Mechanic and Control of Robotic Manipulators
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Lecture No. 20
Introduction to Differential Kinematics

Welcome back to Mechanics and Control of Robotic Manipulator. The last class we started what you call differential kinematics and in the last class itself, I said that the next class we will see what we call velocity propagation. So, this particular lecture is focused on velocity propagation. So, where we can first start with a very general, then we will actually go with specific examples. So, in that sense let us move to the slide.

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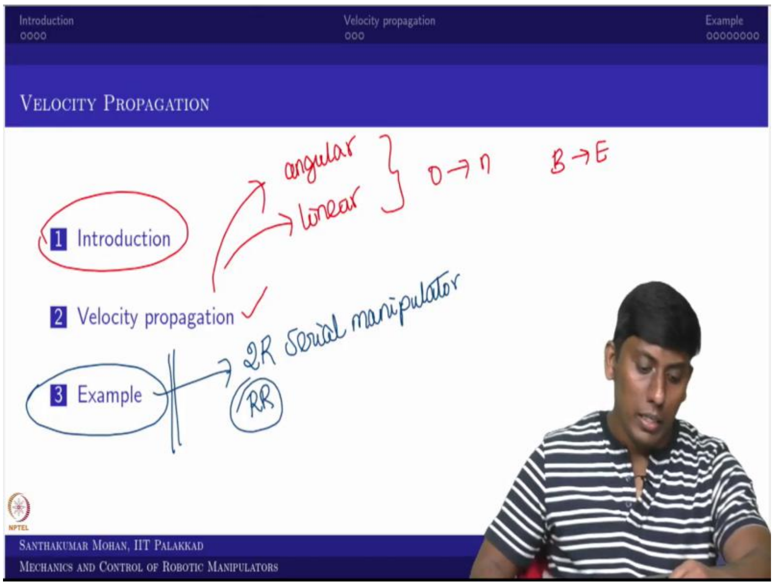

So, we will be going to talk about general what you call introduction, then we will see velocity propagation, this velocity propagation would be, so both angular and as well as linear. So, both linear and angular velocity propagation in the sense, so the base joint to you can say nth joint. So, in the sense of base to end effector we would be doing it. So, that is what we are trying to see.

Then we will be seeing one of the simple examples, later on we will see few other examples. So, the example here we are going to consider is, so 2R serial manipulator. So, I should not call to 2R, because 2R means, it is actually to rotary joint, but it is supposed to be called RR. So, this particular manipulator will take as an example and see.

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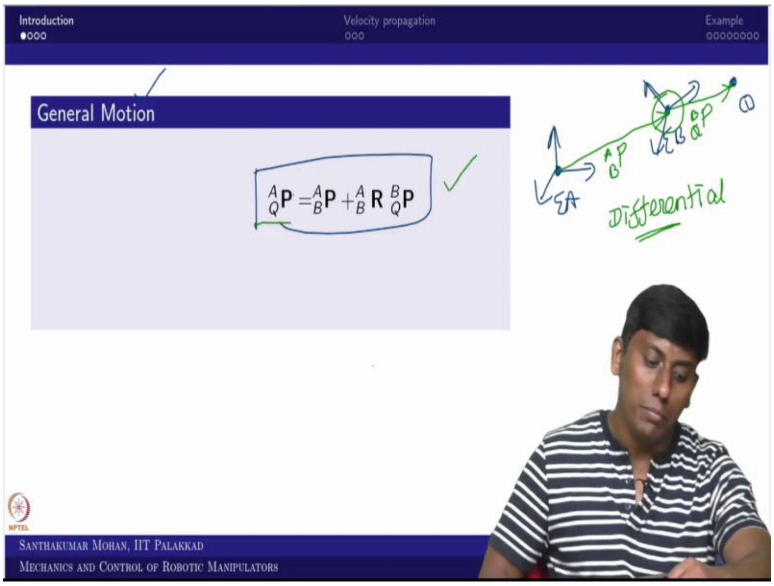

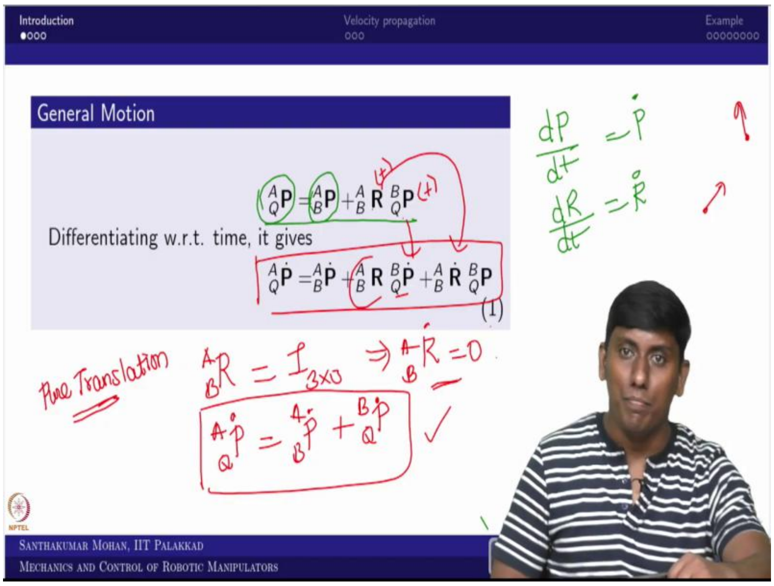

So, let us move the general, general is like what. So, we know like the general spatial motion can be returned in the form of this. So, what that means, so you have a frame you can say A, and there is another frame B which is you can see like this is rotated. So, now there is a point Q.

So, if this is the case, we can write, so this vector Q with respect to B and this vector we call position vector of B with respect to A, if that is known and this rotational matrix information is known, then I can write the; what you call the position vector of Q with respect to A frame in this form.

But now, what we are going to do we are trying to talk about what you call differential kinematics. So, for that what we can do, we can differentiate this equation. So, what it comes we can see so what you can see the dot nothing but so dP by dt I am writing as a P dot. So, d R by dt I am writing as R dot. So, this is the case. So, now if that is the case if you are differentiate this equation with respect to time.

So, this dot will come, and this is also dot will come. So, then this you can differentiate this would be function of both, so in the sense this is also a function of time, and this is also a function of time. So, first we will take the position derivative, then we will actually take the rotational matrix derivative.

So, if you are looking at this is the final equation. So, in this equation we can take it one thing, what the one thing. So, instead of general plane motion if it is pure, so translation. So, if it is pure translation what that means, so the rotation matrix is identity matrix. So, in that sense what it becomes also this P of Q A dot become P of B A dot plus. So, this is identity matrix, so this will come.

So, Q and B since its identity matrix this rotational matrix derivative would be 0. So, that is the case. So, in the sense of what you can see this is two velocity updates, in the sense you have a like one velocity and there is another velocity. So, what you can do, you can take the vector addition that is what this case, but if it is actually pure rotation, so if it is pure rotation, what it will give?

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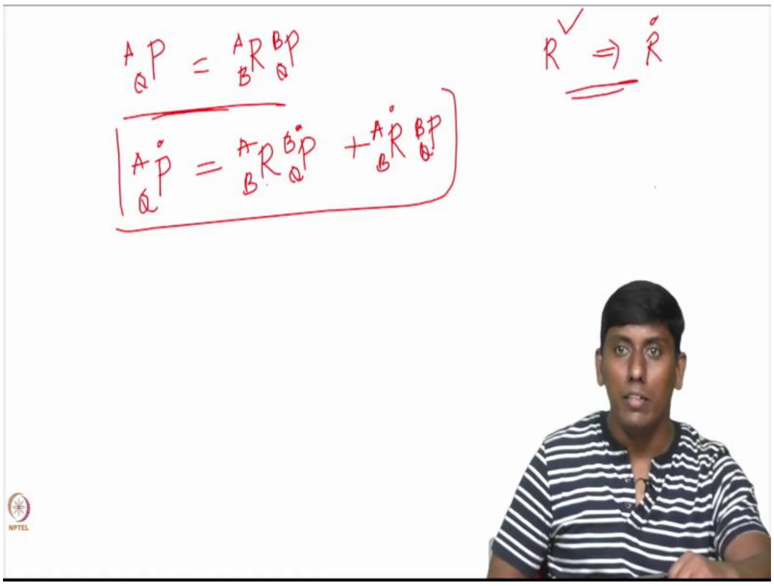

So, if it is pure rotation, so, this is as simplified as this. So, if it is pure rotation it is simplified like this. So, if you take a derivative, so this is work going to come, this vector would be having independent plus, so this R B A dot P Q B. So, now this is for pure rotation, but what we know, we know like R, can I find R dot in some other form. So, this is probably one my concern. So, can I do it?

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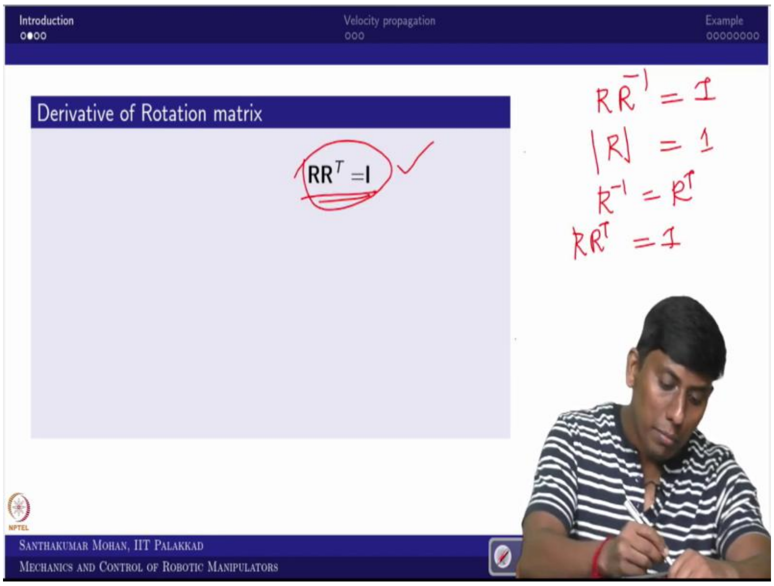

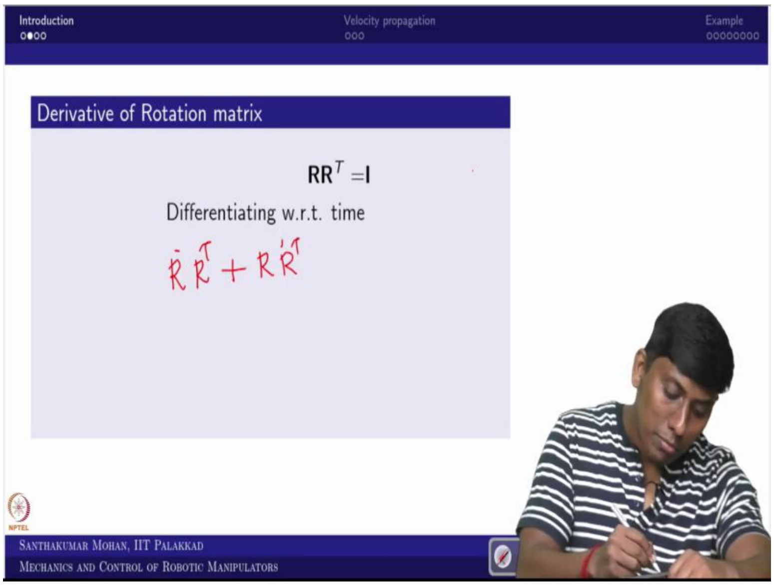

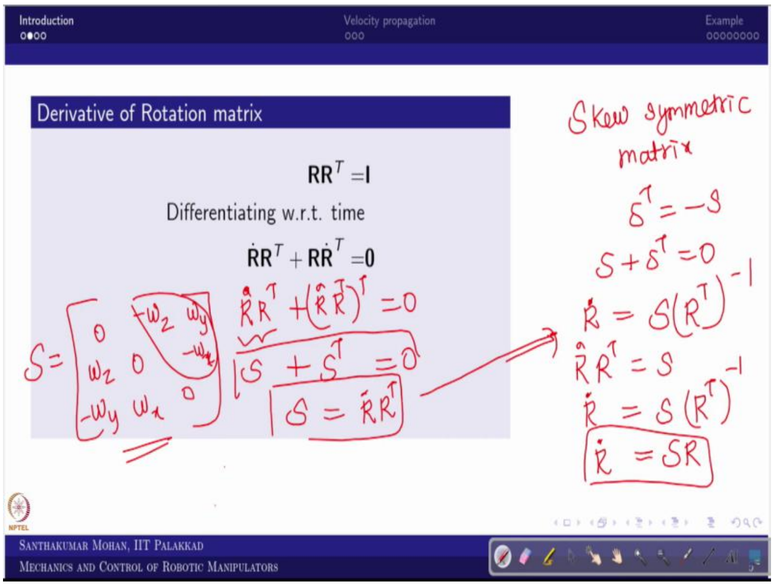

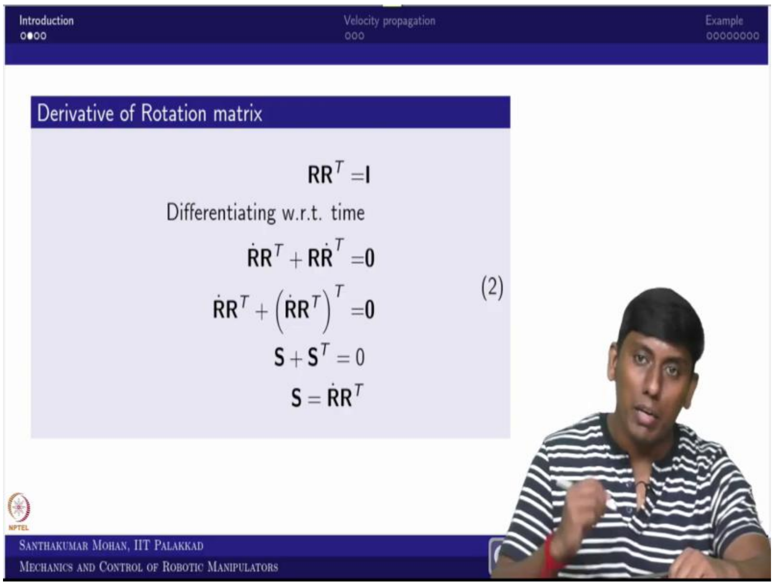

So, for that we are taking one simple idea. So, what we know. So, RR inverse is identity matrix, but we know R is orthogonal matters in the sense you can say the determinant of R is 1. So, in that sense of what you can write R inverse can be R transpose if that is the case what you can write RR transpose equal to I. So, that is what I have taken.

So, now what I can do, I can differentiate this. So, what that eventually give, so differentiating with respect to time of this. So, this will give R dot R transpose plus, so R R transpose dot equal to 0. So, this is what you will get. So, but what this, so you can take this transport throughout, so what I can get RR dot transpose plus R dot R transpose whole transpose equal to 0.

So, what this, so you might have studied about skew symmetric, skew symmetric matrix is nothing but so where S transpose equal to minus of S. So, in the sense S plus S transpose equal to 0, so this is very similar to that form. So, this I can write as a S and this is actual like S transpose it is equal to 0, in the sense of what.

So, S can be written as R dot R transpose. So, in the sense you can write R dot R transpose as skew symmetricity matrix or what you can take the R dot. So, I let us say take this R dot as the skew symmetricity matrix. So, this is pre multiplied, I will just, so then you can this inverse, from this equation, I am writing R dot R transpose equal to S.

So, now I am looking at R dot, so then I can actually take S R transpose inverse, but what is R transpose inverse that is nothing but R only. So, this can be written as this. So, now you need to write the skew symmetricity matrix, the skew symmetricity matrix here is only 3 cross 3 because your rotation matrix has 3 cross 3.

So, in this sense, you can write this is I can write as probably some omega z, then omega y and minus omega x. So, this is 0 0 0 diagonal would be 0 and opposite is positive. So, this is the way I can write, this is some skew symmetricity matrix. So, you can see the diagonal is 0 and the off-diagonal term is actually just the opposite same sign. So, then it will give S, transpose S minus S.

So, now this policy we can do it. So, that is what we are trying to do it. So, in that sense, I am rewriting this equation in your benefit form, you can see I have rewritten this and this I have written as skew symmetric matrix form and where R dot I can find us S into R. So, this way I can do it. So, this is one way.

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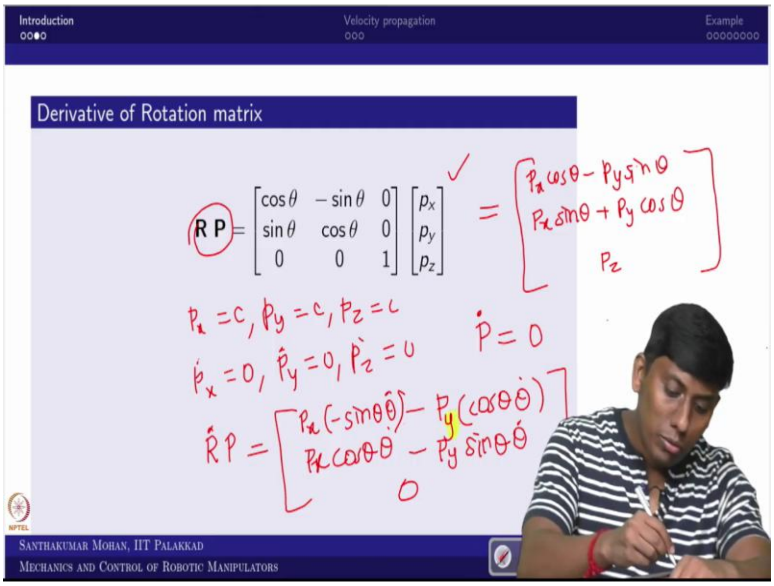

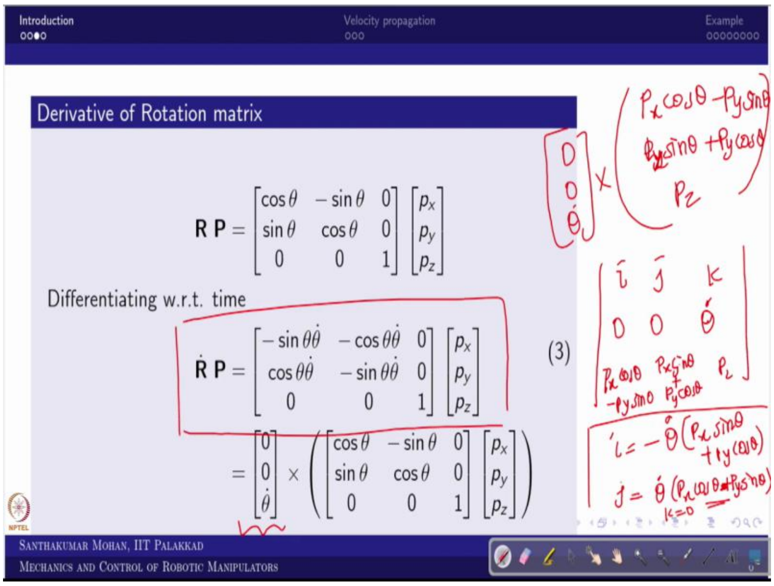

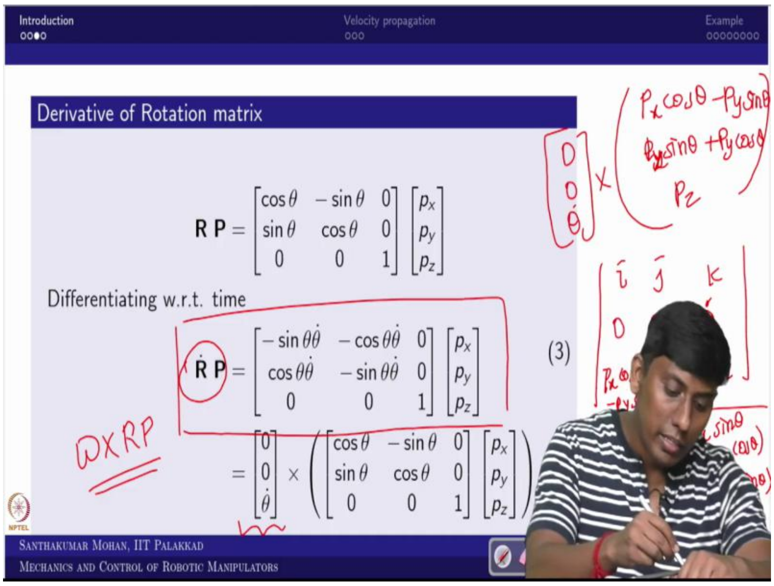

The other way actually, like people used to do is something like this. So, I know R and P, I am just taking a random R matrix, which is a rotated aboard z axis. So, it is having this and the position vector as like this. So, now what this equivalent. So, this equivalent is something, so Px cos theta so then minus, so Py sine theta is the first vector, the second vector is Px sine theta plus Py cos theta and the third one is sample Pz.

So now, you are assuming that the Px equal to constant and Py equal to constant and Pz is equal to constant. So, in that sense what we are trying to say Px dot would be 0 and Py dot would be 0 and Pz dot would be 0. In other way around it is actually a constant. So, the P vector would be 0, that is what we are trying to do.

So, now, if I differentiate this, so R dot P, what it will come So, I can rewrite this, so what would come, so Px dot would be 0, so I am not bothering, so in the sense the Px, so the cos theta is having function, so minus sine theta, so theta dot and this would be minus, this would be a like minus, so minus, so minus I do not know like, so minus Py. So, minus Py, so minus Py cos theta and theta dot I am sorry.

So, then in the bottom Px cos theta theta dot and the second one is minus Py sine theta theta dot and this would be 0. So, like this you will come, or you can do it this differentiation, so that is also there, so I am just doing it first the differentiation of this. So, I will show and come back.

So, now, this is the equation which we obtained. So, this equation, I can rewrite in the other way round. So, what other way around you can multiply this, so it as actual Px cos theta minus Py sine theta, then this is Py sine theta, Px sine theta plus Py, so cos theta and Pz. So, this is one vector if I multiply.

So, this I take a cross product with theta dot. So, what that will give, that will you actually like, so i j k. So, then this would be having 0 0 theta dot and this is a Px cos theta minus Py sine theta and this will give you, so Px sine theta and plus, so Py cos theta and the Pz and you can see this term you take it, so in the sense you take, so I will like multiply.

So, i it will come. So, what will come. So, it is minus theta dot, so Px sine theta plus Py cos theta. So, that is what we are also getting it. So, then the j vector would be, you can see what would be the j vector, j vector would be again. So, theta dot it is minus of minus plus, so that is why I have a taken that way. So, Px cos theta. So, plus. So, Py sine theta. So, like this I will get minus. So, now you can see, the K would be 0 because it is K.

So, now this is what we obtained in the previous equation, the same thing we are obtained in two forms. So, in the sense what you can see, if I actually take our R dot P in the other way around, where so 0 0 theta dot cross with the R P, so that is also like giving the same relation what you did in the previous case. So, in this sense what you can see, so you can write R dot P as something like omega cross R P. So, that is what we are trying to find.

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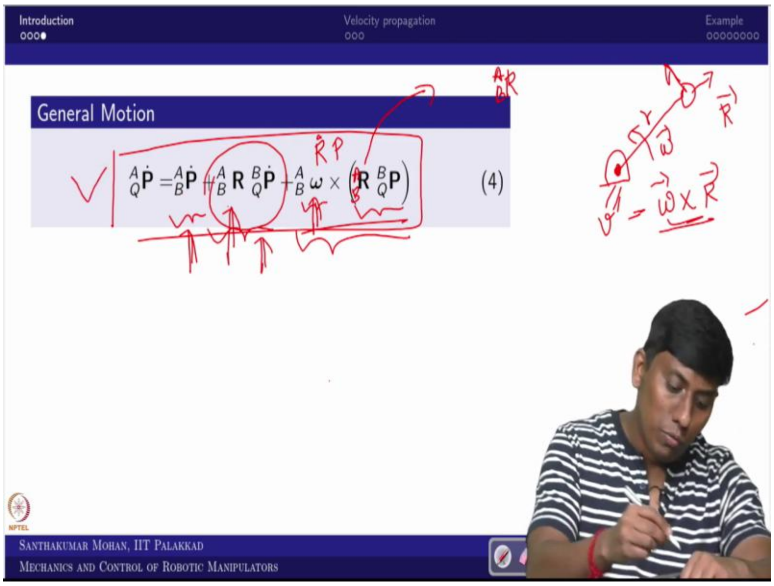

So, you can see right. So, now the previous equation where R dot P, I have written as omega which is angular velocity vector multiply with the, you can say R cross R, you can say R multiply with the P. So, this is one such case. So, in fact, if you recall your general dynamics, so for example, this is a rotating omega and this is the R, so your position vector goes like this. So, which is R vector and this omega with respect to z axis.

So, now this cross product will give the tangential velocity which is what you call linear velocity that would be omega cross R. So, that is what we have actually seen, which is nothing but we if it is pure rotating. So, in this case this is for pure rotation and this is pure translation with rotated frame and this is actually the frame translation.

So, this is pure translation with respect to rotated frame and this is pure translation. So, in the sense this as a slip, slip velocity and this is having a slip velocity, but it is in a different frame. So, you are converting into the frame, and this is a pure rotation. So, that is what the idea, so here I should write this.

So, now whatever I have written this is rotation matrix of B with respect to A that you please remember I missed that syntax here. So, now you can see like this is the general motion, so which we are going to use further and further.

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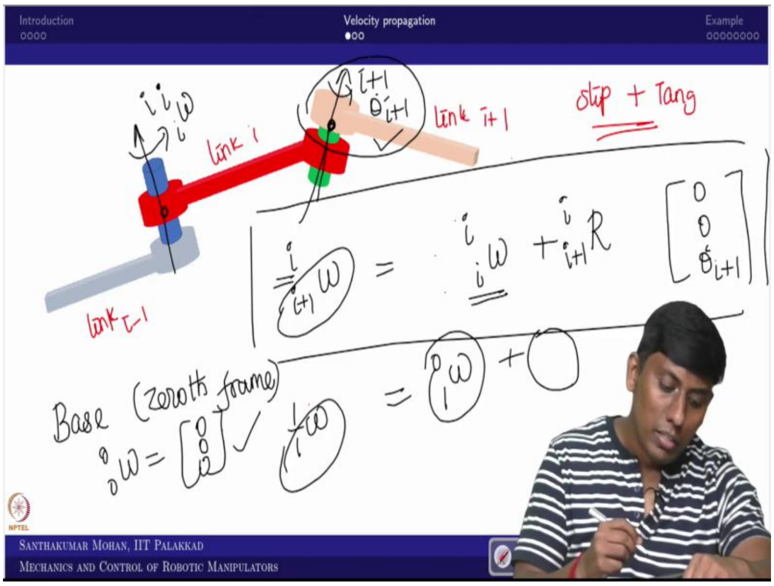

Because we are going to talk about velocity propagation. So, you should remember. So, there will be slip velocity plus, so you would be having a tangential velocity these two components should be known, so let us come to the velocity propagation state. So, you can see like there are three links.

So, I call link, you can say this is link i minus 1, this is link i and this is link i plus 1. So, obviously, what one can expect immediately. So, this would be the axis i and this would be the axis i plus 1. So, now if I separate only this link i, so what I can see that would be having ith axis axis and i plus 1 axis.

So now, I assume that this is fixed, and it is having some angular velocity omega i. This omega I is referred with respect to this coordinate. So, now you can see that this is having an independent motor we assume it is a rotary So, in the sense it would be having. So, theta dot i plus 1 would be there.

So now, if I want to know the; you can say angular velocity of this frame with respect to this frame, so what I need I need an angular velocity of i plus 1 with respect to i frame. So, what that would be, so that would be simple you can see that you know already omega i that is with respect to this plus what you have.

So, there is additional angular velocity at the z axis. So, in the sense you have 0 0 theta dot i plus 1, but that is with respect to i plus 1 frame. So, what you can do, you can take the rotational information. So, this is giving the, you can say angular velocity of omega, or you can say angular velocity of i plus 1 with respect to i, but what one can see, so you know the base, so the base or you can say is zeroth frame.

So, zeroth frame, we can always assume that the angular velocity 0 as vector or we can assume that it is actually like, put it on a mobile frame. So, you know this angular velocity in the sense, so what exactly unknown, so you do not know like, what is the propagation in the sense, what is omega 1 and omega 1.

So, omega 1 with respect to 1 you do not know, but what we have a relation, so we have a relation omega 1 with respect to 0, can I convert that with the something. So, if that is the question comes, so what I can see, so I have already one relation what relation.

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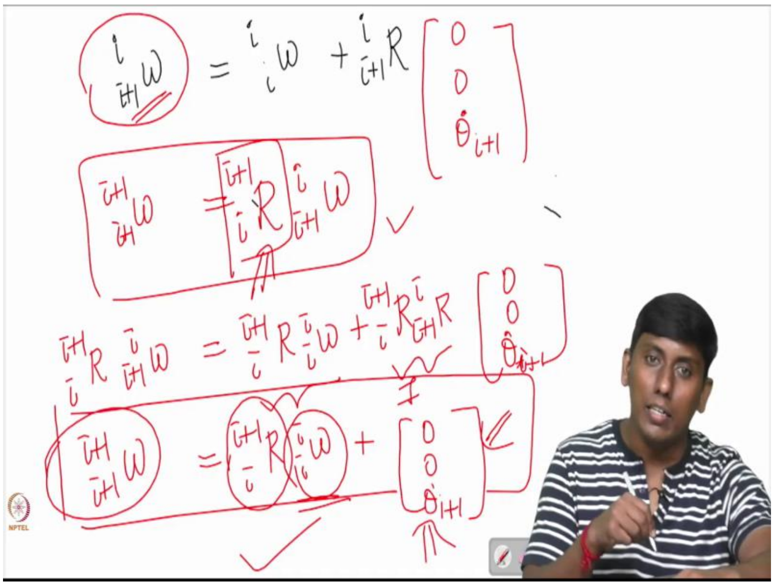

So, omega i plus 1 with respect to i, i got it as omega i plus i and this is a rotational matrix of i plus 1 with respect to i. So, this is I put it as, so I will put it as something. So, I put it this as 0 0. So, theta dot i plus 1. So, now this information I know, so what I can do, i wanted i plus 1 to i plus 1.

So, for that what I have, I have the angular velocity information of i plus 1 with respect to i, now I just to convert that to from the previous frame in the sense, so I just multiplied this with this. So, this will give, so this is a simple transformation, we have already seen, so this is one of the operators like in the rotation matrix we said that the rotation matrix can be used as operator.

So, in that sense, so we can see this conversion can be done. So, now what we can see, you can multiply this, this rotation matrix throughout. So, in the sense, so i plus 1 i, so if you multiply. So, what you will get. So, this become identity matrix and this section like you know because the previous frame and this is nothing but omega i plus 1 to i plus 1 frame. So, in this sense write it this.

So, this is what we obtain. So, this is what we call angular velocity propagation. Once you know base angular velocity you know the rotational information because you must be having the forward kinematic model is available with you and your motor speed would be available if we can start a rotary joint.

If it is a prismatic joint this would be 0. So, there would not be any active angular velocity there. So, then this would be a to like modified, that is what this particular equation is giving. So, you can see like if you know the previous frame angular velocity and the current active velocity then you can find the current angular velocity vector.

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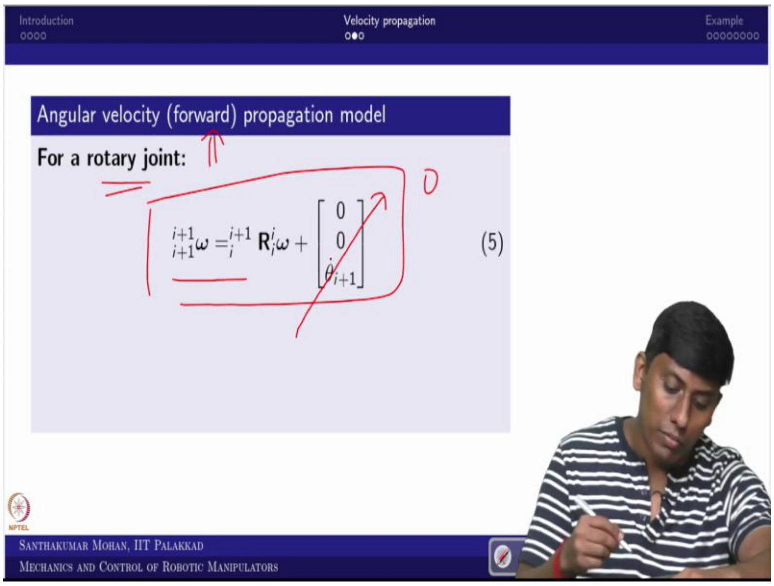

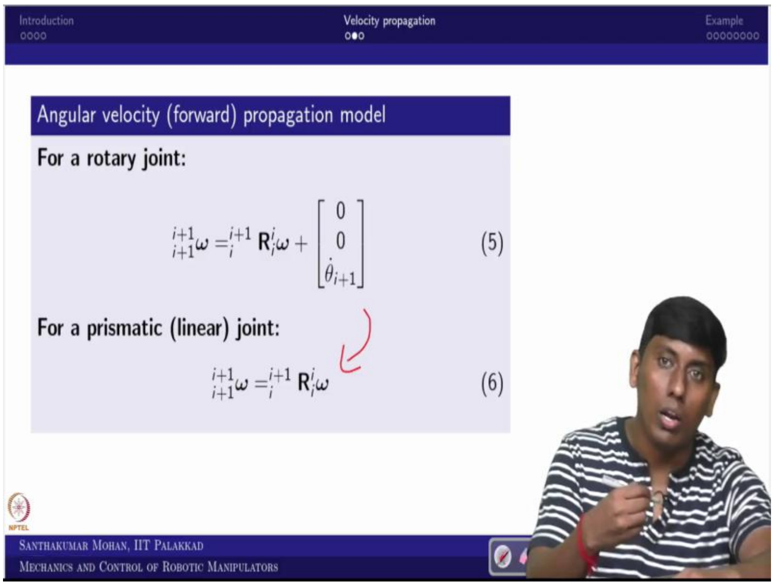

So, that is what we call like angular velocity propagation model, this is we call forward because we start from 0 to i or 0 to e or 0 to n. So, in the sense you can see this is what we derived, this equation we got it. So, now instead of this is a rotary joint. So, if it is prismatic joint this become 0, so that is what we are trying to say, if it is a linear joint. So, this becomes 0 and it would be a simplified equation. The same sense we can a go ahead with what you call the linear velocity propagation.

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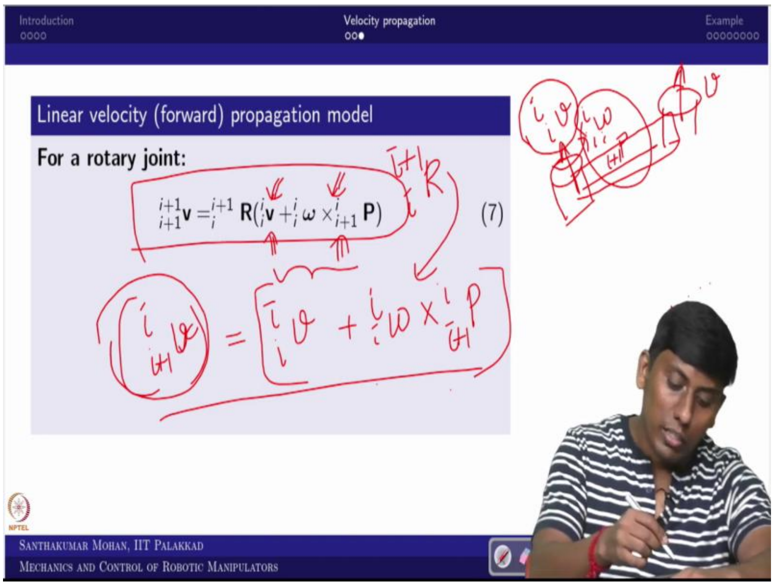

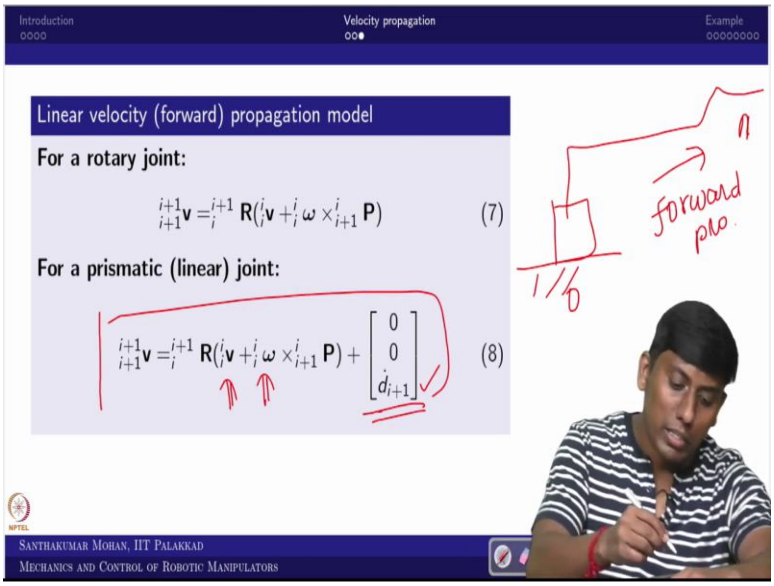

So, linear velocity propagation what one can see. So, this is one link. So, this would be having you can see, you can say joint axis. So, now we know the linear velocity of this and here you can assume that I wanted this linear velocity. So, what you know this is rotating omega i with respect to i and this is the position vector of i plus 1 to i.

So, what this will do, this will give a tangential and this will give a slip. So, that is what you can say this is a slip and this is tangential. These are all information would be in the frame of you can say i plus 1, in the sense I can write. So, V i plus 1 with respect to I can write V i because this is the same part of body.

So, then you can see this would be the tangential component. So, this would be available, and you want to change the coordinate, you multiply this throughout, you can say throughout R i plus 1 to i, so then this if you multiply this equation will come. So, again you can see this is the slip, due to the previous frame and this is the tangential velocity due to the previous frame angular velocity and the position vector, if it is a translation joint or you can say prismatic joint what you will be getting you would be getting additional linear velocity.

So, in the sense that would be in z axis, so that we can just add. So, now you can see this is the linear velocity of the active joint of the i plus 1 frame. So, then you will get. So, now, you can see that if you know; you can say angular velocity of zeroth frame and linear velocity of zeroth frame you can find the propagation of you can say 0 to n both angular velocity and linear velocity.

So, this is what we call you can say velocity propagation. So, this is a forward propagation because we are starting from the, so base to end. So, we are actually starting from 0 to n. So, that is why it is called. So, forward propagation. So, now we will see one simple example with that we will close this lecture.

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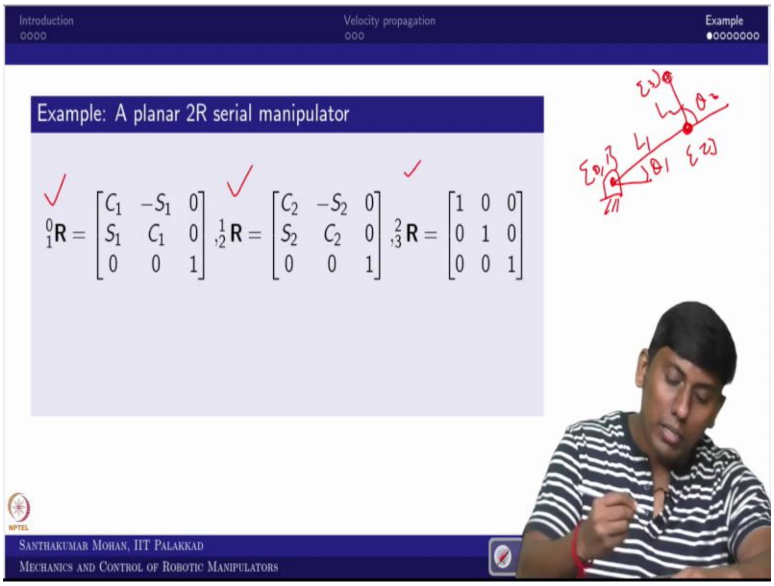

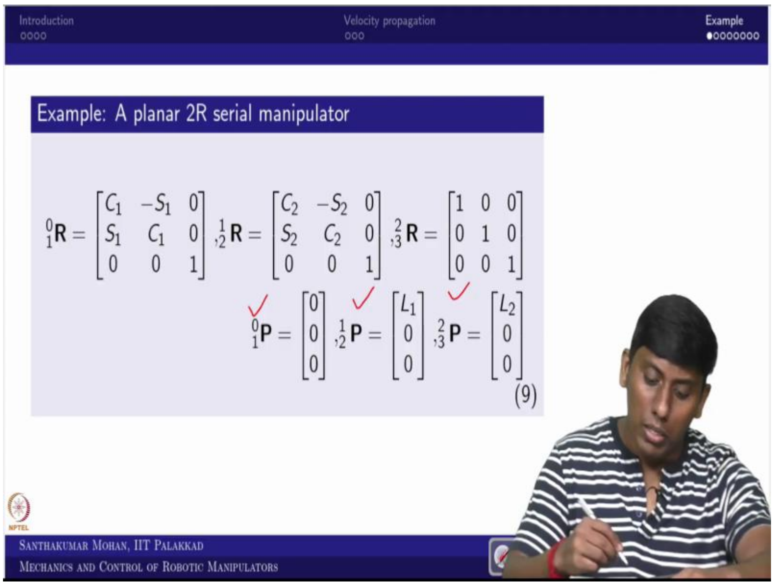

So, what example will take again a 2 R serial manipulator, which is easy. So, in the next class I will show you in the MATLAB. So, now this is theta 1 and the L1 and this is theta 2 and L2 and this is 0 and 1 and this is 2 and this is a 3 right. So, we have already obtained the d h parameter and we got individual rotation matrixes. Similarly, we have already obtained the position vectors. So, now once this all know what we have to do.

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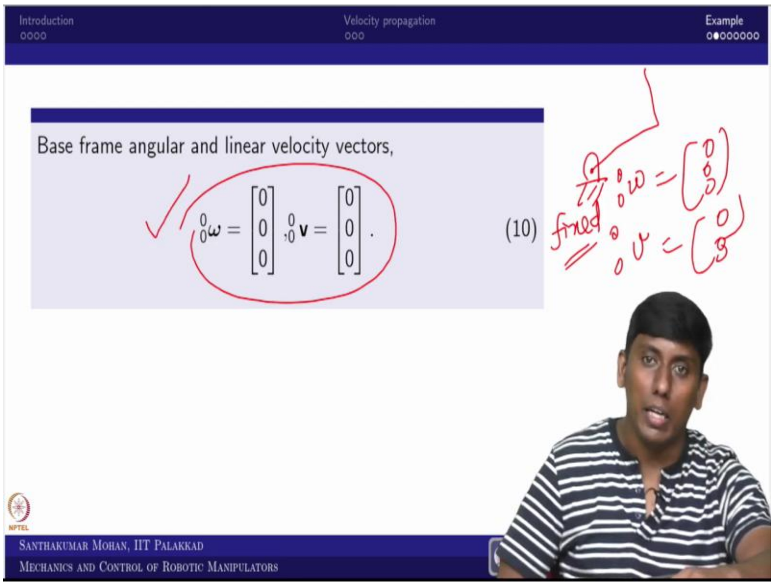

We have to start the; what you call base linear and angular velocity, we assume that the frame is fixed, in the sense this is fixed. So, in the sense omega 0 0 would be simply 0 0 and V 0 0 also simply 0 0 because it is fixed. So, if it is actually fixed, the frame velocity would be 0. So, that is what the assumption we have taken. So, now we can go again.

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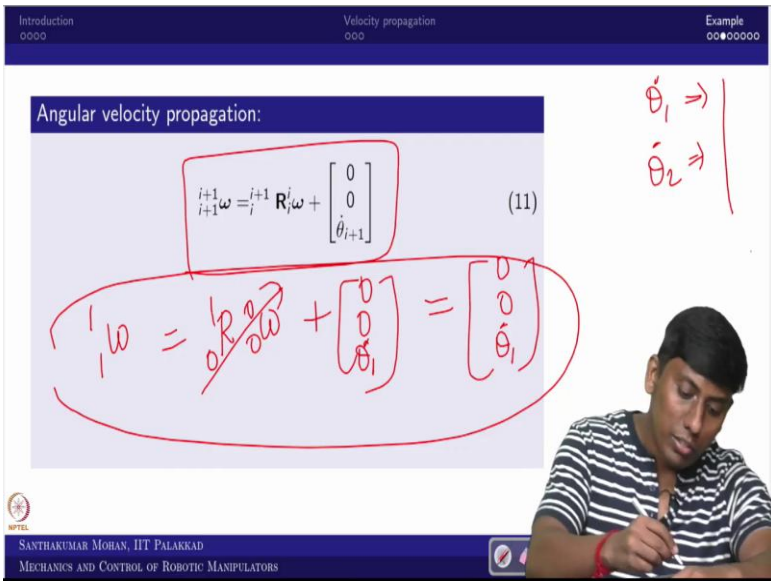

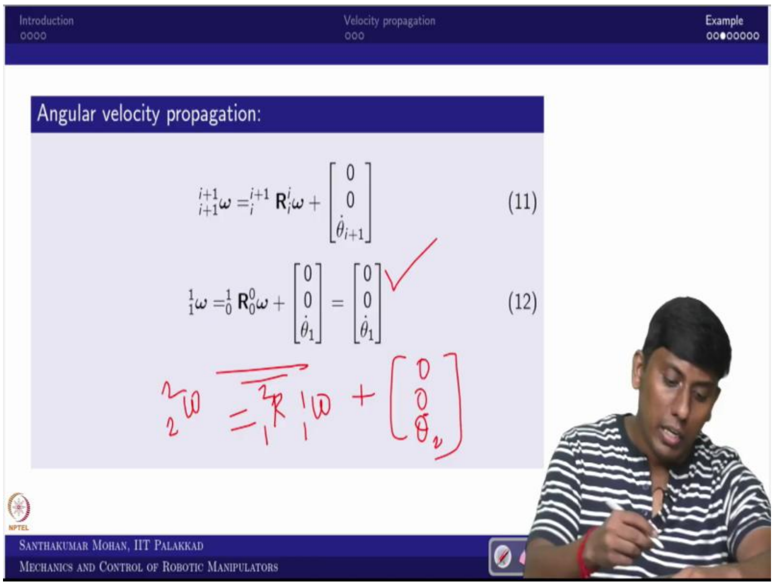

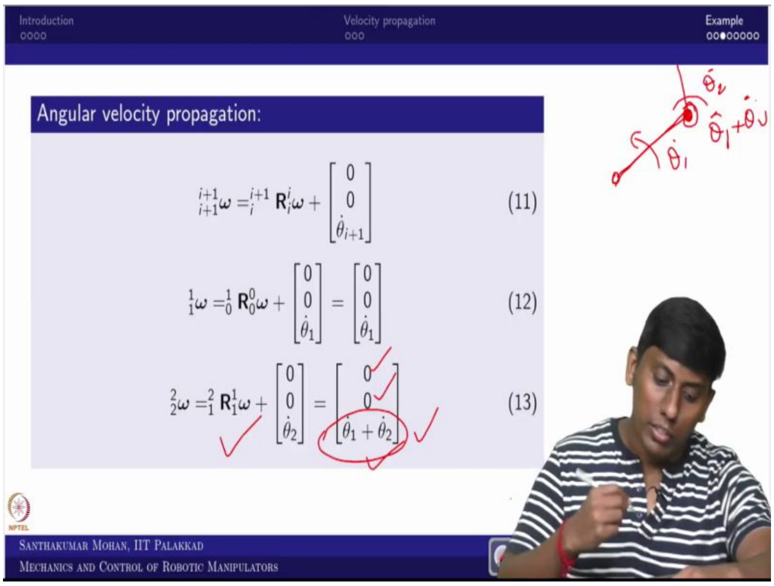

So, this is the model we know and what you can see there is that theta 1 dot exists in the first frame and theta 2 dot is exists in the second frame, these are the angular velocity of the active joints. So, now we can start the first frame in the sense of omega 1 1 that I can write 1 0. So, omega 0 0 plus 0 0 theta 1 dot.

So, now what this would be become 0. So, in the sense 0 0 theta 1 dot, so we obtain the first frame angular velocity. So, that is what we have done. So, now once you know what you can do you can start omega 2 2. So, that would be 2 1 and omega 1 1 plus 0 0 theta 2 dot. So, that is what we have done. So, this is also we have done then finally you will get it. So, once you attain any linear and the angular velocity you have to cross check.

So, now you look at it here. So, this point is having theta 2 dot, but this frame is having another angular velocity, then if you look at it, this is rotate with respect to this and the independent motor is rotating on that particular axis, in the sense the angular velocity on this particular frame would be, so theta 1 dot plus theta 2 dot that is what it is obtained and that would be in the z axis. So, that is what you can see x axis 0, y axis 0 and z axis is theta 1 dot plus theta 2 dot.

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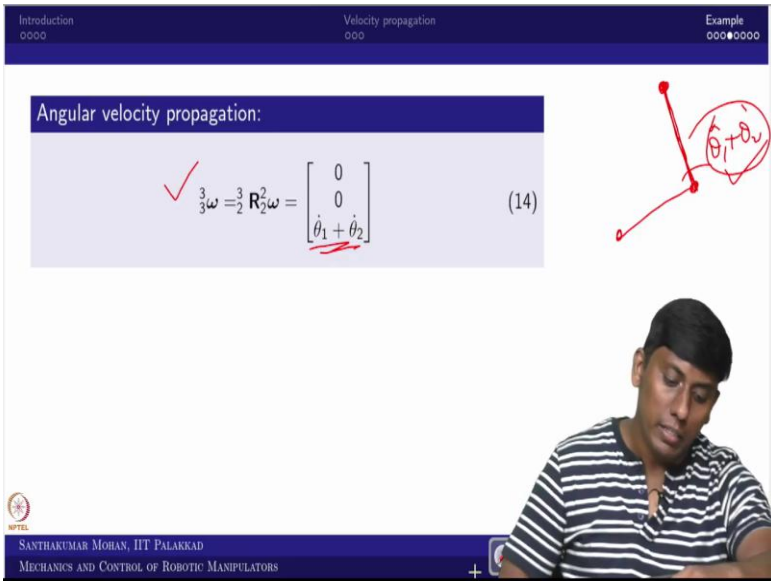

So, the same way you can extend to the third frame which is you can say passive frame which is a tool effector frame. So, this is the angular velocity. So, this is also clear for you because, so there is no active you can say joint here, so then you can see this is a part of the same link. So, this link is having theta 1 dot plus theta 2 dot, this is the frame, and this link is having this angular velocity, then this point also would be getting the same. So, that is what we have found here.

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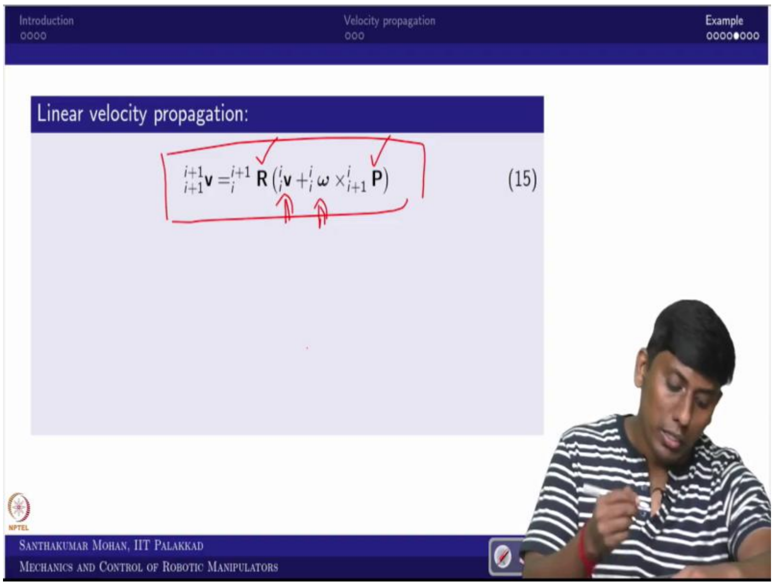

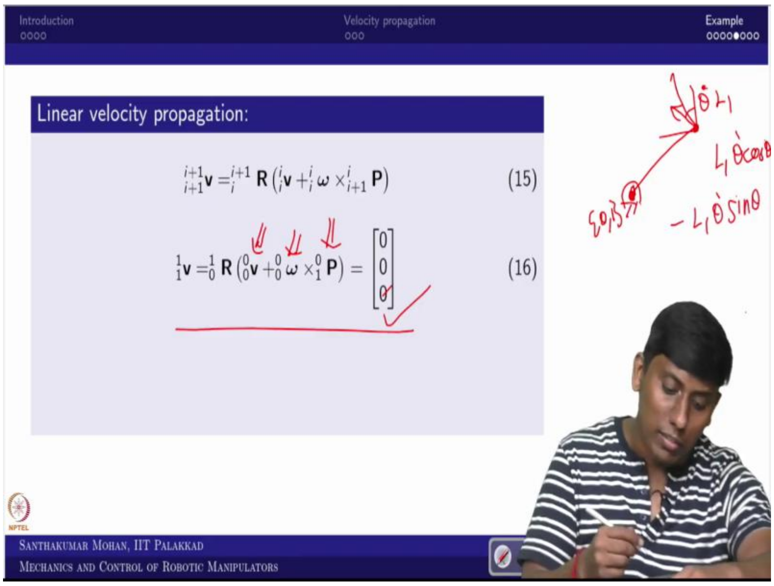

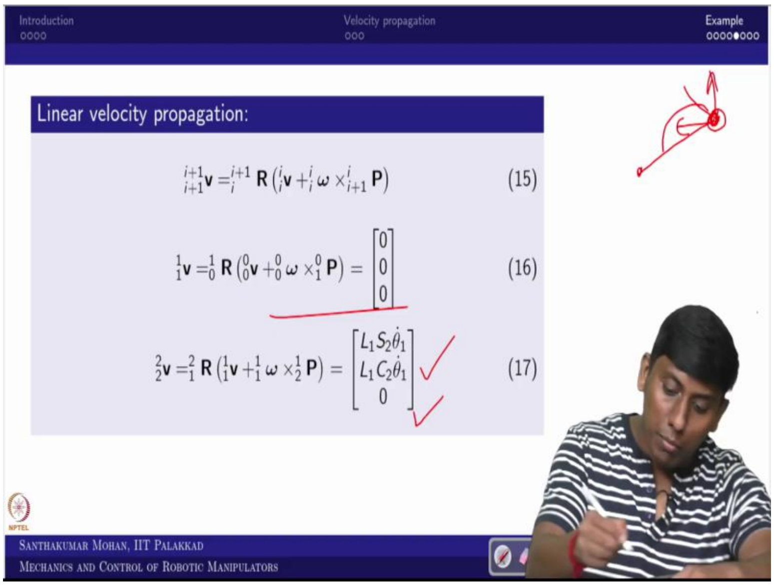

So, now you can see we can go with the linear velocity propagation. So, we know already position vector, we know already the rotational matrix and we know the zeroth linear velocity and 0th angular velocity and we know all the angular velocity propagation. So, we can propagate we want to v1 to 1 v2 to 2 all those things we can find. So, v1 to 1, so this is we know this is we know and the position vector we got this.

So, that you can actually cross check. So, this is 0 and 1 you can see that this is on the same axis, so there would not be any linear velocity that is what proven here, when you come here, what would be the linear velocity you can expect. So, the linear velocity would be the tangential that would be theta dot L1.

So, that would be in, if you are decomposed into two axes. So, this would be coming, so L1, so this can see this is theta. So, L1 theta dot cos theta would be, you can say y axis and L1 theta dot sine theta dot minus that would be x axis that we can cross check. So, now you can see that this is obtained.

So, we have started from what you call the base, and you are seeing it, this is a second frame itself if you bring it to the base, so this is you obtain this frame. So, that is true because you are getting it this is what your x axis and this is what your y axis you are getting it proper, when you are transforming you will get what we have returned in the previous lecture, previous slide.

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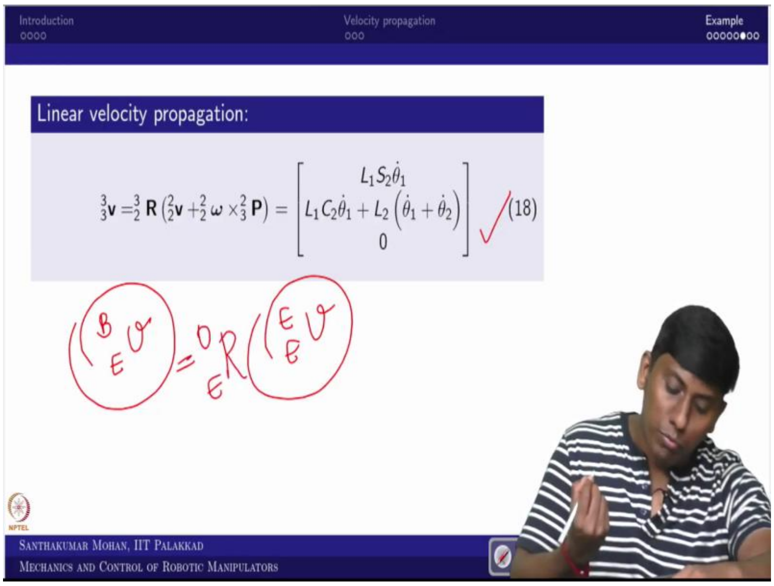

So, now, we can actually try the last one, you get this. So, now once you all obtained what really, we wanted, we really wanted the end effector velocity with respect to base. So, what we have, so we have end effector velocity with respect end effector frame. So, then what we can do, we can multiply that with this. So, this will give.

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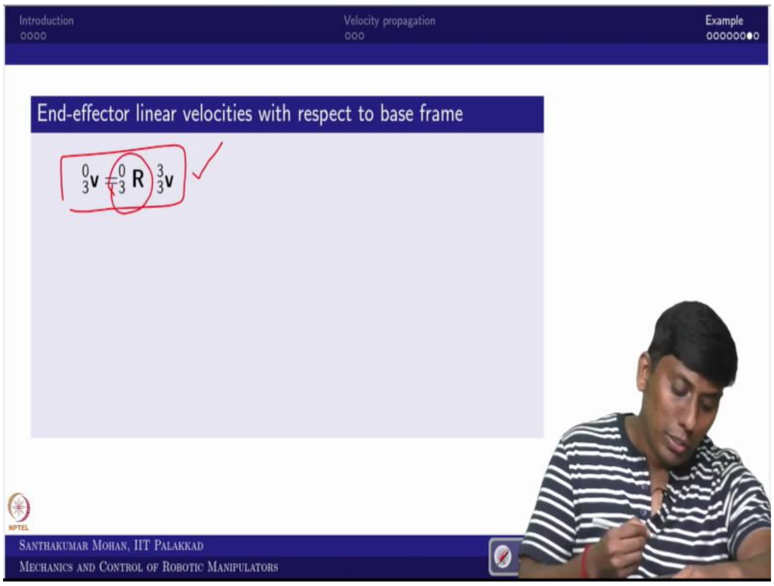

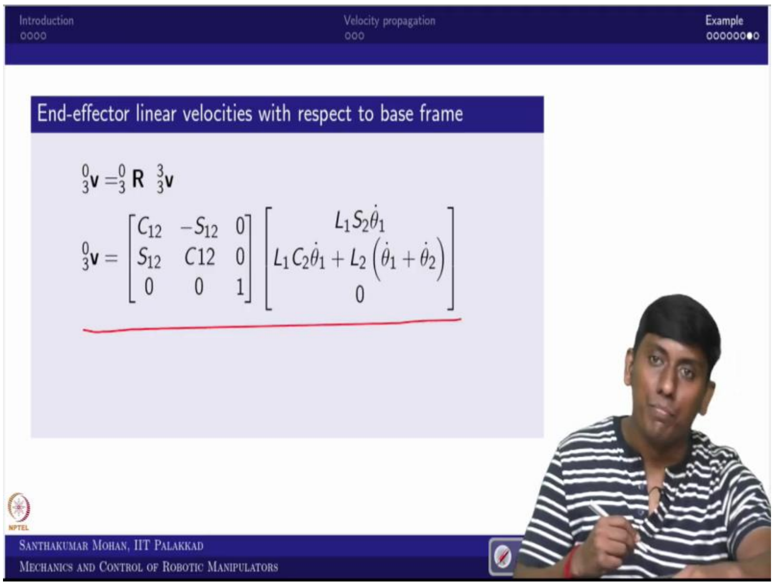

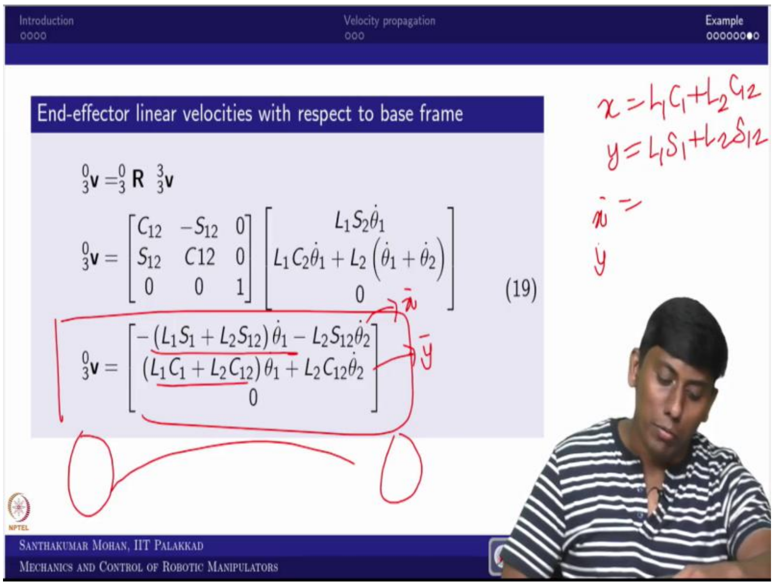

So, this is what we are doing it here. So, in that case, this already we know and this information we know, and we can obtain this. So, now you can see this is the end effector velocity you multiply all those things, this is what you obtained, you can cross check, what is the L. So, L this, so your x is this your y is this.

So, if you take x dot and y dot, so that would be equivalent to this, this is equivalent to x dot and this is equivalent to y dot this is matching. So, we know already in the last class we have seen that this can be separated as end defector velocity to the joint space velocity we can map as the kinematic transformation matrix which we call in velocity form. So, we call as a Jacobian matrix.

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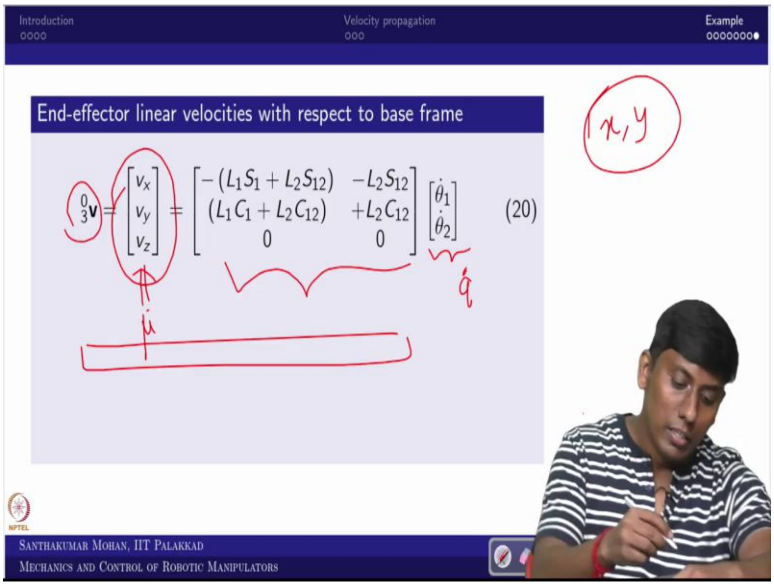

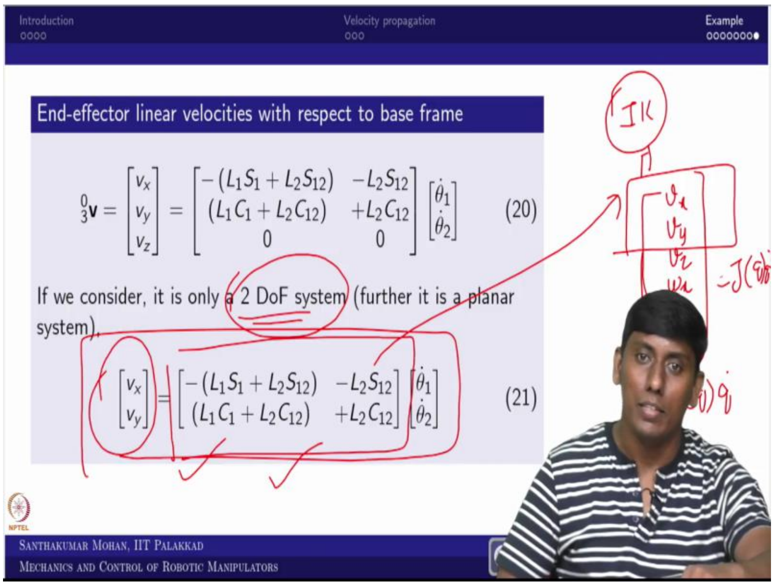

So, that is what we have done. So, Vx Vy Vz and this is, I call as you can say linear velocity component. So, this is the Jacobian matrix and this is what the you call q dot, so this mu dot. But you know this is 2 R serial manipulator, this is working in a planar that to it is going to be x y you can say positional device.

So, in that sense you can make it 2 DoF system only Vx and Vy you can restrict. So, then this would be the original Jacobian matrix which we obtain. So, you can recall in the inverse kinematic model, we have done this. So, the same Jacobian we are obtaining here also. But here what we obtained, we obtained through the help of velocity propagation, we did not partially differentiate. So, that is what the whole idea.

So, originally what you are to write. So, Vx Vy Vz, so omega x omega y and omega z, so this would be Jacobian of q into q dot. So, this is the case supposed to be, so but what we are restricted based on the system configuration, we have restricted this is only active, so then you can reduce the Jacobian matrix in this form.

So, with that I am closing this particular lecture, in the next lecture what we are going to see, we are going to see about the modelling in MATLAB, how we can obtain this Jacobian matrix, how we can obtain individual angular velocity and linear velocity and then we would be seeing the further end, how we can go for a static case. So, with that, I am saying thank you and see you in the next class, in the MATLAB. Thank you.