Mechanics and Control of Robotic Manipulators
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Lecture No. 13
Frame Arrangement and Examples Part-2

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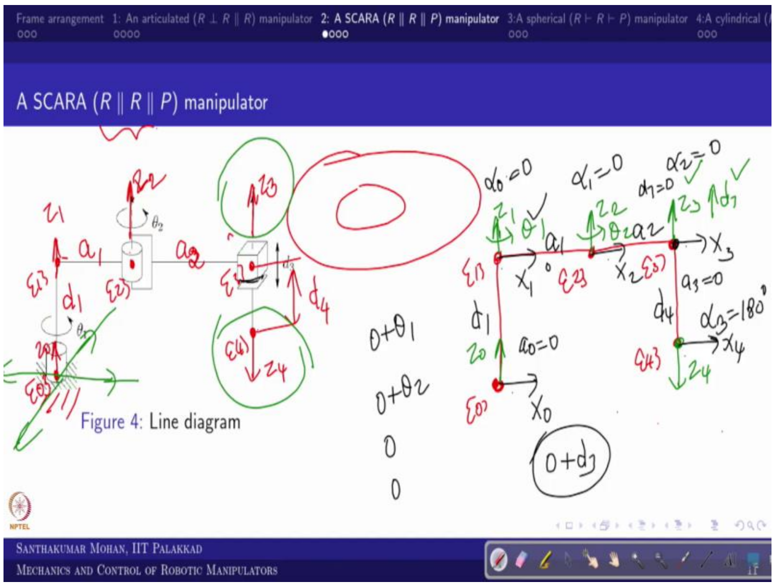

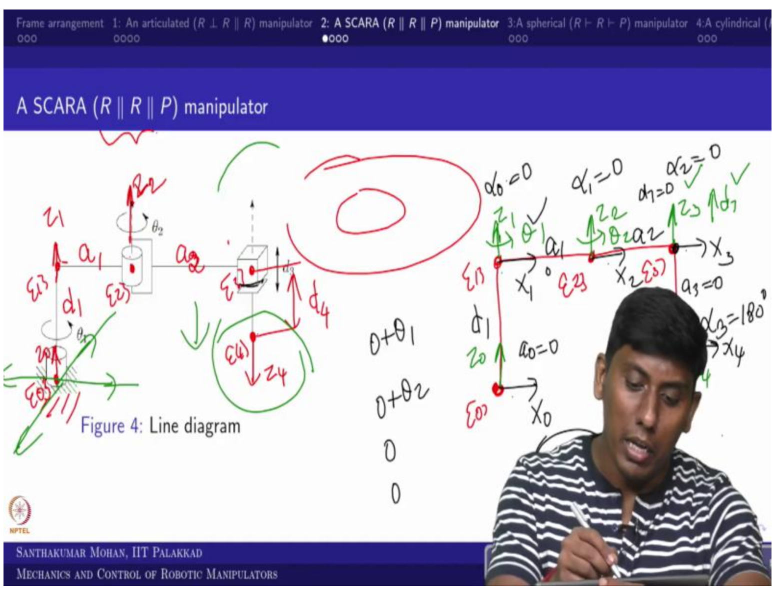

Welcome back to Mechanics and Control of Robotic Manipulator. The last-to-last lecture, we stopped few examples leftover. So, I thought before that I thought of giving the cases, different cases how to attempt. So based on that now we will come back to these examples. So, these examples, so we will start with one of the classical examples what you call, two R are parallel and one prismatic also parallel. So, now there may be a question will come, so what would be the nature of this.

So, this is also like going to swept on a volume, because this is going to rotate in a plane, but this make it what you can say up and down. So, what we did in articulated very similar, but here the workspace would be more like a doughnut and that to like, you can make it is not a doughnut, it is something like you can see hollow cylinder kind of thing it would be obtainable, based on the limits. So, what now what we really wanted to see here, so we will try to find out the frame arrangement and then we can see what would be the end vector position with respect to your base. So, that is what we are interested.

So, if we are seeing this interest what one can see, so we can fix always the Z axis. So, this is Z1 and this is Z2 based on given and the based on this the positive direction is given here, so please be clear. So, this is the positive direction of this. So now what one can see, so these all are parallel. So now we can take conveniently where you want to start what so called the frame starting point. So, we know like conveniently we start this is 0, and we take Z0 also parallel.

And now I am taking the convenient as the last point as 4, this is pointing downward as the fourth axes. So, now what else we need to do. So, we need to fix the frame 1, 2, 3. As we already seen the frame 1, 2, 3 we can conveniently take as per our interest. So now, I am taking this is 1 and this point I am taking as 2 and this point it is moving. So obviously, we can see where it is moving, so on this line. So, the end we have to fix it. So now, we can take it here, this is 0 d3, so we assume.

So now, this is what you call the frame 3. And now you can see like what this as some constant length we call the 4, this point to this point. And this distance, what you call this is between 2 and 3. So this is a3, a2, and this is between 1 and 2, this is a1. And this distance is again d1. So, I am for your benefit, I am just drawing this diagram here. So, here I am drawing it for your convenience. So, what we can see, so, I started this as 0 and, and this is 1, and this as a 2, and this as 3, and this as 4.

So, now in that sense what we can see, this is what Z0, this is Z1, and this is Z2, and this is Z3, this is a distance variable, d3 is variable, in this case theta 2 variable, and this case theta 1 is variable and this case I took it as opposite direction. So, now, what else next case. So, next case is actual like you have to fix the X axis. So, for fixing X axis what one supposed to see, so you have to see the, for example, you want to fix X0, you want to see what is the nature of Z1 and Z0.

So, in this case, all axes are parallel, except Z3 and Z4 are you can see, these 2 are opposite direction, still these two are parallel. So, in a sense what one can do, so you can take any convenient normal axis, you can see that you can take this as X, or this as X, or this as X, or this can be X that is sufficient. In the sense even actual like take anything on the plane you can take it, but you know there is two distances coming, so I am taking this along the X direction. So, in the sense what we are doing it, so we are doing it as, so I am taking in this way. So, this is X0 and this is X1 and this is X2 and this is X3 and this is X4.

So, in that case what would be the DH parameter, so DH parameters in this case, so no rotation except this point. So, in the sense alpha, you can say 4 minus 1 which is like 3 which is 180 degrees. So, remaining all the case alpha 2 is 0, alpha 1 is 0 and alpha 0 is 0. And when you talk about that, you can say the; a distance. So, a0 is 0 and a1 is this, this is a2 and a3 is 0. So, then what else you need to see, so the theta 1 it is the exists which is a variable 0 plus theta 1, this is 0 plus theta 2, the third one is, so there is 0 because there is no active and the fourth one again 0 because the X3 and X4 with respect to Z4 are parallel.

So, when coming to the distance, so 0 to 1 there is a distance that is d1, 1 to 2 there is no distance, so d2 is 0, so 2 to 3 there is no distance along Z3 that is 0. So, d3 is 0 and here there is a distance. So, you can say 2 to 3 it is 0, but 3 to 4 there is a distance that is what you call d4 that is a positive direction, but in addition to that what happened this 2 to 3 there is a d3 which is a variable like this, the sense you have to write 0 plus d3.

So, in order to make it easiest, so what I made it, so in order to make it convenient for you I made a frame arrangement in such a way that what I did, I did this opposite direction. So, in the sense I assume that this is the positive direction which is coming downward.

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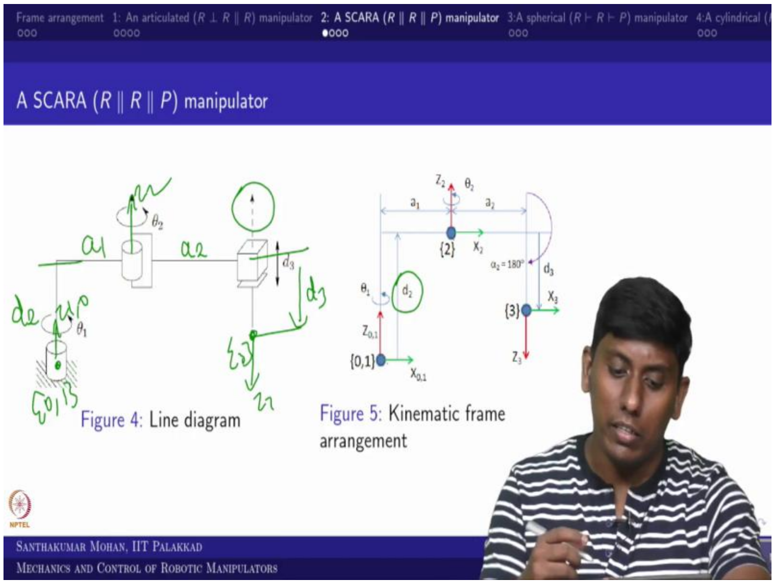

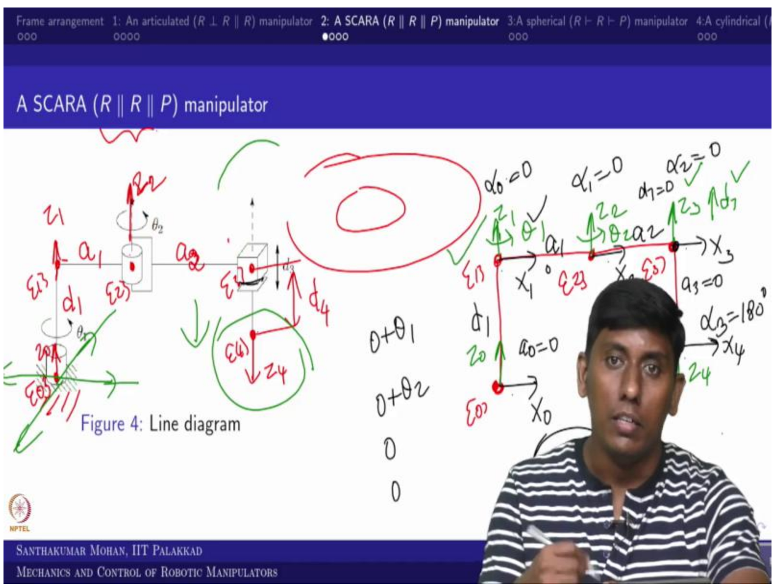

So, that frame arrangement I made it here. So, you can see, if that is the case, this final point I can take it as a3. So, you can see, we have taken a normal way that it will like take 4 frames, but now I simplified, so that is I, that is why I said although this is the positive direction, so what I can do, so whenever I substitute d3, I can substitute as minus.

When I substitute in the actuator end, when I am trying to control, I can put it as that is opposite direction, that is easiest for me. So, this is what the experience and now you can see like whatever else we have done, so the same thing we did, but even then, you can see one additional thing also I make it just for your clarity.

So, what I did, I start at 0 and 1 here. So, this is Z1 and 0 and this is Z2 and this point is Z3, in the sense this distance is d3 variable and this is a2, and this is a1, and this distance is d2. So, that is what you can see, because 1 to 2 I am looking at a vertical distance. So, that is why you can see the frame can be arranged in 2 ways either this side or you can do it this way. So, both are finally at the end giving the same result. So, we can check that.

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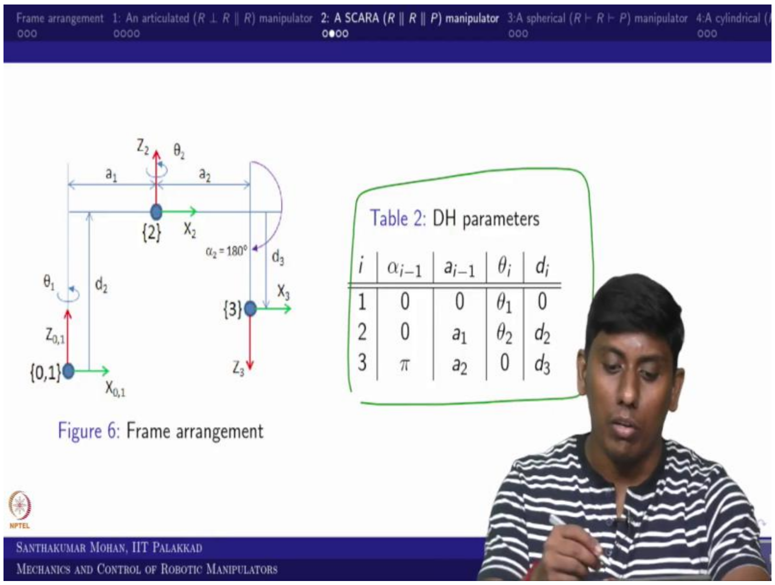

So, how we can do check. So, we can find first DH parameter. So, in this case the DH parameter in this form. So, what one can do?

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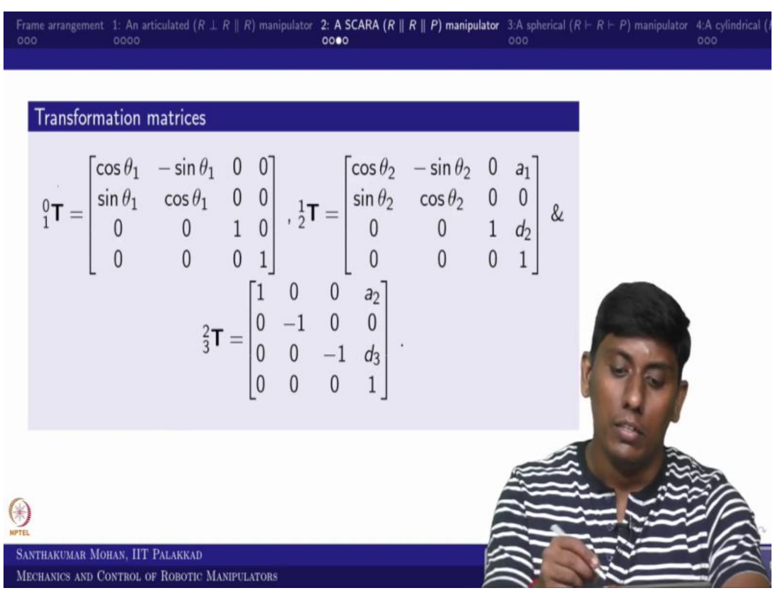

So, you can substitute that into transformation matrices. So, now you will get 3 transformation matrices and you multiply, post multiply one another.

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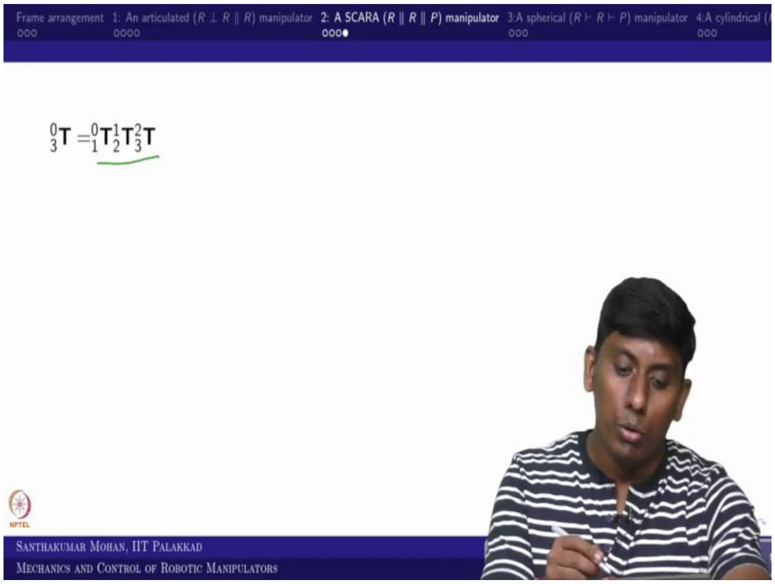

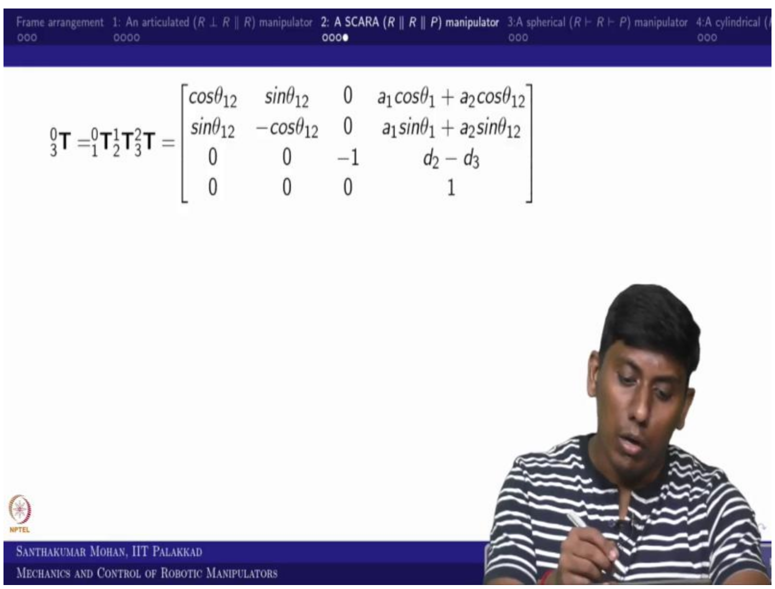

So, finally, what you will get the transformation matrix of 3 with respect to 0 you will get. So, what would be the final end, this.

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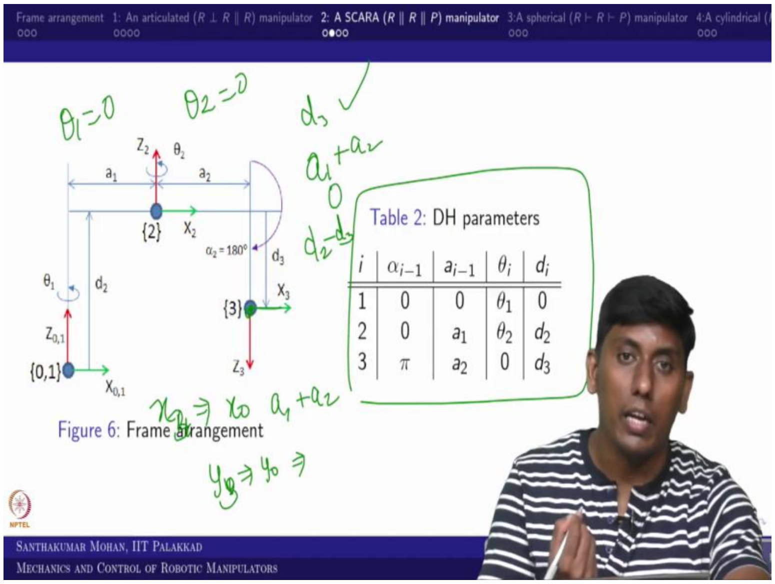

So, now you go back just for understanding you go back. What is your home position, so home position is theta 1 is 0 and theta 2 is 0 and d3 as it is. In the sense what would be the X, you can say four with respect to X0 that would be a1 plus a2. What would be Y4 with respect to Y0 that would be or you can say the fourth or you can say, third frame. The third frame with respect to 0 is X axis it is a1 plus a2 distance, Y axis it is 0 distance and Z axis it is d2 minus d3, this is opposite direction. Whether we are checked, we are getting that or not. We will check it.

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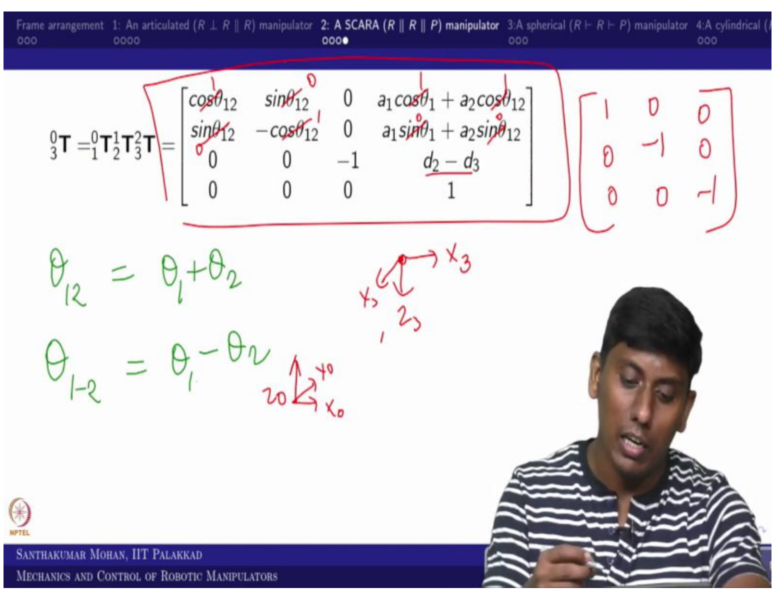

So, what we can do, you put theta 1 and 2 are 0 degree. If I write this way, this is equivalent to theta 1 plus theta 2. If I write 1 hyphen 2 which means theta 1 minus theta 2, that is the idea. So, I hope now you are actually clear. So, in the sense this is cos theta 1 plus theta 2. So, theta 1 and theta 2 0 means this become 1, this becomes 0, this becomes minus 1, this becomes 0, and this become 1, this becomes 1. So, a1 plus a2 exists and this becomes 0, this becomes 0, then this whole term would be 0 and this is actually like exists.

So, further what you need to do, you need to check. So, what one can see the Z3 is coming and Z0 is upper direction, further everything is actual like, you can say this is X3 and this is x naught. As per the right-hand rule this is Y naught, and this is what Y3. In the sense what one can see, the rotation matrix of 3 with respect to 0 by projecting X0 projection like this. So, Y3 projection on 0 is 0 minus 1 0 and Z3 projection is 0 0 minus 1, you get that also.

So in the sense, whether use what kind of you can say DH parameter, finally if you get this end product is as per your understanding and the original way, so then this is what we call a, you can say, a true or actual like correct DH parameters. So, now, we will go one and additional example.

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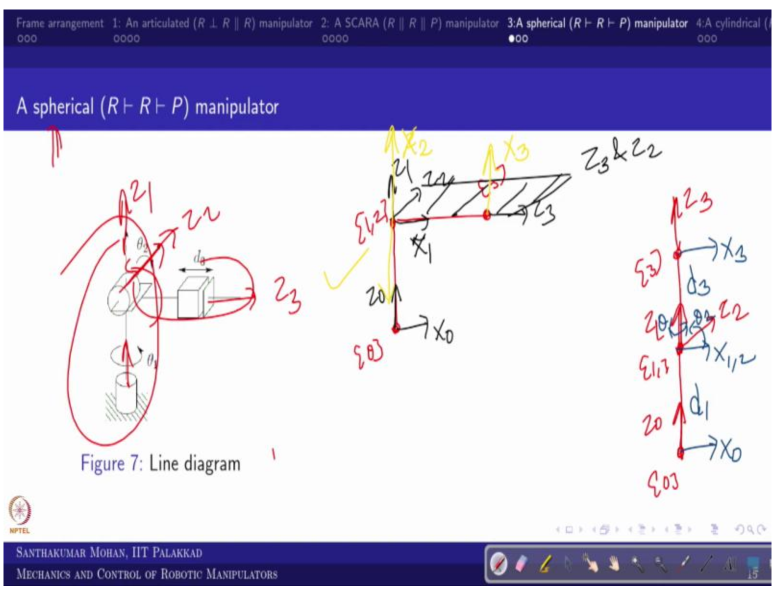

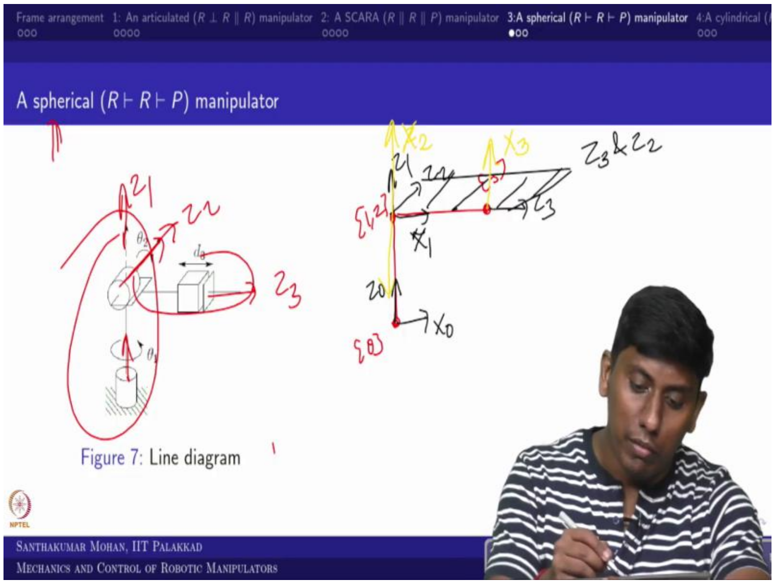

So, what we can do this is R R P, in the sense it is very similar to you can see several of us will be having it. So, in this sense, the first rotary here, the second rotary, and the third vertical is going in and out. So, in the sense this make a, you can say spherical workspace. So, that is what we are calling as a spherical manipulator. So, in this case, it is very straightforward, because this half we have already seen, and this is the only additional part. So, which we can see.

So, now, this is Z3 and this is Z2 and this is Z1. So, in the sense of what we can do, so we can see, this is 0, and this is 1 and 2, and you can see this is 3. So, further what we can see this is a Z3, and this is Z2, and this is Z1, and this is Z0. So, X0 is fixing with respect to Z1 and Z0, these two are parallel and these two are the plane containing the progressing side. So, this is what you call X1, but while choosing X2, what would be the case, you have to check Z3 and Z2 nature, the plane containing is this.

So, in that sense, what would be the direction, it should be up or down. So, I am taking this is X2 direction. So, obviously, the X3 I can keep it like this, so now this is the way we can actually, like fix the frame. I hope I have done that, no. So, what I did, I have modified. This is one way you can do it, the other way what we can try to do it. So, I am actually trying to show, so you can see you rotate this. So, if you rotate this because this is theta 2 is active, if you rotate this what happened, this is 0 frame, this is 1 and 2, and this is 3, where this would be still Z3 and this is Z1 and Z2 and this is Z0.

So, in that case, what is the benefit, you can fix X0, X1 and X2 and X3. So, this is based on the experience now you can see this is, so d1, and this is d3, and this is theta 2, and this is theta 1, these are the variables.

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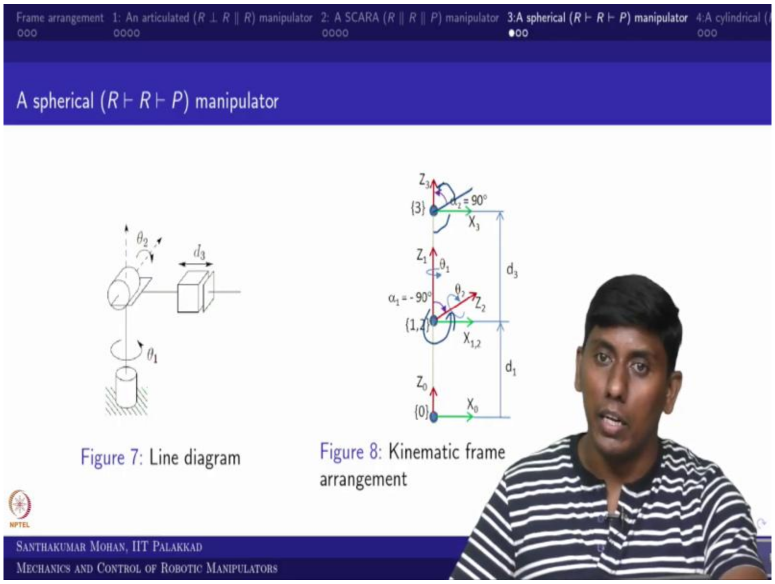

So, that is the frame arrangement which we did here. So, there are 2, you can say twist angle will come. So, 1 is between 1 and 2. So, the second 1 is between 2 and 3. So 1 would be 270-degree rotation, the other one is you can see, so the other one is a 90 degree rotation, that is all. So, now, we can cross check whether whatever we obtained that DH parameter is giving the right result or not.

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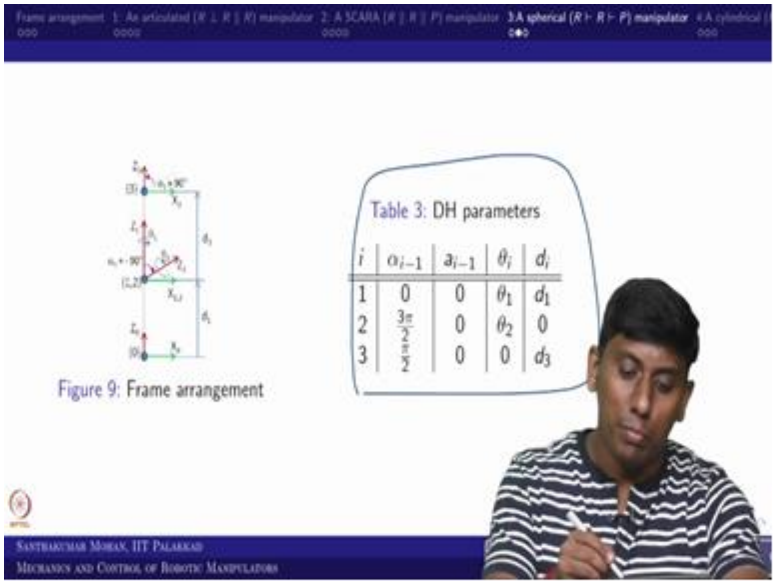

So for that, what one can do, we can first find the DH parameter.

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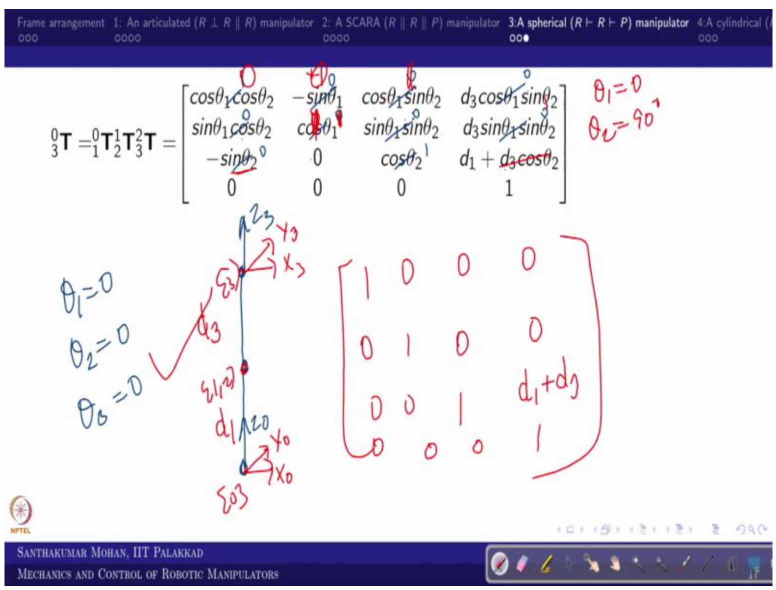

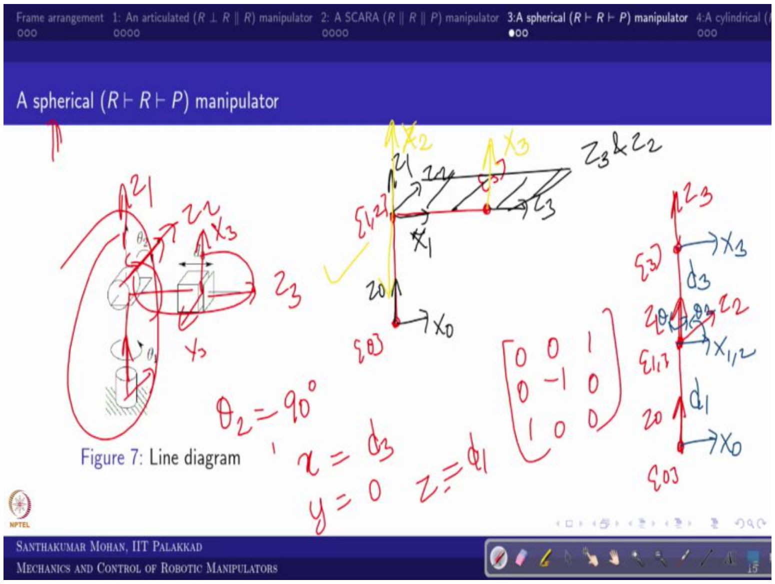

So, we can see this is a way the DH parameter we have obtained. So, based on the obtain DH parameter we can find the final transformation matrix. So now we can substitute theta 1 equal to 0, theta 2 is you have to see, this case 0. So, theta 3 is 0, based on the home position. So, in that sense, what one can see, so this becomes 1, this becomes actual 0, this 0. So, this becomes 0, this also becomes 0. This is already 0, this is combination 0, this is actual like combination 0. This is 1, this becomes 0, this also becomes 0, this remained d1 plus d3.

So, you would have recalled what we did. So, we have taken this way. So, this is Z3, and this is Z0 and I am drawing this is X0 and this is X3 and this distance is d1 and this distance is d3, and this is 1 and 2. So, if you are looking at the third frame projected on 0, so what are the things, so the X axis it is parallel, so 1 0 0. So, Z3 also parallel in this sense the right-hand rule if you take, this is the Y3 and this is Y0, so 0 1 0, so you can see this 1.

So, then the third one is 0 0 1 and what would come the other way around? So, the X axis displacement you can say 0 and 1 there is no displacement, 0 that is we cross checked, Y also there is no displacement and Z axis d1 plus d3. The sense, this matrix is correct. However, however, so however if you want to cross check this configuration. So, this configuration you want to check, so then what would be the theta 2, so theta 2, you have rotated so the sense theta 2 supposed to be 90 degrees.

So, then only you will get in that sense, what would be the X displacement would be d3. So, Y displacement would be 0 and the Z displacement would be, so d1. And the rotation matrix, so it would be, so you can see what we fix. So, the, this is what we put X3. So, in the sense X3 is on the Z0 0 0 1. So, the second one is coming back. So, what would be this, so this is what Y, Y3 comes here, but here, this is opposite. So, 0 minus 1 0. So, the third 1 is 1 0 0.

So, we will cross check that is obtainable here. So, by substituting that, so you put theta 1 is 0, but theta 2 is 90 degrees. So, in this case, this becomes 0. So, this become, what you call, this become 0 and positive rotation or negative rotation you have to see, so what we have taken this is the direction and we have taken that. So, I will cross check that, so where is that. So, so, we have rotated this way.

So, in this sense, so, this is the direction so that is coming. So, in that sense, you can see there is the minus 1 comes here. So, then here also minus 1 comes and, this is 0, this is 1 and here you can look at it the combination is given 1 and this 0 and 0. And this is you can see this is 1 and this is also 1 d3 exists and here this becomes 0 the d1 is exist. So, in the sense, that is also we have cross checked and it is happening.

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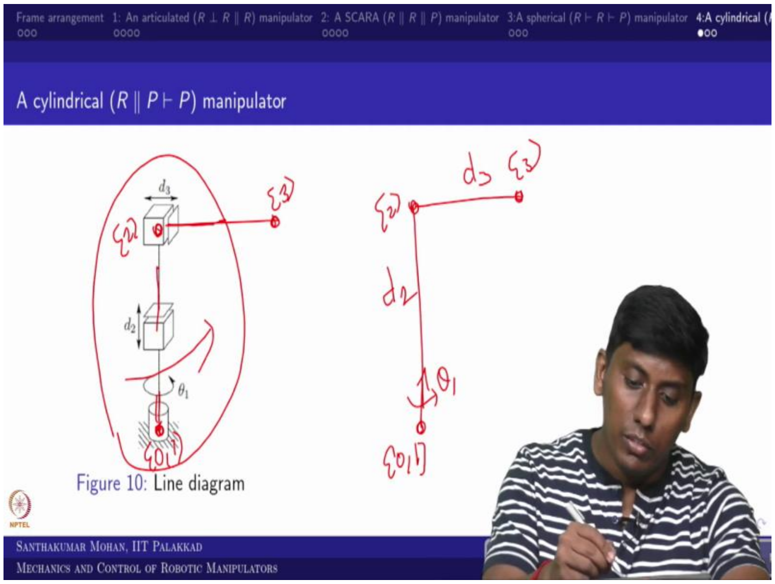

So, now we will move one more example which is we have seen 2 prismatic joint, which is we call cylindrical, which is one of the easiest one which you would have realized, because most of you would have definitely gone through what you call the; you can say workshop practice, where he would have seen the cylindrical drilling machine. So, the cylindrical drilling machine is in this configuration. So, where you can roll, first thing is you can rotate the arm then you can move up and down and then you can actually translate to and fro.

So, that is what the whole idea here. So, here you can see like this is the first rotation, then the translation, finally translation. In the sense what you can fix, so here you can fix 0 and 1 and the end tip you can put 2 because they are intersecting and the last point you can put 3. So, in the sense what you can draw, you can draw it is easiest. So, this is 0 and 1 and this is 2 and this point is 3, this is d3, and this entire thing is d2 and here rotation is theta 1.

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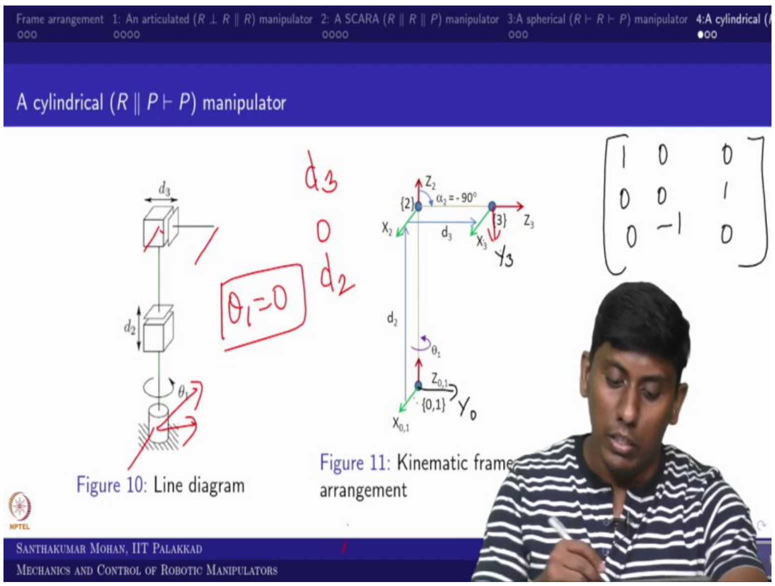

So, that is what we have done you can see. So, x axis you can fix it either way. So I choose an X axis which is coming outward. So, I can see it everywhere, it is coming outward. So that is a benefit. Why, because I no need to change the orientation of the X axis. So, that is what the case. But if I do the other way around, what happened if I would have started, if I would have started here as X. So, then there would be a change and I cannot keep it here, but if I choose this X parallel everywhere either it is going in or out.

So, both are actually going to make sense. For me all the time I put in, so this case I have taken out. So, now what would be the standard, so theta 1 would be 0 is the home position, in that sense the X axis displacement is d3, Y axis displacement is 0 and Z axis that axis displacement is d2. And the orientation is what, so the orientation is 1 0 0. So, then that the second one would be, you can see like 0 0 minus 1, the other 1 is, so I just I will use it here.

So, what does, so the X3 projected on you can say 0, so that is parallel to this, so 1 0 0. So, in this case this is Y0, and you can see Y0 this case means what you will get, this is what Y3. So, Y3 is parallel to Z0 but it is opposite. So, 0 0 minus 1 and Z3 is parallel to Y0, so 0 1 0.

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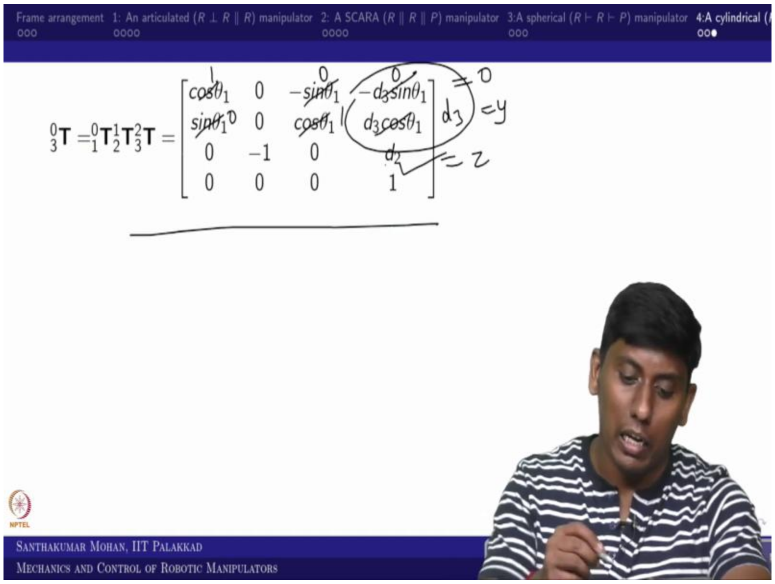

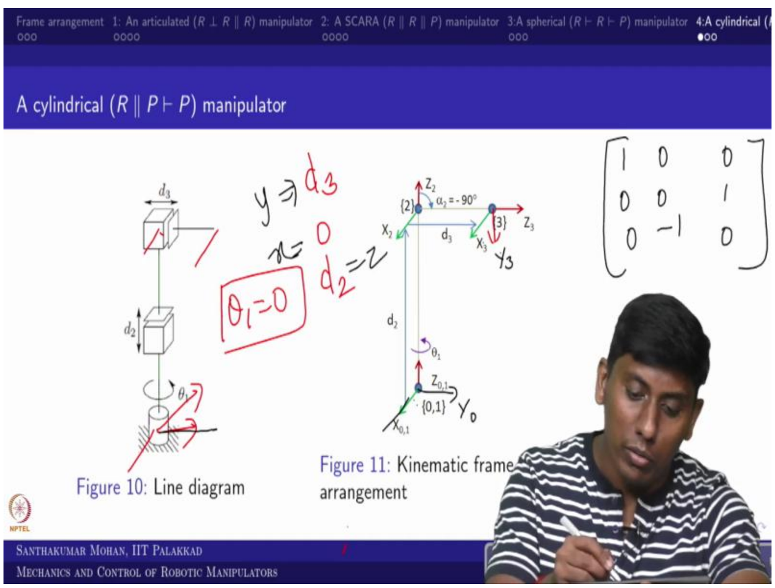

So, these all we can check whether you are DH parameter is giving the same way. So, we can find that, so this is the case. So, you can see like, if you put theta 1 is 0 this becomes 1, this becomes 0, this becomes 0, and this become 1, and this become you can say 0, this become a d3 and this become d2. So, we made a small mistake there.

So, please correct it, because I conventionally we took it So, you can see this is the displacement, which is d3, that is supposed to be in Y axis because we take X this way. So, this is Y and this is X and this is Z. So, I hope now you are clear. So, that is why there is a swap. So, now you can see this is what happening. So, Y d3 and this is Z and this is 0. So, with that you are clear. So, one last attempt we make and then close this particular lecture.

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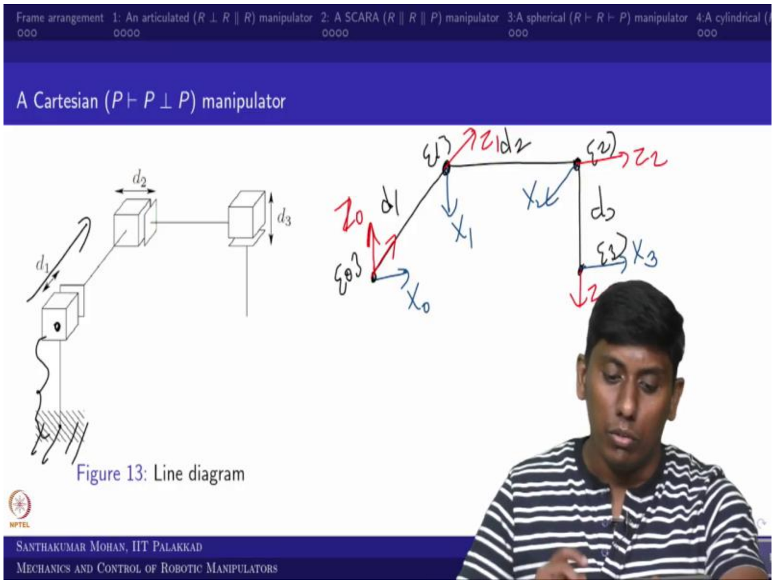

So, what is that so, it is a Cartesian. For fixing Cartesian you have to play a small clever rule. So, what small clever rule, so although you start from the base here, but the first joint is going like this. So, for our benefit, we can start the base here and this distance we can cover it later on. Otherwise, we have to start a passive joint here. So, I am starting here like this, so in the sense, this is 0, and this is 1, and this is 2, and this is 3. So, this is d1 covered, this d2 cover, and this d3.

So, now what would be the joint axis, this is I put it probably I put it this way. So, this is Z0, and this is Z1, and this is Z2, and this is Z3. So, then X axis obviously, the plane containing a perpendicular to that. So, this is X0, and here the plane containing either you can go up or down. So, this is X1 and in this case, either you can come outward or inward. So, this is X2, and I can make it parallel to this X3 .

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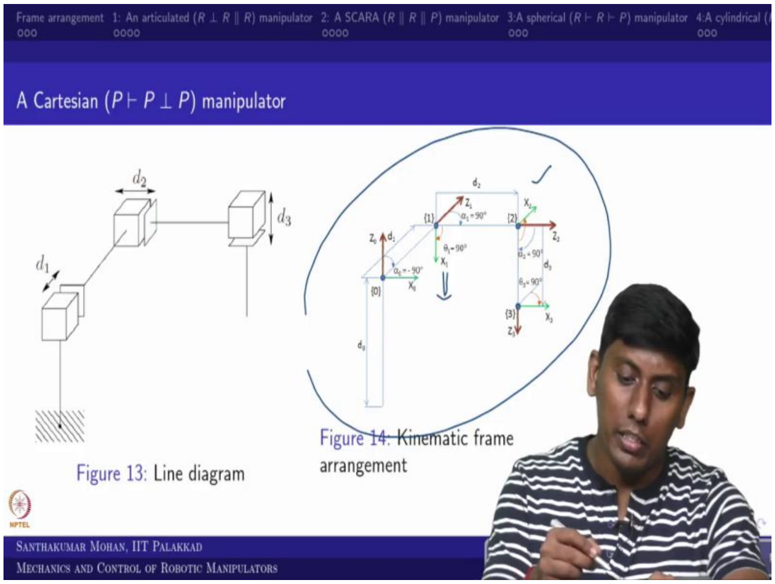

So, now, we can see that in a pictorial. So, you can see like, this is what we have taken. So, X1 we put it downward and X2 is inward, that is the configuration we did. So, now, I hope I left this as exercise.

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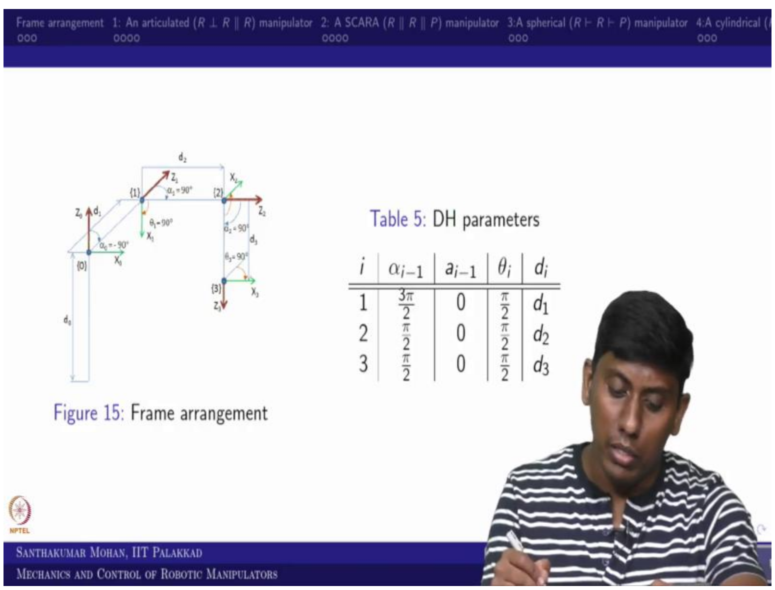

You find the DH parameter and cross check, whether you are getting the same DH parameter, what I am getting it.

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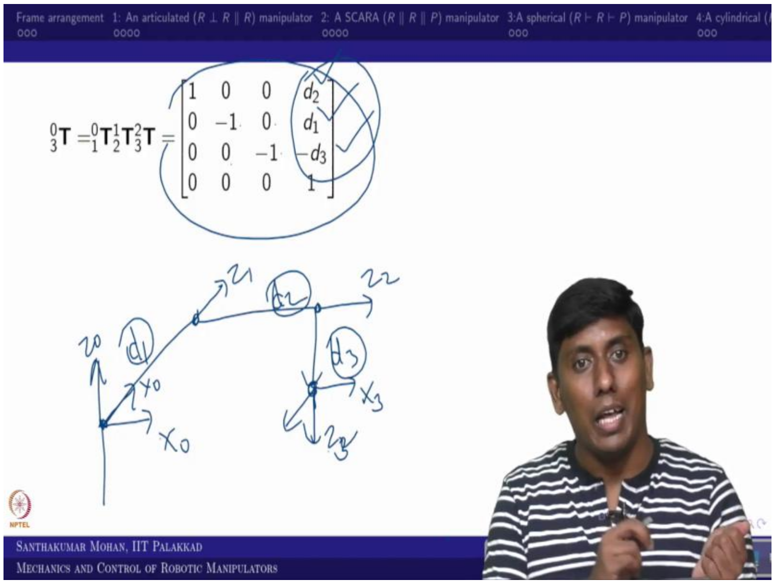

So, you substitute that, you substitute that finally you will get this. So, what one supposed to feel it, so this matrix needs to be cross check. So, what we did, so this is Z0, and this is Z1 and this is Z2 and this is, this point is actually Z3. So, d1, d2 and d3 and this is X naught. So, now based on this configuration, so what one can see, so obviously, the X displacement along X0 is d2 that is coming. So, along Y the d1 that is coming, along Z it is opposite direction d3 minus d3 that is coming. So, the position vector side it is true.

So, coming to the orientation side, this is X3; X3 parallel to X naught, so, 1 0 0 and Y3 is opposite to Y naught, so 0 minus 1 0 and Z3 is opposite to Z naught 0 0 minus 1. So, in sense what we did, we have found everything is in like order, and you have obtained the right, you can say transformation matrix and right DH parameter. Now, you are ready to fly for, forward kinematics and then inverse kinematics everything, even you can go further on kinematic control through differential kinematics. So, what we have done, we have done very simple.

So, we have started with DH representation, the DH representation is full of DH parameter, we have taken few examples, even this example we have taken few cases and then we have taken these example. I hope now you are clear about how to fix the frame and how to arrange it, how to calculate it. So, with that I am closing this particular lecture, and see you then with the forward kinematics in the upcoming lecture. Thank you. Bye.