Mechanics and Control of Robotic Manipulators
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Lecture 14
Forward and Inverse Kinematics of Robotic Manipulators

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Welcome back to mechanics and control of robotic manipulator. So, last lecture, what we have seen actually is a forward kinematics, because we have seen the example. Finally, what we end up, we have end up with tool or transformation matrix of end-effector or tool with respect to base. So, this is what we found.

So, from that what one can see, we can see the end-effector, information of rotational, information of end-effector with respect to base and the positional information of end-effector with respect to base is obtainable. So, that is what we have actually like done that. So, this is nothing but what you call forward kinematics, if you look at it, so that is what we are actually like trying to see here in detail.

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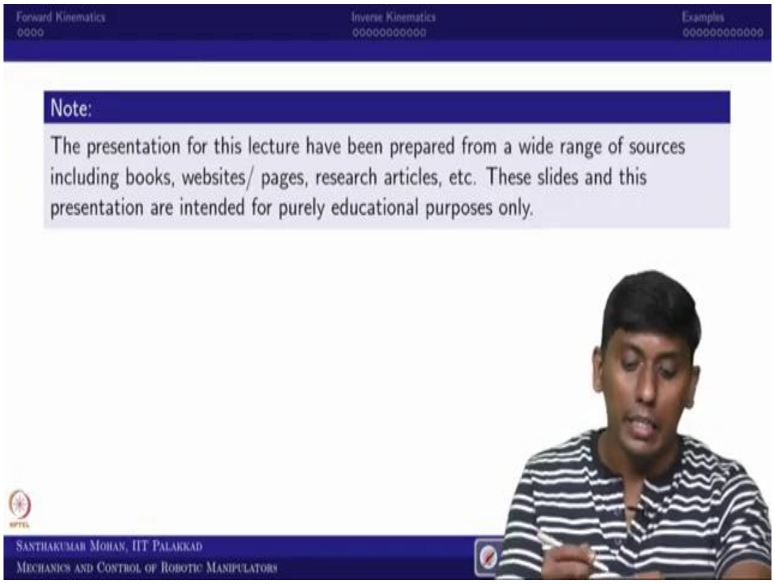

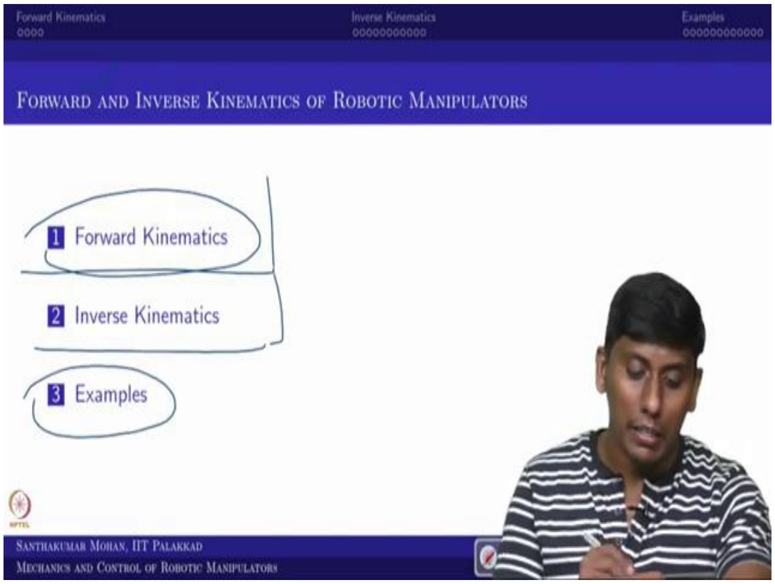

So, what that mean, so we are going to see in this particular lecture forward and inverse kinematics of robotic manipulator in specific. So, first forward kinematics, we will see what is what? And then we will give a small introduction to inverse kinematics, and we will be attempting the example at the end. So, in that sense, what we can see, we will recall what we have understood as forward kinematics in the very beginning.

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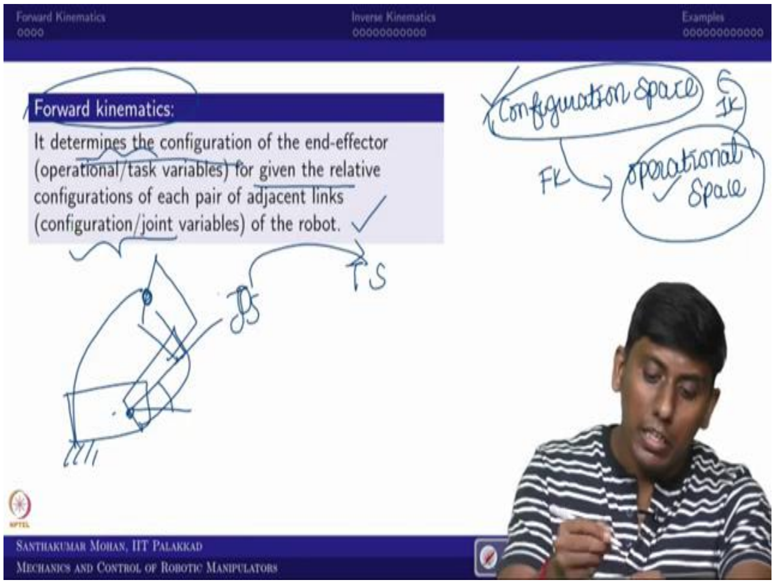

So, we have seen that there are two kinds of spaces. We have seen one is what you call configuration space. So, the other one, what you call the task or you call operational space. So, we have actually like seen these two are the spaces. So, what that mean, operational space means actually like end-effector positions and orientation, in our understanding, configurational space variable means, so it is actually like joint variables, in the sense, joint angle and joint distances.

So, if this is actually like given, if you want to find out, where is my arm for a given, what you call, the joint information. So, can I actually like find what is or where is my arm? That is what you call operational space in the sense of what you call forward kinematics. So, whereas the other way round, I fixed my arm. I am trying to see how my joints are arranged. So, this is nothing but what you call the inverse side, so we call inverse kinematics.

So, that is what we are actually like written in the word. forward kinematics, means it determines the configuration of the end-effector for given relative configuration of each pair of adjacent links. So, we call this is actually like assembly. For example, you take individual link and you actually, like fix it.

So, this to this, this much angle, this to this, this much angle, like that, you fix it. What you will see, this is the end, this is the base. So, the base to end, you will find it in the known relative information of one body to another body. So, that is we simply call this as joint space, and this is we call task space. So, that means, so we will actually like write task space and joint space. So, we know this, and we are trying to find out.

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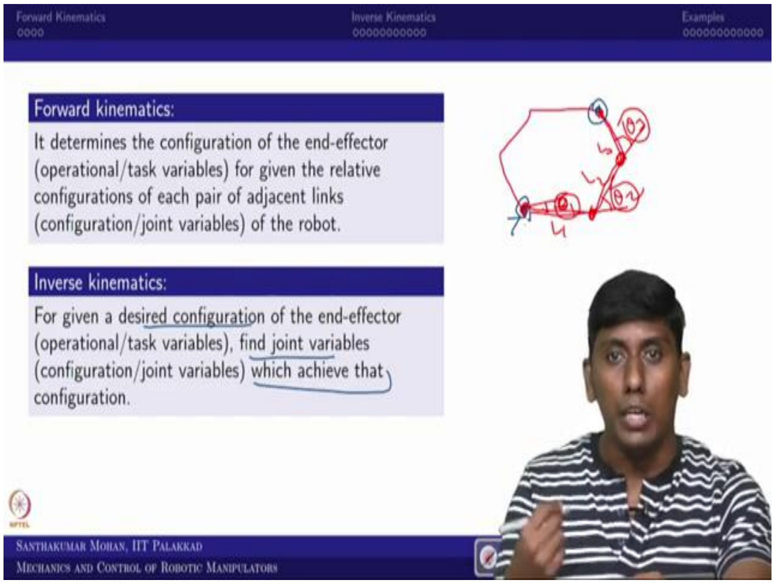

So, what would be the inverse kinematics? It is opposite. So, where for given desired configuration of the end-effector finding the joint variables or relative information of each body-to-body information, in the sense relative information of joint information. So, that is what we are trying to find out. So, this means what we are trying to do.

So, I given a configuration, this is what my base, and this is what my end-effector. So, can I actually like fix it? So, for example, this is 3 R joint. So, I have to actually like, see how to fix it. So, in the sense, so these are the 3 R's. So, now this angle, I assume that minus theta 1 and this angle is actually like theta 2 and this angle is theta 3.

So, in the sense, if I actually like fix L1, L2 L3 are constant. So, I fix theta 1, theta 2, theta 3, I can find it. So, this is what we can actually like see, for given this, this is the way I can arrange or the other way around. Like that, you can keep getting, this is what you call inverse kinematics. So, that is what we are actually like written it here.

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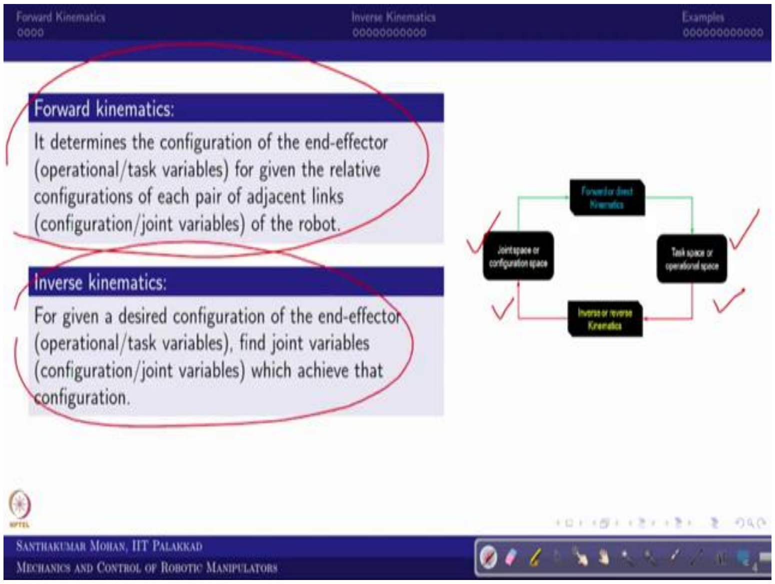

So, what we have written, so this is forward kinematics means so for given joint space, we are trying to find out the task space. Inverse kinematics or reverse kinematics mean for given this, so we are finding this. So, that is what the whole idea, but this particular lecture is going to talk about more about this. And later part, more about the second part in the sense inverse kinematics.

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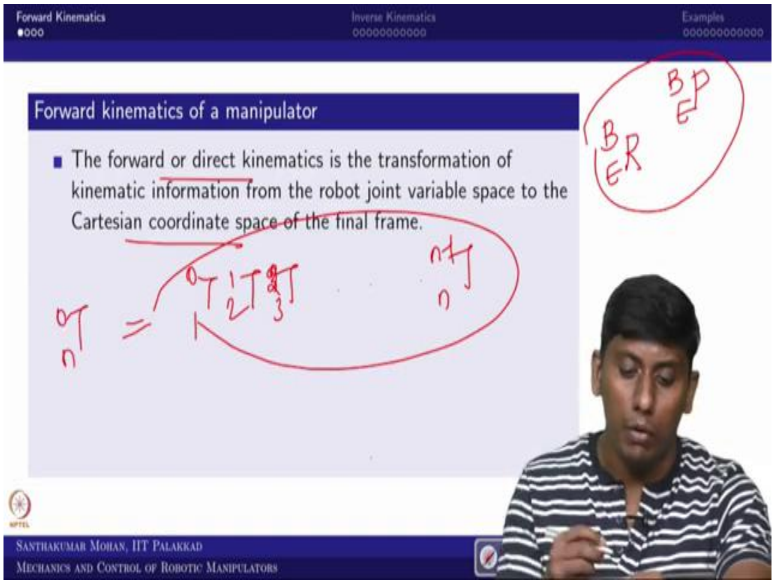

So, this is already we have explained more and more. So, what exactly we are trying to find out? We are trying to find out the Cartesian coordinate space information, we are trying to find out. We are trying to find the position vector of end-effector or the tool with respect to base and the rotational information of the end-effector with respect to base. So, this is what we are trying to find out. So, for that, what we know already? So, if you know the individual transformations, so then you multiply that. So, you can see, so what you will get, so n minus 1 to n; so, you will get t0 to n. So, that is what we are trying to find.

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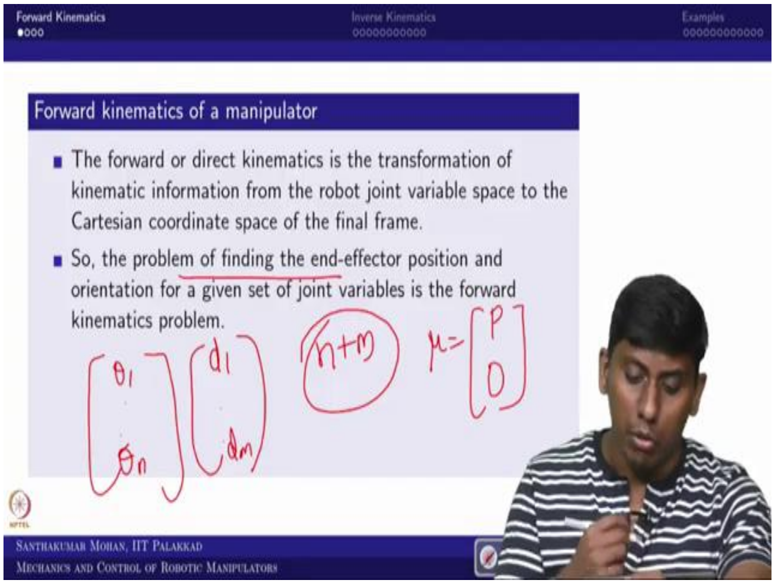

So, this is problem of finding end-effector position in orientation that is what we discussed. So, here what we can say theta 1 to theta n and d1 to dm. So, where n plus m is the total joint variables and these all given, I am trying to find out mu, which is the position vector and the orientational information of the end-effector, that is what we are trying to find.

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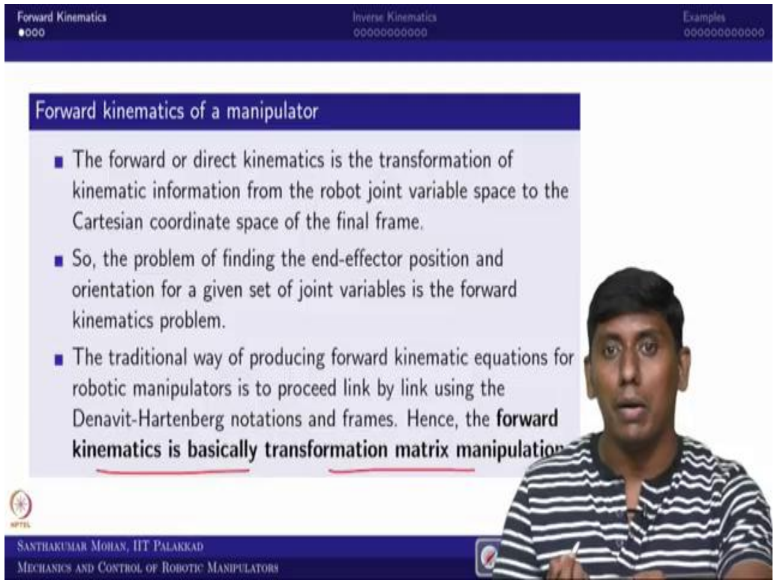

So, this is very straightforward. So, forward kinematics is basically a transformation matrix manipulation; what that manipulation means, simple multiplication, that is what we are trying to see.

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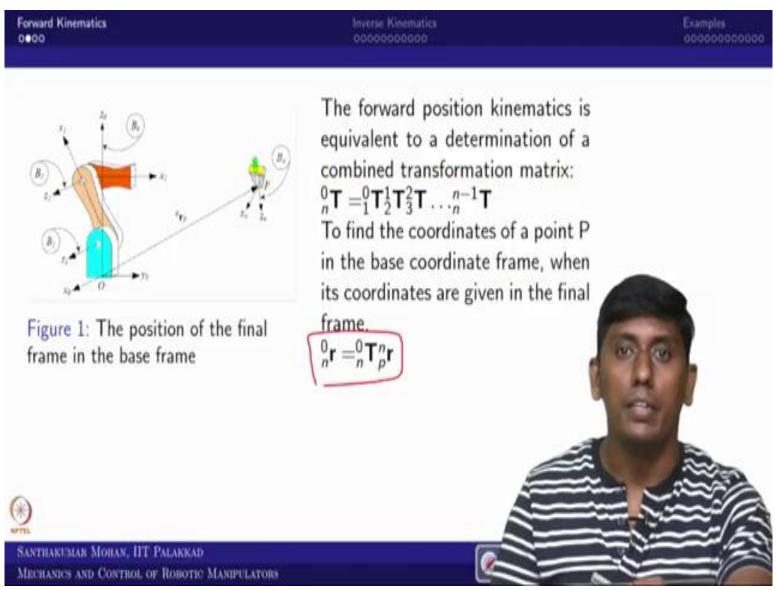

So, for example, this is the real robot, and there are several bodies associated with the joint, and this is giving the end point p and the base point 0. So, you want this and the orientational information. So, what one can do, so we can get this. So, this will give the final end and then you can find if anything else is there. You can multiply with this transformation matrix; you can do it. So, that is the whole idea. That is what we are writing it. So, I hope you understood, what is forward kinematics?

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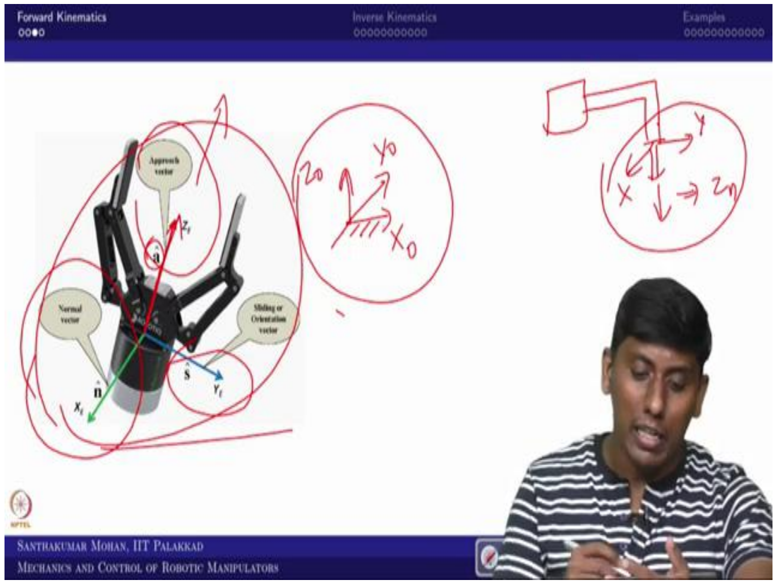

We will go further on. So, now we have seen few examples. We did not bother about the end-effector. So, we did not even fix the final end point certain format. So, in that sense, what would be the difficulty we will end up, for example, I am making a frame arrangement. I will end up with something, and you make another thing, you will get, so equally like both the DH parameter and the transformation matrix anyhow going to give a right result. But what happened one to another the information when we transfer, we have to give, this is what the convention I have taken. This is the end-effector convention I have taken, and this is the base coordinate I have taken because I have taken this way.

So, in order to avoid these all, so we always assume that X0 Y0 and Z0 like this. So, if you take the initial frame as like this, so then we have to come up with some procedure for the end-effector also. For example, KUKA is making something, and FANUC is making something, Adept making something. But these all are supplying robots to some of the industry. Industry cannot learn individual transformation matrixes. So, they wanted to see what is my end-effector? How I can move?

So, in that sense, you need to have the end-effector also certain manner. For example, if I see this is a gripper, which is attached with the end. So, there is one provision. So, what is the direction of approach, for example, so this is the robot which is ending, and this is the tool which is end up. So, what is the direction of this approaching towards? So, this is the direction I can fix it as a joint. So, this is Zn axis. So, in the sense, this is the approach direction. This is approaching the object or approaching the object around. So, then this is the Z axis, we call approach vector corresponding to that whatever I call it as a cap as a unit vector. That is what you call approach vector.

Now what else? There are two other axis you need to define. So, this normal to this, so normal to this means there are two directions. So, one is the gripper or the tool which is sliding. So, that direction you can take it as a sliding or orientation vector, which come from the Y axis and the normal to this, what you call simply normal vector, that is what X axis. So, now, in this case, so this is normal, and this is the sliding direction. So, in the sense, this is X and this is Y. So, that is the way.

Now the base also you make it standardized and the end also you make it standardized. So, in the middle, you take any number of conventions, finally, you want base to end information. So, that would be in a uniform form. So, that is what this particular attempt, so in order to make it a standardization.

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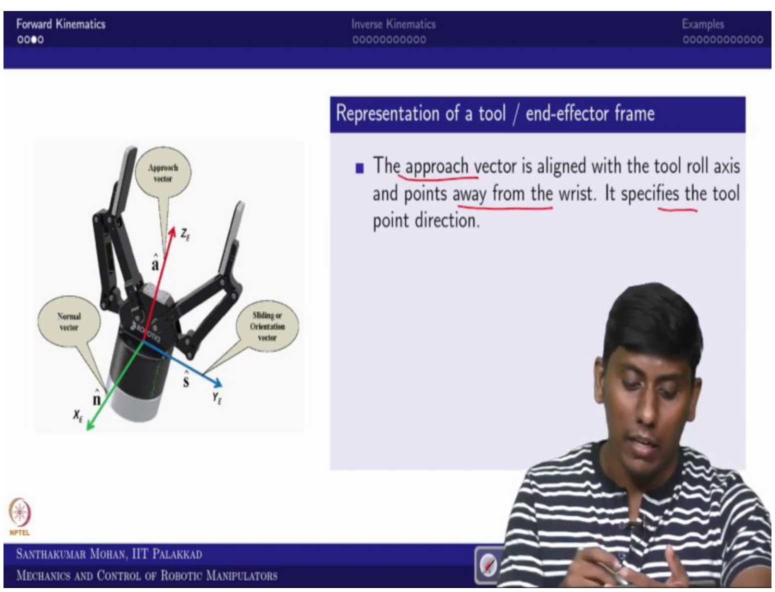

So, what we made, the representation of a tool or end-effector frame. So, what we made, first one is, we make it approach vector, which is aligned to the tool roll axis, if it is a tool. Otherwise, we can take it the orientation of the wrist, how it is away going, how it is approaching the object, that would be the tool point direction, or that is the gripper direction or that is the end-effector direction that is what we are going to fix as Z axis.

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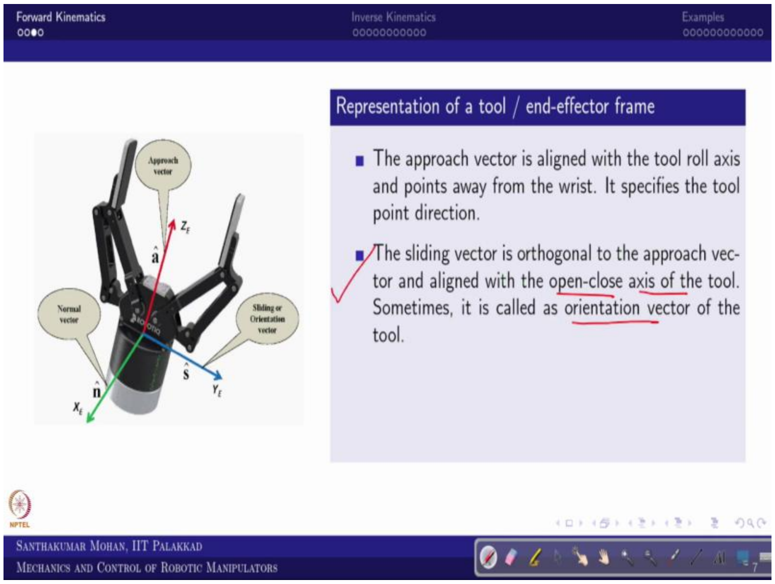

So, similar way, what we can see, the second thing is the sliding, which is how the gripper is opening and close, or what is the tool pitching. So, that direction, what you call orientation vector or sliding vector, we call. If you take a gripper, how the open and close of that tool are, you are pitching it, so how that is making it, so that is what the sliding vector. So, what would be the left, the normal to that.

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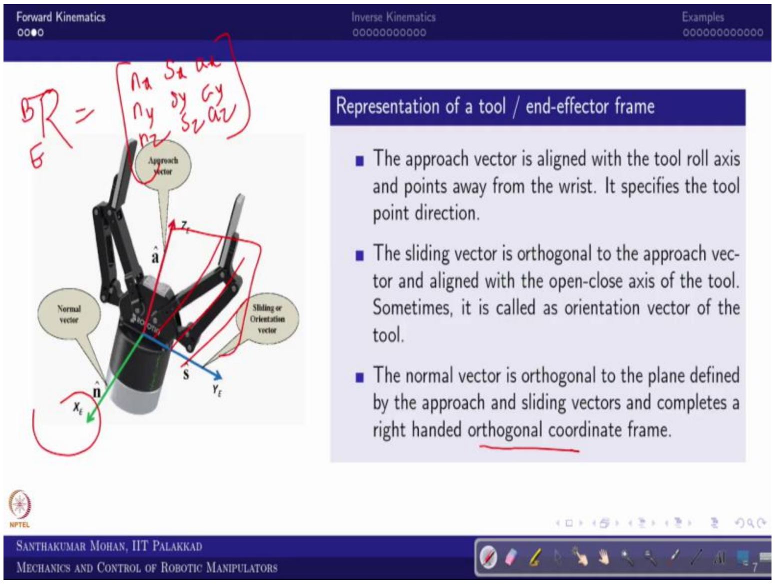

So, normal to this plane? So, that is what the X axis which we are going to fix. That is complete the right-handed orthogonal coordinate. So, what we can see, so if you write your transformation matrix, so this would be normal X, normal Y, normal Z, then this is sliding X, sliding Y, sliding Z, then this is approach X, approach Y and approach Z. The Z rotation matrix of the end-effector with respect to base. This way we can write the information. That is what we are doing in the next slide.

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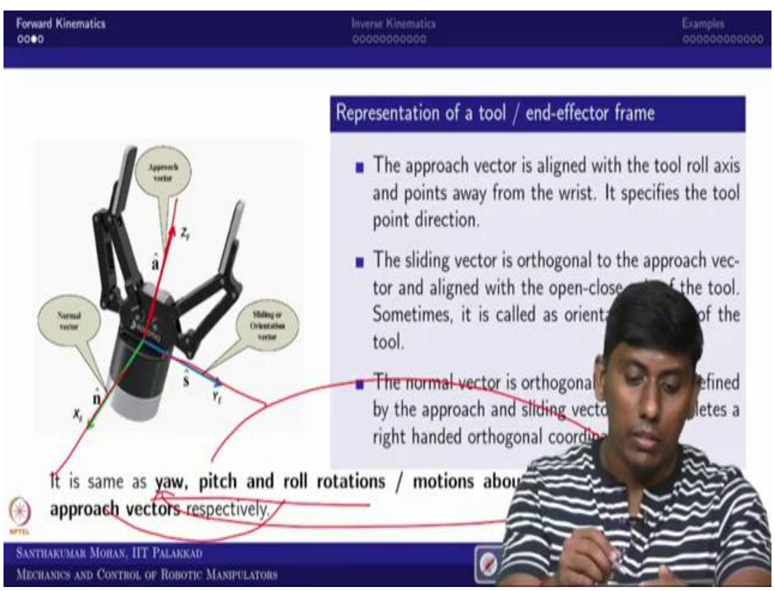

So, now you can see that if that is the case, what you can see, this is roll axis, this is the pitch axis and this is what you call the yaw axis. So, that is what we are writing it. So, the normal would-be yaw, the sliding would be pitch and approach would be the roll. So, this is localized coordinate. So, the bank is the roll and like pitch and then sliding, all those things can be brought in. So, now you are holding your gripper. So, you want to move this. This is yaw, so this is roll and this is pitch.

So, now all these three combined, but how we are making it, so you are yawing which is in X and you are pitching, that is in Y and rolling in the Z axis. So, this is much more convenient for the industrial manipulator community, so that the standardization can be brought in. I hope now you are clear.

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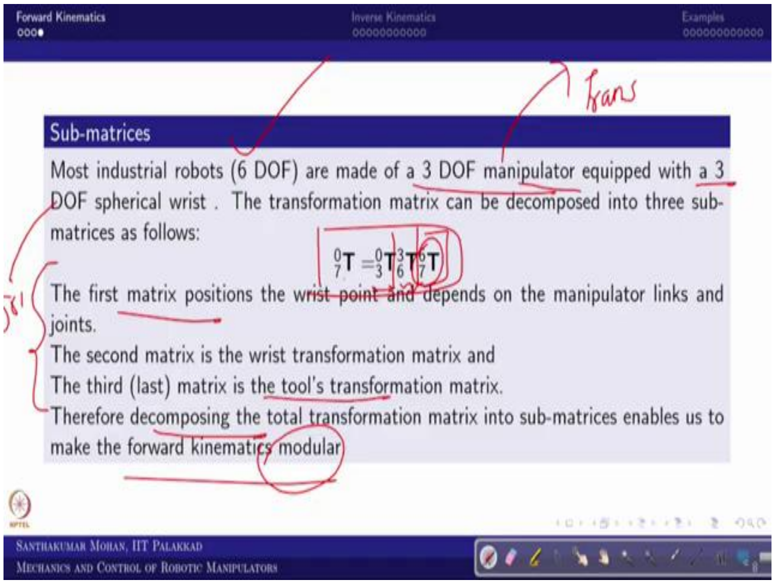

So, based on this, what we can see, if you take a 6 DOF arm, so that would be like 3D of manipulator, so which make a 3D of manipulator with a 3D area of spherical wrist, it can configure the translation and this configure the orientation. So, if you do this, what we can see the transformation matrix of, the 7, with respect to 0 can be break into, first 3 would be the major axis, the second 3 would be a minor axis, further you can call tool, if it is more than 6 axis, joint axis then that you can call redundant axes. So, that is what we are calling it.

So, now that the last one is defining what the tool orientation. The middle one is saying how the tool is oriented with respect to the wrist, or with respect to base, how the wrist is oriented. And the first 3 is, so with respect to base, how much or where the wrist is placed. So, this decomposition would be really beneficial when we go for inverse kinematics. So, that is what the whole idea. This decomposition, what it is giving, it is giving a modularity. That modularity is making much, much easiest when we go for forward to inverse kinematics.

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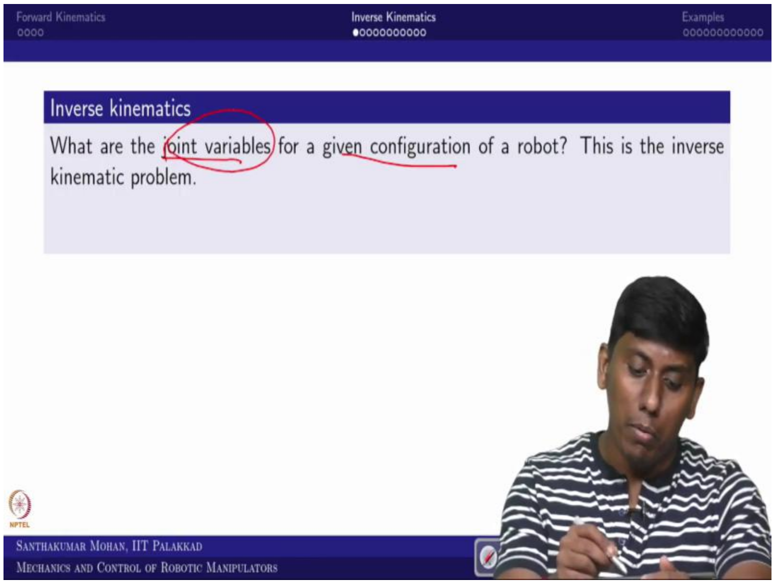

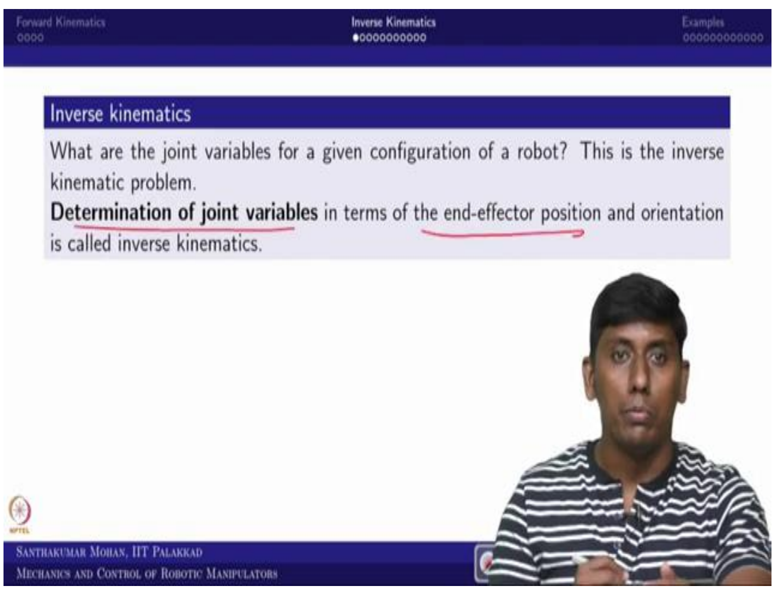

So, now I just attempt what is inverse kinematics. So, inverse kinematics is nothing but what are the joint variable to meet for a given end-effector. So, what that means, for given configuration of robot, what are the joint variable I need to give it. So, this is the inverse kinematic problem.

What we are trying to determine the joint variable for given end-effector position and orientation.

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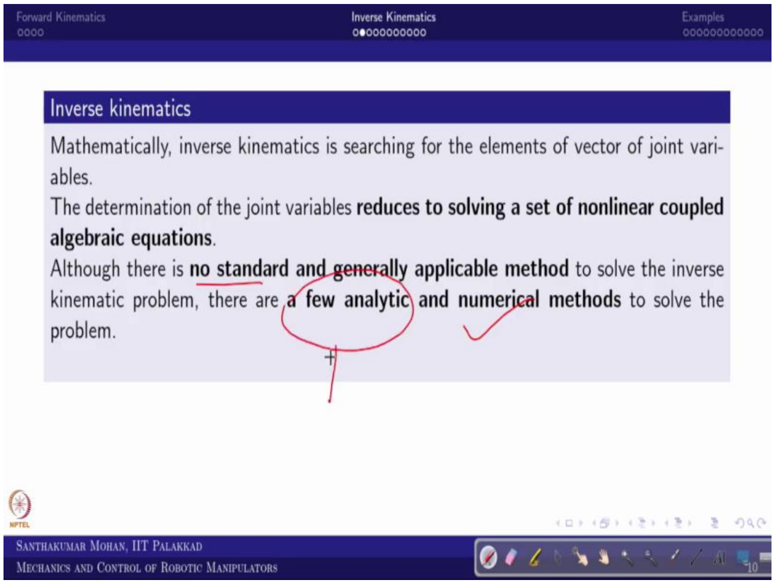

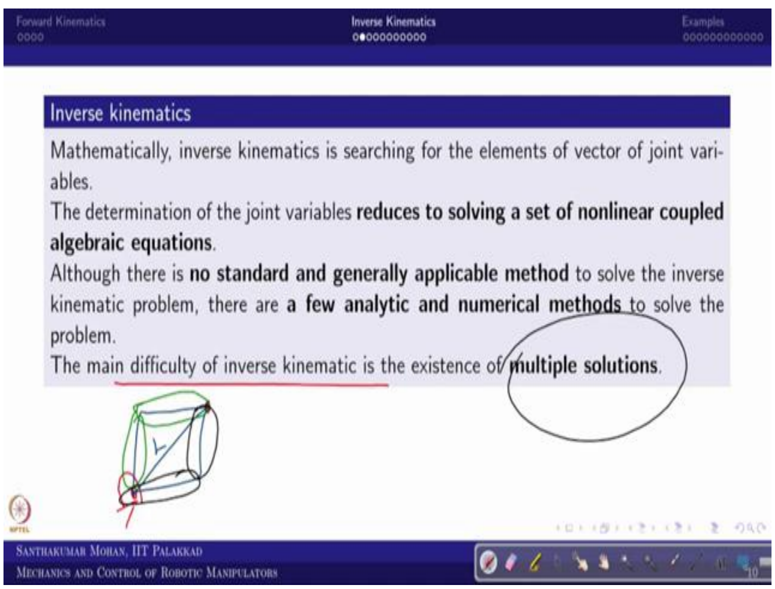

So, this is straightforward; mathematically, what we are trying to do, we are trying to search the joint variable, what we are trying to do, we are reducing or reduces the solving of set of nonlinear coupled algebraic equation; why it is so? I will write that in later on. So, in that sense, the forward kinematics is very unique, but inverse kinematics is not unique. There is no standard or generally applicable method, so there are several methods, but one of the best one is numerical, if you go higher and higher, but the analytical method is very, very useful for control purpose.

So, that is what we are seeing, but the difficulty here is what, for example, this is 2R serial manipulator. So, this can be configured like this or like this. So, even you take this position L, so this can be configured in this form, or it is in the other form. What you can see, there is a possibility of multiple solutions. So, if it is a multiple solution, it is always problematic. So, that is not unique, and solving method also, there are several ways. So, that is what we are trying to address here.

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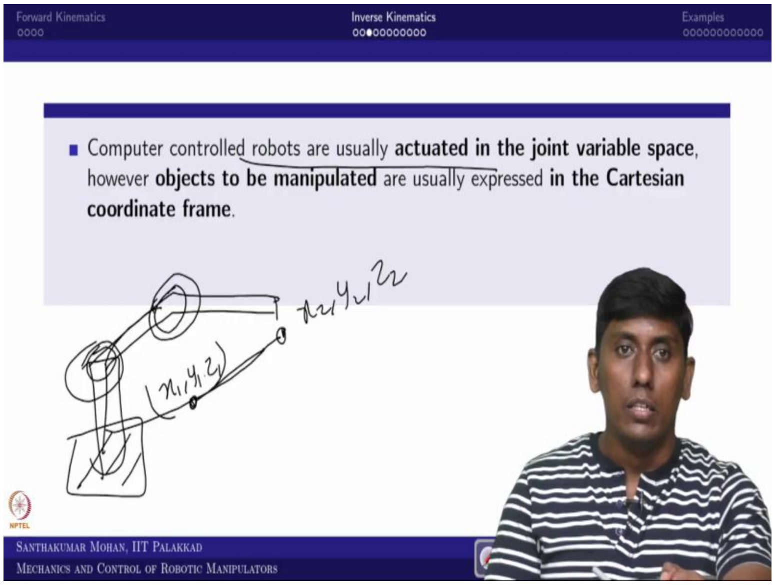

If you are thinking about computer control, so one of the easiest ways is you can derive it based on numerical solution, but numerical solution is always end up with some of the closest solution. Need not to be the optimal or all possible solution. So, now, we will think about a robot, so I am assuming that it is a simple robot, which is articulated. It is something like a SCARA robot. I want to reach this point. So, the robot will not know.

So, indirectly inside the controller somebody has to feed. So, what the user would be giving, this is X1, Y1 and Z1. So, this need to move to X2, Y2, Z2. So, this is what give, even the path would be given. So, now then what you have to do, you have to see what inverse kinematics can do, so that individual joint and you can move it accordingly. So, that is one of the important things which you need to know.

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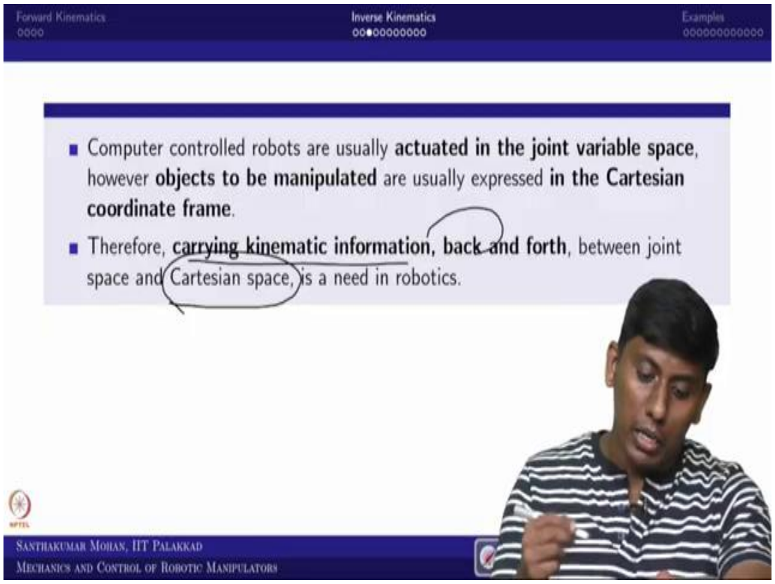

So, what you need to know, you need to know the kinematic information that to back and forth, you would do forward and inverse, then and there, then you will get the Cartesian space, what you wanted. So, that is what the important aspect here.

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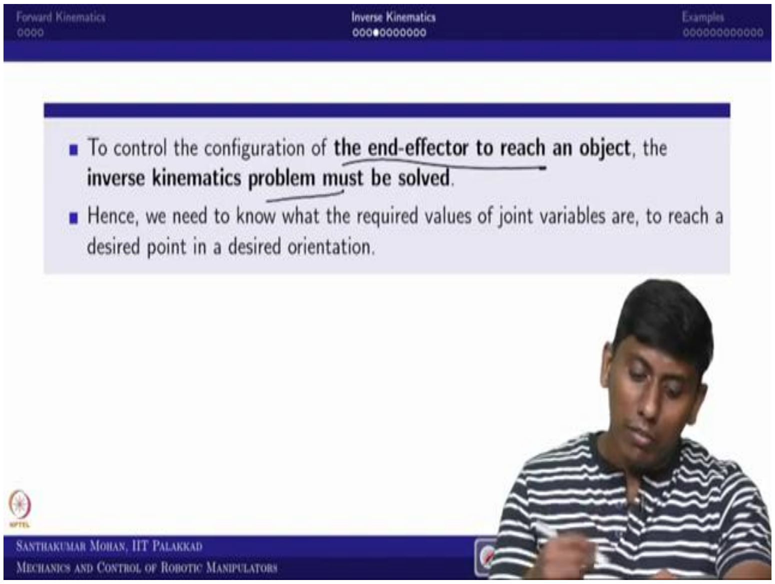

So, for going that what we are trying to reach, we are trying to reach the end-effector, the end-effector to reach an object, so this is supposed to be solved. So, there are several, several questions will come, what are the questions?

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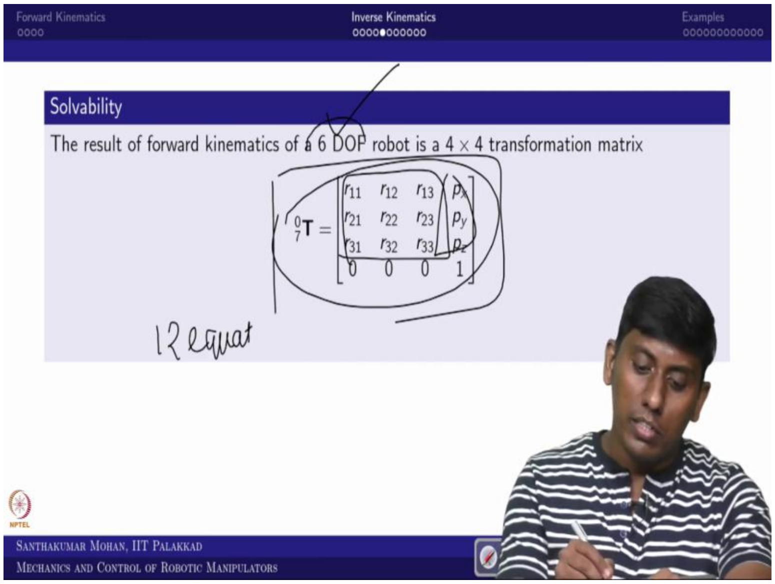

So, this is the final matrix. We assume that this is a 6 DOF of robot with the 6 axes, we conveniently take all our rotary axes, but finally, what you will get, you will get this equation. So, how many useful forms? So, these are 9 plus 3. So, total how much? So, 12 equations. So, you will get 12 equations, and how many unknowns, because it is 6 DOF arm, so theta 1 to theta 6. So, how many unknowns? 6 unknowns.

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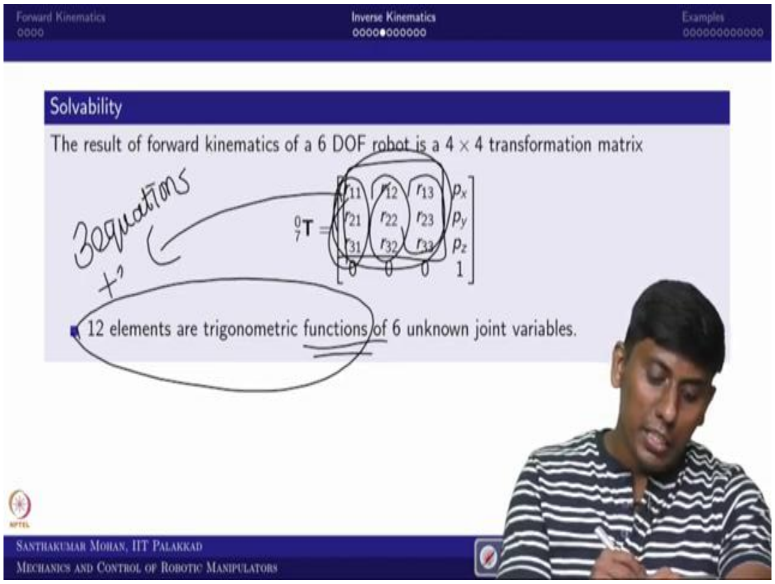

So, these are the cases, but these are trigonometry functions, and further, you know, these are orthogonal matrix. This matrix is orthogonal, and what that mean, it is orthonormal vectors, out of this, what you can get, we will get only useful form is 3 independent equations. What you can see, so 3 equations can be obtained from the rotation matrix, and 3 equations can be obtained from the position vector.

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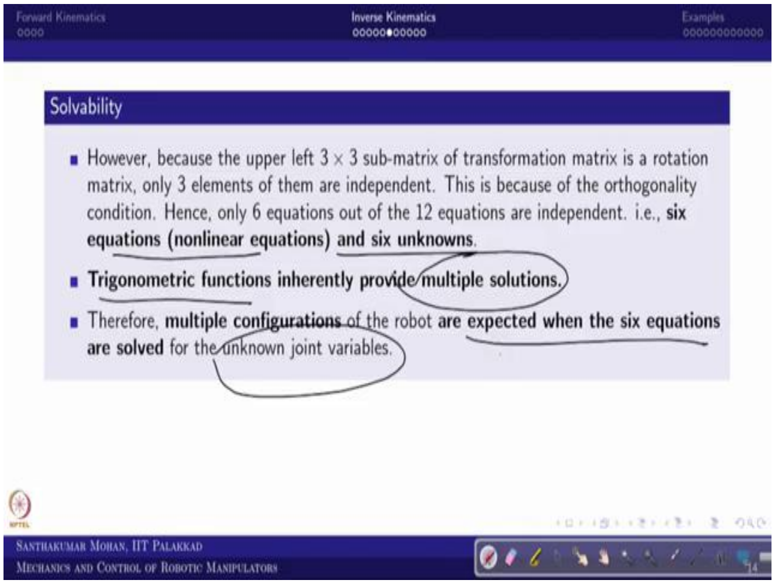

So, what you can see, so it would be 6 unknowns. Finally, you end up with 6 nonlinear equations. This nonlinear equation is trigonometry inherently. So, then what you can see, it will provide multiple solutions. So, if it is multiple solution, these 6 equations supposed to be solved for finding unknown joint variable. You are trying to find out all the configurations, so that is what we are trying to expect. So, in that sense, analytical is better, numerical will not be end up that way.

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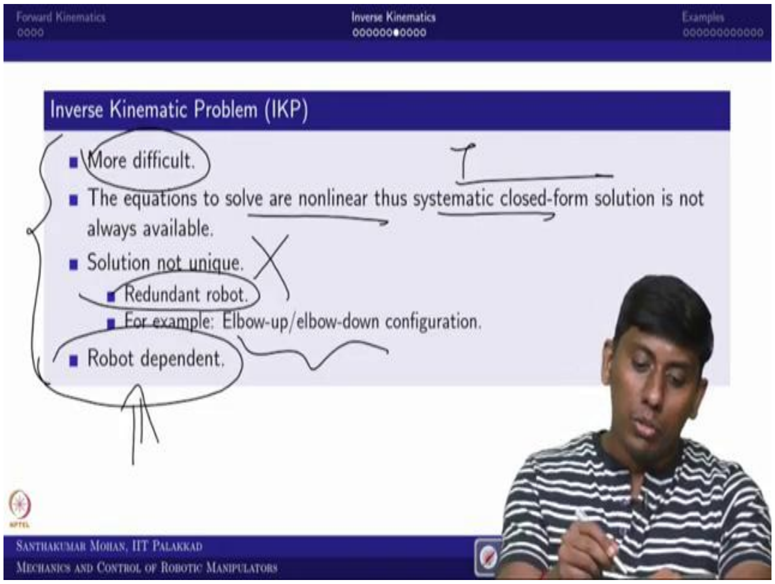

So, now coming back to the overall as inverse kinematic problem, it is definitely more difficult, not like forward kinematics. Forward kinematics is simple, matrix manipulation. You just multiply all matrix, finally you end up, but this is not that way, it is more difficult. The equations to solve are nonlinear. That is systematic closed-form solution is not always available, and the solutions are not unique because the link length are different, and each robot are different and the solution also like, would end up with elbow up and elbow down. So, further if the redundant robot, the solution are not easily obtainable and the solution of inverse kinematic solution is robot-dependent. You cannot generalize, so that is what the whole idea.

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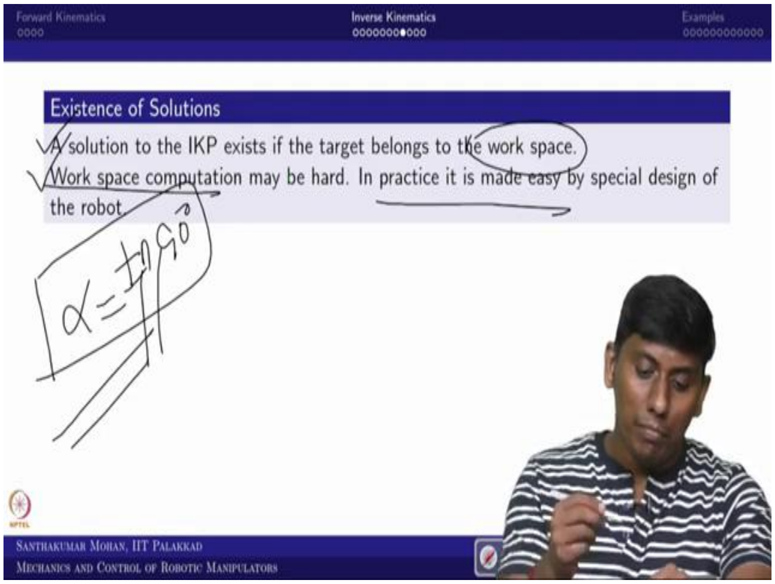

So, even for attempting that you should know whether the solutions are existed or not. What is the easiest way? We have already seen what is workspace? So, if the, you can say, given point is within the workspace, then the solution is existed, but how you obtain is different. So, for that you need to find the workspace, but workspace computation is very much hard.

In practice, it is made easy with a special design of robot, so that is why I said these alphas are always multiples of 90s, so multiples of 90s alphas, so that what happened, your configuration in such a way that it would make some kind of idea. Further, if we will come back, what is the necessary and sufficient conditions? So, that we will see, this is one of the necessary conditions, but is it sufficient to find the analytical solution or not? That we will go back.

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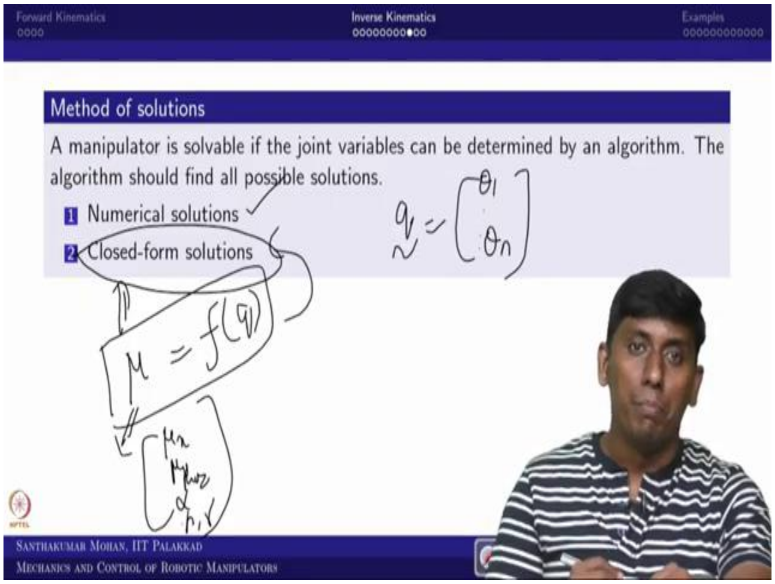

So, for that, what we are trying to see? So, the method of solution how to solve. So, obviously you already know, one is analytical, other one is numerical. So, even that numerical solution can be obtained through optimization or general numerical method. But the closed-form solution is we simply call analytical, why it is called closed-form? Your end-effector equation would be found as this. So, q is position vector – sorry, joint variable vector. So, it is theta 1 to theta n, we assume that all are rotary joint. This mu is, so your X axis position to final alpha, beta, gamma. So, this is what we call. So, now if we get this kind of relation, then we call it is a closed-form. But you know already. The closed-form solution is not so easy.

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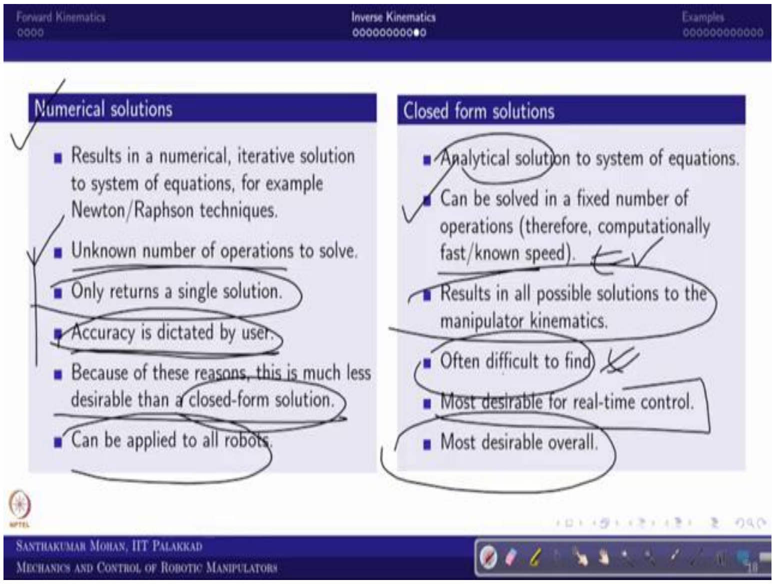

Analytical solution is what we call closed-form. So, numerical solution, what we call, it is iterative, that is what you call numerical solution. It is having unknown number of operations to solve, but this is straightforward. It can be solved in a simple fixed number of operations. It is very, very easy. But existence of closed-form solution is one big question, but this is easy. It is fast, when you are trying to do a control perspective, this is better, but this is for computational benefit it is better. Further, this returns only one solution.

So, it may not be optimal or that is the closest solution. But in this case, all possible solution can be obtained. This is like accuracy is very much questionable because what resolution you want or what first you want, according to that the solver would be defined. So, the accuracy would be very, very, pragmatic based on the user defined. This is one thing often to find difficult, but this is easy, because easily we can write the numerical solution.

But this is not that way, because of these all reason, it is much less desirable as compared to closed form. But if you think about redundant robot, the closed-form solution is not exists. In that sense, you have to go for the numerical solution. It can be applied to any such robot. But this is certain, specific robots. That is what I said, the existence are there. This is most desirable in the real-time and overall, it is desirable.

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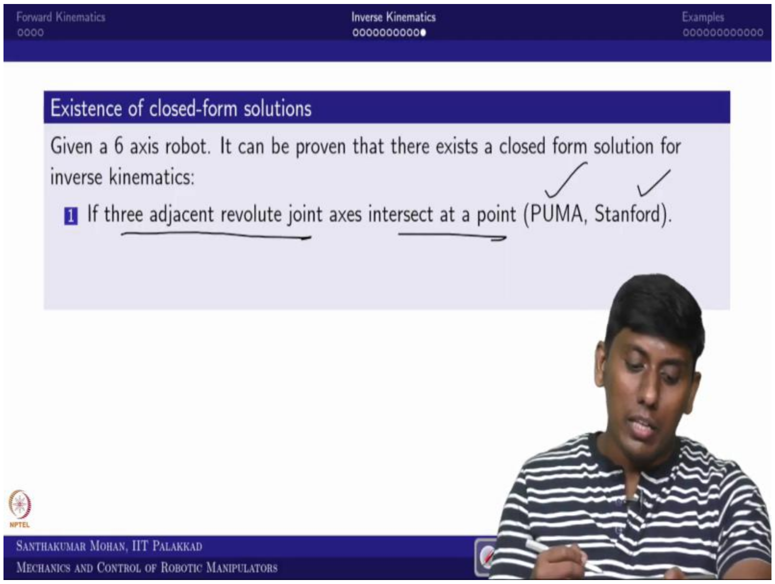

So, for understanding that I am putting the necessary and sufficient condition. The existence, you already know that the solution actually exists. You have to come up with the 6-axis robot, and the given point supposed to be within the workspace. Further, what you need to know, any one of these conditions suppose we fulfill, what that? So, three adjacent revolute joints intersect at a point, which is very easy because most of the industrial robot would have a spherical joint, the three final orientational axis of the wrist would intersect at a point. So, the common is Stanford arm, or PUMA arm and all, one of the examples, some of the examples.

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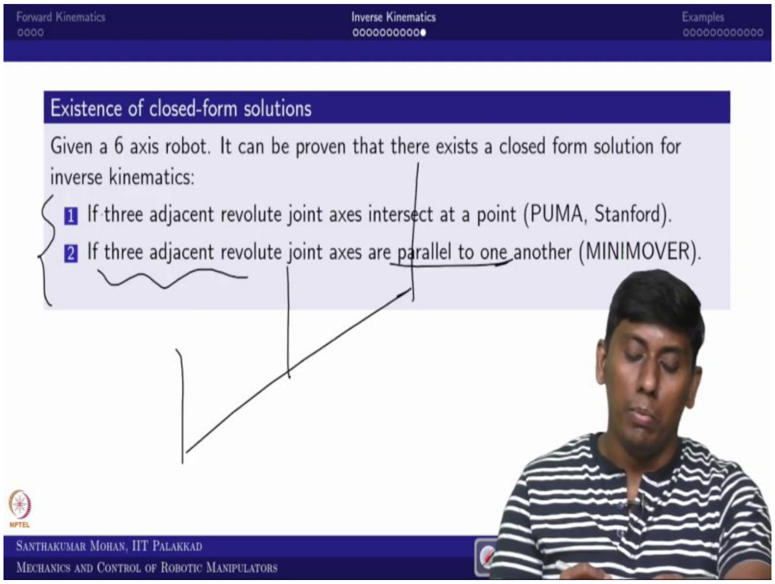

So, then what is the other possibility either this or that? So, what that mean? So, three adjacent revolute joint axes are parallel to each other. So, these three axes are adjacent axis. These are parallel. Then also the solution exists. So, what you can see, the necessary condition is, it is supposed to be 6 axis robots and the given point supposed to be within the workspace. Further, so that is the sufficient. So, the necessary condition is either one of this condition should be fulfilled. So, that is what we are seeing it.

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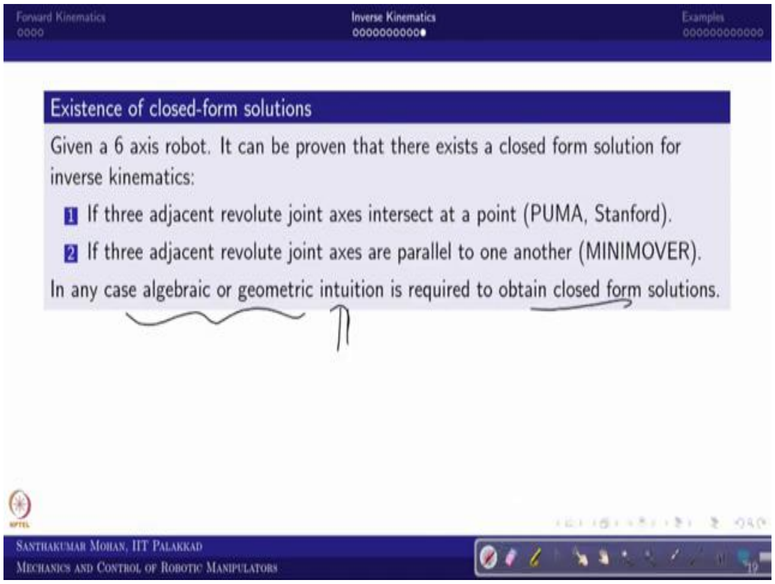

So, in that case, either you can get algebraic or geometric intuition can be used, and then find the closed-form solution. So, I hope you understand the difference between forward and inverse kinematics. The next lecture, we will be seeing the examples of inverse kinematics. We will take few such cases which we have attempted in the previous lecture, that we will take and see how to get the inverse kinematics. And with that, we will close the kinematic part, then we will move to what you call differential kinematics. So, with that, I am closing this session. Thank you. See you then. Bye.