

**Mechanics and Control of Robotic Manipulators**  
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**Lecture No. 12**  
**Examples Related to Frame Arrangement**

Hi, welcome back to Mechanics and Control of Robotic Manipulator. I hope the last lecture was interesting. Before going to continue the remaining example, I just want to give you a, like again emphasize on the frame arrangement. And I will be taking a 4 such complex cases and sub cases, and I will show like what is the difficulty in fixing frame some cases with respect to Denavit-Hartenberg parameters. So, that is what we are trying to see.

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Frame arrangement 00 Examples 0 0 0 0

DISCUSSION ON FRAME ARRANGEMENT

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So, in that sense what we are trying to go, we will try to go back, what we have seen in the last lecture as the initial part, the frame arrangement. I am not going to, you can say see in detail, but I will come back with 4 such cases where these cases are actually giving two axes, but in a different orientation how we can approach one to another, that is what the whole idea.

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### Frame arrangement

- Joint axes are assigned to Z-axis of corresponding frames
- Fixing the frame location:
  - If  $Z_i$  and  $Z_{i+1}$  are intersecting at some point, then that intersection is the frame location (i.e.,  $i$ ).
  - If  $Z_i$  and  $Z_{i+1}$  are parallel axes, then fix the frame at the convenient point on the  $Z_i$  axis.



### Frame arrangement

- Fix the  $X_i$  axis on the common normal between  $Z_i$  and  $Z_{i+1}$  or axis perpendicular to the plane containing these  $Z_i$  and  $Z_{i+1}$  axes.
- Link length cannot be negative, so direction of  $X_i$  should be properly assigned.



So, in that sense you can recall all those these things. So, we know like Z axis always associated with joint and the fixing the frame also like, if it is intersecting the intersection point would be the  $i$ th frame and if it is a parallel you can take convenient. Similarly, you can see like the X axis always taking a common normal, if the common normal is not there then the axis perpendicular to the plane containing the Z axis. And you can please remember that the link length cannot be negative.

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Frame arrangement  
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Examples

Case 1

$d_0 = 0$   
 $a_0 = L$   
 $\theta_1 = \checkmark$   
 $d_1 = 0$

NPTEL

Frame arrangement  
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Examples

Case 1

$d_1 = 0$

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Frame arrangement

Case 1

Examples

$\bar{a}_0$	$\alpha_0$	$a_0$	$\theta_1$	$d_1$
0	0	$L$	$\theta_1$	0

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So, now, we will come back certain example. So, I got a case, where the  $Z_1$  axis given like this and the  $Z_2$  axis also given in this. So, in this case these 2 are parallel. So, such cases we have seen in the last example, what we have done, so we have taken a convenient place and draw the you can say common normal. So, then what we have done, we can fix your  $X_1$  axis and this is the point 1.

So, in that case what you can see, the  $\alpha_0$  would be or in the sense 1 minus 1. So,  $\alpha_0$  would be 0 and  $a_0$  would be this length  $L$ . And you can this is active or not that based on that you can decide. So, now  $\theta_1$  would be a variable and the  $d_1$  in this case 0. So, this is very, very easy.

But now imagine, now imagine there is a case which is given. So, this is the case which is given, so where these two are non, you can see parallel line. So, it is slightly away in the sense, there is a motor here and the motor here like that. So, in the sense it is connected this kind of number, so how we can do it. So, it is again the same you can push this and what happened last case we said the  $d_1$  becomes 0 in this case, the  $d_1$  would be this distance.

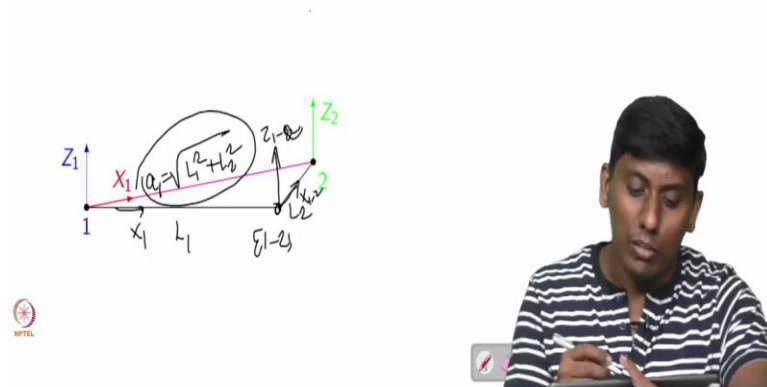
So, that is what we are trying to see here. So, you can make a common normal and now, you can see that this is  $X_1$  direction, and this is what 1, and this is what 2. And now you can write it, so this is  $a_0$  and this is  $d_1$ . So, this is you can write as a  $d_1$  and we assume that this is rotation, or this is rotation, you can make it. So, now, if I write  $d_1$  or this, so it is slightly different, why?

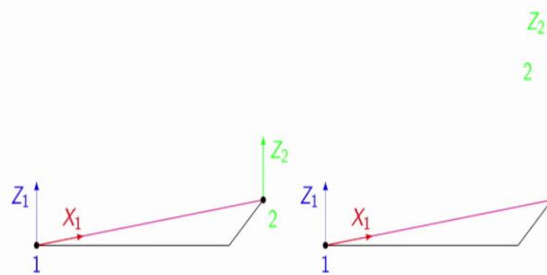
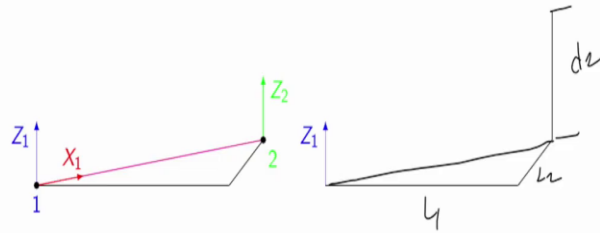
Because we are always seeing this with respect to previous. So, in the sense of what we supposed to see.

So, if I am writing this frame, so if I am writing that two, so in the sense this is  $i$ ,  $\alpha_i - 1$ ,  $a_i - 1$ ,  $\theta_i$  and  $d_i$ . So, when we are writing here whatever number, this number with respect to previous we see and this number with respect to progressing we see. This joint parameter is to see progressing, this link parameter will see with respect to previous. So, in that sense, so this  $d_1$  and this  $a_0$  is not right. So, that is what the whole idea, that is why I just plotted here.

So, in that sense what we can see it, so we will come back here. So, what would be the case for 2, so the distance along  $X_1$  and the angle about  $X_1$  in this case it is 0, the distance along  $X_1$  is  $a_1$  and the angle with respect to  $Z_2$  if any that is  $\theta_2$  and the distance along  $Z_2$  from 1 to 2 along  $Z_2$  that is what you call  $d_2$ . So, now, I hope you are clear on this. So, based on that, we will see the next case.

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Which is little complex, what complex. I have taken the same plane, in the sense earlier also it is parallel, but now the parallel which is having off set it in both X and Y. So, in the sense you can see like this is in offset in 2 axis. How can we attempt? So, now it is you can draw a common normal, then there is a multiple common normal. So, what would be the best common normal, you will draw connecting this.

So, now what you can fix your X axis, you can fix your X axis this way. So, that is what we are actually going to do, you can say we draw the common normal and you fix at the X1 like this. Now, you can actually get it, and this is 1 and this is 2. But so in that case what you can see, this is L1 and this is L2, so then this would be square root of L1 square plus L2 square that you are to

be remember, this would be  $a_1$ . So, that you have to be very clear. So, in such cases what one can do, so we can do a passive plane, or you can say passive axes. So, I call this 1 to 2, this is I call Z1 to 2. So, this I fix it X 1 to 2 and this I put X1.

So, then what happened, so instead of having a straightforward 1 joint Information, I will be taking a passive joint of Z 1 to 2 that I will take because of that what happened, this L1 and L2 would be addressed, not like the square root. So, this is one such say simple example, you can think about it. Now, this move up what one can think, we can again address what we did earlier. So, this is what we did.

So, we address this  $d_2$  and this is L2, and this is the L1, is there you can draw common normal or you can go via passive frame. So, this is what the whole idea. So now, we can see X1 we fix it, if we want to go 1. So, now, similar way we can see the case 3.

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Frame arrangement  
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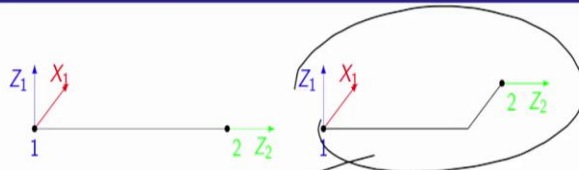
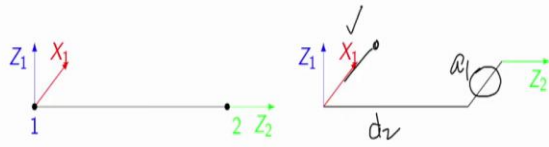
Examples  
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○  
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Case 3

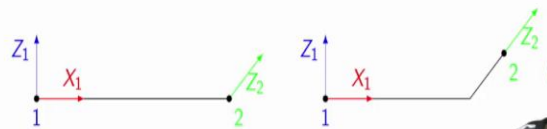
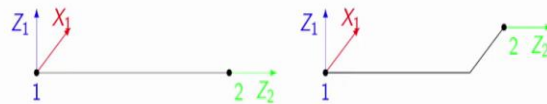
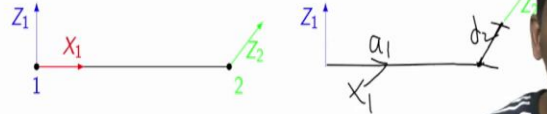
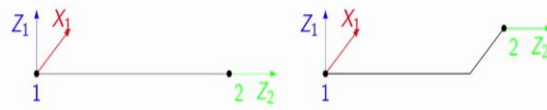
$d_1 = \sqrt{2}$   
 $a_1 = 0$   
 $\theta_2 =$

$Z_1$   $X_1$   $Z_2$   $X_2$   $d_2$

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What case 3, so these 2 are not parallel, but intersecting. So, then what is the easiest way, you can extend and where the intersecting point that would be the frame 1. So, that is what the rule of thumb. And the plane containing this  $Z_1$  and  $Z_2$  they are normal to that would be your  $X_1$ . So, the direction can be this or this. So, we are taking conveniently which is we are putting inside. So, that is what the whole idea. So, now, in this case it is not big, but this distance what it covers, it covers as  $d_2$  and you have  $\alpha$ .

What would be  $\alpha$  in this case, so  $\alpha_1$  would be 90 degree and  $a_1$  would be 0 and  $\theta_2$ , would be based on the nature. So, now, you are clear this distance is this distance is  $d_2$ . So, that is what we are writing it here, the same situation, but it is slightly offset in both X, Y like we

have seen in the sub cases. So, how we can do, so in this case it is nontrivial, it is not like a trivial case. So, what then one can think, so you can make this. So, we can do one simple aspect. So, you can make one passive.

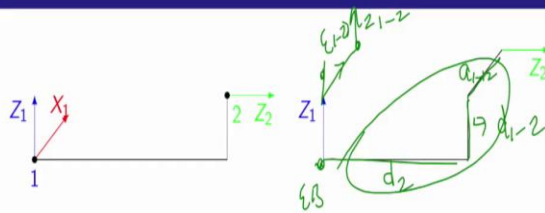
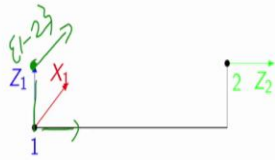
So, what we can do, we can make it  $Z_1$   $Z_2$  as a passive joint and then I can make it, so this is  $X_1$  and this is  $X_1$  to  $Z_2$ , this is easy for me. So, this is the way we can make you can say passive frame arrangement, otherwise this will not be directly address. But in this case, you can see this is along  $X_1$ . So, you can call it as this is  $a_1$  and this is  $d_2$ . But still, if you want to make a passive frame that is more beneficial to understand, but this case even you can avoid. So, that is what we have seen.

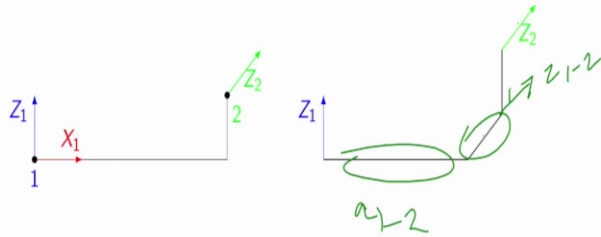
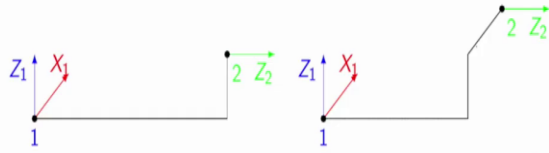
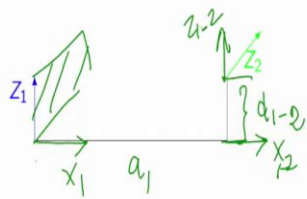
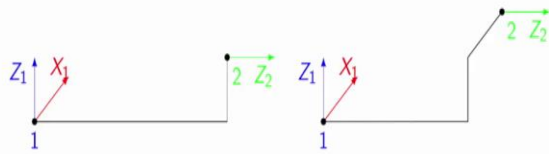
So, now further cases in the same, so we are putting it, so like this, so what one can see here, the  $X$  would be coming either outward or inward, but this distance who would cover, nobody will cover. So, but while looking at here, the  $X$  is coming towards this. Based on our you can see case this is  $Z_2$ , and this is the plane containing and this is  $X_1$  direction. So, this is actually covered in you can see in  $X_1$ , so that is why you should not see directly from to here. So that is why you have to be very careful what is your joint axes direction and what way it is arranged.

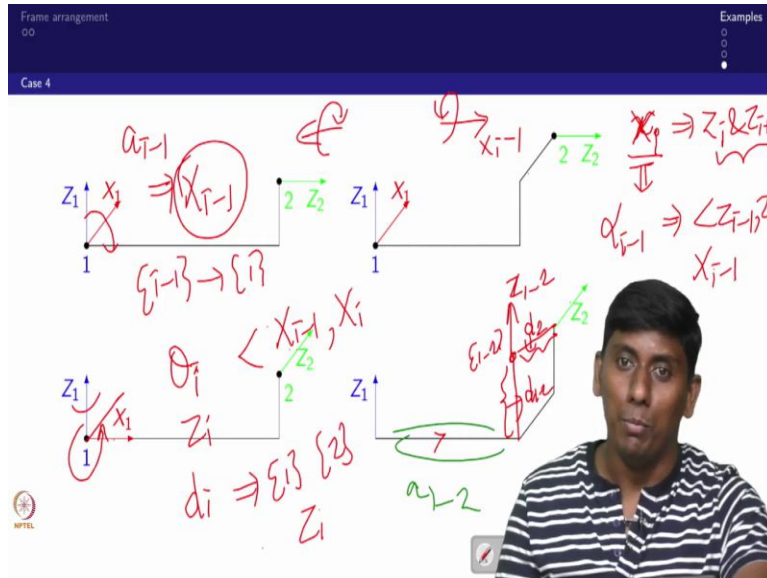
So now, we will go the fourth in the same sub cases. So, what we can see here there is a distance, which is again you can see you can fix  $X_1$  here and this would come under  $d_2$  this come under  $a_1$ . So, like that you can cover it. So, that is what we have seen.

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The diagram illustrates a frame arrangement for a cylindrical link. It shows two coordinate frames: a fixed frame and a frame attached to the link. The fixed frame has axes  $Z_1$  (vertical),  $X_1$  (horizontal, pointing right), and  $Z_2$  (diagonal, pointing down-left). The frame attached to the link has axes  $Z_{1-2}$  (vertical),  $X_{1-2}$  (horizontal, pointing right), and  $Z_2$  (horizontal, pointing right). Distances  $d_1$  and  $d_2$  are indicated. The slide also includes the text 'Frame arrangement', 'Case 4', and 'Examples'.







The fourth the case is very complex. So, that is what is going to give that real passive frame arrangement, what is that you have seen right. So, this is, on this and this, but when you draw, when you draw the plane containing this, so this is the plane containing. So, what would be the direction of X, so which is piercing in or coming out. So, if that is the case, this distance is not covering in X and this distance is covering with Z2, this is equivalent to  $d_2$ , but this distance that it will cover.

So, now people can say, so you can draw a common normal, no. The common normal would be the direction this. So, then what one can do, so one can do is easily, so you can make a passive, in the sense you make it a passive frame here. So, this is 1 to 2 and you fix your Z1 here, 1 to 2, and then actually address this. So, in that sense, you can see still X1 here, so X1 to 2 here, but this would be covered as, so  $d_1$  to 2. So, like that.

So, now, you are I hope understanding what is the benefit. So, now, this is straight away, you cannot do it, so you have to come up with the passive frame. The passive frame can be done several modes, so one of the easiest modes, you extend the same distance and then you keep it, this is 1 to 2, that is the easiest option. And you keep your X1 as whatever way you wanted to do it, either this or this, then you can rotate all those things, you can do it.

So now, the same way we can see even in the case for some other complexity. What complexity, 3 distances have come. So, one in X, one in Z, one in Y. So, how we will address? So, in this case, so is there any way to do it, yes. So, what way we can do it. So, this is anyhow covering

with  $Z_2$ , so what I can do, I can actually, like, command or something like this and this much distance I covered. So, in the sense what I am trying to do, I am trying to keep this as  $Z_1$  to 2. So, in that sense, so, what would be the direction of  $X$ , it is a parallel, so I can keep.

And this distance, so this is 1 and this 1 to, this distance will come, this distance will come as  $d_1$  2, this distance will come as  $d_2$ , and this will come as a 1 to 2. So, like that you can cover it. So, this is what we can see. So, right now I am straightaway writing because I have experience. So, but you have to actual like come up with this. So that is why I am giving these cases, sub cases, so that the next time when I show some examples, you will immediately realize why I have taken the frame in that way.

So, now in that case, so this is the way we started. So, now, we can go the furthermore complex example. So, this is parallel, right now we have taken the other way around. So, then what you can see, so in this case, so 1 distance you can directly cover in  $X$  axis, because this is the plane containing  $Z_1$  and  $Z_2$ . So, obviously the plane perpendicular to that that would be  $X_1$ . So, this distance you can cover as  $a_1$ , but this distance you have to cover with a passive frame. What would be the choice, so I can take this  $Z_1$  to 2. So, in that sense, what I can take, still I can take  $X$  1 to 2 this way.

So, this distance will come as  $d_1$  to 2. So, like that you can make a passive frame and then you can extend. And this passive frame will not be having any active joint this distance automatically will adjust when you multiply, you can say 1 to 2 via 1 to 2. So, now, the same way the last complex example is coming into a picture. So, this is not really a complex, because this is we have done and the same way we can do it here. What, so this distance would cover from here and this distance would come as  $a_1$ .

So, in this case  $a_1$  to 2 and this distance you have to cover. So, this you can take it as  $Z_1$  to 2. So, then who would cover this. So, in that sense, so we have to see not this way we can do it. So, then what one can do, so you can use it as we did here. So, this is 1 to 2 and keep it this as  $Z_1$  to 2. So, then this is cover as  $a_1$  to 2 and this is  $d_1$  to 2 and this distance as  $d_2$ . So, like that you can make it. So, this distance as  $d_1$  2 and this distance is  $d_2$ . So, these are the; you can say complex cases. So, I hope now, you are actually able to understand what exactly supposed to be known. So, again I am giving a small tip again and again.

So, if you talk about X axis, so if you are talking about  $X_i$ , so you are to see the nature of  $Z_i$  and  $Z_{i+1}$ . If these two are perpendicular or intersecting or it is plane containing, so then this would be perpendicular to that, if these two are parallel, then you have an easiest choice you draw a common normal, conveniently you fix some point as a frame, then you can extend. So, now when you write  $\alpha_{i-1}$  or  $\alpha_i$ , so you have to see that the angle between  $Z_{i-1}$  to  $Z_i$  about what  $X_{i-1}$ . So, then the direction of rotation also matters.

So, for example, this is what  $X_{i-1}$  direction, so then the rotation is this way, if this is the case, so the rotation is like this. So, that positive rotation you have to understand. For example, in this case, so what would be the  $\alpha$ ? So, here you can see like this is the direction, so it is rotating, this case it is 90 degrees. But if you look at here, so this is the way. So, what you can see like it is rotating, so 270 degrees to make it. So, this is what the further end.

And when you talk about a minus 1, this would be distance along  $X_{i-1}$ , from where, so  $i-1$  frame to  $i$  frame along  $X_{i-1}$ . So, what is  $\theta_i$ , so  $\theta_i$  is angle between, you can see  $X_{i-1}$  to  $X_i$  with respect to  $Z_i$ , that you are to be see. Again, the positive rotation with respect to the  $Z$  direction. So, what would be the  $d_i$ , so  $d_i$  is distance between so 1 and 2 with respect to what the  $Z_i$ , again the positive or negative based on the direction of  $Z_i$ .

I hope you are actually clear on this. So, with that I am ending this particular short lecture. So, we will see in the next lecture, the detail example which we left out in the last lecture, so that we would take up and continue it here. Until then, see you, bye. Take care.