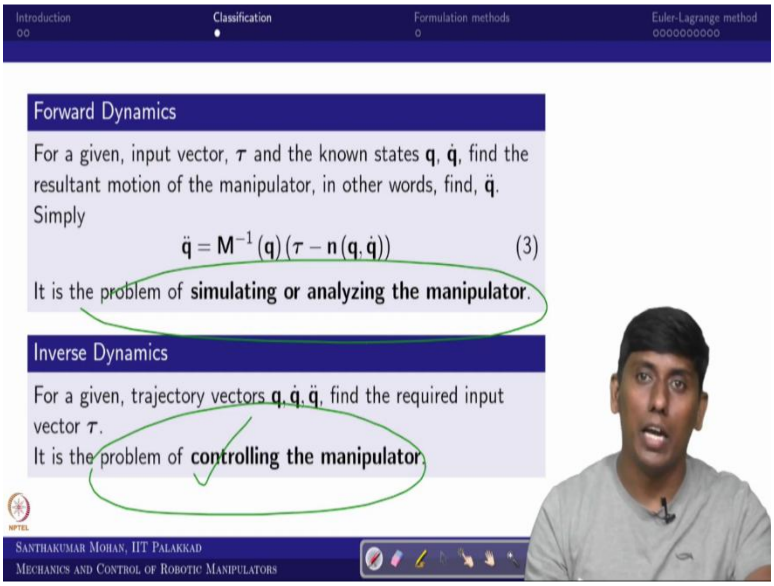

So, now, we will see what are the subclasses of robot dynamics. So, we have seen tau is function of, so q, q dot and q double dot. So, these are the thing. So, now, this is one space, which is motion variable, which we simply call system state. So, we call state variables. So, the other side is what you can see the input variable in the sense it is the cause of motion, or you can just simply call the causes. So, can we map this causes to motion. So, in the sense you take, you can say a serial manipulator, you connect a motor, and you provide a torque how the system is behaving.

So, if you look at that, so what you can see in the sense you are given an input vector, you are trying to find the you can say q dot, q, q double dot and q we are actually trying to find the resultant motion. So, in the sense, so these are known states q dot q double dot q all are like we are trying to find out, in the sense the input is known, and we are trying to find out. This is what you call forward dynamics, this is very simple.

So, you have a motor and give a torque and see the system are you have a linear motor, so you are giving a force and trying to see how the system is behaving. In the sense it is forward dynamics, so what is this trying to do it is analyzing, in real time it is analyzing in numerical background it is simulating.

So, that is what the whole idea, which is very similar to differential kinematics, but differential kinematics is mapping between what, so task space velocity to configuration space velocity, but here it is mapping the input vector to what you call the unknown q double dot then you can integrate then q dot will come. So, that is why I said that known state q and q dot you are trying to find out q doubledot.

This is forward dynamics, it is very close to open loop you can say analysis system does not control, you are giving input and see what is the system output. In the sense there is a football in front of you, you just go and kick. So, how the ball is rolling that is what you are trying to see. So, this is something like you have one ball and you are kicking it, so the ball is rolling, that kind of thing. So, this is what you can see simulating or analyzing the manipulator.

So, obviously, the contemporary of this is this opposite side, where the q, q dot and q double dot would be known, and you are trying to find out what is torque. So, in the sense these are given as a trajectory. So, in the sense it is time-based trajectory. So, which is twice differentiable in the sense q dot, q double dot would be known, in the sense you take a car, and you are trying to follow this with respect to time.

Or in the other sense, so, you have a ball that ball need to go at a certain goalpost. So, this kind of thing in the sense, so how much force or what direction you provide a force the ball will roll here. But the other thing is it is open you are just hitting and seeing the sense you given own input known force and seeing the rolling of the ball, which is forward dynamics, whereas the inverse enemies the balls supposed to go in certain manner. So, what force and direction you need to provide.

So, that is what it is more or less a small kind of controller, but it is still open loop controller, it is a feed forward. So, that is what we are trying to see. So, this is a problem of controlling manipulator, but this is a problem of simulating or analyzing the manipulator. I hope you will be clear now what is the classification of dynamics.

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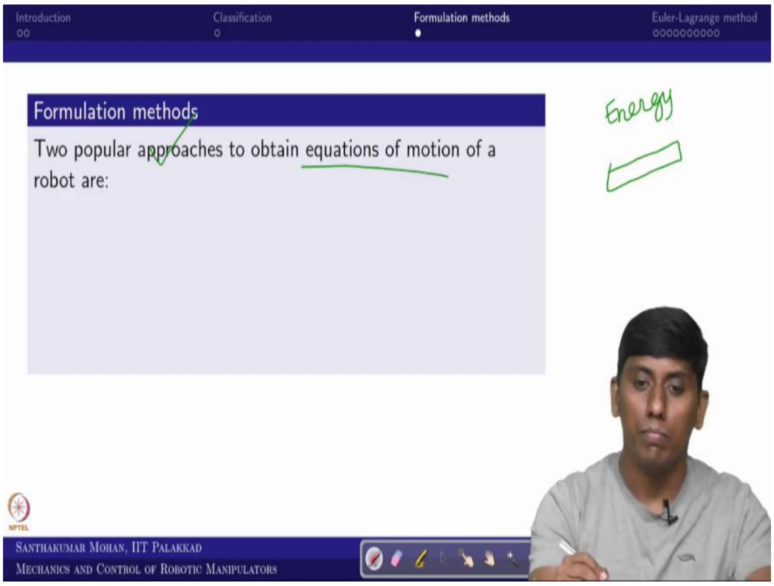

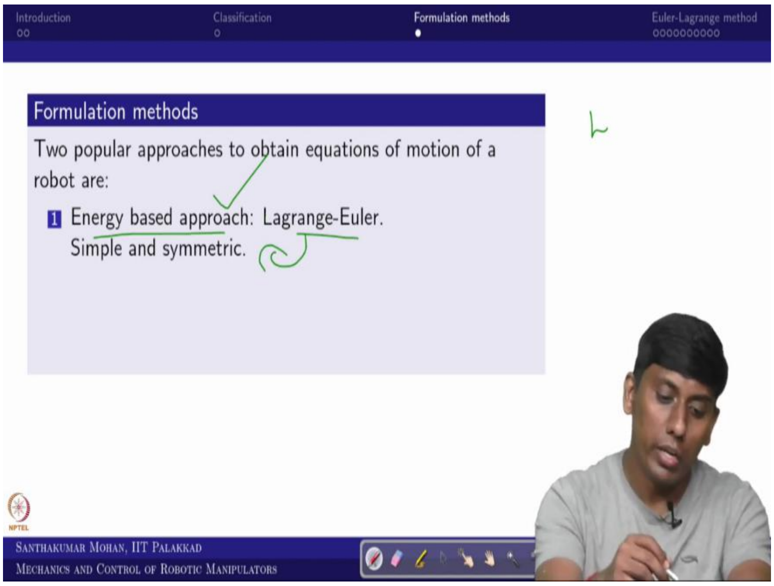

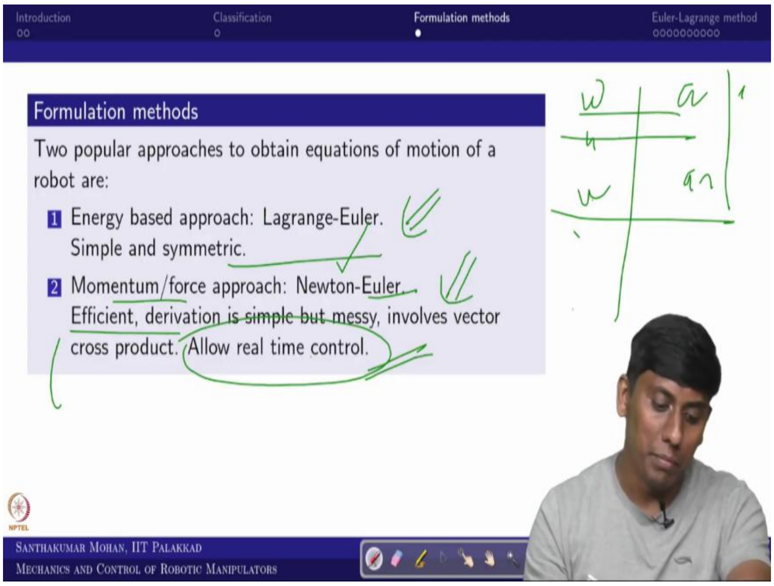

Now, we will see what are the formulation methods. So, there are several methods as I already mentioned, but we will be taking two popular methods or approaches which would be useful for the equations of motion or equations of motion of a robot our manipulator which is very common in rigid body dynamics. So, one is energy approach. So, the other one is like equilibrium approach. So, that is what we are seeing it, or some people call that is momentum approach.

So, one is energy-based approaches Lagrange Euler, where Lagrange is modify the Euler equation and come up this is very, very simple and symmetric in nature and it give you much, much better result for a small you can say size of manipulator with very less number of degrees of freedom. Whereas the degrees of freedom are increasing, in the sense the active joints are getting increased. So, the Euler Lagrange is not so easy, or Lagrange Euler is not so easy.

And the second thing is, the second one is momentum force approach where Newton's you can say second law and Euler’s you can say equation of motion can be used. So, this is very efficient in terms of computational aspect or implementation side because the derivation is simple, but this would be consist of partial derivative and time derivative, but here there is no derivative involved. So, that is why most of the people uses for real time control the; you can say Newton Euler approach rather than the Lagrange Euler approach.

Only issue is this would be having a vector cross product, but it is messy you have seen already the velocity propagation you have written, so omega 1 to omega n then v1 to vn. So, similarly in this case it is supposed to go a 0 to an and then you have to go for the body, then you have to go for a cg to or you can say ac 0 to ac n. Then you have to go for inertial force inertial moment, it is lengthy, but computational aside, although it's a lengthy it is very efficient and easy to implement.

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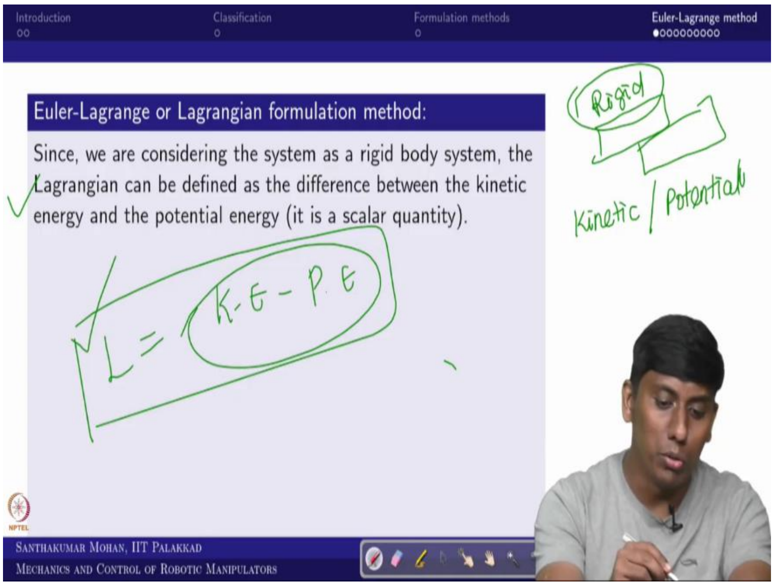

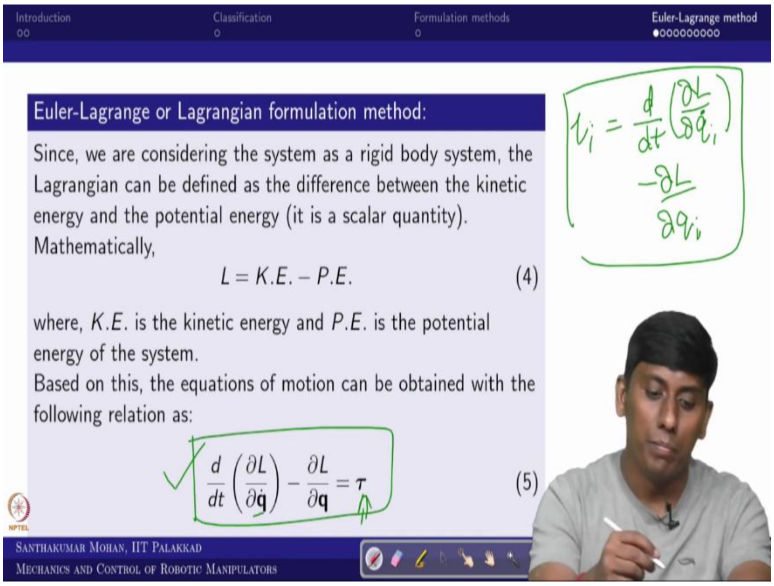

So, let us go to one such method in this particular lecture. The second method we will see in the next lecture. So, Euler Lagrange is as I already mentioned, it is energy-based approach, but in the beginning of the lecture itself, I said this particular course, we are talking about only rigid body. So, in the sense we are talking about rigid body dynamics, so since the system is rigid, so what are the energies would be there, so only two kinds of energies would be there. So, one is kinetic, the other one is potential.

So, these two energies, is there any difference between these two energies, so then there is a motion exists, even this energy difference are all they have written, so kinetic energy is always apparent. So, kinetic energy minus potential energy, if there is a quantity that quantity is a scalar if there is an existence of this quantity, then there is a motion exists. So, based on this L value, we can calculate the equation of motion that is what the Euler Lagrange or Lagrange formulation method some people call Lagrange Euler method also.

So, that is what we are seeing it. So, now this energy difference what we are trying to use. So, further that the equation is coming based on very simplified manner, the detail is already given in some of the lecture handouts you can see how this equation has come. So right now, we stayed away take this equation, this equation has come from the modified energy you can say equation, so you know if this is 0, so that would be the energy conservation, but right now we are giving the input that some modification has done from the energy method.

So, now, you can see that yellow is a scalar, but the q dot is coming in the other sense. So, tau i can write as time derivative of, so doh L by doh qi dot this is a dot minus doh L by so doh qi. So, that is equal to tau i. So, this you can recall this is a vector form I have written but this is industrial element you have to do it.

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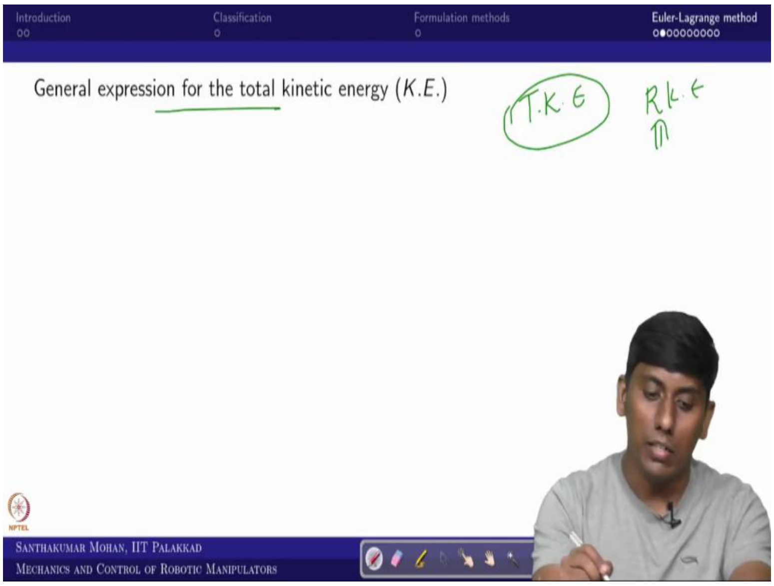

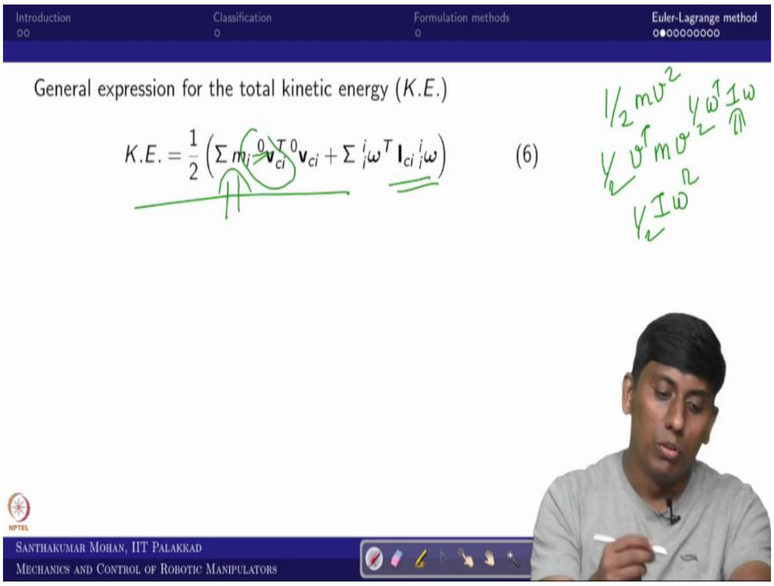

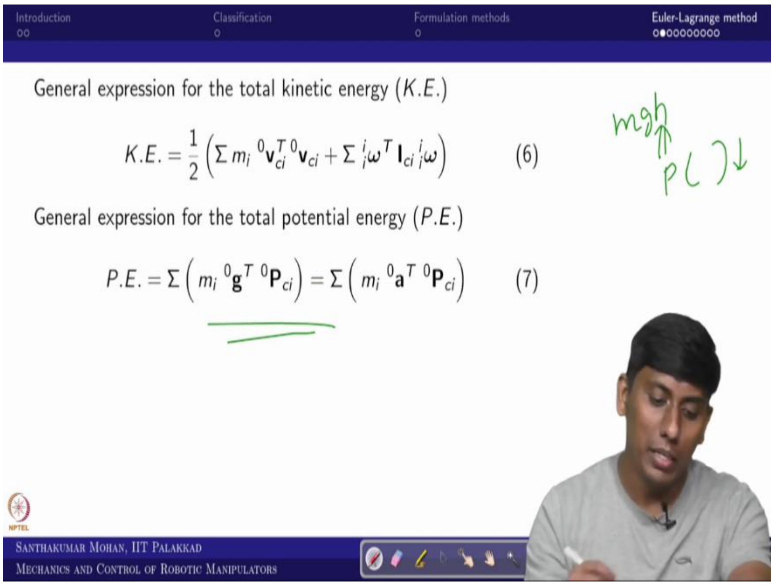

So, just to give idea, so I am just giving a general expression. So, where the kinetic energy would be having two parts. So, one is translational kinetic energy this is due to the mass. So, then the rotational kinetic energy that is due to the second moment of inertia. So, are you can say simply inertia? So, that is what we are trying to write. So, you know half mv squared is a scalar form that you want to write it in a vector form. So, V transpose m into to v. So, half would be common.

Similarly, if you write I omega squared is the scalar form. So, the same thing you write it vector form omega transpose I is the matrix and omega, so this one. So, this is a tensor, the tensor we will be using it, so that I hope you are already gone through what is tensor. So, this is a inertia tensor and this is the you can say translational inertia which is a mass as a scalar you can take it out.

So, one thing you should be knowing this is ci with respect to what you call zeroth frame, in the sense their absolute velocity and absolute you can say what you call acceleration would be using further on. But here it is only corresponding angular velocity, but this is absolute that is what one is supposed to be known. So, let us move further. So, this is what the; you can say potential energy, which is mgh but this h is you can write it in a position vector the vertical axis which axis as you are considering the gravity that axis you are to take it.

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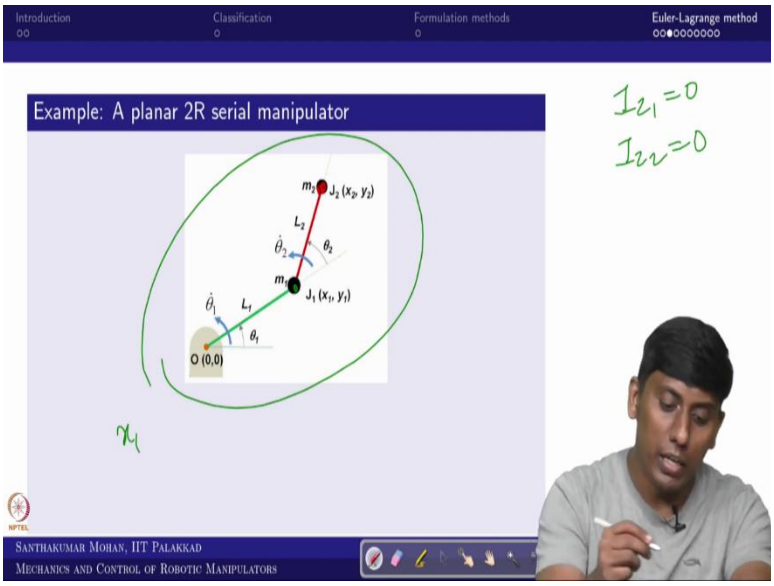

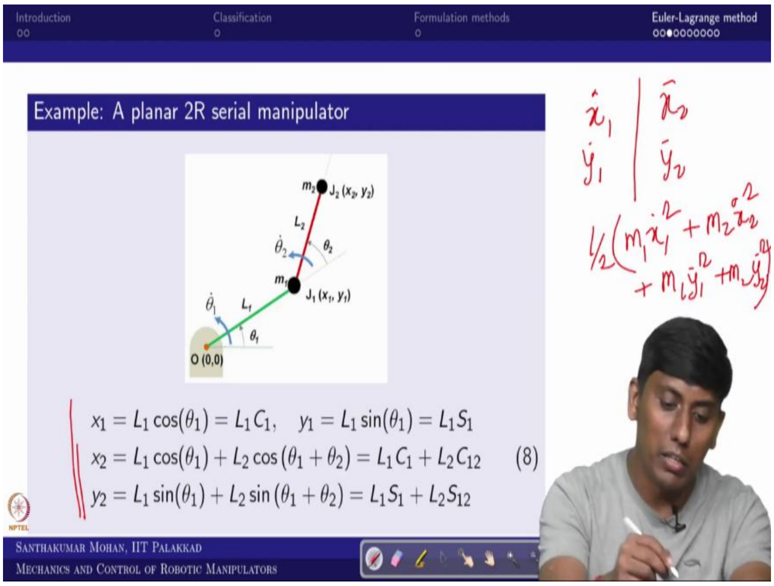

So, now, in that sense, we will take it one simple example, I assume that this is a mass concentrated in the sense inertia of these two are 0, this is inertia tensor is 0 tensor. So, you no need to bother about I omega squared now. So, now, if you take this, so what one can easily see, so what is the x1, so x1 is here, what is x2 that is also clear here, so that we can derive it.

So, this is straightforward. So, similarly, you can see this is also once you obtain what you can do, you can do x1 dot so y1 dot similarly, x2 dot and y2 dot and half m 1 x1 dot squared plus m2 x2 dot squared plus m1 y1 dot squared plus m2 y2 dot square. So, this you can calculate and then you can find it.

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Kinetic energy

$$K.E. = \frac{1}{2}m_1\left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{1}{2}m_2\left(\dot{x}_2^2 + \dot{y}_2^2\right) \quad (9)$$

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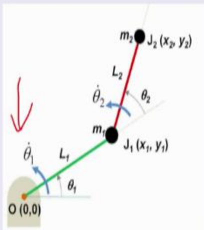
Example: A planar 2R serial manipulator



$$x_1 = L_1\cos(\theta_1) = L_1C_1, \quad y_1 = L_1\sin(\theta_1) = L_1S_1$$
$$x_2 = L_1\cos(\theta_1) + L_2\cos(\theta_1+\theta_2) = L_1C_1 + L_2C_{12} \quad (8)$$
$$y_2 = L_1\sin(\theta_1) + L_2\sin(\theta_1+\theta_2) = L_1S_1 + L_2S_{12}$$

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Kinetic energy

$$K.E. = \frac{1}{2}m_1\left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{1}{2}m_2\left(\dot{x}_2^2 + \dot{y}_2^2\right) \quad (9)$$

where

$$\dot{x}_1 = -L_1S_1\dot{\theta}_1$$
$$\dot{y}_1 = +L_1C_1\dot{\theta}_1$$
$$\dot{x}_2 = -L_1S_1\dot{\theta}_1 - L_2S_{12}(\dot{\theta}_1 + \dot{\theta}_2) \quad (10)$$
$$\dot{y}_2 = +L_1C_1\dot{\theta}_1 + L_2C_{12}(\dot{\theta}_1 + \dot{\theta}_2)$$

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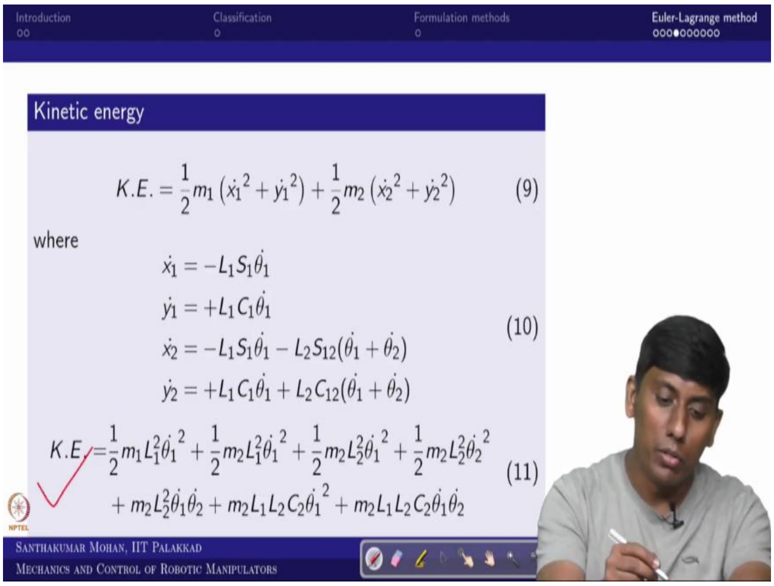

That is what we are trying to do, you can see. So, far that you need to know x 1 dot y 1 dot x 2 dot y 2. So, you know already x 1 and x 2 y 1 and y 2. So, you can take time derivative, so, this is what the time derivative, so that would be function of you can say the joint space variable. So, now, you can substitute that you will get the kinetic energy in this form, and you can take the potential energy.

So, here you can assume that the gravity is pulling down. So, in the sense that gravity direction coming down, so you can see in that sense, so y1 is equivalent to h of the first body, so y2 is y equal into the second body. So, that is what we are trying to write.

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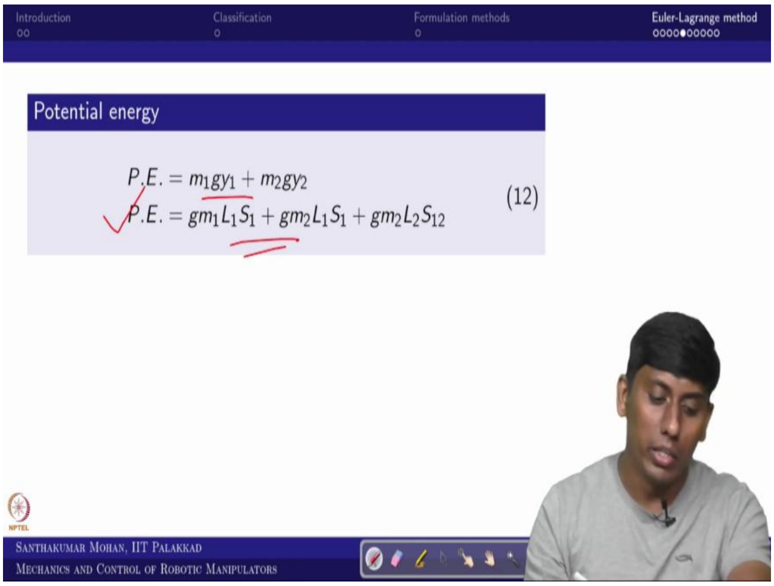

So, you can see like m m1 g y1, so m2 g y2, so that original value is substituted. So, now what we got potential energy we got it. Previous slide we got the kinetic energy.

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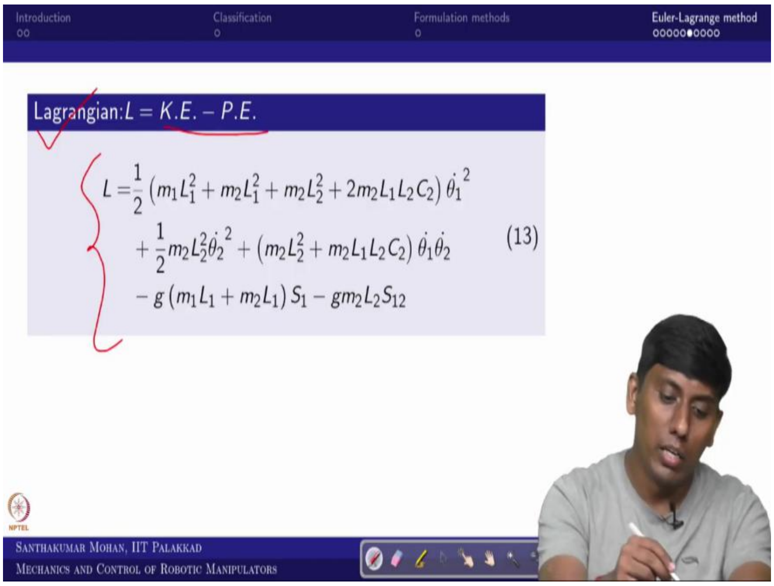

Now, we take the Lagrangian which is kinetic energy minus potential energy this gave this a scalar. So, now we can substitute the relation.

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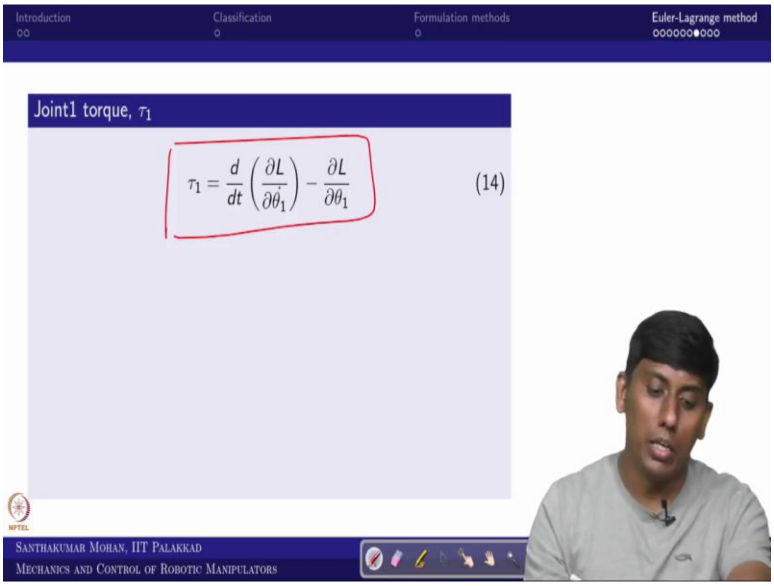

Lagrangian: $L = K.E. - P.E.$

$$L = \frac{1}{2}\left(m_1L_1^2 + m_2L_1^2 + m_2L_2^2 + 2m_2L_1L_2C_2\right)\dot{\theta}_1^2 + \frac{1}{2}m_2L_2^2\dot{\theta}_2^2 + \left(m_2L_2^2 + m_2L_1L_2C_2\right)\dot{\theta}_1\dot{\theta}_2 - g\left(m_1L_1 + m_2L_1\right)S_1 - gm_2L_2S_{12} \tag{13}$$

Joint1 torque, τ_1

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} \tag{14}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \left(m_1L_1^2 + m_2L_1^2 + m_2L_2^2 + 2m_2L_1L_2C_2\right)\dot{\theta}_1 + \left(m_2L_2^2 + m_2L_1L_2C_2\right)\dot{\theta}_2 \tag{15}$$

Joint1 torque, τ_1

$$\tau_1 = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} \tag{14}$$

$$\frac{\partial L}{\partial \dot{\theta}_1} = \left(m_1L_1^2 + m_2L_1^2 + m_2L_2^2 + 2m_2L_1L_2C_2\right)\dot{\theta}_1 + \left(m_2L_2^2 + m_2L_1L_2C_2\right)\dot{\theta}_2 \tag{15}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = \left(m_1L_1^2 + m_2L_1^2 + m_2L_2^2 + 2m_2L_1L_2C_2\right)\ddot{\theta}_1 + \left(m_2L_2^2 + m_2L_1L_2C_2\right)\ddot{\theta}_2 - 2m_2L_1L_2S_2\dot{\theta}_1\dot{\theta}_2 - m_2L_1L_2S_2\dot{\theta}_2^2 \tag{16}$$

$$\frac{\partial L}{\partial \theta_1} = -g\left(m_1L_1 + m_2L_1\right)C_1 - gm_2L_2C_{12} \tag{17}$$

So, what relation if you want to have a tau 1. So, tau 1 is time derivative of a partial derivative doh L by doh theta dot 1 minus doh L by doh theta 1. So, first we calculate doh L by tau theta 1. So, for that you can just recall this is what. So, theta 1 dot is in this term, so this term and these terms are 0. So, in the sense this 2 goes so, this goes out and here you can see like the theta 1 dot is coming only here. So, this goes out in the sense theta 2 dot remain here theta 1 dot remains. So, we can see what is that partial derivative.

So, similarly, you can take doh L by d theta 1. So, unfortunately here, so theta 1 is here, so theta 1 is here. So, in these two equations nothing, so you can take it 0 and these two equations or these two relations are coming. So, that is what you can take it. This is time derivative I have taken. So, this is partial derivative we take a time derivative and then you can see this is what you can see as doh L by doh theta 1. So, now, this is known, and this is known, so you just directly subtract this.

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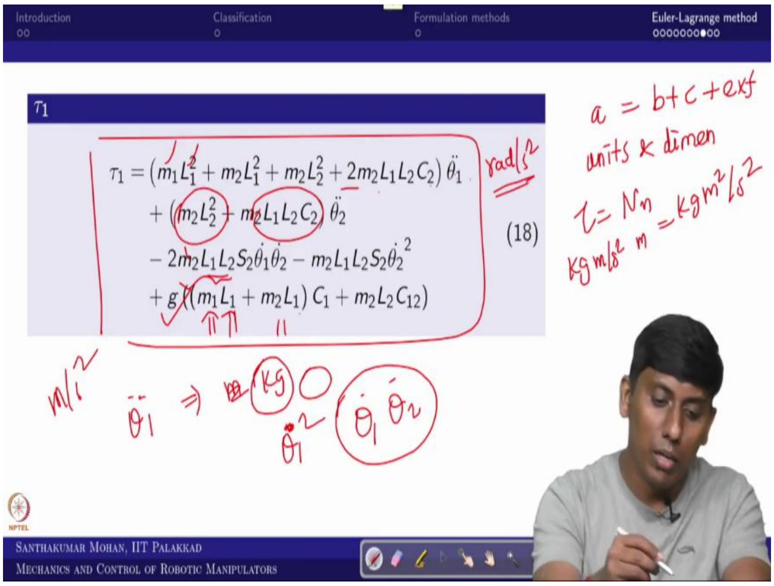

So, that is what you have tau 1. So, tau 1 once you are obtained, so whenever we do you can cross verify. So, what cross verification you have to see any equation for example, a equal to b plus c plus you can say e into f, so what is supposed to be the units and you can say dimension should be equal. So, in this case it is a scalar which is all 1 cross 1 the unit supposed to be match. So, this tau is a Newton meter.

So, then, so inside any term you take it would be Newton meter. So, what this will give, this will view radians per second square. So, in the sense you can y see what is Newton meter you can write, so kilogram meter per second squared into meter in the sense kilogram meter squared per second squared. So, here, this is kilogram meter squared and second squared is there. So, this is kilogram meter squared and second squared like that you can see.

So, in that sense, if theta 1 double dot comes, so then you have to see the other side. So, some kilogram should be there some mass and 2 distances are 1 distance squared should be there, that you can cross check. If theta 1 dot squared comes or so theta 1 dot theta 2 dot comes, so then you can see that this is already equivalent to again radian per second squared. So, you can see the same thing supposed to be there.

You can see 1 you can say mass and 2 you can say distance you can get it. So, similarly, here the gravity is there, which is meter per second squared, then you can see like only 1 meter should be there in the sense 1 length and 1 mass. So, you can cross check. So, like that you can always cross check and then you can a get it.

So, when we write in this in a matrix and vector form, even 1 more check is there, but whenever you get the equation immediately you cross check the coefficient of acceleration or coefficient of velocity squared our velocity product is getting that ratio. So, now, in that sense, you can go to the next one.

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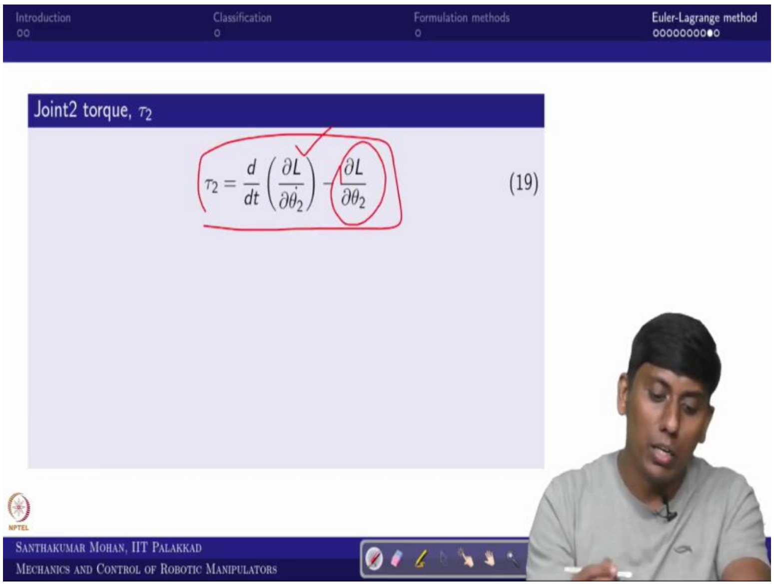

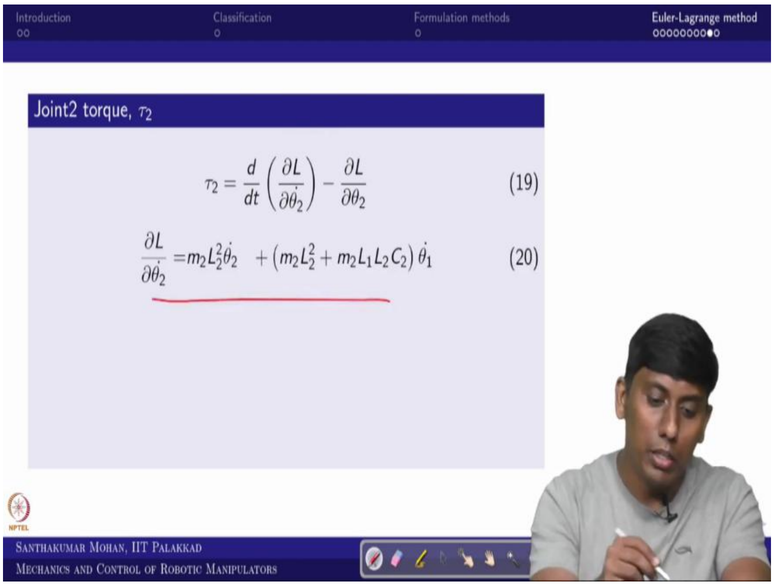

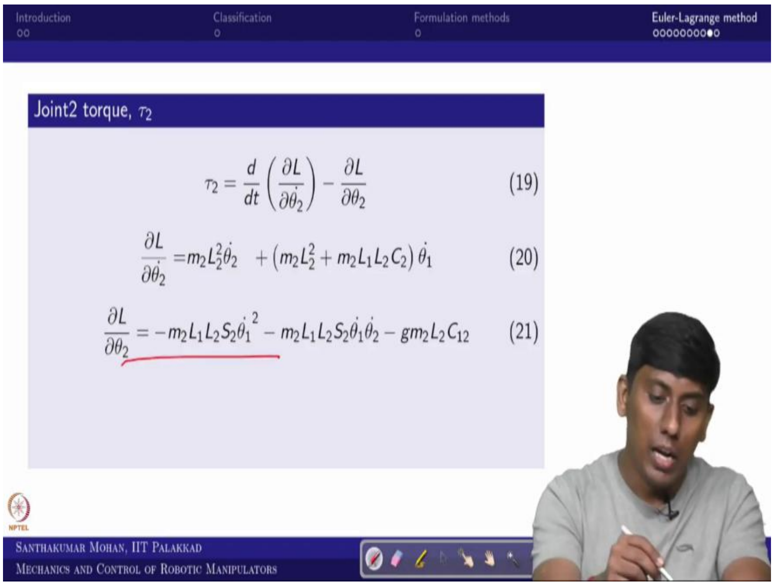

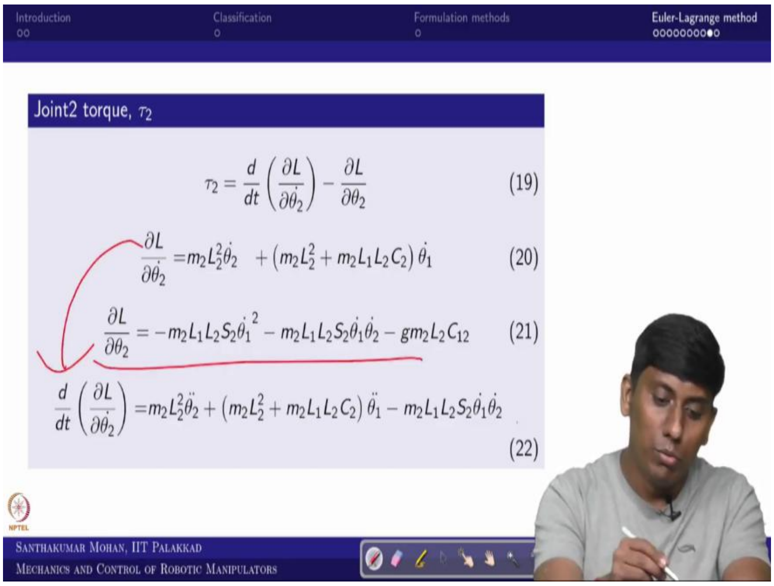

So, that tau 2, so again doh L by doh theta 2 dot and doh L by doh theta 2 you can get it. So, this is the dot you can say partial derivative, and this is the you can say partial derivative with respect to the theta 2 and now, you can see this you can take a time derivative and this minus of this will give tau 2.

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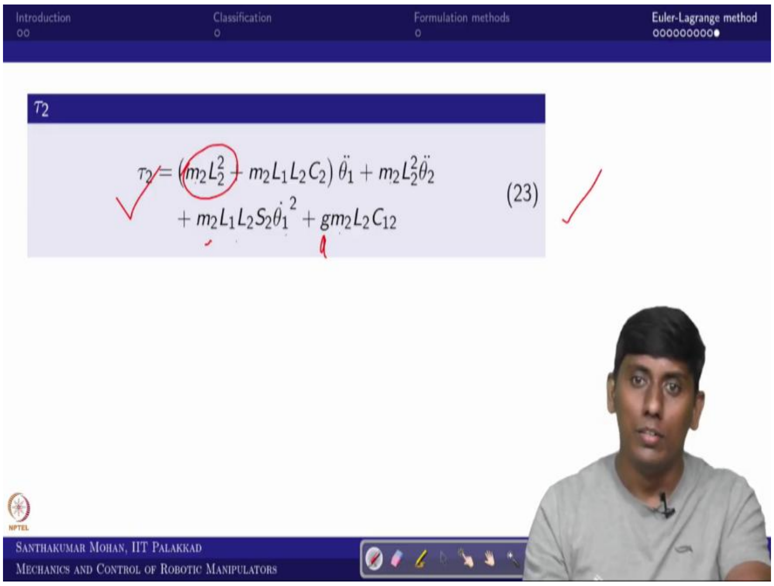

This is what the case. So, again you can see this is 1 kilogram and 2 distance 1 mass and 2 distance 1 mass and 2 distance 1 mass and 2 distance and this is again 1 mass in 2 distance, and this is g is there. So, 1 mass and 1 distance. So, you can cross checked. So, this is the way we have derive it. The same manipulator, we can see what is, you can say Newton Euler method, and we can see this example, and see whether the equation is coming as similar to this particular method.

So, in that sense the next lecture we would be seeing the other method call Newton Euler method. And we will see they are also the same example. So, with that we will close this dynamics part. Then we will go to the equations of motion, where we will write in a state space form. And then in the, you can say simulation side, we take more example, and we will understand. So, with that I am closing this particular lecture. See you then. Thank you. Bye.