Mechanics and Control of Robotic Manipulators
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Lecture No. 11
Frame Arrangement and Examples Part 1

Hi, welcome back to Mechanics and Control of Robotic Manipulator. So, last class what we have seen basically like a Denavit-Hartenberg representation, especially we arrived the term called arm matrix, even the arm matrix can be obtained 2 ways, 1 is standard, the other 1 is non-standard. So, we are going to use a non-standard, the non-standard is nothing but the mix of i minus 1 and i. So, where the link parameter would be in the i minus 1 axis whereas the joint parameter would be in i axis.

So now, in this particular lecture we are going to see, as I already mentioned in the last lecture, we are going to see something like a real manipulator, but although I said real manipulator, we would not be seeing the real manipulator in the picture. So, we would be taking real manipulator configuration and trying to derive the; you call Denavit-Hartenberg parameters for given configurations. So, this is what our intention here.

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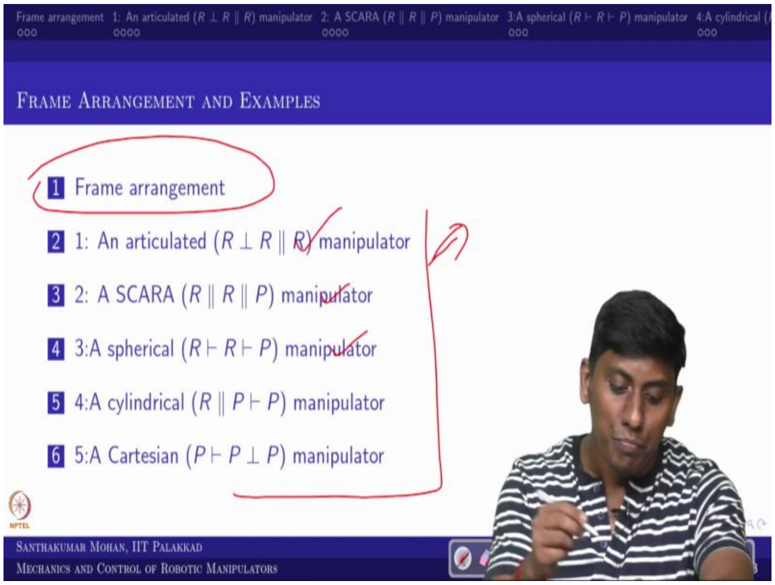

So, if that is the case what we are going to cover we are going to see, so how to fix the frame, because there are number of frames we are going to fix. So, there are supposed to be some procedure or norm, which we are going to follow. In addition to that, we are going to see there are 5 such examples, which are coming from the; what you call in real time. So, these are existing manipulators which we are seeing the first 3 axes for our own benefit. So, even then actual there is an additional benefit also we would be having that probably you would be coming to know when we attempt to probably two lectures later. So, that is the whole idea.

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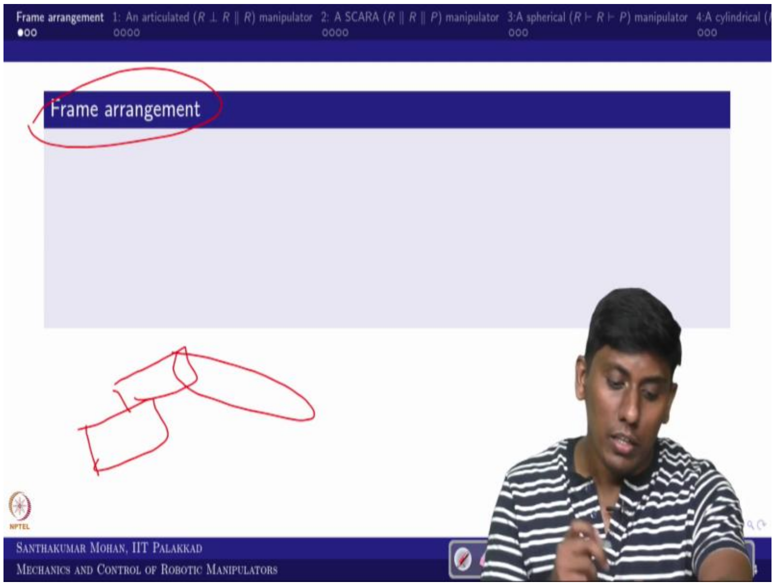

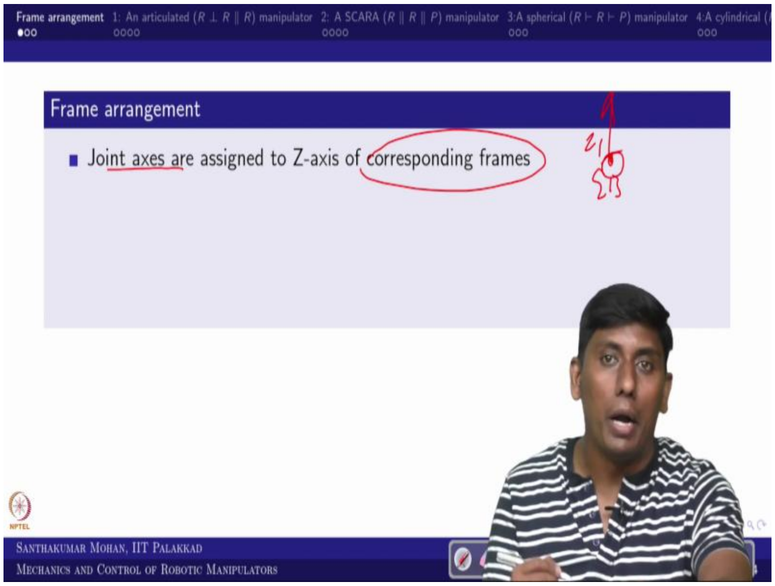

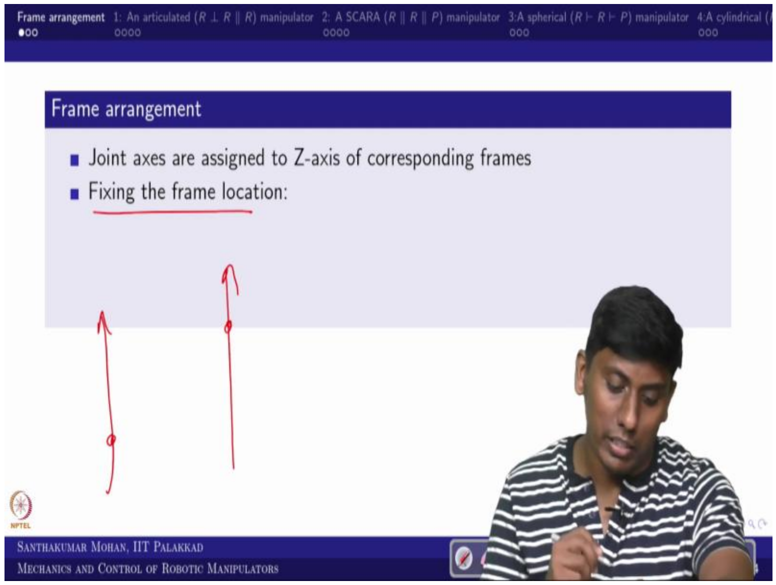
Frame arrangement 1: An articulated ($R \perp R \parallel R$) manipulator 2: A SCARA ($R \parallel R \parallel P$) manipulator 3:A spherical ($R \vdash R \vdash P$) manipulator 4:A cylindrical
Frame arrangement
Joint axes are assigned to Z-axis of corresponding frames
Fixing the frame location:
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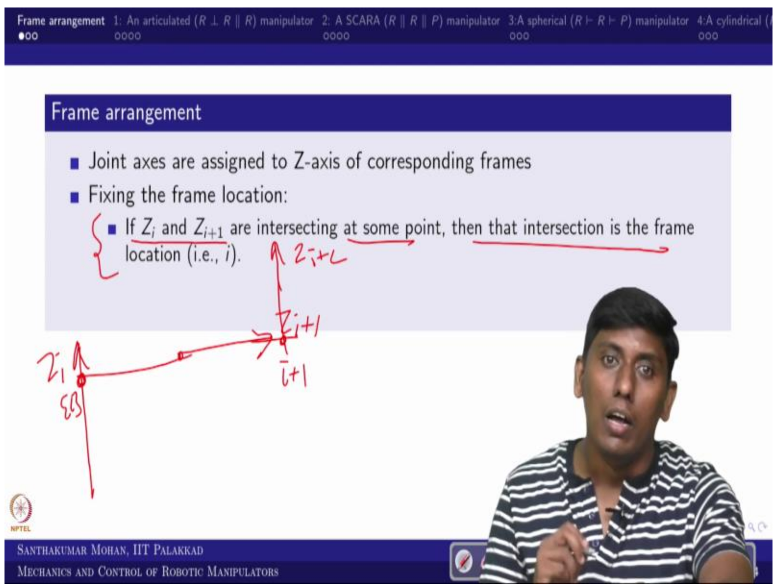
Frame arrangement 1: An articulated ($R \perp R \parallel R$) manipulator 2: A SCARA ($R \parallel R \parallel P$) manipulator 3:A spherical ($R \vdash R \vdash P$) manipulator 4:A cylindrical
Frame arrangement
Joint axes are assigned to Z-axis of corresponding frames
Fixing the frame location:
If Z_i and Z_{i+1} are intersecting at some point, then that intersection is the frame location (i.e., i).
Z_i
Z_{i+1}
$i+1$
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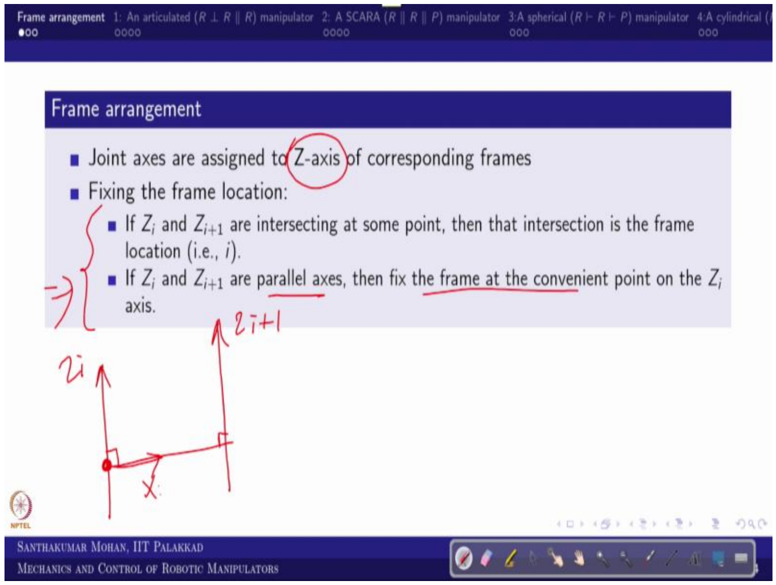

Let us go to the what you call frame arrangement, why we are thinking about frame arrangement, for example, you have several body, so how you can fix your joint axes, and what way we can fix the joint axes, if we fix the joint axes, where you can fix your frame, and where you can fix x axis, all those things you need to follow.

So, for that, we are going to follow certain small set of procedures. The first procedure what we are going to follow is, so you all know like joint axes always assigned to the Z axis or the Z direction so, which is corresponding to the concerned frame. For example, if I say, so the frame 1, so the joint axes would be Z1. So, like that we are going to write, so if you do this, so how to fix this 1. So, in the sense the frame location, in the sense the fixing the frame location is always a concern. For example, you have 2 parallel axes, that to fix you 1 and where to fix 2.

So, there are several options. So, for that we need to provide some kind of additional set of rules. So, that is what we are going to call if Zi and Zi plus 1 are intersecting at some point, so then that intersection is the frame location of i. So, for example, this is what Zi and there is Zi plus 1 which is going. So, you have to see where these 2 intersect this is what the 1. So, now additionally 1 more going what you call the Zi plus 2. So, where these 2 intersect this is the point i plus 1 like that we can keep going.

So, this is what this rule says. So, if these 2 are not intersecting, neither intersecting nonparallel or if it is simply parallel how we can fix it, so far that we need to go some other cases. So, if it is neither intersecting nonparallel, then we have to do some kind of virtual frames. However, if it is parallel, then the; you can say frame can be fixed at a convenient location, at the convenient point on the Zi axis. Now, you can see these two are your Zi and Zi plus 1. So, now, you can fix it anywhere on a convenient basis.

So, now, in that case, you can see that this is the common normal which you are going to draw. So, then you are Xi can be fixed accordingly. So, this is what the whole idea. So, now, what we have seen we have seen what is Zi or what is Z axes arrangement, then based on the Z axes, so we can find the fixing of the frame. So, what else we need to know, so we all know like this would be having to axes system.

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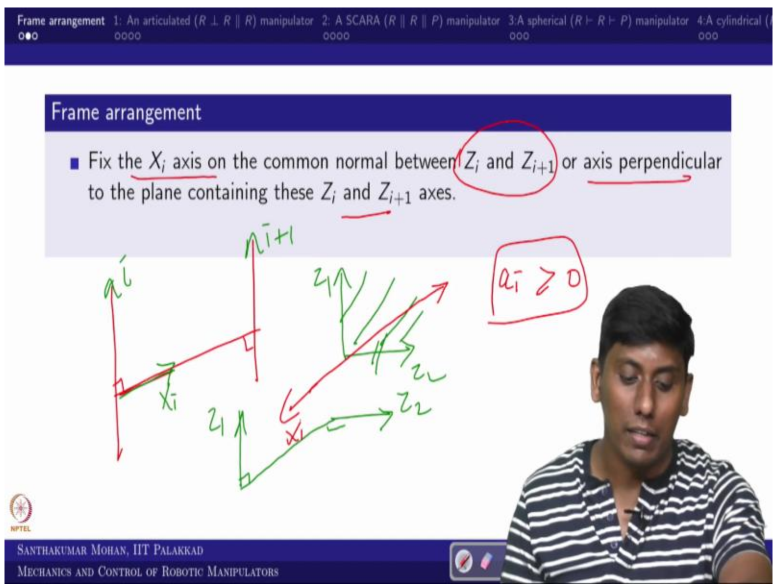

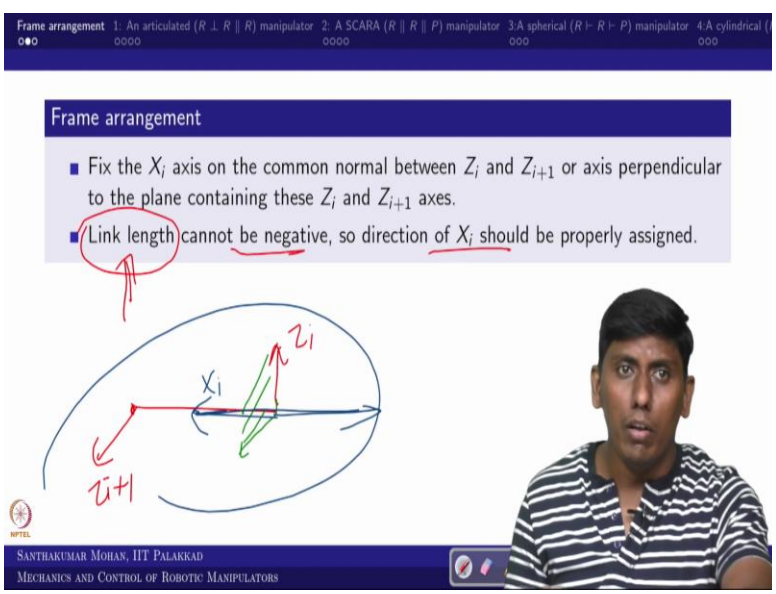

So obviously, then we have to see how to fix X. So Xi fixing is, always easy, because it is based on the nature of Zi and Zi plus 1. If it is parallel or you can say neither parallel, non intersecting, you can always draw the common normal between these two. And then you can fix along that as your axis. So, then that to like in the progressing side then that would be your Xi. This I say ith axis and this is i plus 1 axis you can take it.

If, if these two are intersecting and it is perpendicular, for example like this, so this is Z1, and this is Z2. So, then how you can draw common normal, so then we can see that the common normal may be like this but instead of doing that what we can do it. So, we can draw the plane, this is the Z1 and this is parallel to Z2. So, this plane is going to consider as your axes plane and you can see normal to this plane.

So, your Xi axes, so whether it is pointing inward or outward that is choice of yours. So, provided if there is a length then that length always in the progressing site, so that the link length always positive or 0, that is the way we can consider. So, now you get to know, so when we see the example, you would be getting much more clear, that is what we say. So, the Xi axis, axis perpendicular to the plane containing Zi and Zi plus 1 axis.

So, what else we need to know, so we supposed to know the link length is a physical parameter that cannot be negative. So, in the sense this is a geometrical parameter that cannot be negative, how the mass cannot be negative the same way the link length cannot be negative. So, in that sense Xi always properly assigned in such a way that this link length is considered as a positive one. For example, now, so, this is Zi and you have Zi plus 1 here, so what is the choice of Xi.

So obviously, the plane containing the perpendicular to the plane containing, so in that sense of what would be the plane, so this is the plane containing. So, what would be the direction of Xi, so either this side you can go or this side we can go. So, now, what one can see, so the link lands supposed to be positive, so you can fix Xi this way, this is what this particular rule all about saying.

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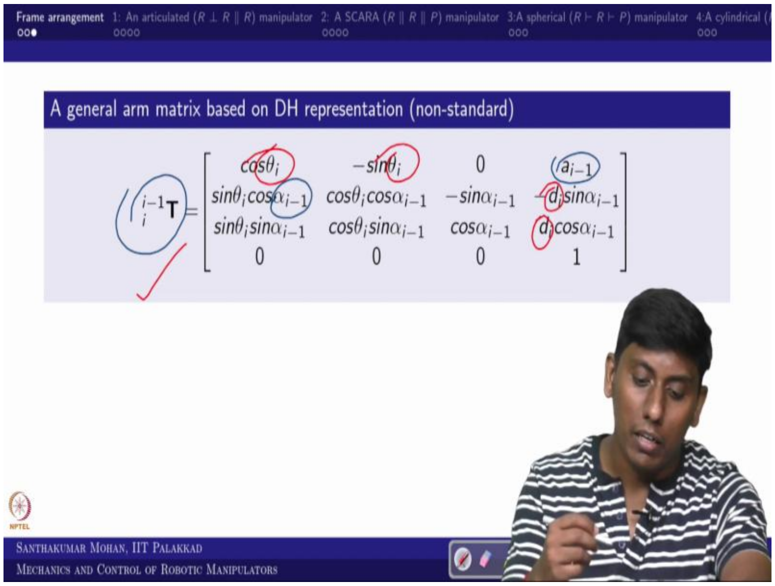

So, now we will move to the real example. So, before that you know, you just to recall what we derived as arm metrics based on what you call non-standard form. So, why we call non-standard, you can see these are an a i minus 1 frame and the other what you can see these are in what you call in i representation. So, that is why it is we call non-standard there is no other specific reason to call non-standard, but this gives more intuitiveness, so that is why we are following this.

$$ {}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} cos\theta_i & -sin\theta_i & 0 & a_{i-1} \\ sin\theta_i cos(\alpha_{i-1}) & cos\theta_i cos(\alpha_{i-1}) & -sin(\alpha_{i-1}) & -d_i \sin(\alpha_{i-1}) \\ sin\theta_i sin(\alpha_{i-1}) & cos\theta_i sin(\alpha_{i-1}) & cos(\alpha_{i-1}) & d_i cos(\alpha_{i-1}) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

So, now, we just recall this matrix because, we are going to substitute this Denavit-Hartenberg parameter of individual frame and then we are going to get something else out of that. So, for that what we are trying to do, we are trying to find out, like some example we will take and we will substitute and get it how it is going on.

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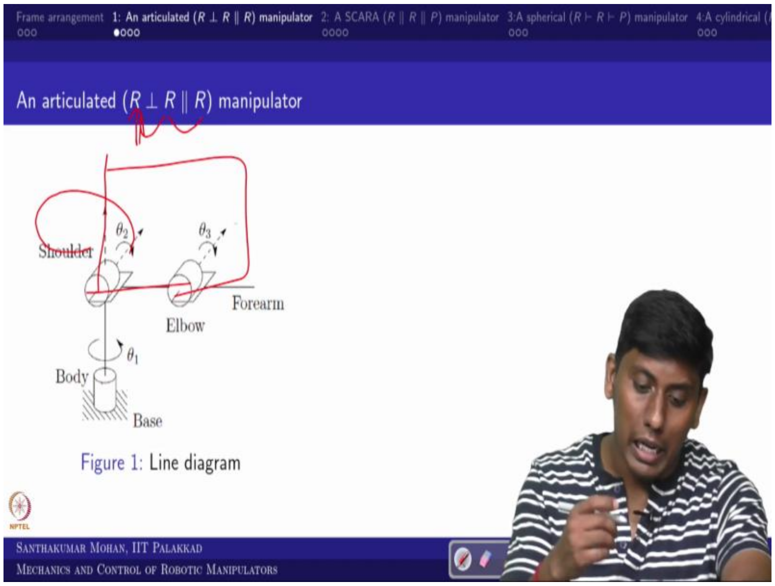

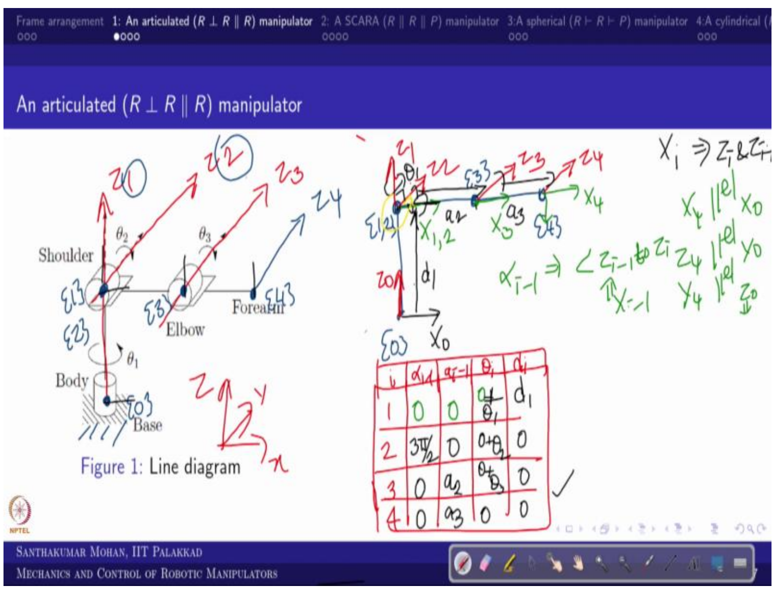

So, for that one classical example, what we call an articulated manipulator, what it means articulation or articulated, it is going to articulate and especially, it is having all rotary joints. So, how the rotary joints assigned, so first rotary joint is vertical, that is second rotary joint is perpendicular to that, the third rotary joint is parallel to that. So, in that sense what we can get, so these two axes make a planar motion, that planar motion, you can swept with respect to this axis, you will get a 3-dimensional case.

So, that is what the whole idea, I hope now, you are getting somewhat clear idea. So now, we will see how to fix the frame. So, for that I am using it here. So, what one can see, so this would be like your Z1 axis because the first rule says that the joint axes supposed to be Z axis, so here this is the first joint, so that way I fixed as first two directions also they have given, so that is what Z1 axis and this is Z2 axis and this is actually like Z3 axis.

So, what is the next rule of thumb, so you are fix the; what you call frame. So, what you can see, it starts from 1 to 3, so you have to obviously start from 0 and end with 3 plus 1. So, then you can see like you have to fix the frame point, the first fixing frame, frame point you can choose, but now we assume that this is the base, so I can start from here as zeroth frame. So, now the second frame location, you know, it is like easy, so what easy, you have to see that if take Z2 and Z1, whether it is intersecting or not, in this case it is intersecting. So, the intersection point is your frame 1.

So now, Z2 and Z3 are parallel, so you can take a convenient point, so I am taking the same point as 2, which is giving much more simplicity for me. So, further what you can see, so you have, you call the forearm, which is the final tool. So, that tool Z axis you can choose, so now I assuming that this is parallel to this, then you can see conveniently, you can take anywhere on the Z3. I am taking where these two intersect, this is the point I call 3, this intersection is just the link intersection. So, now this is the point 4, which is that is the end.

So, now the same thing I am drawing it here, just for your benefit. So, I am just drawing it here. So, what we did this is 0 and this is 1 and 2 and this is 3 and this is 4. So, what else we know, I am drawing this. So, this is Z0, and this is the Z1. And you know, this is Z2, and this is Z3 as per the configuration and the final one is my choice and the beginning also my choice. The beginning choice I took it as this because we always take this is what the X and this is what Z because the Y would be going inside. So, that is what the right-hand rule all says.

So, because of that consideration I have taken. So, now what else we need to see, so we need to fix the X axis, for fixing x axis what we can see, the Xi you have to see based on Zi and Zi plus 1 coordination. So, if I take that way, so what is the X0 configuration that needs to be seen Z1 and Z 0. So, what is the Z1 and Z0 nature, these two are parallel that to in the same line. So, in the sense you can take X0 anywhere either orthogonal to the Z0.

In the sense you can see like, in this plane anywhere you can take, so I am taking it a very simple idea. So, what that, so I am actually going to take what we used to do as a conventional, so X0.

Now coming to X1 what we need to see, so X1 would be the plane containing Z1 and Z2 perpendicular to that, that is what the X1 direction or the axis. So, and the direction what you can feel it, so that is the progressing direction, so that I can take this. And Z2 and Z3 are parallel, so I can actually take X2 in that way. So, similar way I can take this X3 and X4.

So, what it is giving, it is giving X4 is parallel to X0 and you can see Z4 is parallel to Y0 and Y4, so what would be the Y4 direction, so you can take a right hand rule again. So, this is thumb and this is forefinger will go in, then only you can get it. So, that is what the case. So, then you can see Y4 parallel to, you can say Z0, but opposite direction. So, that is what you can see it. So, now we will come back. So, we can see what exactly we have done, that I am trying to show with a DH parameter.

So, what are the link length and what are the geometrical parameter you can expect here. So, you can see that there is one length here. So that length somewhere, and there is another length you can feel, and there is another length. You can physically see this to this there is a physical length, this to this there is a physical length, and this to this also there is a physical length. Now, how we are naming it that length, that is coming you can say in few seconds later.

So now, we will try to draw what you call, you can say like simple DH parameter. So, we will try to see that. So, I am just drawing it that. So, I am trying to give that in a tabular form. So, how many will come so, there are, so four because 1, 2, 3, 4 and this is I call i, this is alpha i minus 1, then ai minus 1, and this is theta i and this is di. So, what we are actually trying to see what is alpha i minus 1, what is alpha i minus 1, this angle between Zi minus 1 to you can say, I will write to Zi with respect to Xi minus 1 this is what you have to see.

So, if you see here, so is there any angle between Z1 and Z0 with respect to X0? No. So, is there any distance along X0? No. But is there any angle with respect to X0 or with respect to Z1 where the angle between X0 and X1 is there, theoretically no. But what happened this is one of the active joint which can have theta 1, that sense 0 plus theta 1. So, what else you know? So, the distance between zeroth frame to first frame or 1 frame along Z1, what is the distance? that is what you call joint distance that is d1. So, now, you got some, you can say idea, so how to find the dh parameter, let us go to the next one.

So, what is next one? So, alpha 1 we need to find out, so in the sense alpha 2 minus 1. So, what that, so it is angle between Z1 and Z2. So, theoretically it is 90 degree but what direction, so you can see like X2 you can, you can see X2 fix it here. So, when you fix it X2 here what is the direction, it comes this way. So, in the sense what you need to do, so you need to rotate this to 270 degree to make it parallel to Z2. So, the Z1 supposed to rotate about X1 rotate to 270 degree to make it parallel to Z2.

Otherwise, what you can write, you can write minus 90 degree. So, I always prefer to go in a conventional way. So, you can write that is, you can say 3 pi by 2 in the sense to 270 degree. And there is no distance along X1 so 0 and this is again active joint. So, in the sense that is 0 plus theta 2 and there is no distance along Z2, so that is 0. Now, coming to third frame, so the frame 2 and 3 how much difference it is. So, there is no angle, because X2 along X2 are about X2 there is nothing an angle. So, that is 0 in the sense Z2 and Z3 are parallel.

So, what is a, so a would be realized along X. So, in this case X2 is when see there is a distance along X2, where it is traveling from 1 to 3. So, this is what we call in this case a 3 minus 1. So, this is this is a2 and this is d1. So, this is I call a3. So now what you have, so this is again an active joint, so in the sense 0 plus theta 3 and there is no distance along Z3. So now, coming back again, so Z3 and Z4 are parallel, so there is no angle.

Along X3 there is a distance that is a3 and what else you know, so there is no active, so it is 0 degree and there is no distance it is 0. So, you got this. So, now, we will actually go the same thing in a pictorial way, what I did it already.

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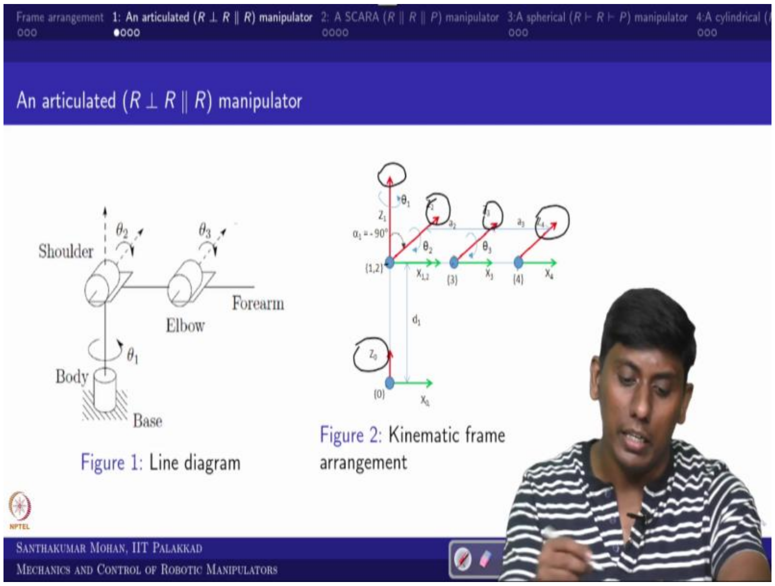

You can see, so this is what 0 and this is 1 and 2, and this 3 and 4, and we have fixed Z1, Z2, Z3 which are given and the Z0 and Z4 are choice. So, I keep it as per my want. And X0 to X4 we have to fix it. So, we have fix it based on the rule which we have used. So now, based on this what we have found, so we have found what you call the DH parameter. So, the same DH parameter what we have 10, we can cross check here.

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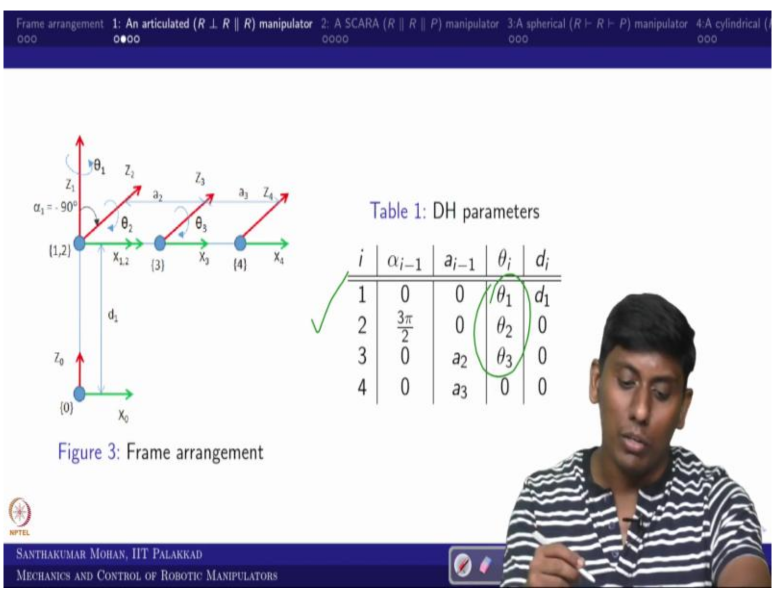

So, you can cross check here. So, this is the you can say frame arrangement, which we obtained as per our understanding. So, based on this what we got, so we got the DH parameter, only thing

I did not put zeros anywhere, because 0 plus theta 1 would give theta 1 only. But you have to keep it in your mind. So, theta 1 is the active joint. So, that is not realizable here, but that is power joint. So, that is why we are putting theta 1 as a variable.

So, in this particular case all are actual like this, all 3 thetas are variable. So, now, we will see this we can put it in arm matrix, what would be the transformation matrix, you can recall the second slide and then you can substitute this.

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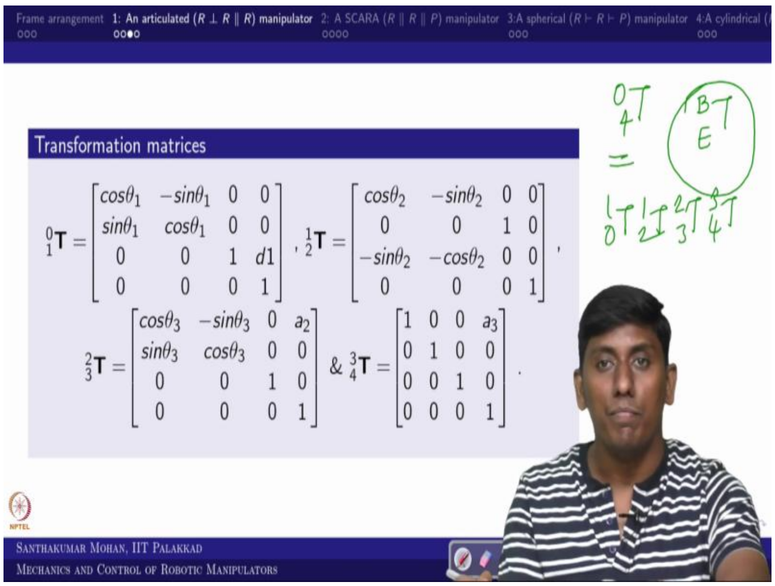

So, you will get T 0 1 as this. So, then what else you will get T 1 2 you will get. And in the sense transformation matrix of 2 with respect to 1 transformation, transformation matrix of 1 with respect to 0, like that you will keep getting it. So, up to what, so 4. But what we are really interested, we are really interested to find the transformation matrix of 4 with respect to 0. In the sense of what I can write, transformation matrix of the end with respect to the base. So, this is what we interested, what we know, we can post to multiply this in the sense T 0 1, T 1 2, T 2 3, T 3 4 you multiply that would be equal to this.

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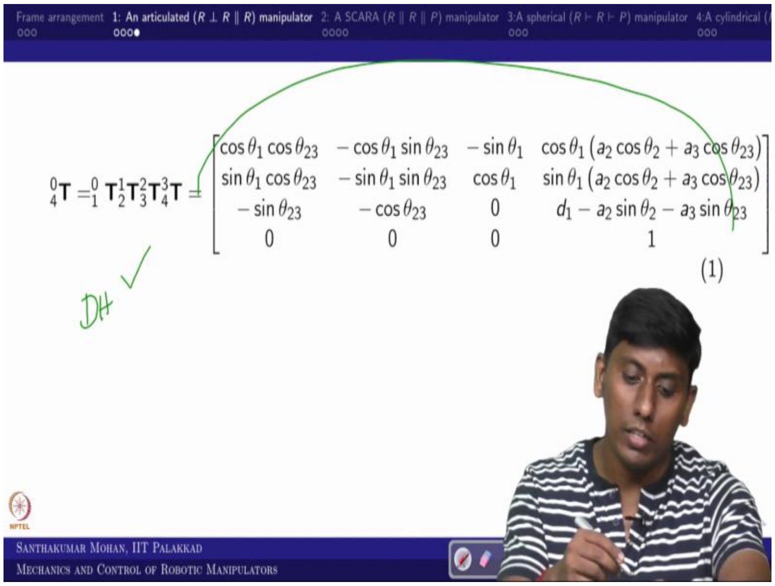

So, that is what we are going to do. So, this is what we are going to do. Now, if you substitute and get it finally, these matters will come. So, now, the task is not ended here. Why, once you obtained this you out cross check, whether whatever you obtained your DH parameter are, you can say correct or not. So, how you can cross check, there is a procedure which we use to follow as a student or as a researcher. So, what is the procedure?

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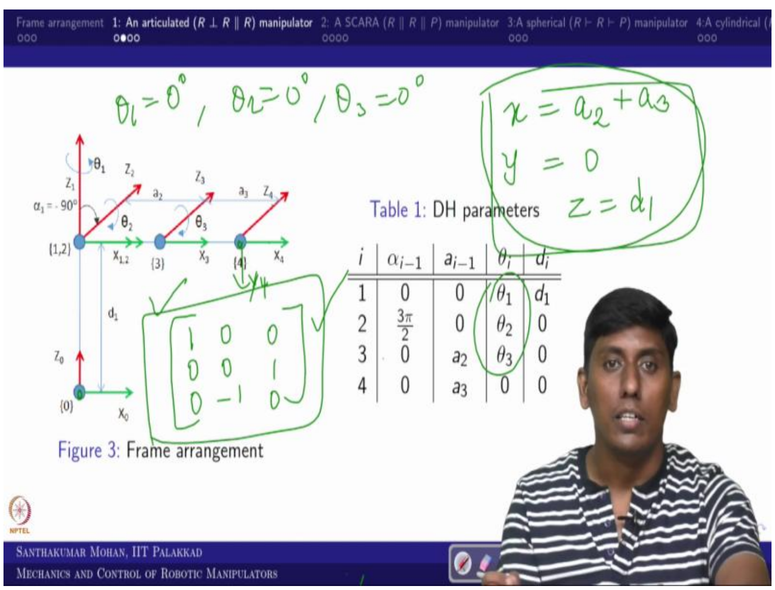

So, you can recall here, the frame arrangement. So, what we have arranged, we have arranged everything in this form. So, if you assume that this is the home position, so what would be the theta 1. So, we have obtained theta 1 is 0 degree and theta 2 is actually 0 degree and theta 3 is 0 degree. If you take this all are zeros. So, what would be the X displacement or you can say the X position of 4 with respect to 0 what would be that, that would be you can see X4 is parallel to this, in the sense a2 plus a3.

What would be the Y displacement or you can say Y position of 4 with respect to 0 frame. So, that would be 0, there is no displacement. What would be the Z equivalent, you can see that this and this point is along Z0 what is the distance that is d1. So, whether we are getting this or not. So, similarly, what one can see, so based on this the rotational information you can see. So, if you project X4 on 0 frame what it would come, X4 and X0 parallel, so it will come 1 0 0. So, Y4 is here, so if you project Y4 what would be that, that would be 0 0 minus 1.

If you project Z4 to 0 frame, so that would be 0 1 0. So, this is the rotational matrix and this is the position vector on the home position, we can cross check whether you are getting this once you are obtained the matrix what you found here.

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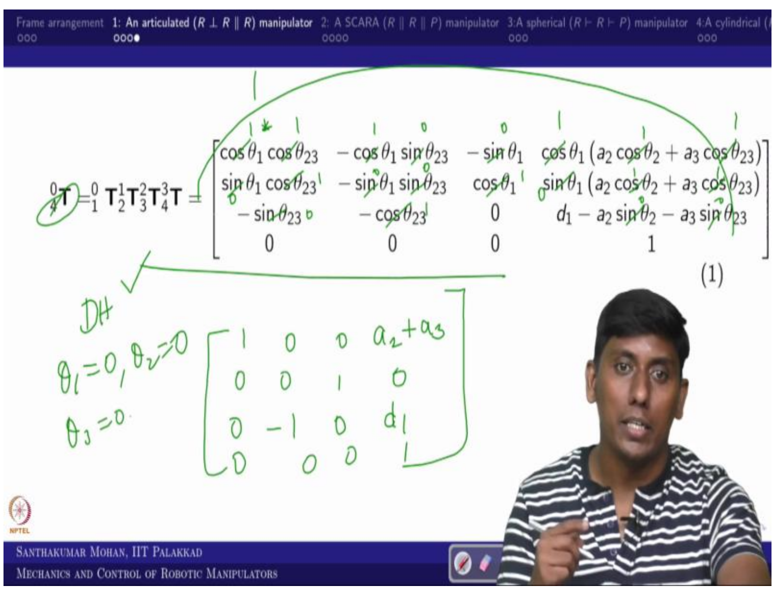

So, how we can substitute, so you substitute, so theta 1 equal to 0, theta 2 equal to 0, and theta 3 equal to 0. So, what become, this become 1, this become 1, 1 multiply 1 is 1 and this becomes 0 and this become 1 and this becomes 0 in the sense you can see, this is giving 1 0 0, this is matching. So now, you can actually check here this is 1 and 0, this is 0 and 0. So, in the sense 0 0 and this is 1 but minus is there, so minus 1, this is also matching. So, here you can see 0 and this is 1 because theta 1 0, so cos 0 is 1 and this is 0.

So, coming back here, this become 1, this also become 1, this also become 1. So, what left, so a2 plus a3 left, but here this itself is 0. So, no need to see that because this is 1, this is 1. So, 0 multiply with anything is 0. And finally, you can see this is actually going to be 0, this is also 0, what left d1. So, in the sense what you obtain, so you obtained the 1 which you have seen as a home position that you are crosschecked.

So, that is what I said, this is the last task. Once you obtained DH parameter you derive this, you can say the final transformation matters, in the sense transformation matrix are 4 with respect to 0, you calculate and substitute your home position and cross verify. The similar kind of example we will see in the upcoming lectures. So, until then, see you, bye. But you please keep it in your mind the frame arrangement is one of the critical points, you keep remember the thumb rule, because once it comes into practice, it is easy. So, thank you. See you then, bye.