Mechanic and Control of Robotic Manipulators
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Lecture No. 19
Introduction to differential kinematics

Hi, welcome back to Mechanics and Control of Robotic Manipulator. So, far what we have seen is kinematic model, especially we call geometrical model, in the sense the position model of, you can say robotic manipulator, especially serial manipulator we have seen. This week onwards we are going to see differential kinematics what that means, so one step further.

So, we would be seeing everything in the time derivative of the position, in the sense velocity model we are trying to see. So, before going to see the velocity model we will see what is the basic principle behind this. So, what are the types of differential kinematics and in that what matters we end up, although in the last class we have seen Jacobian matrix, we will be seeing the Jacobian matrix in detail and then we will be seeing how the Jacobian matrix can be used as a useful tool. So, that is the whole idea in this particular lecture.

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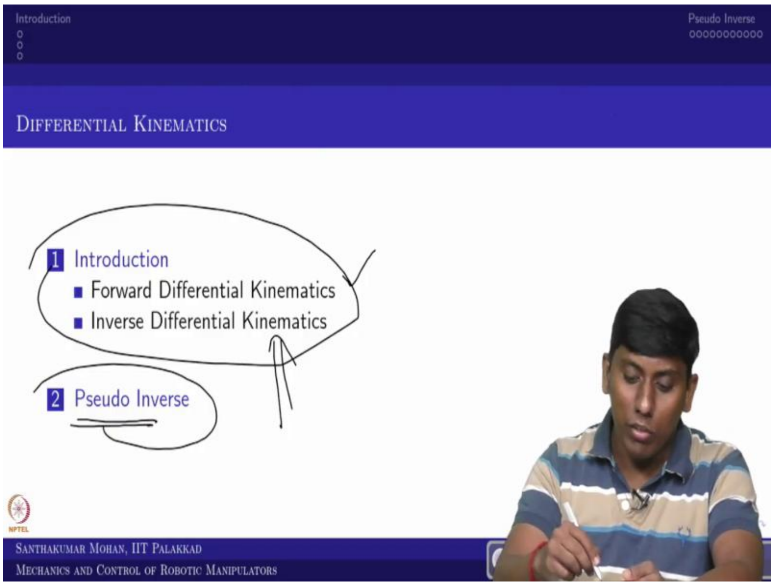

So, in the sense that we can see, we are going to talk about the forward and inverse differential kinematics, these are the two types and then we would be seeing that the inverse differential kinematics would be end up with a Jacobian inverse. So, there we need if it is a rectangular, we may require pseudo inverse. So, we will be talking about pseudo inverse in the second half. In the first half is very straightforward.

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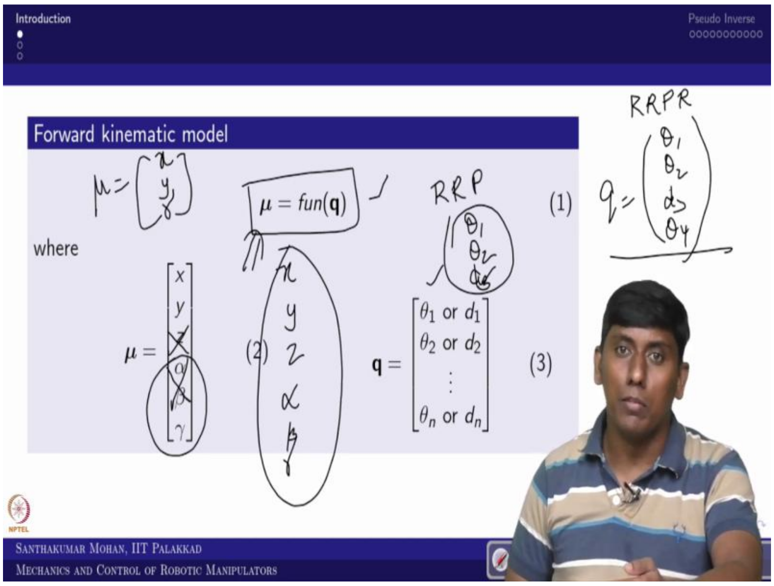

So, we know the forward kinematic model as mu equal to a function of q. So, where q is a vector which would be a function of or you can say vector consist of theta 1 to d n in the sense, so it would be a rotary joint or a translation joint. So, then theta 1 or d1 would be there, for example I have, so RRP. So, then theta 1 theta 2 and d 3 would be the variable.

If I have RRR, so PR, then it would be theta 1 theta 2 then d3 and theta4, like that I would be having a vector. So, this is what you call q vector. So, if it is 4 variables, then you can see 4 variables you can control in the sense mu would be you can write as, so x, y, z. So, alpha beta gamma would be what you call the three angular orientation that is what we have written.

But if it is in a plane, so if it is in a planar manipulator, this two like z would not be coming and these two also would not be coming. This is the mu vector would be reduced to x y and gamma. So, like that you can actually get, this is very straightforward you recall, so whatever we have seen so far as a serial manipulator these all-forward kinematics solution we have obtained. But if you differentiate this with respect to time.

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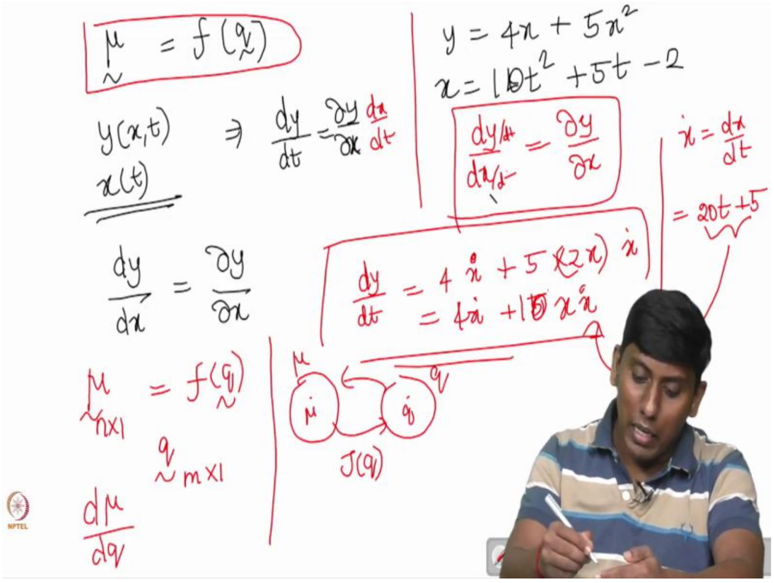

So, what you are trying to do, you are having a mu as a vector, and this is a function of so q is another vector. So, now, I am just giving idea. So, y is 4x plus 5x square, where x is you can say, so probably 10 t squared plus 5t minus 2. So, now if the equation is like this, so what you can see y also function of t, but it is a function of x and t. So, x is function of t.

So, in the sense I want to differentiate y with respect to x, but differentiation in the sense, time differentiation. So, then I want, I wanted dy by dx, this I cannot do it what I can do. So, doh y by doh x into I can write this way. So, what I can write. So, dy I can take it by dt. So, which would doh y by doh t, doh x and multiply with dx so dt. So, in the sense.

So, that is what we are trying to do. So, in the sense I actually wanted like dy by dx, but I cannot do it, what I can do you can see this dt, dt all go away in principle, so the doh y by doh x is equivalent. So, that is what we are trying to see this, in the sense this is dt and this is also by dt.

In the sense here what you can. So, the dy by dt can be written as first I will say that this 4, so doh x by doh t. So, in this sense, so 4 into x then x dot, so this would be a 5 into 2x into x dot. So, in the sense what you can see it, so this is 4x dot plus you can write a phi so 10 phi into 2. So, 10 x x dot, so this is the value will come.

So, now you can see what is x dot, so x dot is nothing but dx by dt, so what that value here, so it would be a like 20t plus 5. So, now I substitute that here. So, then I can actually get it. So, this is if it is a scalar, you can do it. But now so, mu is a vector and q also a vector in the sense mu is n cross 1 and f of q where this q is m cross 1. So, now what you are interested, you are interested is d mu by dq, it is not straightforward. So, far that the Jacobi has given a method. So, what that, that is mapping two spaces.

So, this is mu space and this is q space. So, where mu dot and q dot relation we are mapping between one to another, either this or this. So, this mapping what we call kinematic transformation matrix, this matrix in the velocity form, this is what you call Jacobian matrix. So, that is what we have written. So, very simple example again I am doing it.

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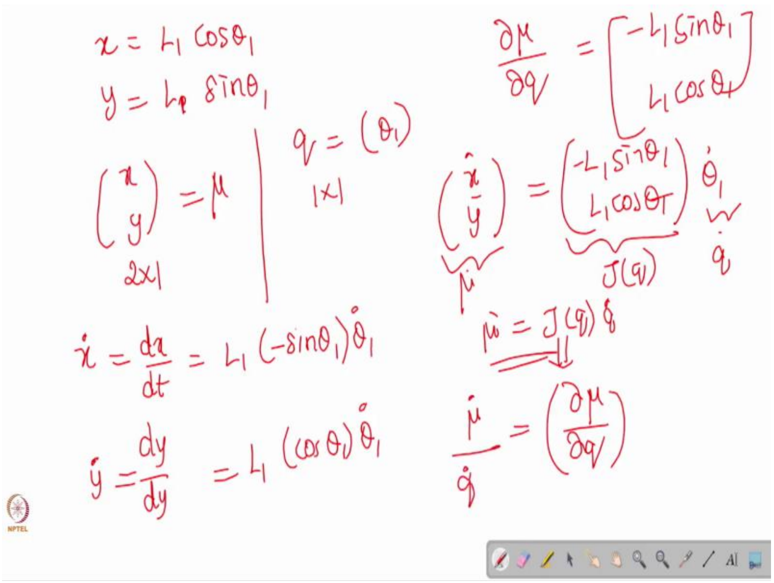

So, for example, x is L1, so cos theta 1. So, y is L1 sine theta 1, in the sense it is simple pendulum kind of thing, where L1 is constant. So, what would be this, so I can write x and y as mu. So, in the sense this is 2 cross 1. So, where q is only theta 1 which is 1 cross 1, what I wanted, I wanted I, so doh mu by doh q in the sense I can write this.

So, I will write x dot which is nothing but dx by dt which is L1 minus sine theta 1 and theta 1 dot. So, dy by dt which is I call y dot, dot means differentiation. So, cos theta 1 and theta 1 dot. So, if I take a partial derivative. What it will come? So, it would come, so this minus L1 sine theta 1. So, this will come. So, L1 cos theta 1. So, now you can see these two are same.

So, in the sense, so x dot y dot you take it one side. So, what you can see, so L1 minus L1 sine theta 1, then L1 cos theta 1. So, this whole multiply with the theta 1 dot. So, what is this, this is nothing but q dot, what is this, this is nothing but mu dot. What is this, this is a matrix in this case it is, so simple 2 cross 1, but what this, so this is a Jacobian matrix.

So, this is J of q. So, in that sense what you can see that mu dot I can write as J of q into, so q dot. So, this is a way we can relate. So, now, this you can actually see this is a partial differentiation of mu with respect to q and this you can write mu dot divide by q dot.

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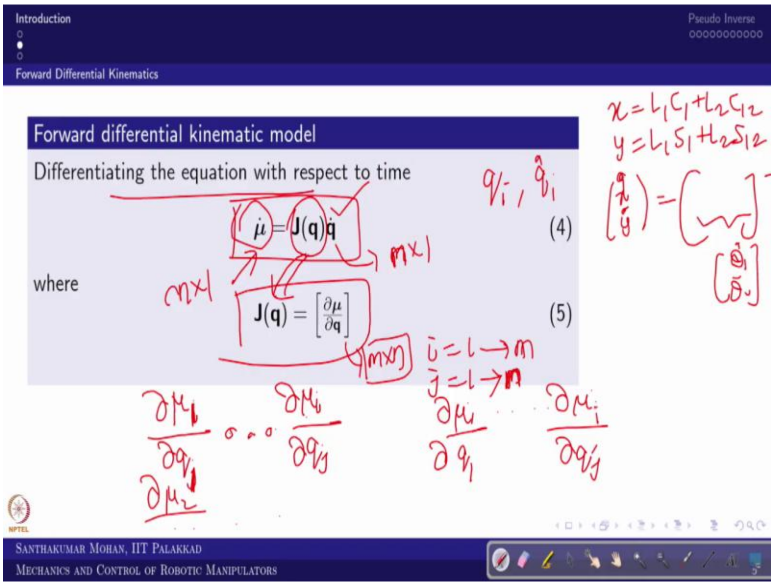

So, the same thing what we are writing in the other form. So, differentiating with respect to you can say time, differentiating the equation with respect to time, you can see that mu dot we have obtained as J of q into q dot, where this J of q is matrix of you can say partial derivative, why it is called matrix.

So, you can rewrite this equation again. So, x is L1 C1 plus L2 C12. So, y is L1 S1 plus L2 S12. So, if you differentiate this with respect to time, you can see x dot y dot will come. The other side it would be 2 cross 2, multiply with you can say theta 1 dot and theta 2 dot. So, this is what you call Jacobian matrix that is a partial differentiation. How you can write a partial differentiation, you can write doh mu you can say i divide by dough q j.

So, where you can write this would be, so doh mu i, where I can start with the 1. So, this goes till doh q j. So, the second row like it is doh mu 2 which goes like this and the final one is doh mu i goes doh mu i, so q1 to doh q j. So, what this means, i goes 1 to n, j goes 1 to m. So, this would be m cross 1 that is what we have said, this is n cross 1.

So, some book would have written this is m that is n. So, you can write it that. So, that is a uniform format, then this would be m cross n as the size. So, now, this is what you call differential kinematics. Now, if I know the q dot, I tried to find out the J of q, if the qi and q dot of i are known, what I can find, I can find the mu, in the sense it is straightforward.

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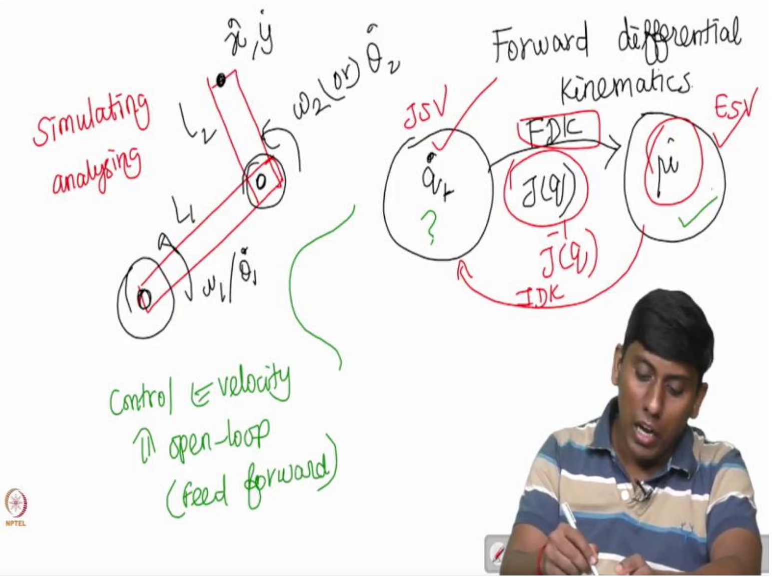

You connect you can say is, in the sense you take a serial manipulator. So, you take a motor, and you fix the rotational speed omega 1 and omega 2. So, in the sense or theta 1 dot or theta 2 dot. So, you fix it and L1 and L2 are known. So, you are trying to find out what is the x dot and y dot. So, in that sense what this is, this is straight forward. So, which we call forward, so differential kinematics. So, this is what forward differential kinematics.

What that means, so you have q1 or q dot that is known and J of q is known, or you are trying to find out what is mu dot, you are trying to go, this is what you call forward differential kinematics. So, now the other way around, so the mu dot is known, and you are trying to find out this, so then what you are trying to do the J inverse of q. So, this is what you call inverse differential kinematics.

So, now I hope you got idea. So, it is very straightforward as what we have seen as forward kinematics and inverse kinematics. So, here this is joint space velocity, and this is the end of vector, or you can say task space velocity, you are mapping two velocity space and the mapping matrix is here is Jacobian matrix.

So, where you have q dot is known and you are trying to find out mu dot that is forward differential kinematics, in the sense you have a motor and you are running, in the sense what you are trying to see, if you are doing in a numerical so you are trying to simulating the system, if it is the other way around in the real it is analysing the system.

So, the other thing is you know this, and you are trying to find out this in the sense you want to follow some end effector velocity profile and see that what your joint space velocity vector. So, in the sense you are trying to control, but in velocity level, but without any feedback. So, it is open loop. This means it is a feed forward. So, that is what you can see it is inverse differential kinematics. So, that is what we are trying to see. I hope you can see.

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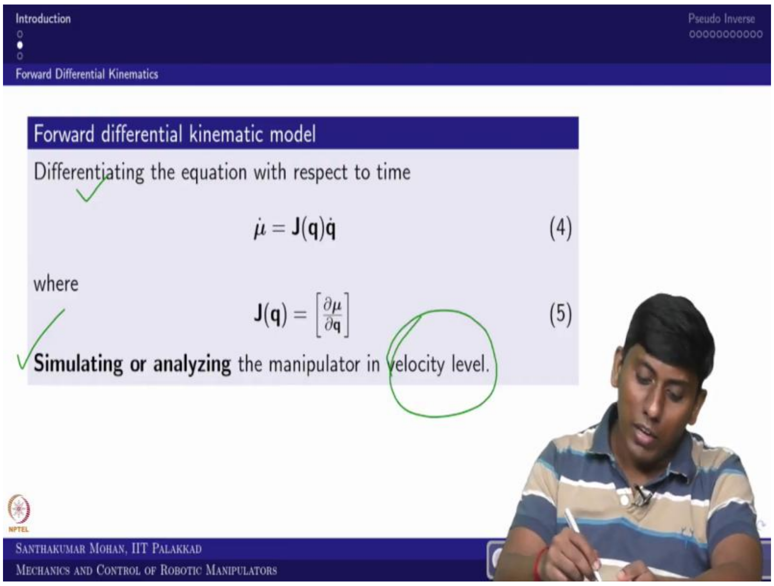

So, this is the forward differential kinematics, where this equivalent to simulating or analysing the manipulator in velocity level.

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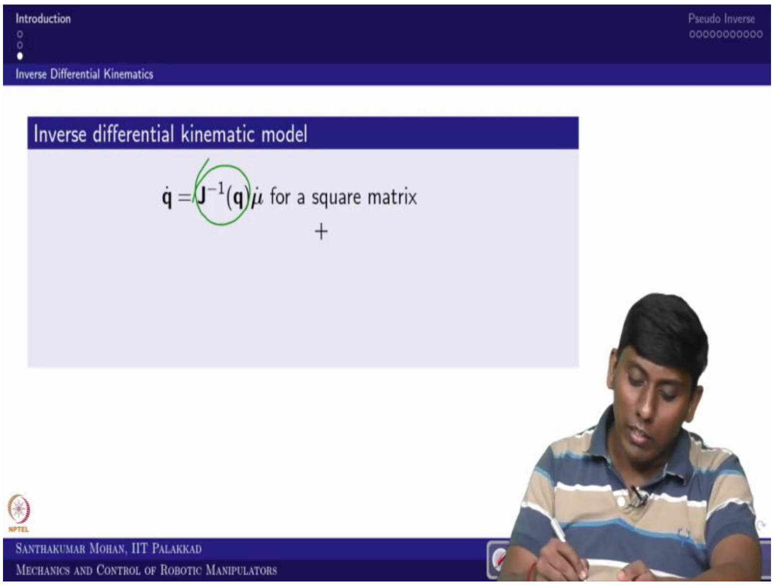

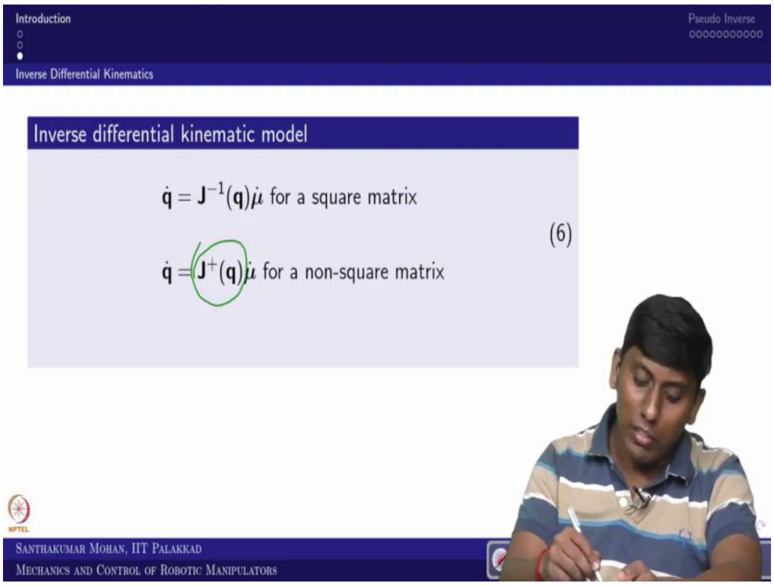

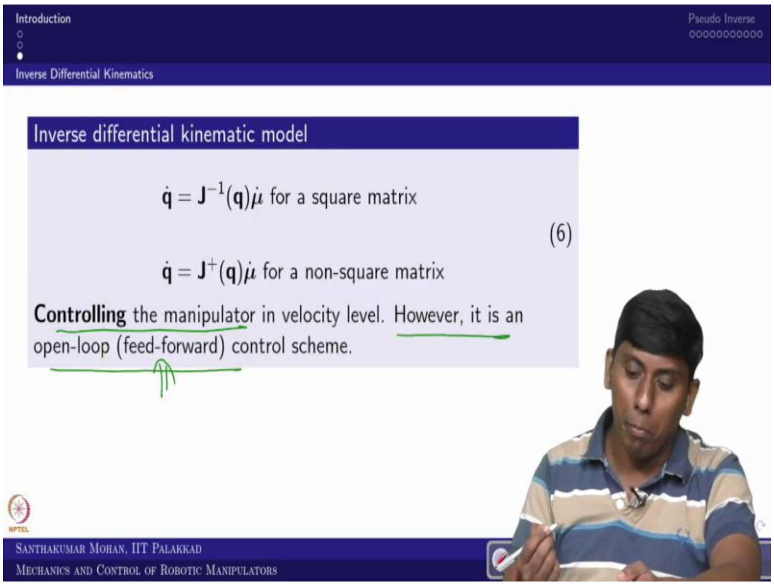

Whereas the inverse differential kinematics you can easily recall the J q inverse. So, in the sense it is like if it is a square matrix, if it is a non-square matrix, you would be using a pseudo inverse, which we have written as J plus. So, what this is equivalent, this is equivalent to controlling the manipulator in velocity level, however it is in an open loop control scheme in the sense it is a feed forward control, you please recall that, so it is a feed forward open loop.

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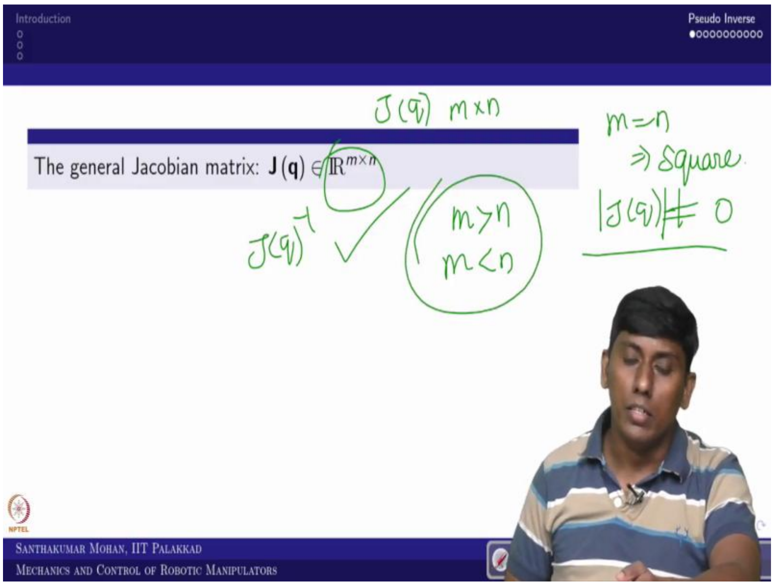

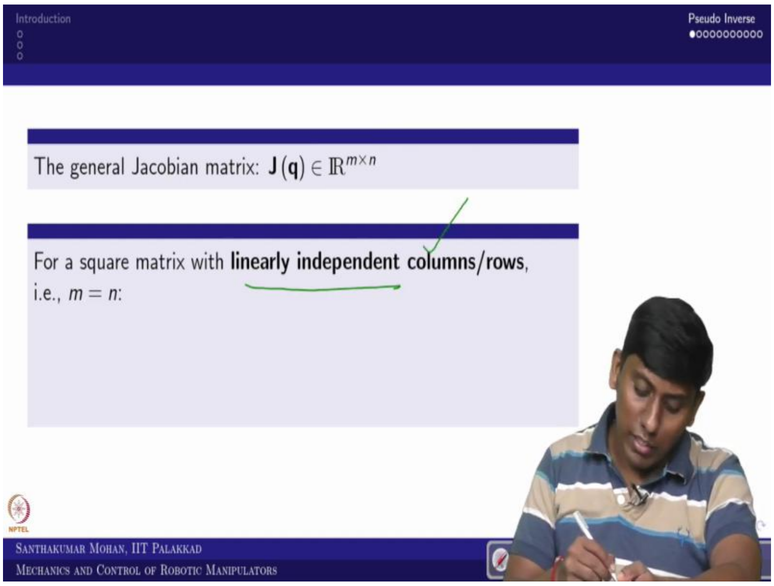

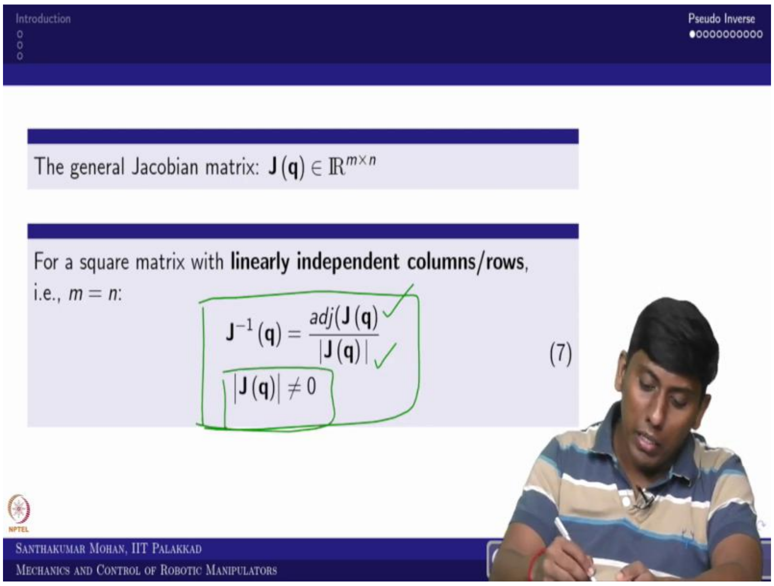

So, now, we will actually see this J. So, this J of q is having m cross n. So, now if m equal to n, then that is actually giving a square, where the determinant of J is nonzero. So, then you can see that the determinant is nonzero, then the J of, so q inverse exists. But if it is actual like m is greater than n or m is less than n, then how you will solve it.

So, far that we will take Moore-Penrose pseudo inverse. So, what that this is depend if it is a square matrix it would be having equal linearly independent columns and rows, this is means m equal to n you can easily use the J of you can say Jq inverse is simple adjoint a divided by J you can say determinant provided the determinant is nonzero.

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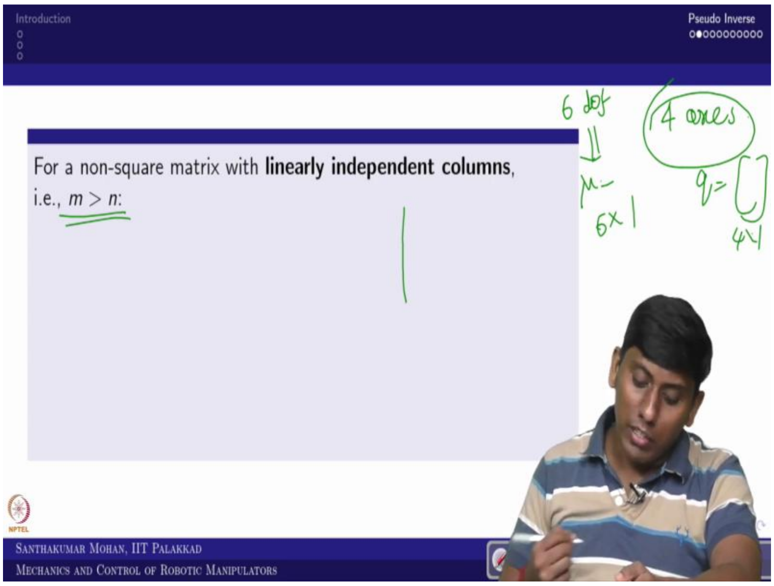

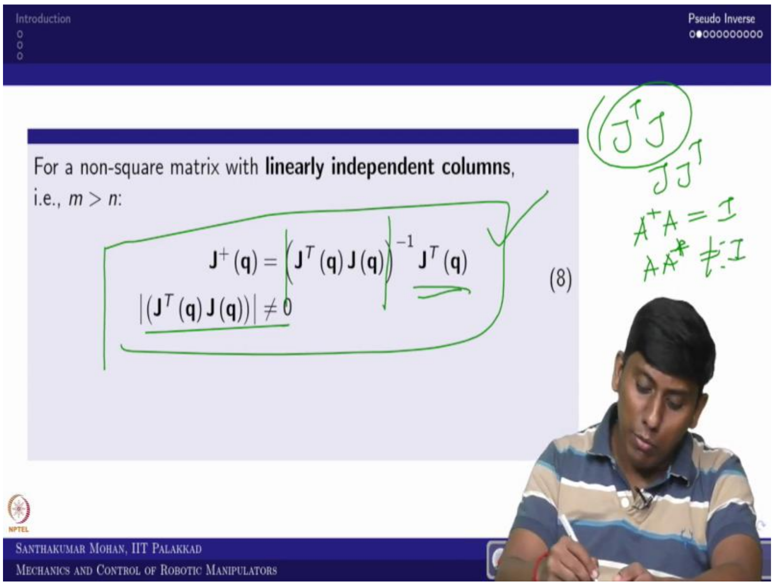

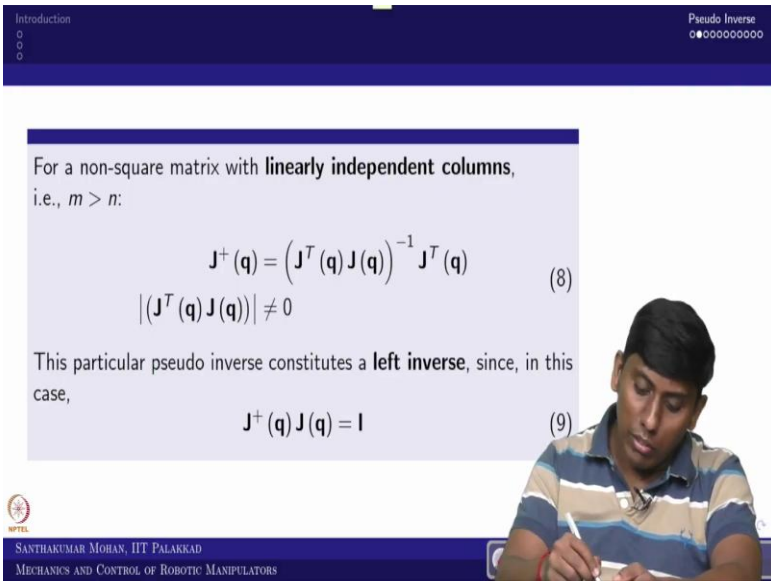

But if m is greater than n. In the sense, so, you have only 4 axes, but you have actual like spatial in the sense 6 DoF system, but 4 axes in the sense the q is having only 4, but the mu is having 6 cross 1. So, if that is the case what you have, you have 6 equations, but 4 unknowns, it is actually over specified, then what you can do you can do the left inverse. So, this is the left inverse where we have a square matrix J transpose J or J into J transpose.

So, we take J transpose in J and take inverse and multiply with the J transpose. So, this is what you call left inverse. So, provided this determinant is non-zero. So, why it is called left inverse? So, this inverse what you are taking, if you are multiply prior prefix or pre multiply that will give identity and this will not give will not give identity matrix. So, that is the idea. So, this will not give identity matrix. So, then you can see what the other way is around.

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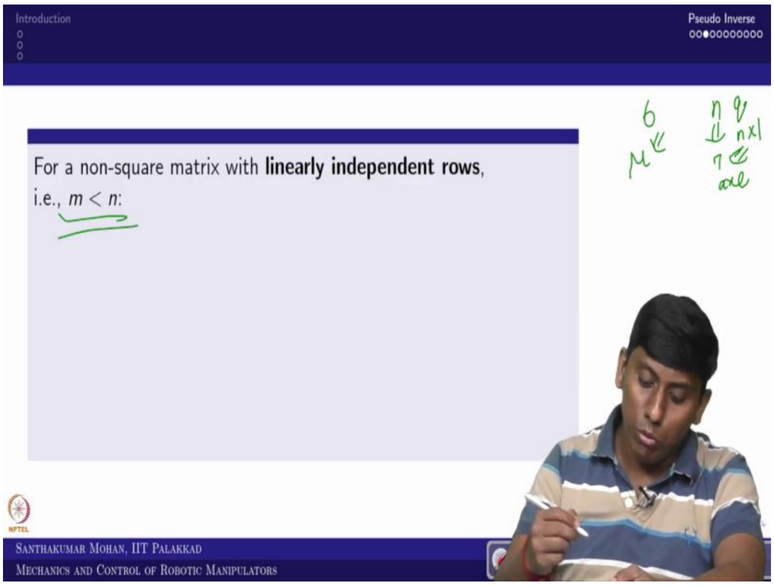

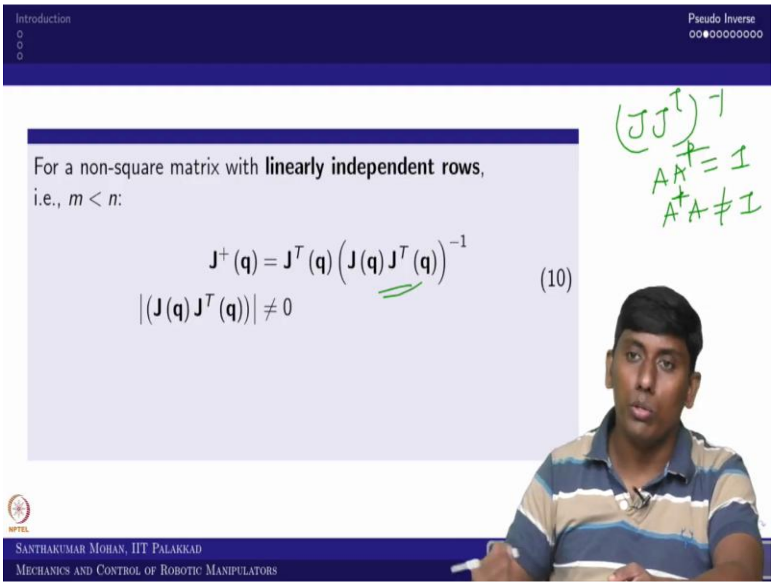

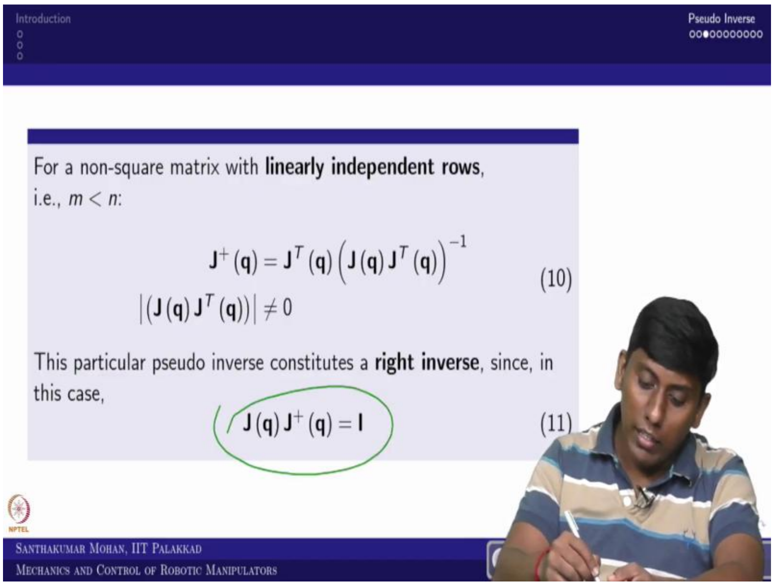

So, the other way around is like, if is m less than n in the sense, so it is for example, 6 is the mu vector, but you are n is more than 7 axes. For example, it is 7 axes in the sense q is like n cross 1 that is 7 axes in the sense this is having a 6 equation, but the seven unknowns in the sense it is under specified you have multiple solutions possible.

So, in that case what you can do you can take the right inverse, so where you take J J transpose inverse, so in this case, so what right inverse? So, A A plus equal to I, but A plus A is not equal to I. So, that is what we are saying it, so you can see.

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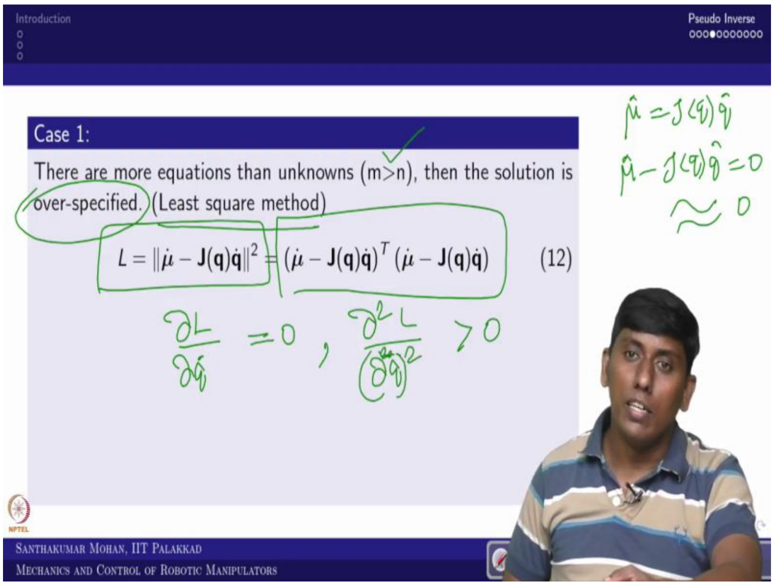

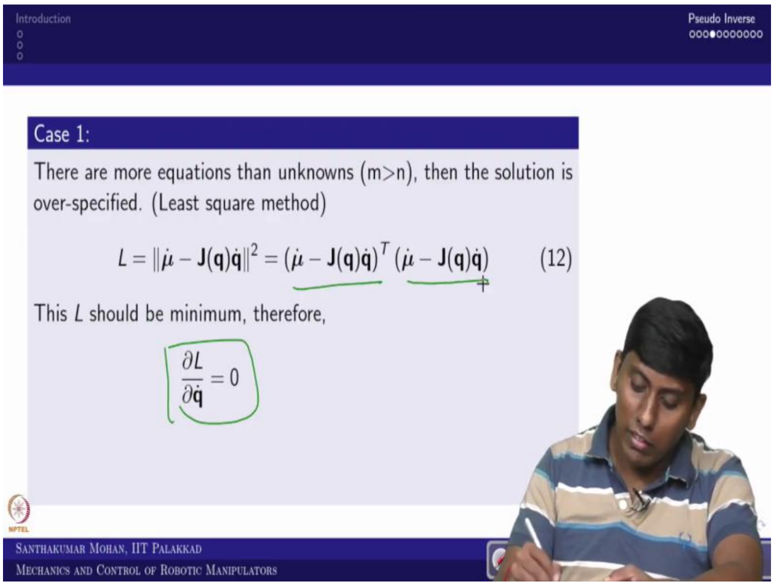

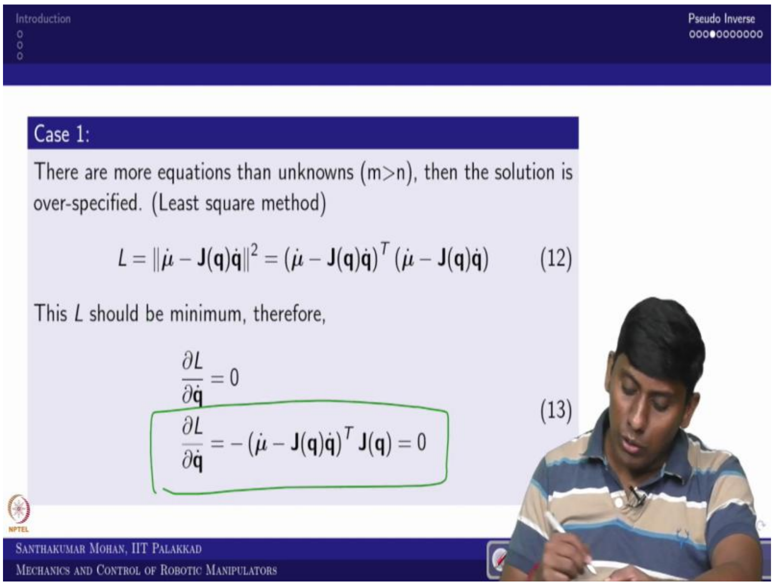

So, the other thing is how this has come. So, far that we are taking a case 1, where it like m is greater than n. So, this can be solved with the least square method, in this case the situation is over specified, what the case, so you have mu dot which is J of q into so q dot, this I am assuming as 0, so mu dot minus J of q, so q dot is 0.

So, this is what I wanted, but this is not possible, but what I am trying to see this 0 is not obtainable, but close to 0 or what is the minimum I can get it. So, if that is the case, I can take a norm, second norm. So, that second norm I can write it in this form. So, now if this is minimum what one can see, the doh L by doh q dot supposed to be 0 provided the doh squared L by doh q dot should be greater than 0.

So, that is what is supposed to be known. So, in that sense what one can see, you can do the first the partial derivative. So, in this case what would be the partial derivative, so this would be retained and this could be taken. So, now you can see this. So, in this case if you substitute equals 0, so do not say that this is non-zero, you pre multiply everything.

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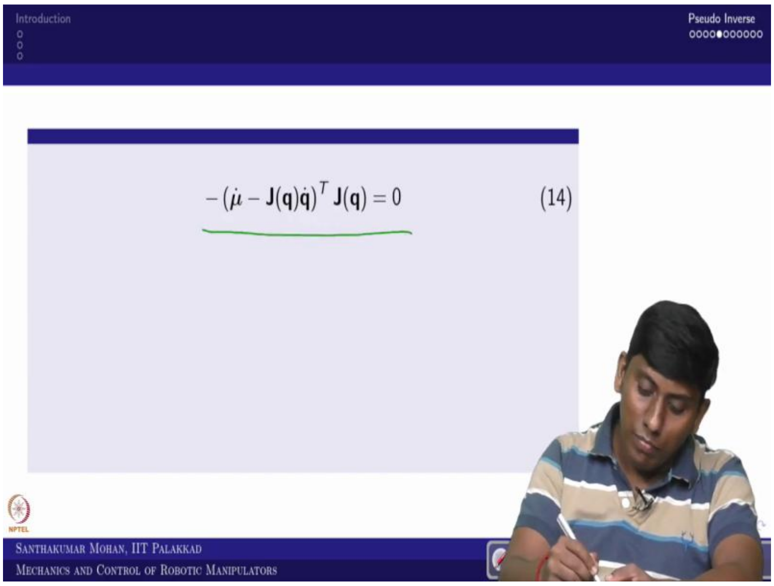

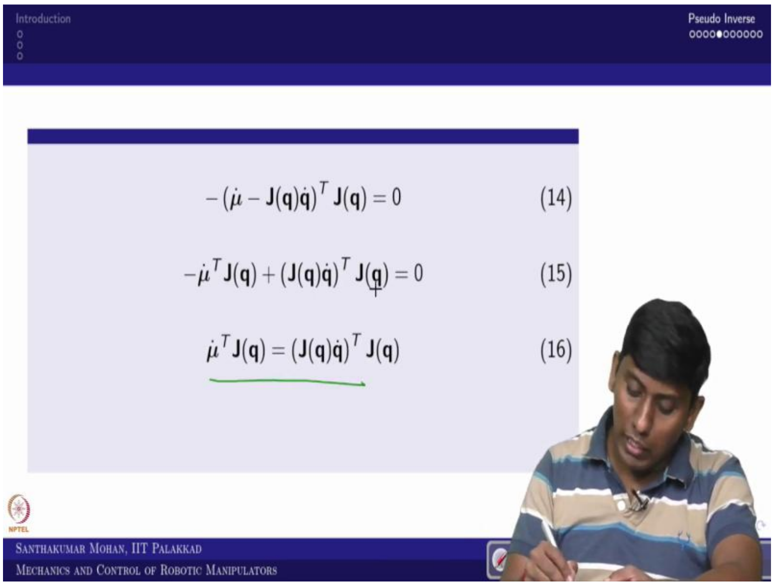

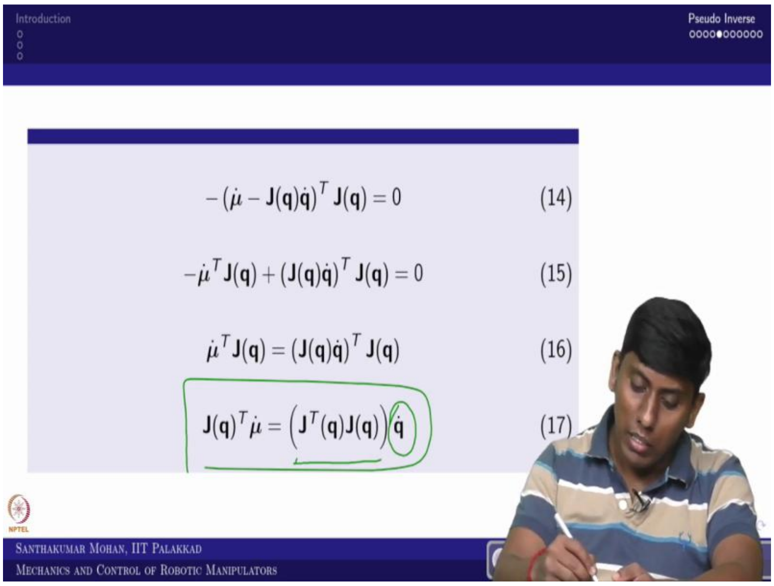

So, then what you will get, so you multiply that. So, you multiply this you will get it. So, now you rewrite this equation, you in the sense you would take transpose through out, so you will get this. So, now this is the rewriting equation and taking a transpose you will get this. So, now it is very, very easy. So, in the sense you take a transpose this come, so now, you want it q dot, so then this whole matrix take inverse and multiply with this.

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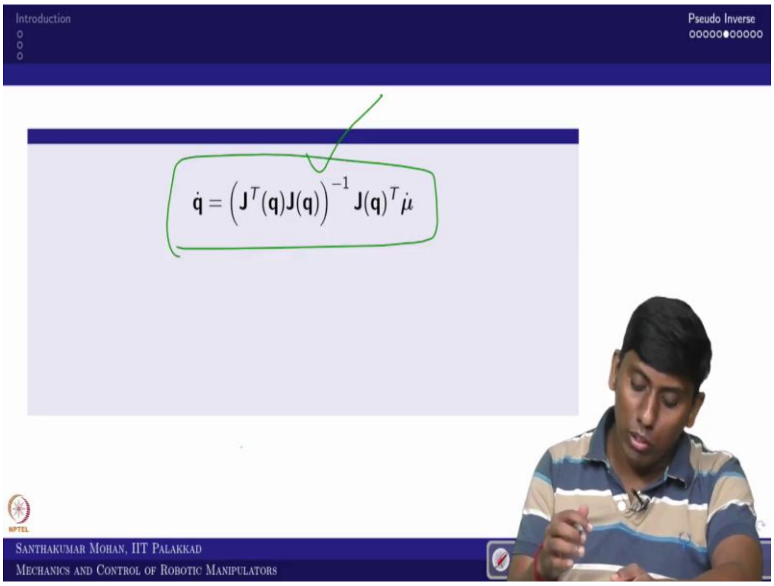

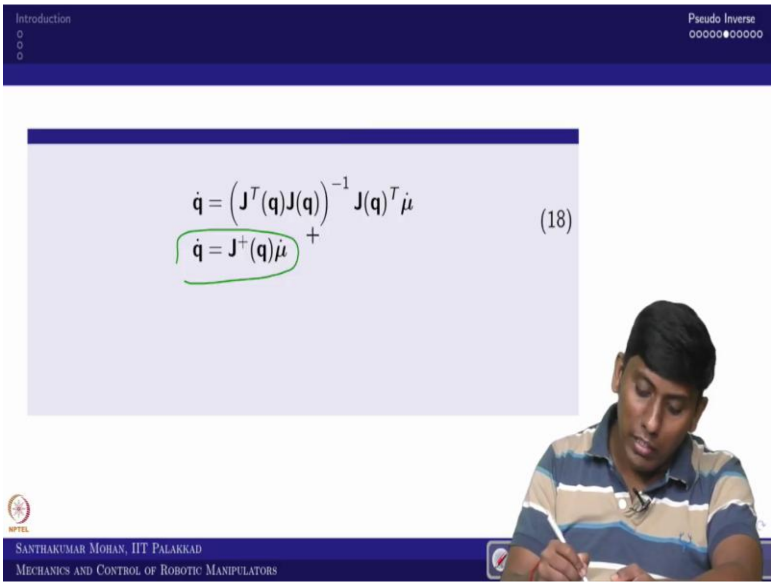

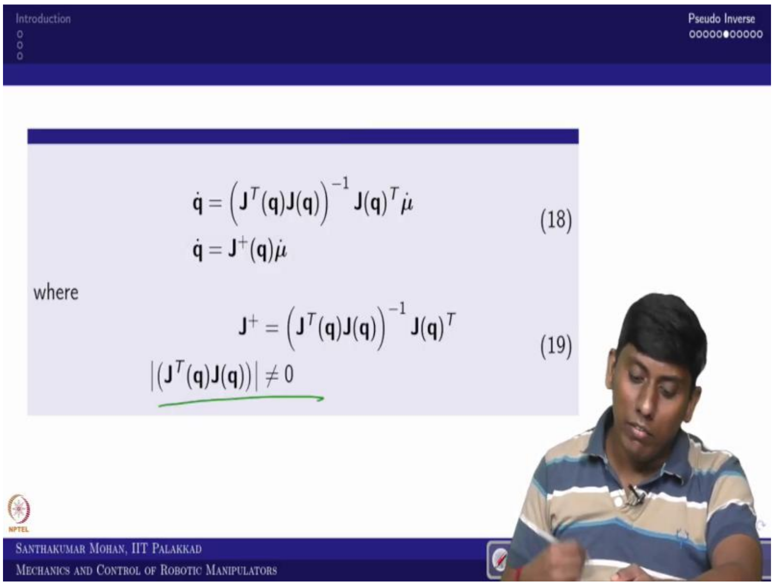

So, this gives the left inverse. So, in this case this is over specified, and it will give a chain like theoretically no solution, but it will you come closer to the actual solution. That is what we have say this is a pseudo inverse provided this determinant is nonzero. We take the next case.

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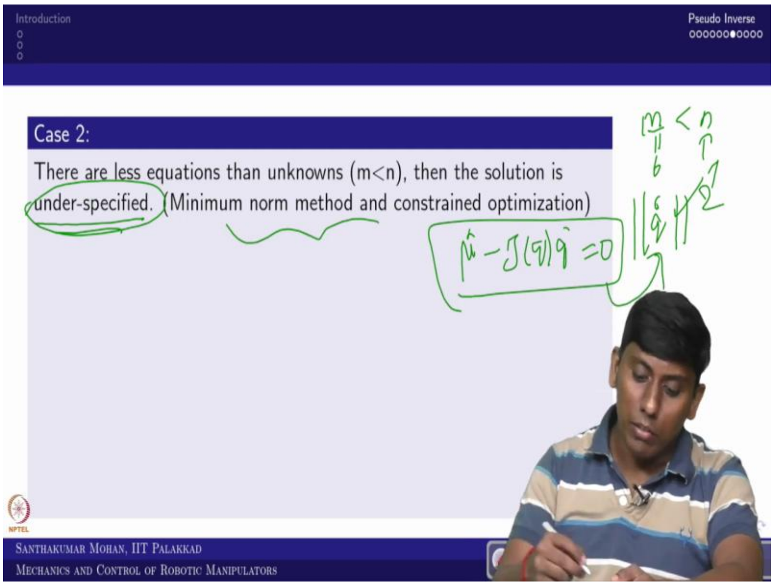

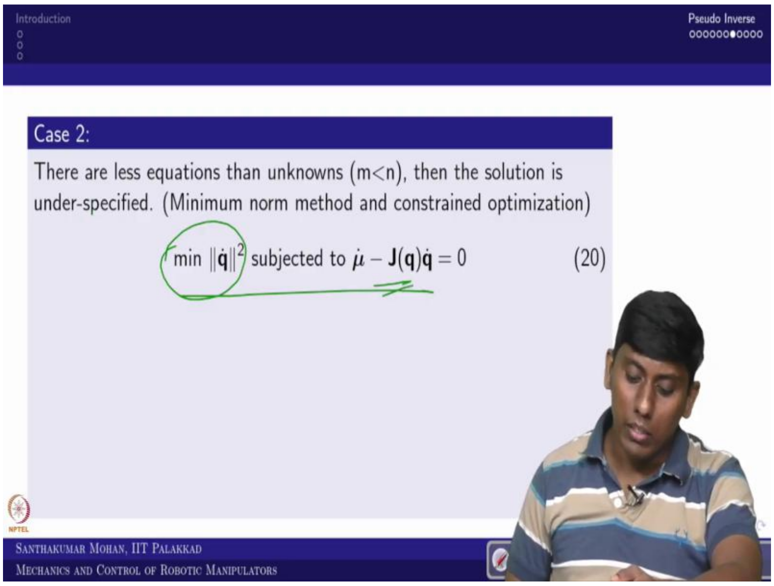

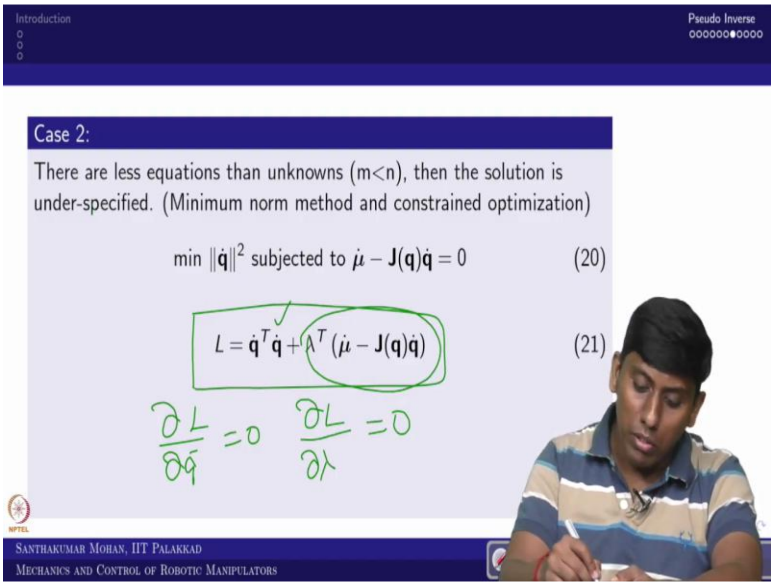

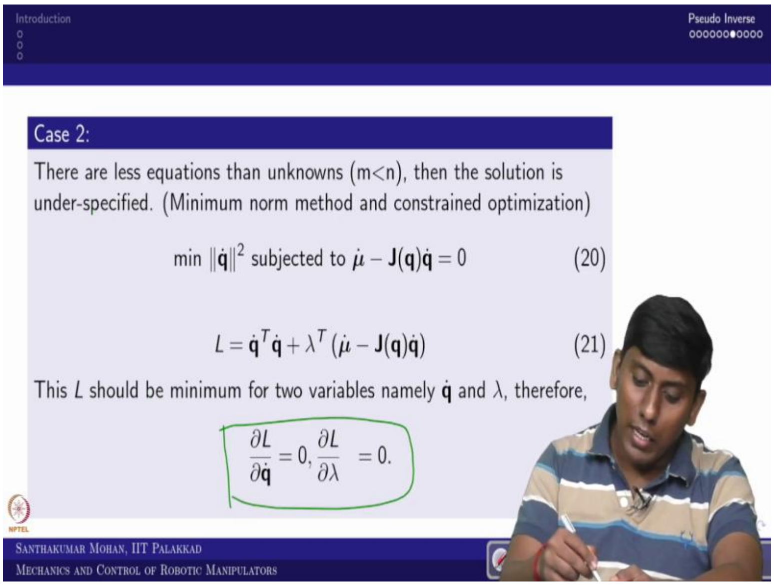

So, the next case is underspecified where m is less than n. In this case this is 6 and this is 7. So, in this sense it is actually like underspecified you can use a minimum norm method with the constraint optimization. So, what that means, so the q dot would be exists but multiple solution, so you minimize the q dot. So, in this sense you take 2 norm provided the mu dot minus J of q into q dot equal to 0, this is your constraint and you are trying to minimize this.

So, that is what we are doing it. So, minimizing this with the help of this constraint equation. So, then you can see the Lagrangian multiplier will come into picture, your optimization function or objective function would be not only this, this constraint also welcome. Now, there are two variables, so doh L by doh q dot also like 0 and doh L by doh lambda also would be 0. So, two equation two unknowns we can solve the unknowns.

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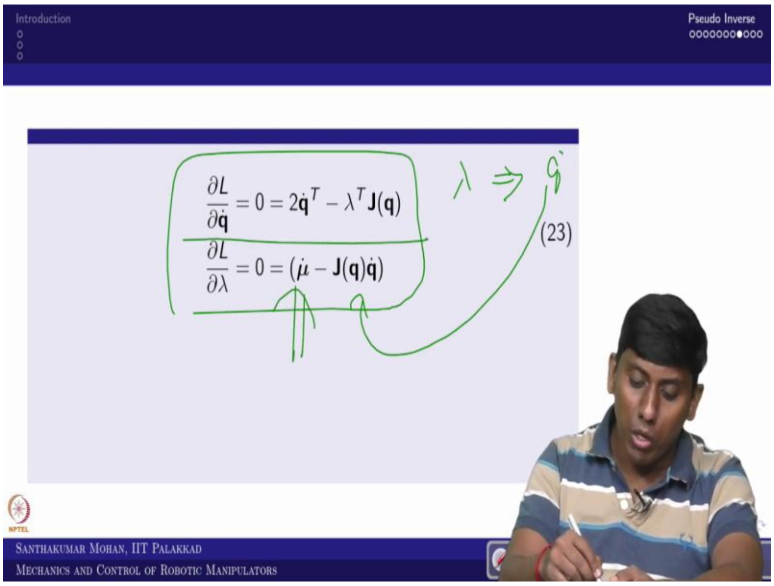

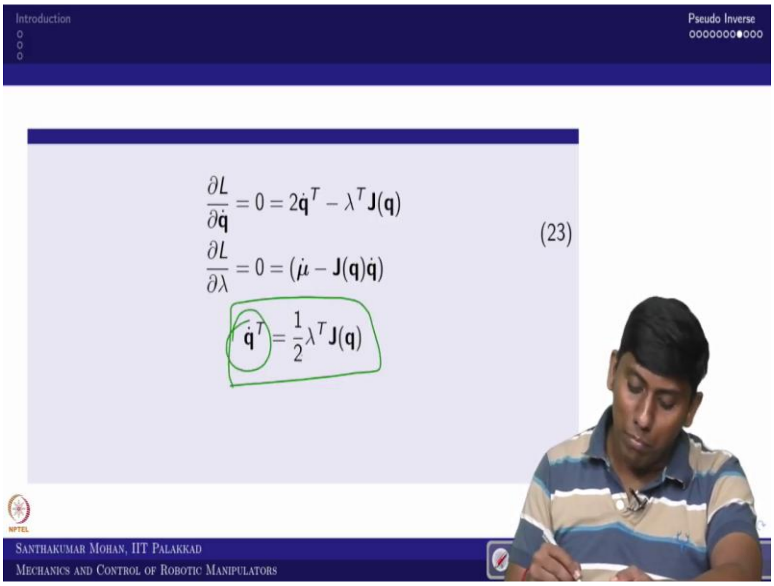

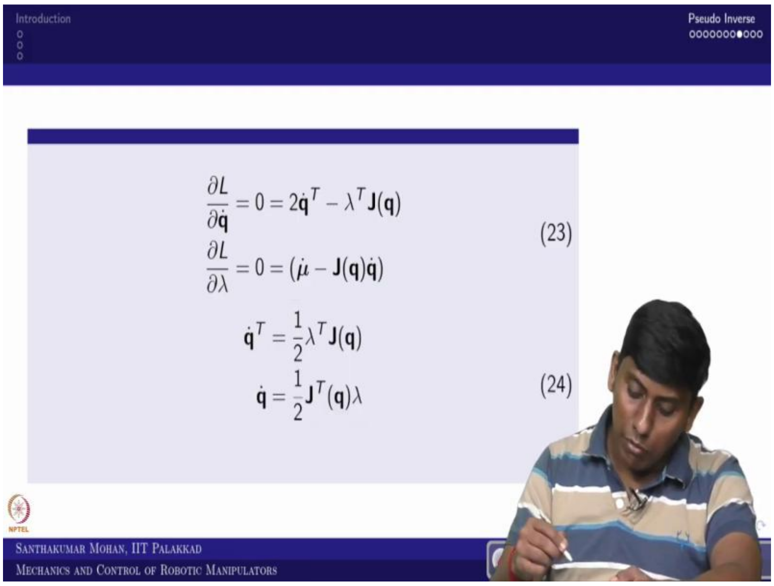

So, first I can take the partial differentiation and rewrite this into one of the meaningful. So, what I can write, I can write lambda in the form of q dot, then that I can substitute here, then I can find the lambda in the order of mu dot so that is what first I take it this. So, the q dot I have written in that lambda form that I substitute into the second equation.

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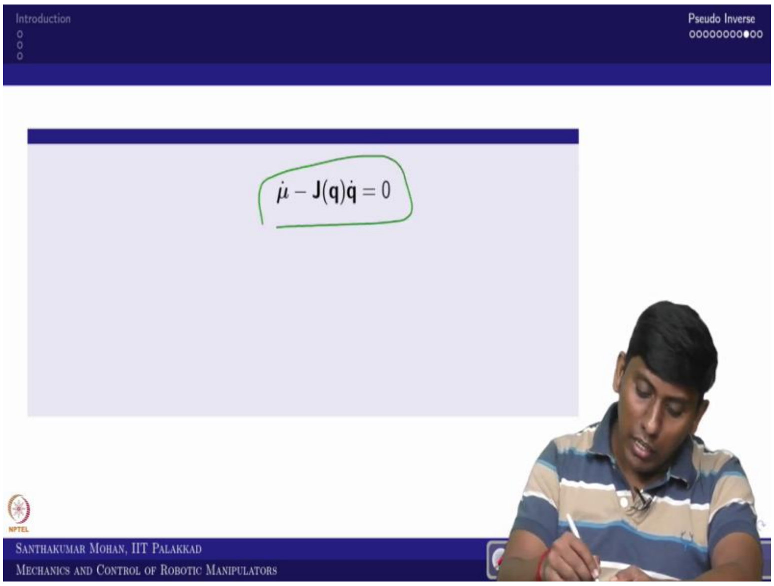

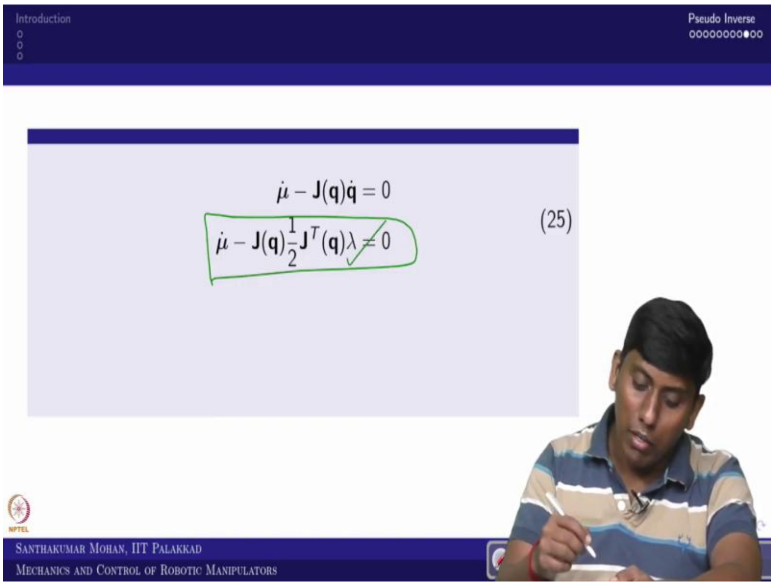

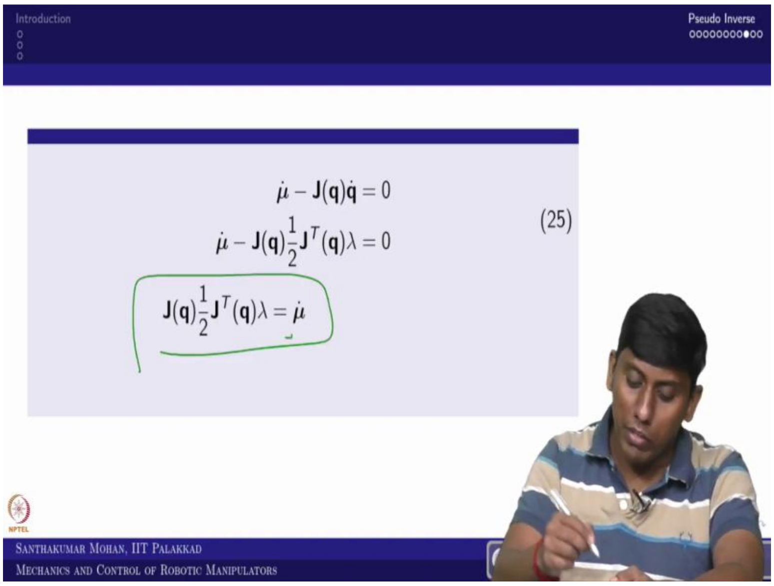

So, the second equation I substitute here, I substitute that, so it this will give. So, now I rewrite this where the lambda would be obtained in the form of mu dot, so that is obtained. So, now this I substitute into that previous equation.

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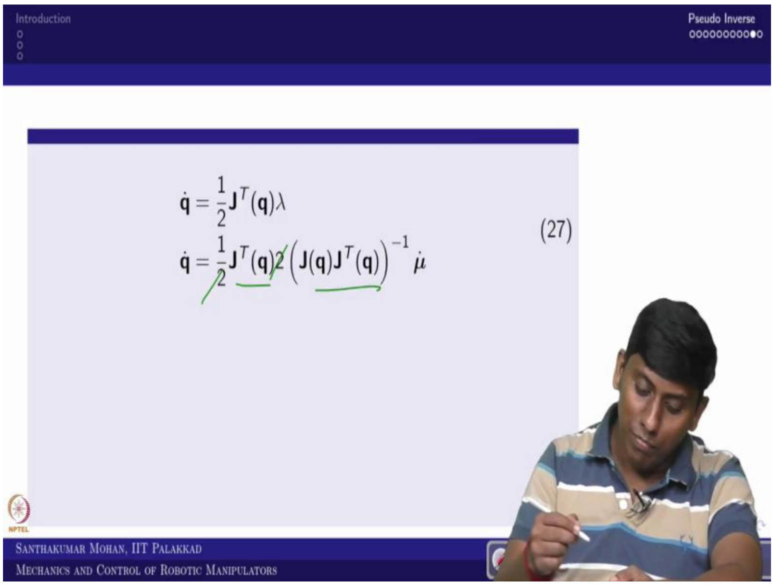

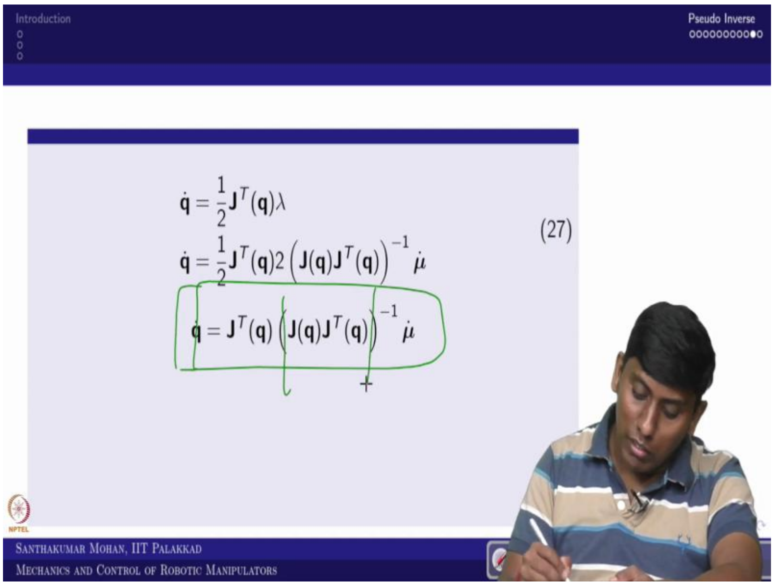

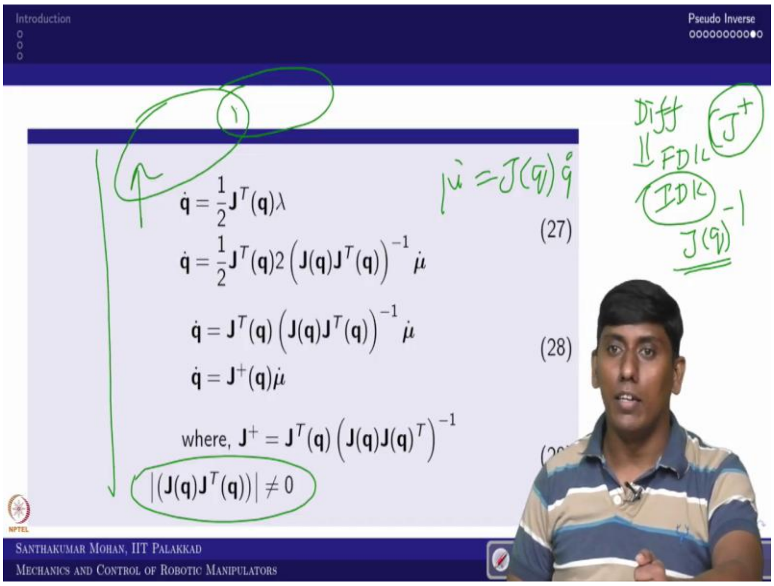

So, what that equation this equation if I substitute lambda, so this 2 and 2 will go. So, you can see this is look like a right inverse. So, that is what you can see. So, provided what you can see, so this determinant is nonzero. So, that is what we have actually written. So, now what we have done we have taken left inverse and right inverse, so everything is done, and we have seen.

In the other way around this class, we have talked about differential kinematics, that we have seen forward differential kinematics and inverse differential kinematics, where the inverse differential kinematics J of q inverse is coming, if the J is a rectangular matrix, so you have to use the pseudo inverse and in this class, we have seen what is the pseudo inverse form, we can use it.

In the sense Moore-Penrose pseudo inverse we have used and the detail why it is left inverse and right inverse we have explained. So, in fact, this detail explanation is available in one of our previous conducted NPTEL course. So, I probably provide the link in the handout, you can go through in detail and then you can come back.

So, with that I am closing this particular session and the next lecture we will be seeing about velocity propagation. So, velocity would be propagated one joint to another joint how it would be propagated from here to here. So, that we would be seeing. So, based on that we can end up with a final you call mu dot in the order of J of q into q dot and one joint to another joint in velocity relation also, we will be obtaining. So, that is what the next lecture until then. See you, bye. Take care.