Mechanics and Control of Robotic Manipulators
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Lecture No. 22
Manipulator Statics and Workspace Singularities

Welcome back to Mechanics and Control of Robotic Manipulator, this particular lecture is slightly deviate from the last 2 lectures. So, till now what we have seen kinematics and differential kinematics, but here we are going to talk about what you call statics.

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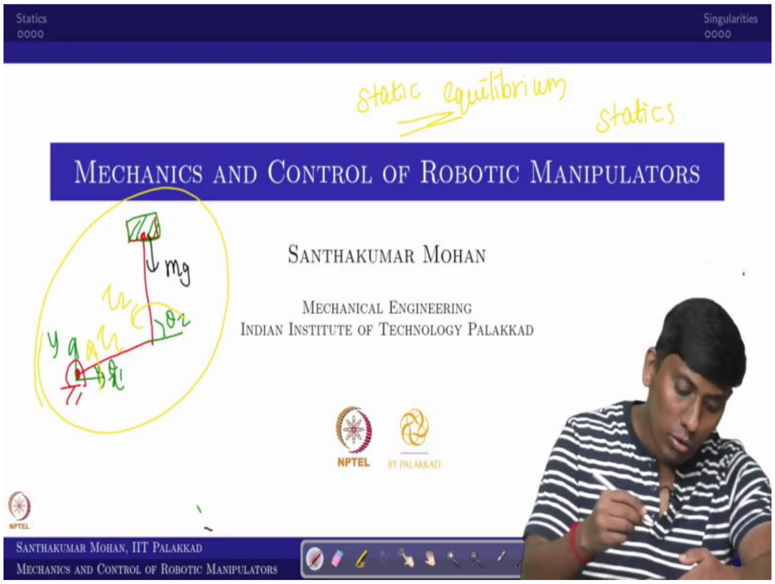

What that mean, for example I have a 2 R serial manipulator. So, this 2R serial manipulator is having an end effector, this end effector is carrying a load. So, I assume that it is a mass. So, it is like y and x and this is having a gravity. So, now, it is pulling down. So, now, what idea is coming here, so if I want to keep this location as theta 1 and theta 2 frozen, what I need to provide a torque.

So, what torque I have to provide here as tau 1 and tau 2, so that this configuration would be you can say standby, in the sense it would be static equilibrium. So, this is the case we are going to attempt, in the sense what we are going to see we are going to see the statics as the main ingredient in this particular lecture.

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So, let us move to the original content. So, we are going to talk about statics. So, we are going to give a relation. So, the relationship, so between end effector effects, so I call the end effector or task space efforts, so efforts to what you call the joint space you can say inputs. So, that is what we are trying to see if you recall the previous slide also you can see like, so if I give you the payload here, so what would be my inputs tau 1 and tau 2 that is what we are trying to see.

So, this relationship I am trying to find out, in the sense you have end effector forces and moments, so that I am trying to relate as tau and small f. So, this relationship I am trying to bring it here. So, that is the whole idea of the statics. So, then you know already, so the Jacobian may be singular certain instant, so why that is happening and what instant it is happening. So, then we will bring that call singularities.

So, what kind of singularities are existed in the serial manipulator, these all we are going to see in this particular lecture, but I am going to give a small disclaimer here. So, we are not going to see that singularity in detail, because this course is mechanics and control, we are not going to see the singularity in detail just to giving an introduction and a small you can say idea about singularity and types of singularity.

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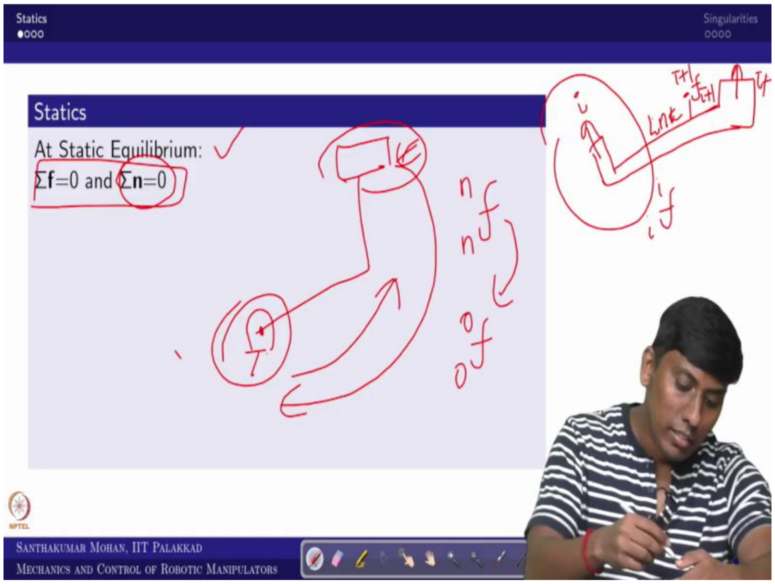

Let us move to the static case. So, you all know like static equilibrium means, the sum of you can say all forces would be 0, similarly sum of all moments would be 0. So, now, these 2 conditions are exists, so I take random link. So, I am taking a random link. So, this is random link, which is I call link i. So, in the sense this is ith frame, and this is i plus 1 frame. So now, if I have a force, force at i plus 1 to i plus 1 can I get the force of i.

So, in the sense we are trying to come backwards, why it is backward, because we are talking about the serial manipulator, the end would be clearly known, how we said that the velocity propagation from the base because the base velocity and acceleration would be known, we would be doing a forward propagation the same way the end side is your task end that would be known.

So, from there we are trying to come back. So, that is why it is a backward propagation in the sense if you have fn, so can I find the f 0 0 or not? So, this relation we are trying to derive that to like we assume that that configuration is frozen, in the sense there is no work has been done. So, that is what the whole idea in the sense we are going to use the static equilibrium.

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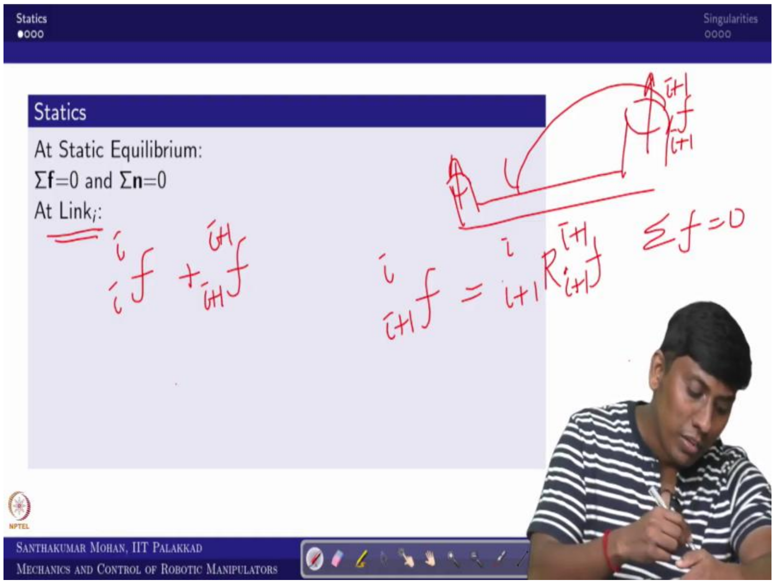

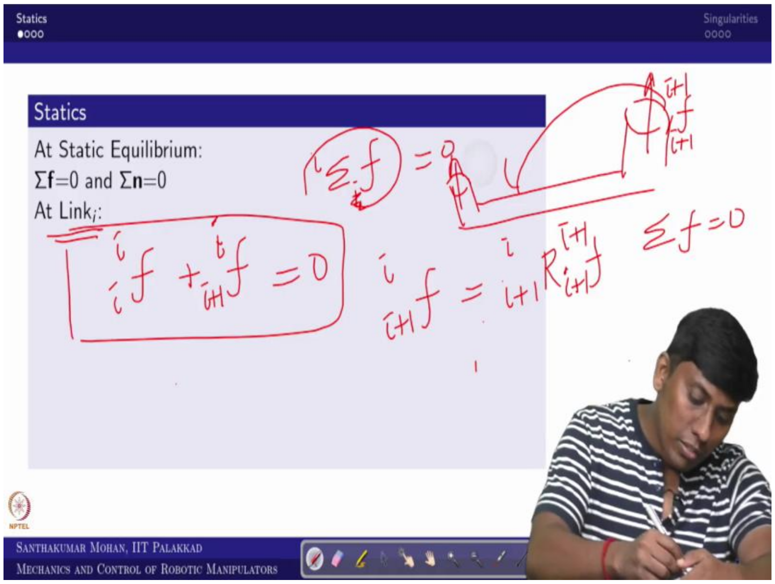

If we take a link i in the last you can say picture I was showing, so you take the link i. So, now you can see that there is one force acting, so this force is like i plus 1 to i plus 1 frame. If I want to convert here, so what I need to do, so I can multiply that, with respect to this. So, this will give what, so f i to i plus 1. But what one additionally you should know this is statically equilibrium, in the sense static equilibrium, in the sense the cumulative forces would be 0.

So, if that is the case here, what are the forces would be acting, so 1 force would be i to i. So, then there would be another force would be i plus 1 to plus 1, but we are writing it right now, we are seeing everything in the ith frame. So, then what I am writing this ith because we are seeing an ith frame. So, this supposed to be 0. So, wherever we are putting plus sign or minus sign that is based on your representation, but what we are saying the sum of f with respect to i frame, with respect to i frame, so that we are equating to 0. So, that is what the whole idea.

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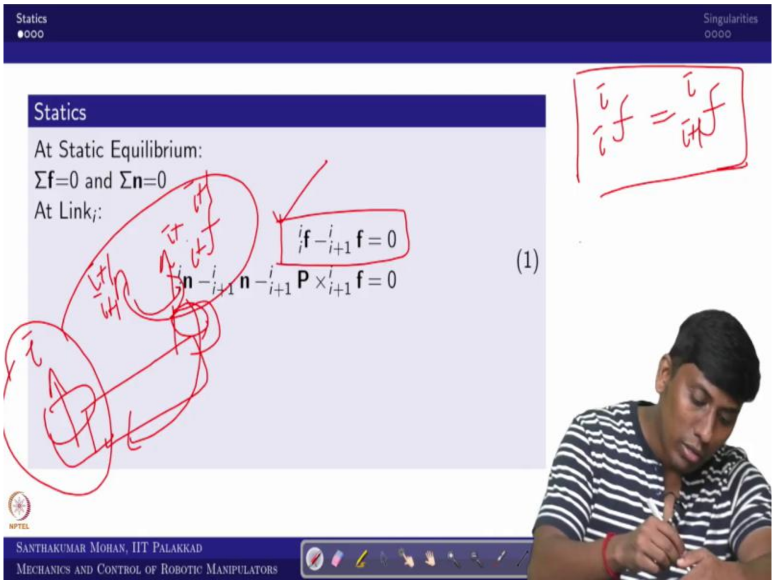

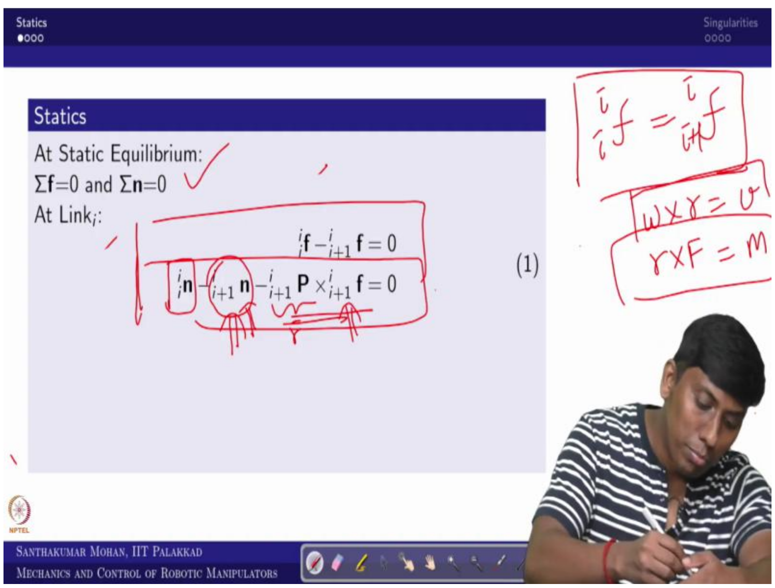

So, if that is the case, so what you will get you will get this relation. So, that is what I said the sign is up to us, we assume that f i to i supposed to be equal to f i plus to i, so then only this equation would be valid. So, we are taking this way and then we are using this equation. Similarly, the moment, so the moment also like you can see like this link, this link would be having another you can say link having ith joint and i plus 1 joint So, this would be having an angular moment, and this would be having a force vector i plus 1 to i plus 1. So, these 2 would be generate the torque at ith frame.

So, that torque how I can write it, I can write it as in this form. So, what that form, so, you can see like this is what we wanted, but that is represented from the previous you can see, or you can say proceeding joined angular moment. And the proceeding or you can see, you can say before joint force is also acting. So, now, you should be clear omega cross r would be linear velocity and r cross F is the moment. So, the direction should be very clear.

So, now, you can see this is equal to r cross F. So, this is what the static equilibrium condition. So, whatever we have written this we have written in this form. So now, we can write this as a propagation model, in the sense backward propagation model.

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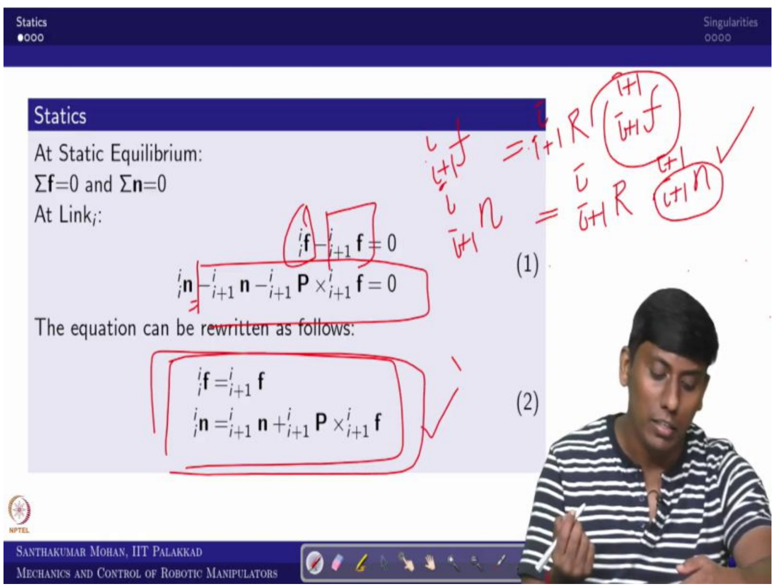

So, for that what we are rewriting, we rewrite this equation in this form. So, where we are equating this equal to this. Similarly, we are equating this all equal to this form. So, in that sense these are the two equation, but what I said this i plus 1 to i plus 1 the force and similarly the i plus 1 you can say angular moment with respect to i plus 1 frame is available. So, in that sense I can multiply that with this matrix, this will give so, i plus oneth you can say angular moment with respect to ith frame would be obtained.

So, similarly here if I get, this is force vector of i plus 1 with respect to ith frame would be obtain. So, this is what we have used here. So, now, you can see this is the backward propagation model because we start from n to 0. So now, based on this we can derive the force vectors and angular moment, right now we are not going to use that, but we have some relations.

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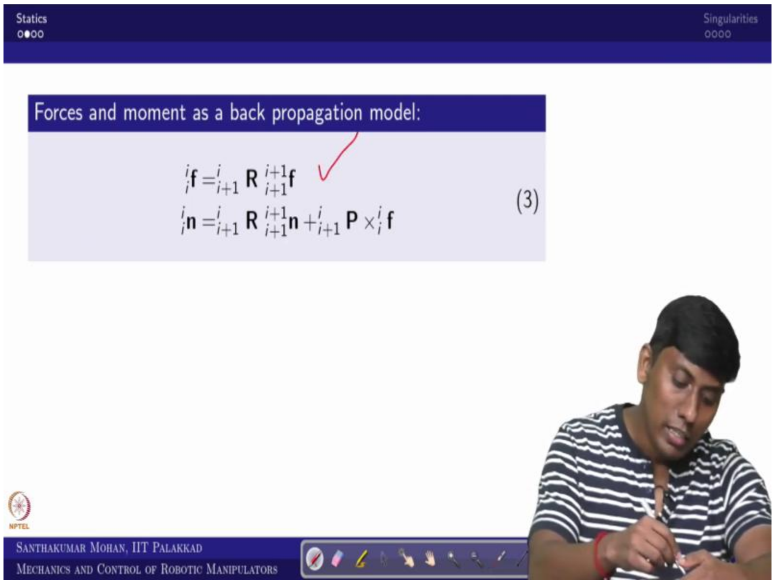

So, what relation we have, so we have these relations, what we have written already.

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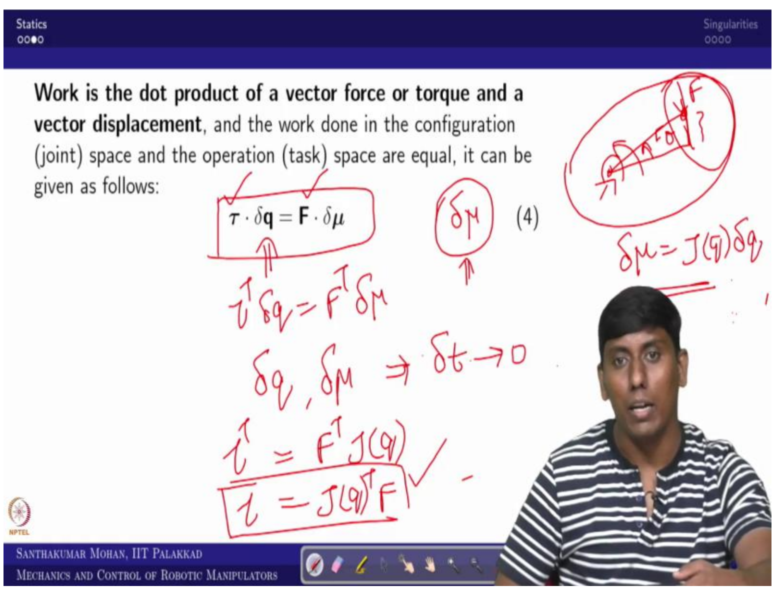

So, what we have is this virtual work. So, work is a dot product that we know, as a vector force or torque and a vector displacement. But what we can see that right now, I am just giving an idea. So now, this is a simple pendulum, I am putting a force here. So, how much this force displaced, so that would be corresponding to the displacement happened on the you can say the input side. So, in the sense, the total work done in virtually is 0.

So, but what we can write it in the other way around, whatever is work done at the end effector level, that is supposed to be equal to the, you can say the configuration or joints space level, if that is the case, just joint space torque vector written as tau and the end effector force, and you can say moment written as capital F. And you know delta mu is the change in displacement in mu vector and the delta q is change in displacement in the joint space or you can say joint angles.

So, now this we know, but the dot product I can write as a transpose. So, this I can write, and we assume that this delta q and delta mu happened on the delta t time and further this delta t is very small very close to 0 our delta t tends to 0. So, what we can write, we can write delta mu as J of q into delta q. So, this relation we have.

So now, if you substitute what it will come, so, the tau transpose is F transpose into J of q where you take transpose throughout, so what you will get, so tau is J transpose of q into F. So, this is what we are going to call as a relation. Now, if you know the end effector forces and you are Jacobian matrix from 0 to n is known, so then what you can see you can find these static you can say efforts at the joint whatever required word that you can calculate.

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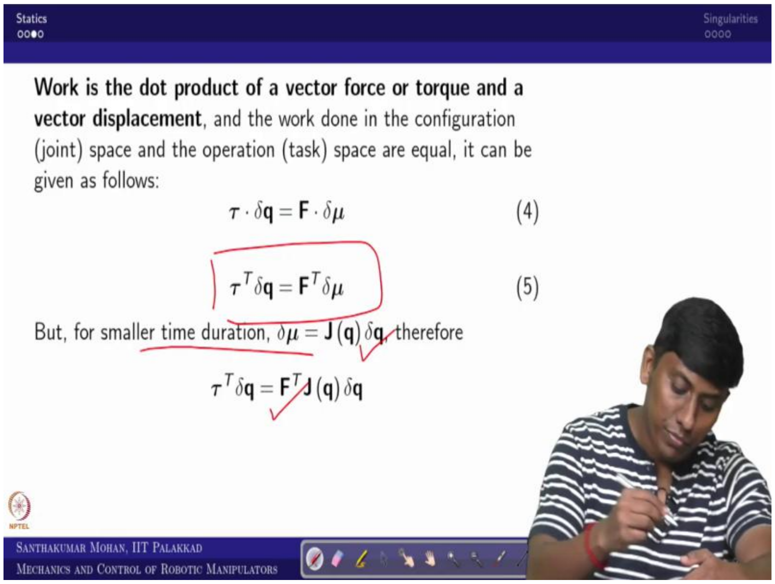

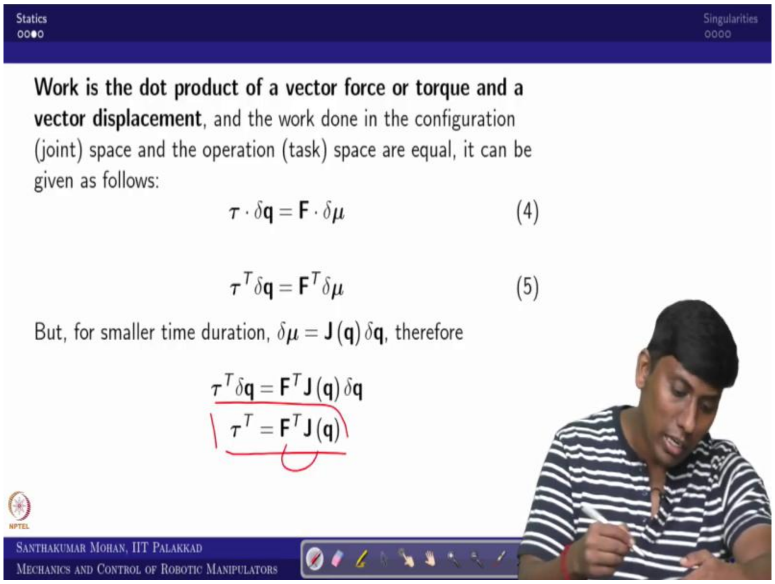

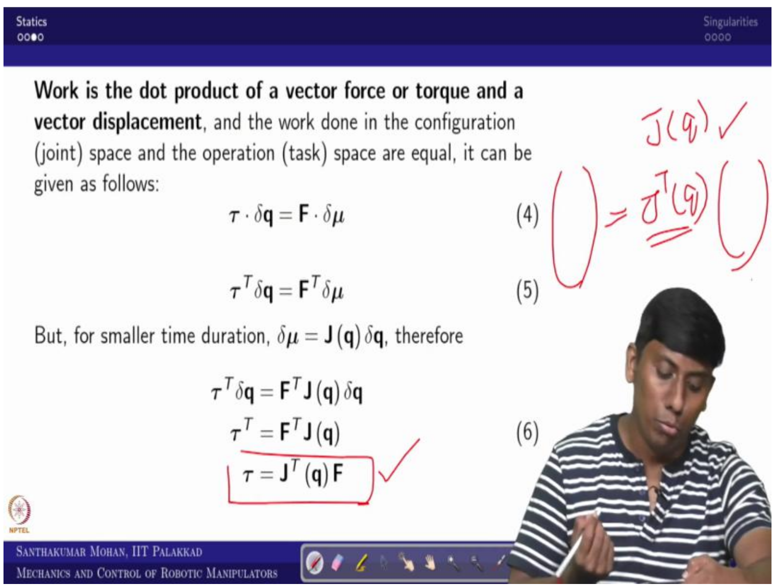

So, that is what we have a detailly returned you can see and for a smaller time duration you can write this. So, if this is valid, this can be done. So, once this is done, so, this is what the final relation can you take transpose throughout, so this whole transpose would become tau and you take whole transpose then this would be interchange with that transpose. So, that is what we get.

So, this relation you know, because J of q can be obtained with the velocity propagation, so now J of q transpose only require. And you will know end effector forces you can calculate what would be the input required to maintain that static equilibrium.

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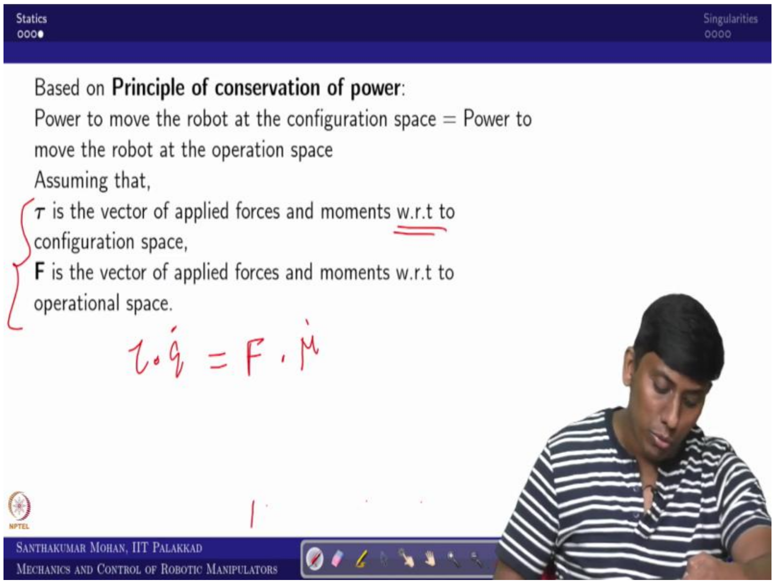

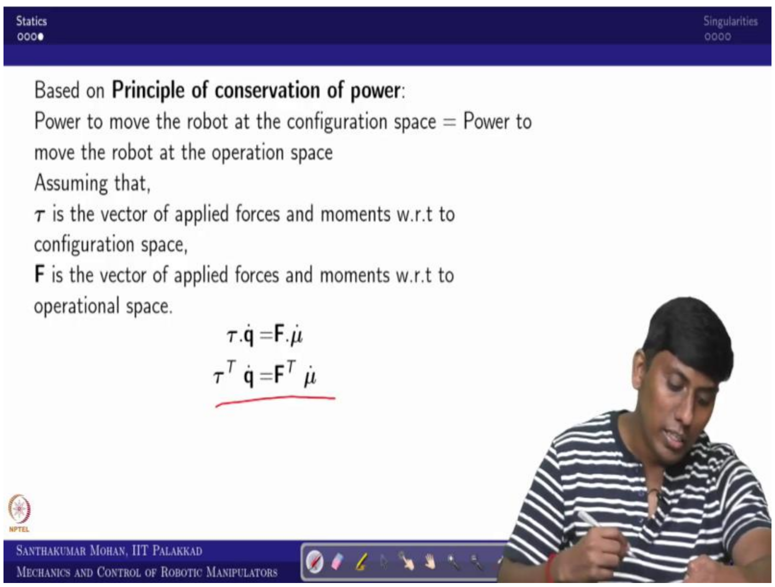

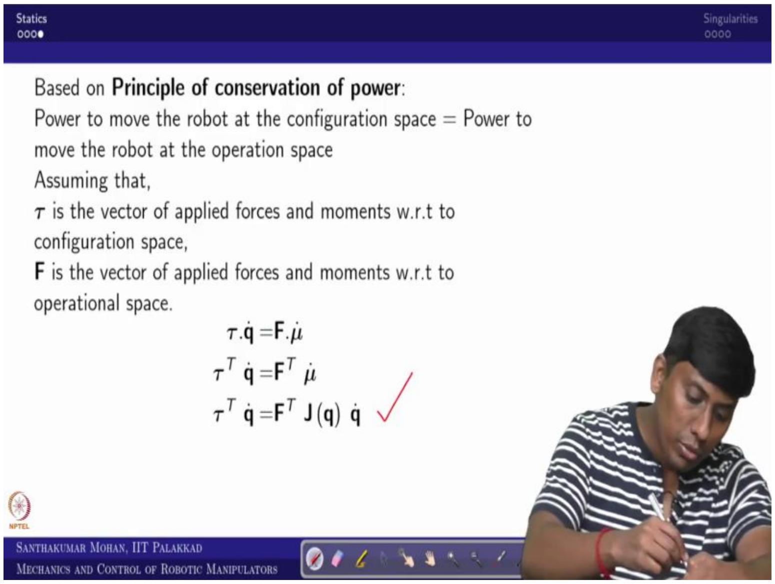

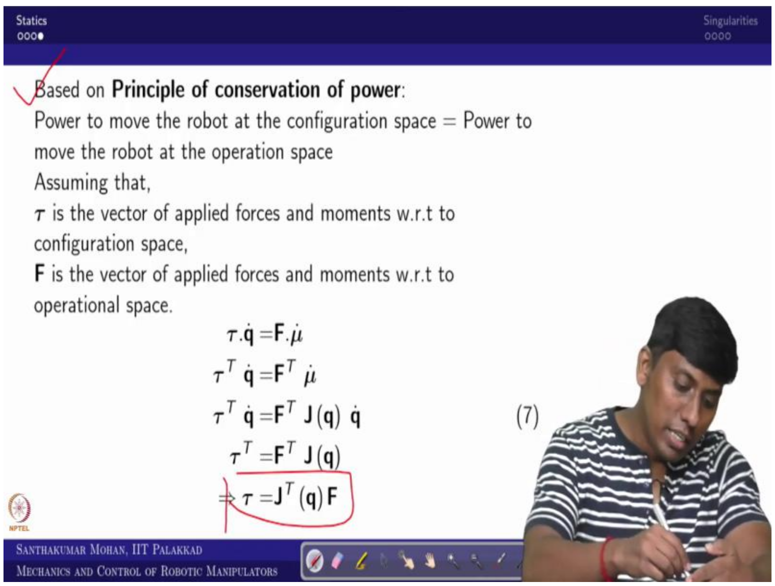

The same relation some people would have written based on the conservation of power. So, what that means, the; you can see the power happened, we assume that there are no losses in the sense of whatever power happened at the output end in the sense task end, that would be equal on the configuration end. So, that is what the case, so that is what we have written. So, in that sense, we can write that as a vector product or dot product. So, that dot product we can write it as, so tau dot you can write as a q dot which is written as F dot this is F so F dot mu dot.

So, now, this relation we know and again we can use the dot product as vector basis. So, this is what the relation and then you know mu dot already J of q into q dot this relation we know already from the forward differential kinematics, and you substitute now finally, you will get the same what you call relation. So, here, so, this is what we obtained and finally, we will get this. So, this is the other way of finding, but we always use principle of virtual work. So, this is conservation of power. So, both are giving the same what you call relation, but this relation we can use it.

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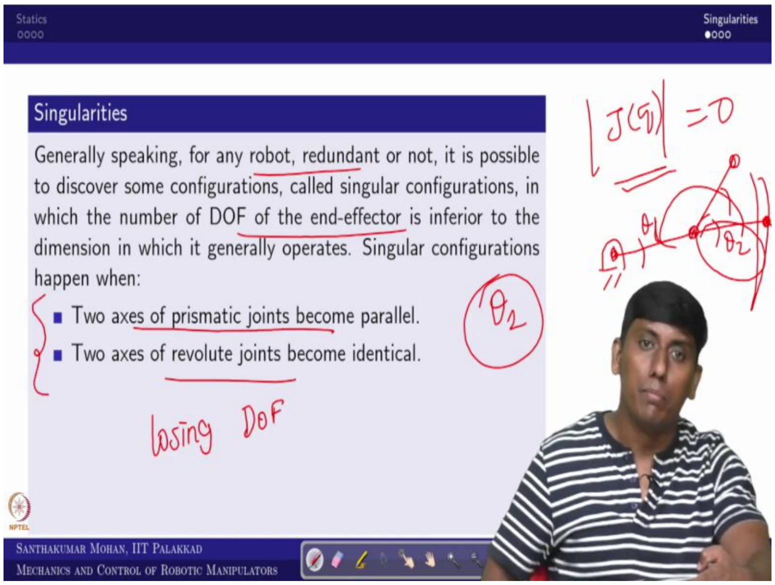

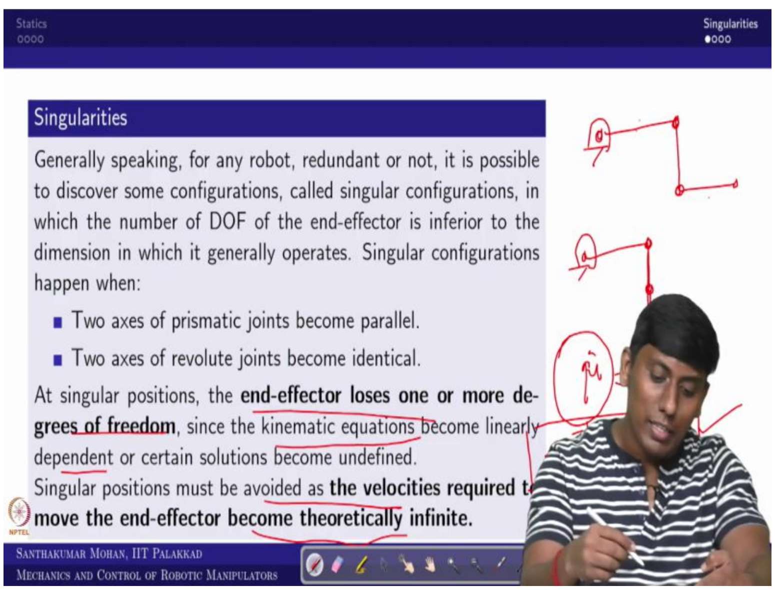

So, now, so, one thing we supposed to know what the other thing. So, based on the J of q if the determinant is 0 what we can say it is singular. So, when this will happen, so whether the robot is serial that too redundant or not, it is possible some cases where the Jacobian matrix the determinant would be 0. So, what cases it can happen if the prismatic joints are there that become parallel or if it is revolute joints are identical or it is collinear.

For example, you take this as a serial manipulator, which has two R. So, now, we are rotate this make it parallel. So, what happened you are having theta 1 and theta 2, but now, this theta 2 is 0. So, now, when I rotate theta 1, the theta 2 is invalid, in the sense that theta 2 is lost its degree because I want to rotate this. So, when this can happen, this can happen is at extreme in the sense, so what pace extreme case or it can come some other collinearity. So, that is what we are doing it.

So, in the sense what happened the end effector would be inferior, the dimension, the sense if you want to write this would be a function of only theta 1. In the sense what you are you are losing the theta 2. In the other way around you are like losing the degree of freedom. So, one or more you are losing it. So, that can happen due to some situation. So, that is what we are writing it. So, now you can see that the end effector losses one or more degrees of freedom, since the kinematic equation become linearly dependent.

So, now, you see that this is a 2 R serial manipulator even I am making it a 3 R. So, now you can see these 2 are collinear. So, this also can happen the other way around. So, you can see this is, so this is another configuration. So, another configuration where you can see that these 2 joints are collinear, this is also like reduce the dimension. So, that is what. So, but what indirectly it is giving, you see you have a relation, so mu dot is J of q into so q dot.

So right now, you have some, some end effector velocity. So, you are trying to find this as a q dot, so if the singular cases what happened the q dot would be infinite why, because the J q inverse you can say J inverse of q become a singular in the sense, so you can see that the q dot become what you can say the J q inverse would be infinite so the product would be infinite.

So, in the sense of for example, you keep, so you keep your hand. So, you can extend, so now you can see that your elbow is lost. So, if you are doing it your elbow cannot have any you can say control that. So, now if I want to fold at this extreme case, I cannot create any force because its required infinite efforts. So, that is what the whole idea. So, that we can discuss a little bit in detail in upcoming lectures.

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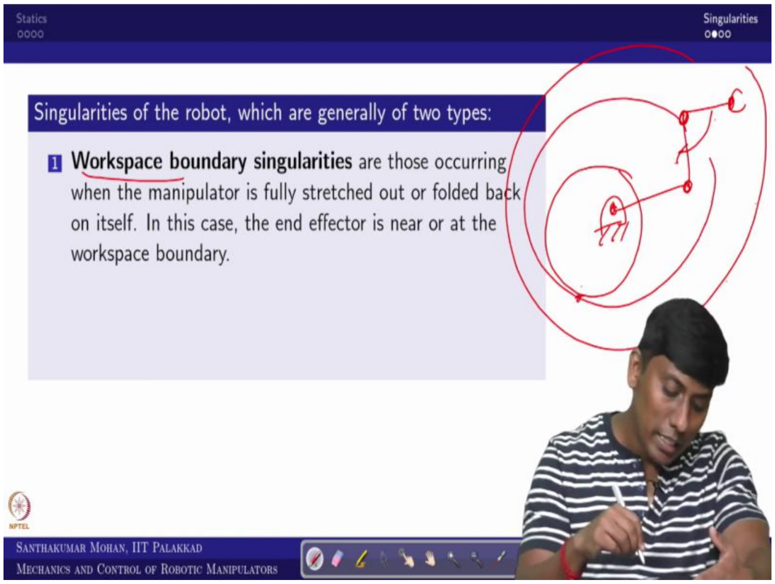

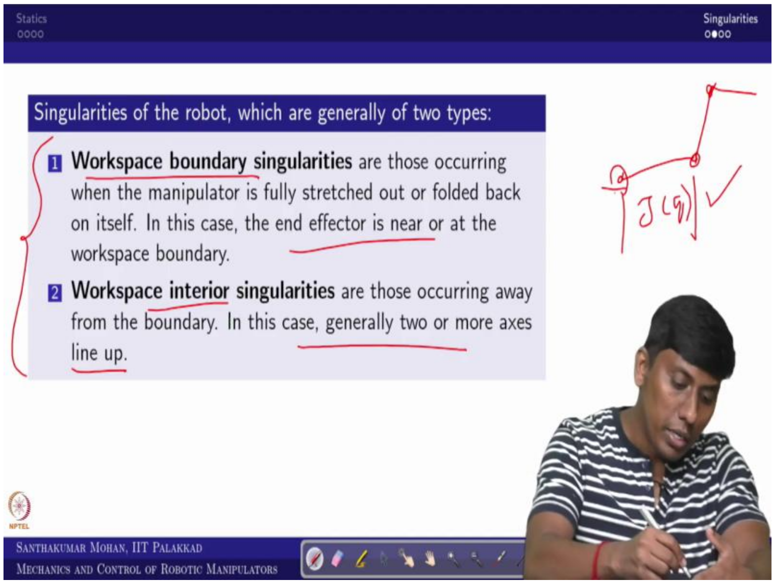

So, right now, we will see what kind of singularity can come. So, there are 2 kinds of singularity in serial manipulator. So, 1 is workspace boundary singularity. So, for example, I am taking this is a serial, 3 R serial parallel or you can say planar manipulator. So, this is the end effector. So, this can happen, like at the extreme where this is the workspace, the workspace extreme it can happen, this point and this point, it can be singular because it can be completely folded, the other way around, it can be happened these 2 can be folded still that would be within the workspace.

So, that is what you call interior singularity, the other one is boundary singularity. Workspace boundary singularity is extreme, if you are taking a 2 R serial manipulator, it has happened only the workspace boundary singularity, but if I add the third rotary joint that would have interior singularity. So, when this can happen, 2 or more axes lineup int the sense co linear. This is the extreme case it cannot go beyond; it cannot go beyond like that.

So, these 2 are the 2 types of singularity, how we can calculate the singularity. So, you have already the clue. So, if you have a Jacobian matrix, you can take the determinant and equal to 0 you will get the singularity condition. If it is a rectangular matrix, then you can use the J, J transpose determinant or J transpose J, so based on the size.

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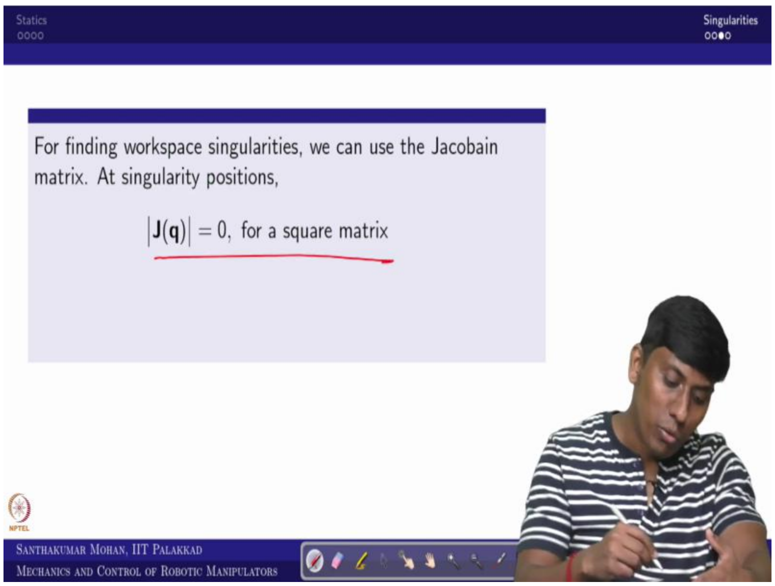

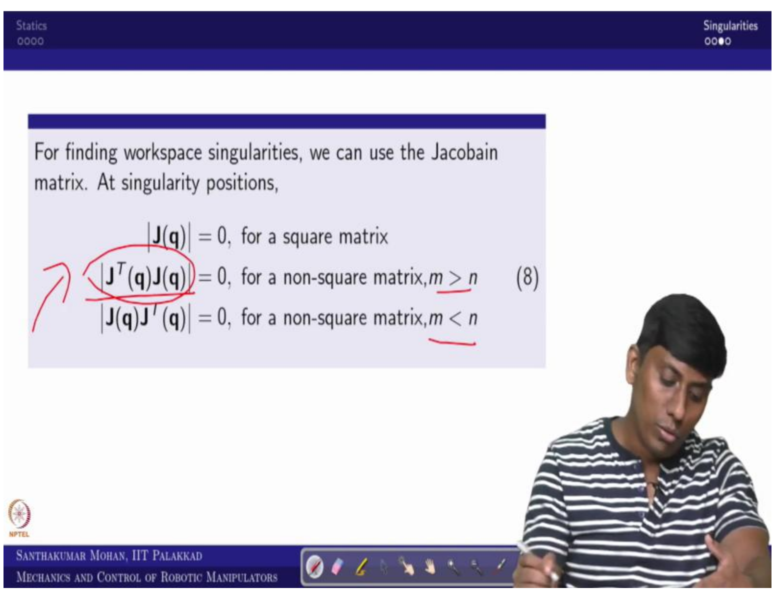

So, that is what we are trying to see. So, we can find this for a square matrix, if it is rectangular matrix, you can take J transpose J determinant or J J transpose determinant based on the condition. Indirectly which rix size is giving a smaller one that is what we are going to take it. So, that is what the whole idea.

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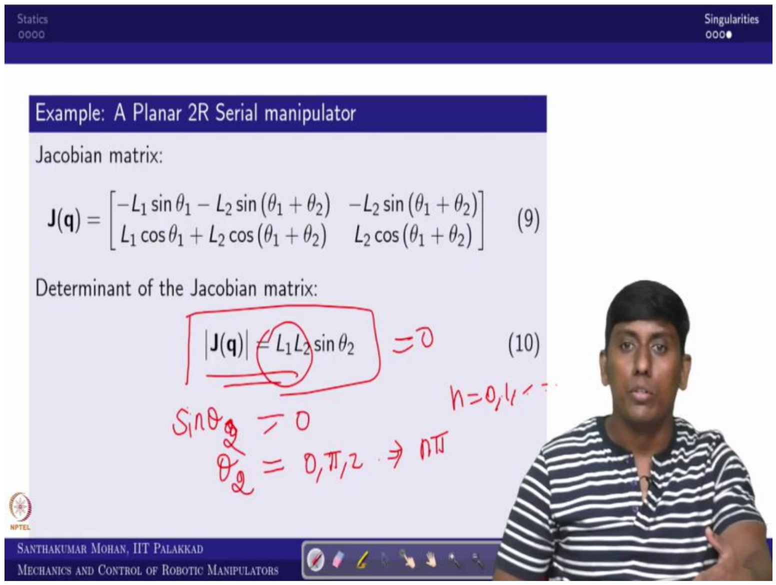

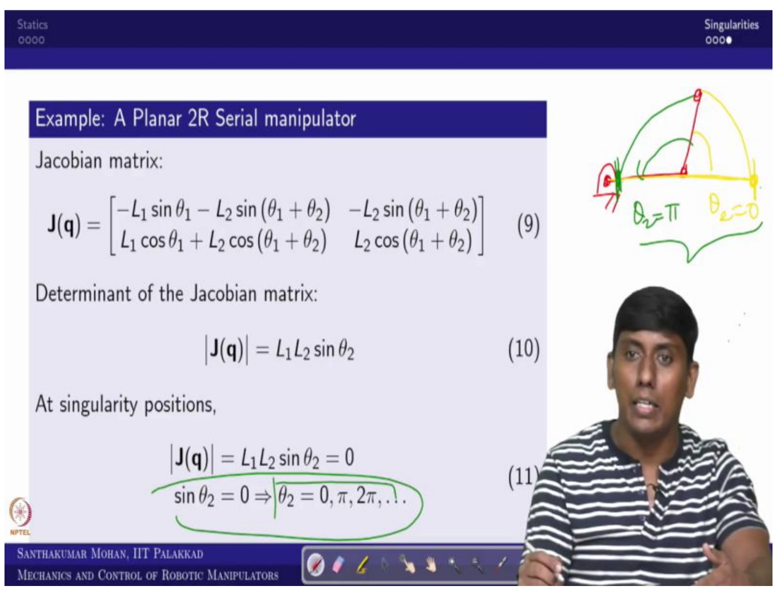

So, I am just showing a small example for here, at 2 R serial manipulator, you know already the Jacobian matrix. So, now you take a determinant, so what it will come the determinant would be something. So, finally, we will end with this as the residue. So, now you equal to 0, so L1 L2 is link length that cannot be 0. Then you can see sine theta 2 would be, this is the reduced form. So, this would be 0, then theta 2 is either 0 or pi or 2 pi in the other sense I can say n pi. So, any n, n start from 0 to you can say any number 0, 1, 2, 3.

So, this is what the condition, so that is very clear. So, when this happened, so very 2 simple case we can take. So, for example, this is a 2 R serial manipulator. So, I am trying to show that when this can singular, so you rotate this and make it. So, what happens this is 0-degree theta to 0 degree. So, what it says it is extreme this is the whole space the boundary. So, now the same thing you rotate the other side. So, it is come here. So, in the sense it is coming here, what that so theta 2 is pi degree. So that is also like in this case, it is the boundary.

So, these two boundaries singularity is realized from this case. So, when theta 2 is 0, or pi, 2 pi is again 0 equivalent and 3 pi is again equal to pi. So now, these are the 2 situation of singularity condition for a planar to serial manipulator. So, we will see more detail in 1 or 2 example we will see in upcoming lecture. So, with that, we are closing this particular lecture. And the next lecture, we will see few examples. So how we can find the singularity and how we can find these static conditions. So, with that, I am saying thank you. See you then. Bye. Take care.