Mechanics and Control of Robotic Manipulators
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Lecture No. 09
DH Parameters

Hi, welcome back to mechanics and control of robotic manipulator, we have already seen so far as actual like we have started from what is position vector, rotation matrix, then we have come back to what you call compound rotations. In that a few different methods we have seen finally, we ended with what we call the DH parameter.

So, this is what we have seen so far and this particular lecture, we are going to talk about more about DH representation. What that mean? So, we would be actual like representing one body to another body, or one frame to another frame, how we can transfer. So, what sequence and what would be the final outcome?

So, in this sense, we are actual like representing the frame i minus 1 to i or we are trying to get the transfers, or you can said there transformation information of i with respect i minus 1 this is what we are trying to find.

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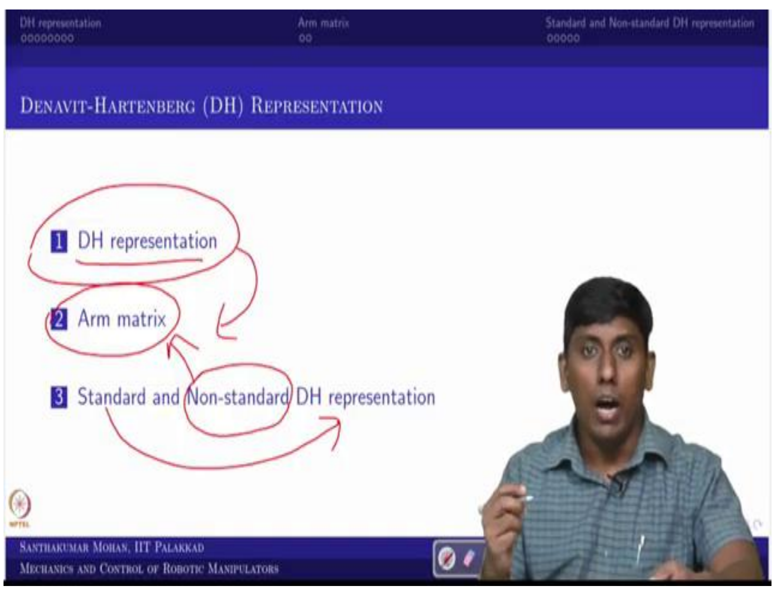

So, if that is the case, so what we can eXpect from this lecture. So, we would be talking about DH representation, finally, the outcome what you call the transformation matrix, but this is related to you call robotic arm, so people call arm matrix and even this arm matrix we can obtain two way. So, one we call actual like the standard representation. So, that is actual like all the variables would be in the ith entity, where the other one is what you call nonstandard, where it would be mixed up i and i minus 1. But in this particular course, we would be keeping as non-standard.

So, obviously you should know what is standard and non-standard. So, that we will actually come back at the end. So, we will start with the DH representation. So, for that we would be taking a random body in the sense random i minus 1 body, or random link i minus 1 out of actual like a set of bodies. So, that is what we are trying to take.

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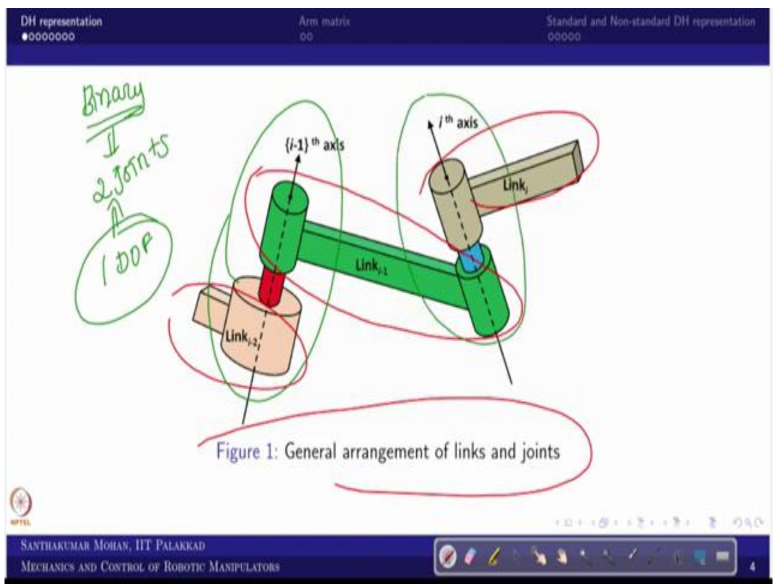

So, you can see right now, this is the general arrangement, we are trying to take the link i minus 1 which is connected link i and link i minus 2. So, obviously, you know like as per the Denavit-Hartenberg representation, what it says? So, any link is a binary link and each binary link would be, the name itself binary link. So, it would be having two joints and as per our case or as per the Denavit-Hartenberg approach. So, each joint would be having 1 DoF system or 1 DoF joints.

So, in that sense what we can see the link i minus 1 to link i minus 1 is connected with the link i sorry joint i minus 1 and the link i and i minus 1 is connected at the joint i. So, this is what we are actual like trying to see.

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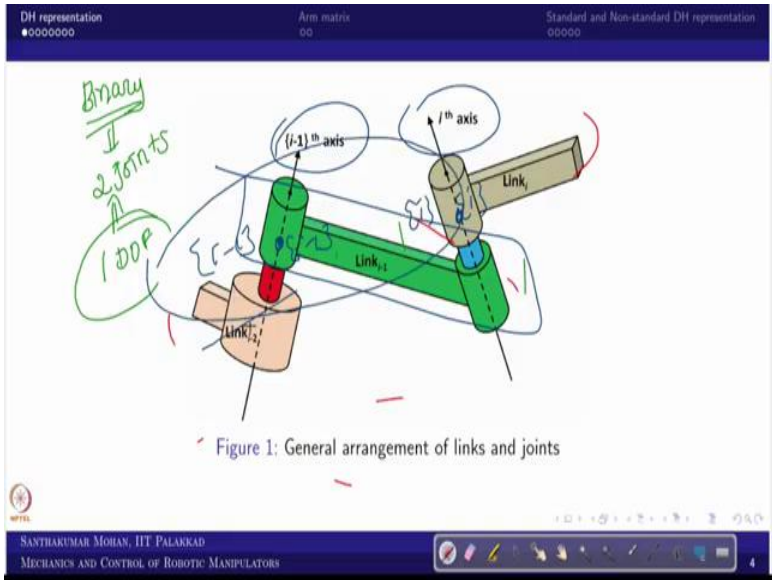

So now, what we are trying to say like if we pick this link. So, if we pick this link. So, what that would be consist that would be consist i minus 1 axis and ith axis. Can we get the relative information of i frame? So, we assume that ith frame somewhere here and i minus 1 frame somewhere here. Can I get ith frame information with respect to i minus 1? Can I get this or not?

So, that is what we are interested, so this information can we get. So, for that first word we need to know axis, we do not want i minus 1 which is connected to i minus 2 link. So that we are cutting out.

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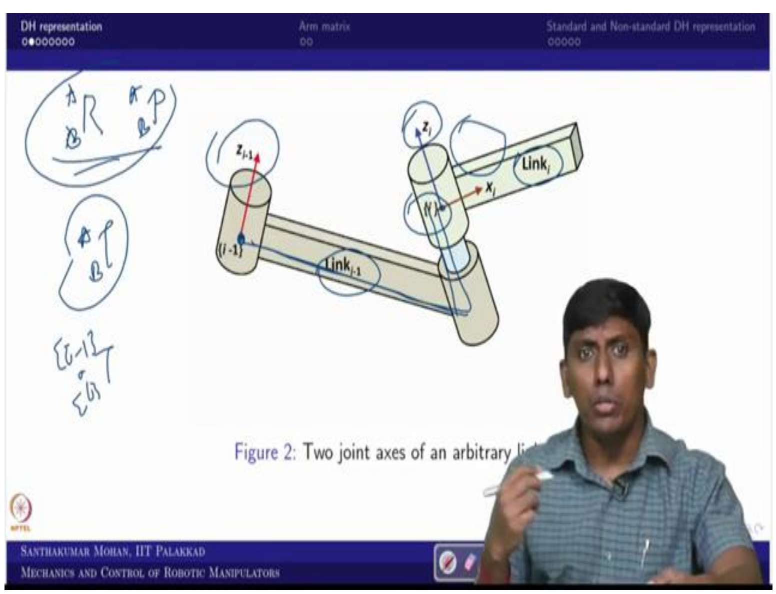

So, now what we are taking? We are taking a link i minus 1 and link i specific we want to see what is Z i minus 1 and Z i axis, because you know all the joint we fiX it with respect to the Z axis. So in the sense Z i minus 1 and Z i already be fixed. Now what we know, we do not know like how they X i goes because X i is supposed to get the information from Z i and Z i plus 1.

So, right now we assume that the X i goes this way. So, now we assume that this is the i minus 1 frame point and this is ith frame point. So, I want actual like travel or in the other way around, I want to know the information of i with respect to i minus 1, in the sense we have done like this. So, the rotation matrix of B with respect to A and the position vector of B with respect to A we have done.

Now, we are actual like combining this and giving as a transformation matrix of B with respect to A. Now, what is we are intended? We are intended that, so the transformation matrix of i with respect to i minus 1, this is what we are interested. So, for that what one can do? So, there are two ways we can do.

So, one simple idea what we can see, for example, something happened on the highway. So the highway is probably little far from the current place. So, you want to know the information what happened at the highway. Probably, people say that there is a new board, signboard that is put in that there is a spelling mistake. Somebody says something it is A is missing, someone else says that no A is added, somebody say that no, no it is E added something.

So, these all actual like what the information you received from someone else. So, instead of that what you can do? You can actually travel to the highway and see what is there, you can see the direction board and see what really the error. So, the similar sense what we are trying to do, so instead of getting the information of i, whatever the associated information of i frame, we do not want to get it from some other source, we travel to there.

So, if we are doing the travel we have to make several subsets, what that? So, you are Z i minus 1 and Z i is not parallel. Similarly, you do not know what is X i minus 1 your X i minus 1 and X i are not parallel and you are i minus 1 to i there is at least two distance we can see one distance along this and another distance along this axis, in the sense, we want to try how to get this particular information.

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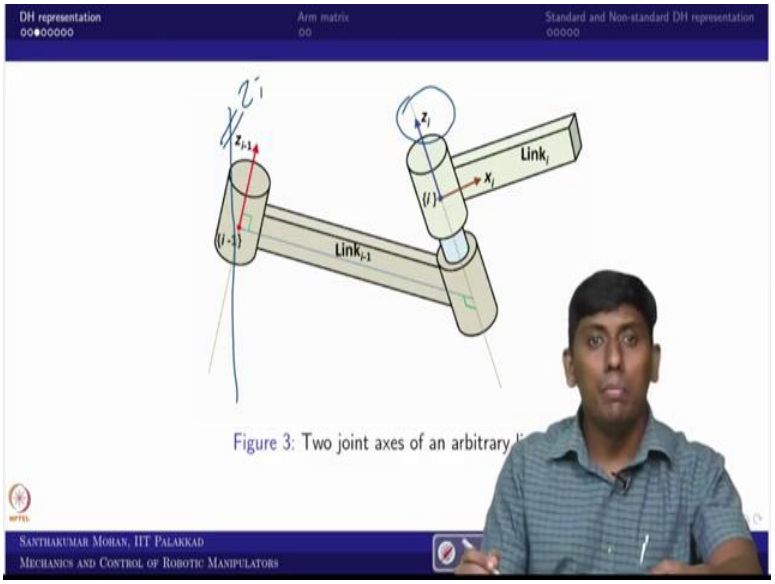

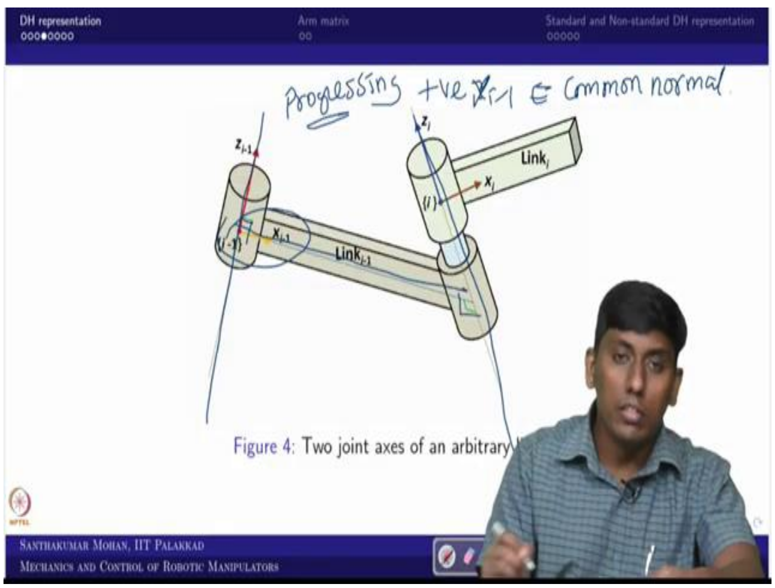

So, for that what we are trying to do first? So, we are trying to make Z i minus 1 and Z i parallel, but what we wanted, we wanted the information of i with respect to i minus 1. So, what we can actually like try to do, you can draw the parallel line which is parallel to you call Z i and then you can actual draw the common normal between these two lines. And this what we can say? This common normal would be the line of X i minus 1.

So, now that is what we are trying to do. So, you can see that we have fixed the X i minus 1, which is actual like along the common normal that to like the direction of X i minus 1, we take it as a progressing side. So, the progressing direction what we call the positive direction of X i, X i minus 1. So, this X i minus 1 is actual like come from where the common so common normal. So, here what is the common normal? The common normal is this is Z i minus Z i this is Z i minus 1 you draw the common normal. So, this is what the line of X i minus 1, this is we have done now.

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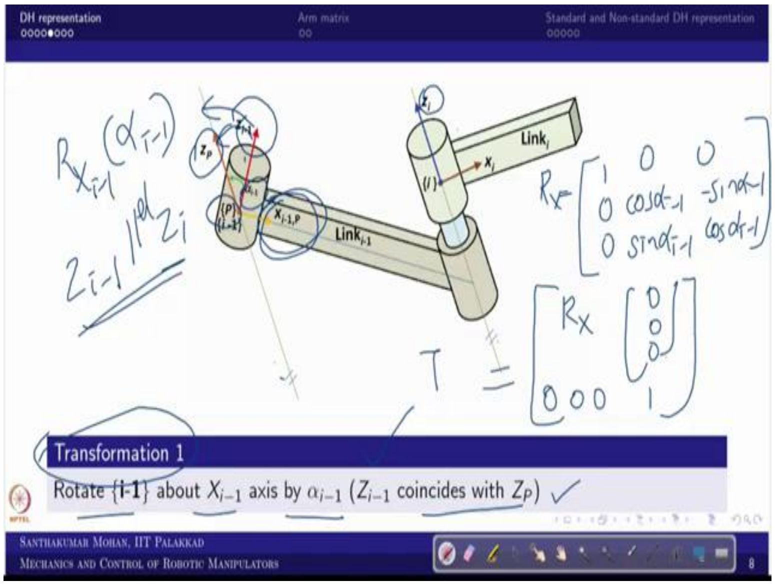

So, what else we can do? We can make it a parallel because we are interested to i, so we can a draw the parallel line to Z i with respect to the frame i minus 1. So, this is the parallel line. So, now what we are taking it, so we are from here, we are taking it one we make it parallel. So in order to make it Z i minus 1 and Z i parallel, so how much angle you have to rotate with respect to X i minus 1. So, that is what we already seen as alpha i minus 1. So, this is what we did.

So, now what you can see like you have something called Z i parallel and Z i minus 1 now by rotating alpha angle with respect to X i minus 1, it become another axis called Zp. Now, since it is Zp has come, so I am keeping that as a new coordinate. So, now because of that, we are keeping it XP parallel to the X i minus 1. So, now, what you have done?

One transformation you have done from Z i minus 1 to Zp with respect to X i minus 1 done. So, what that so have rotated about X i minus 1, what angle alpha i minus 1. So, what you did? So, you made it Z i minus 1 parallel to Z i. So, this is the first operation, so that is what we have written as a transformation 1 rotate i minus 1 frame about i minus 1, X i minus 1 axis with an angle of alpha i minus 1 which makes Z i minus 1 coincide with Zp.

So, now, this is actual like clear the first operation, which is rotation matrix. So, I will write this rotation matrix here what we have done, so it is actual like rotated about X. So, then you can write so this is actual like alpha i minus 1 and this is minus sin alpha i minus 1. So, then this is sin alpha i minus 1 and this is cos alpha i minus 1, this is the rotation matrix, if you want to write it in a transformation base.

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha - 1) & -\sin(\alpha - 1) \\ 0 & \sin(\alpha - 1) & \cos(\alpha - 1) \end{bmatrix}$$

So, Rx and there is no positional change in the sense you can write 0, 0, 0 and this is actual like 1 and this is 0, 0, 0. So, this is what you did the first one translate a sorry transformation, so which is rotation, so that is what we did first operation.

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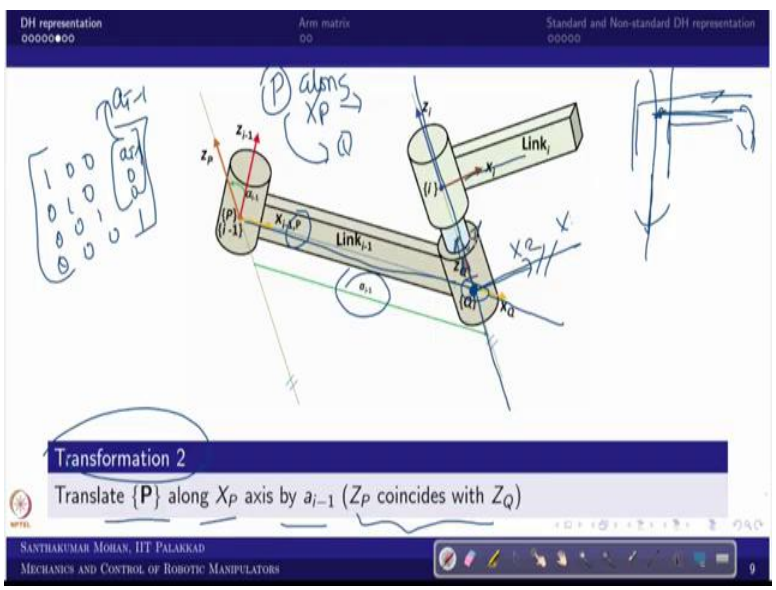

So, let us move to the second one, the second one is what we can do, so now you can see the p and i are distance apart. So, I am taking this extension XP and hit where this in the sense, so you have actual like this road. So, this is your highway, which is very good road and this is a normal street road or probably state highway. So, when you hit, so you are actual like done to reach probably every other aspect would be available. But probably the village road may not be available you feel.

So, similar way, so what we have done? We have make it parallel and then we are trying to reach in the sense you are somewhere here, you come back here and now you are traveling and hitting that road, the same way you have made a common normal, but if I travel along the common normal, what I will end up, I will end up with the Z i axis when I hit that point I am keeping as Q.

So, now what I keep there is a ZQ, which is parallel to Z i and ZP and ZQ are parallel, but what difference between Q and P. So, there is a distance that distance where you can realize you can realize with XP. So XP to what you call Q or P to Q along XP the distance made, what that distance here we marked which is nothing but a i minus 1. So, in the sense of what we did here, we have translated the point P in such a way that that meet the point Q.

So, now the second transformation has done, the second transformation is simple vector. So, what vector you have translated along X i minus 1. So, these are the position vector change, the remaining 1 is actual like identity matrix. So, then you can see this is what the second operation, this is i minus 1. So, a i minus 1, so that is what this term.

So, now, what we did, we did the second transformation. So, we will go the second transformation what it says. So, translate P along XP axis by an distance of a i minus 1, which makes ZP coincide with ZQ. So, now, coming to the third operation, what one can expect the XQ and Xi are angle apart. So, what I can do, I can make a parallel line with Xi. So, this I can label it as XR. So, now, I can actually label it as ZR, so that is what we are going to do in the next picture.

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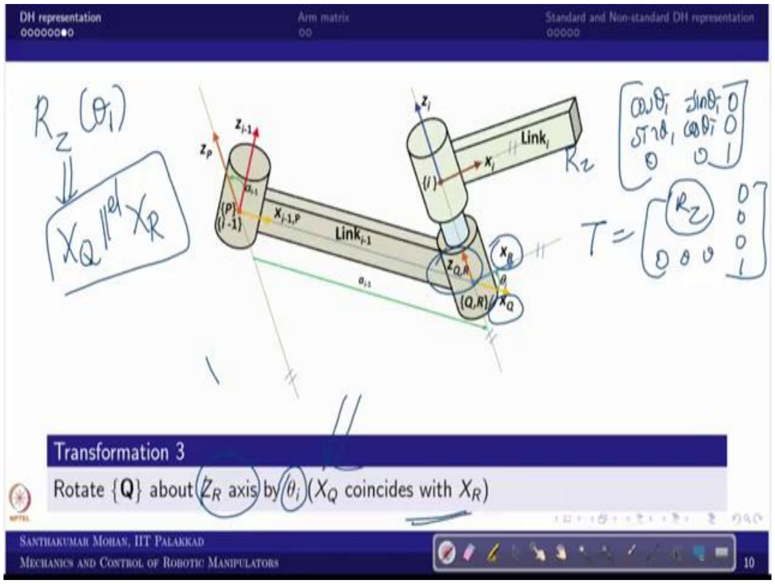

You can see that I mark it, so, Xi parallel at a point Q, which I am denoting as XR. So, now XQ and XR are angle apart, how much angle, theta angle with respect to what, the ZR. So, now what we can do, so ZQ and ZR parallel although, but XQ and XR are non parallel. So, you make it parallel.

So, in this case what you did the rotation about Z, how much angle theta i. So, what you did, you made it, so XQ parallel to XR, so, this is what the operation, so, what would be the rotation matriX in this case? So, cos theta i minus sin theta i 0 sin theta i and cos theta i and 0 0 0 1. So, this is the rotation matrix and you can put it in a transformation way. So, then this would be RZ and 0 0 0, 0 0 0 0 1, this is what you can write.

$$\boldsymbol{R}_Z = \begin{bmatrix} cos\theta_i & -sin\theta_i & 0 \\ sin\theta_i & cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

So, now, this is actually 3 cross 3 and this is all will 4 cross 4, what exactly we did the transformation, the third transformation, where we rotate the Q axis or Q coordinate, the coordinate we rotated about ZR by an angle of theta i, so, that we can make it XQ coincide with XR. So, this is the third operation. So, now, we will go to the next one.

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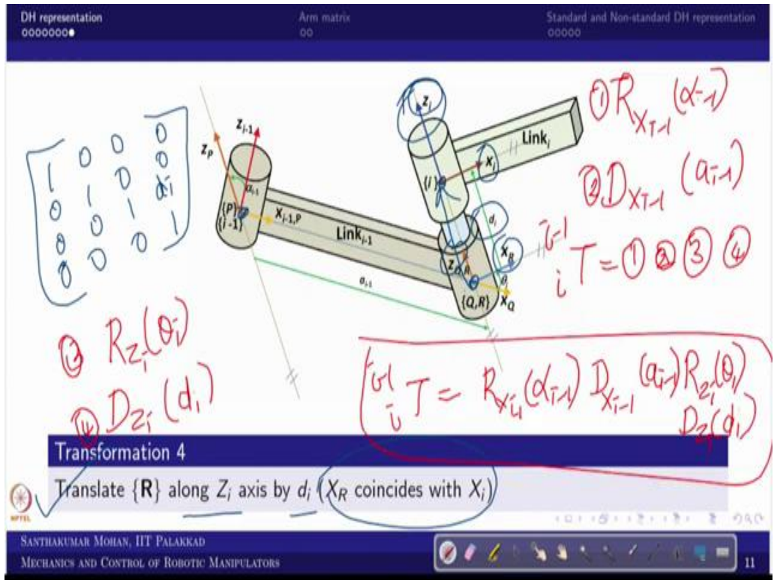

So, what the next one, so you can see like now the I and R only distance apart, how much distance, it is along Zi, how much distance, it is di distance. So, now, what you can see the translation we have done in the sense we travel from R to I along Zi. So, in the sense what you can see this is identity matrix because there is no rotation and all because the XR and XI parallel, ZI and ZR parallel, in the sense these are identity.

So, in addition what we did, so, this di in Z axis rotated, sorry translated. So, now, this is what the final transformation matrix. So, what we did the transformation, so, translate R coordinate along Zi axis by a distance of di, which makes XR coincide with XI, or the Q coincide with I.

So, now, what we did we started with i minus 1 via P, Q, R and reached i. So, this is the way but what we did every instant we did every instant with respect to moving frame we rotated moving frame we translated. In the sense what we have, we have 4 transformation matrix. So, one is a rotation, I will write it in the other way around.

So, I will write first. So, first thing is the rotation, so, rotation I write this, so, rotation about XI minus 1 which is alpha I minus 1, then what we did the translation we did along XI minus 1 which is like ai minus 1 distance. The third thing what we did, the rotation about Z, So, theta i then finally, we translated along Zi, so, di distance, these are the 4 operations.

But this is the first operation, this is the second operation, this is the third operation and this is a fourth operation, but as per our convention if we do any operation or any transformation with respect to moving frame, so, what are we supposed to do? We have to post to multiply in the sense your transformation matrix of i with respect to i minus 1 would be posed to multiply of each operation.

So, the first operation, the second operation, third operation then fourth operation in the sense what one can see, the Ti with respect to i minus 1 would be rotation about Xi minus 1 of alpha i minus 1 angle then translated along Zi minus 1 with ai minus 1 distance then rotation along Zi with the theta i and finally the translation along Zi di distance. So, this is the final matrix, you know individual matrix, you multiply.

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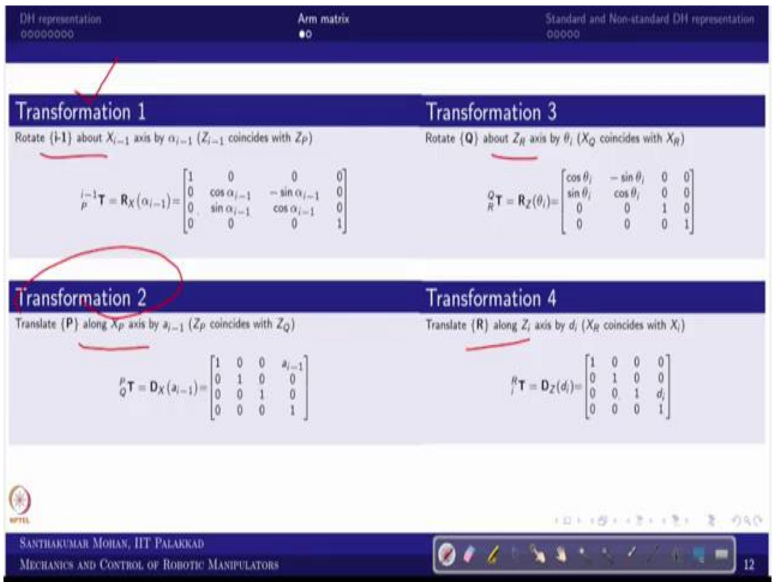

So, for benefit of yours, so, I have seen that I have consolidated this, the first one is transformation, rotation about Xi minus 1 then the transformation which is translation along X P then we have done the rotation about ZR, then finally, translated along Zi. So, these are the four matrix.

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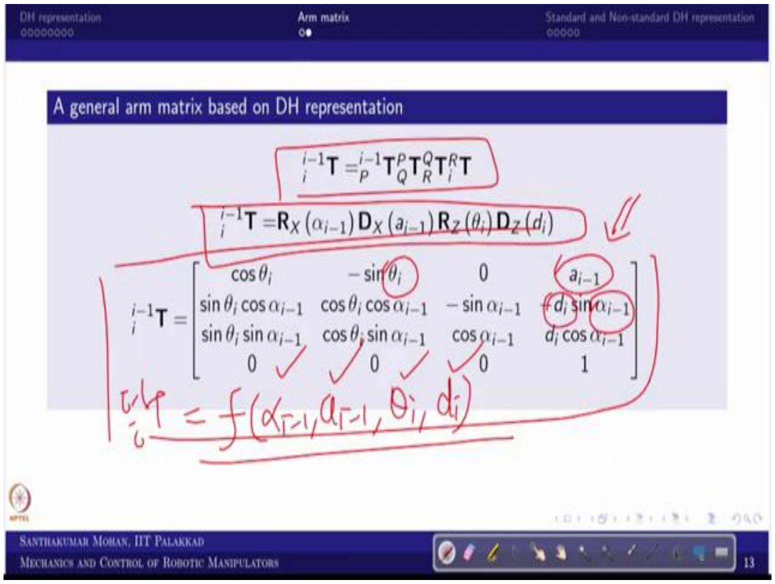

So, you post multiply that is what we did, you can see this is a post multiplication. So, this is the individual matrices. So, finally, you will end up with the matrix which is what you call transformation matrix. Now, you look at this transformation matrix, how many parameters are there, only 4.

So, ai minus 1, so alpha i minus 1 di and the theta i these are the four parameters in the sense the transformation matrix of i with respect to i minus 1 is function of alpha i minus 1. So, a i minus 1, so theta i and d i. I am writing in the order also, first we did, second, third, fourth. So, now this is what we call arm matrix. So, now finding this arm matrix there are several ways.

$$ {}^{i-1}_{i}\boldsymbol{T} = \boldsymbol{R}_X(\alpha_{i-1})\boldsymbol{D}_X(a_{i-1})\boldsymbol{R}_Z(\theta_i)\boldsymbol{D}_Z(d_i) $$

$$ {}^{A}_{B}\boldsymbol{R} = \begin{bmatrix} cos\theta_i & -sin\theta_i & 0 & a_{i-1} \\ sin\theta_i cos(\alpha_{i-1}) & cos\theta_i cos(\alpha_{i-1}) & -sin(\alpha_{i-1}) & -d_i \sin(\alpha_{i-1}) \\ sin\theta_i sin(\alpha_{i-1}) & cos\theta_i sin(\alpha_{i-1}) & cos(\alpha_{i-1}) & d_i cos(\alpha_{i-1}) \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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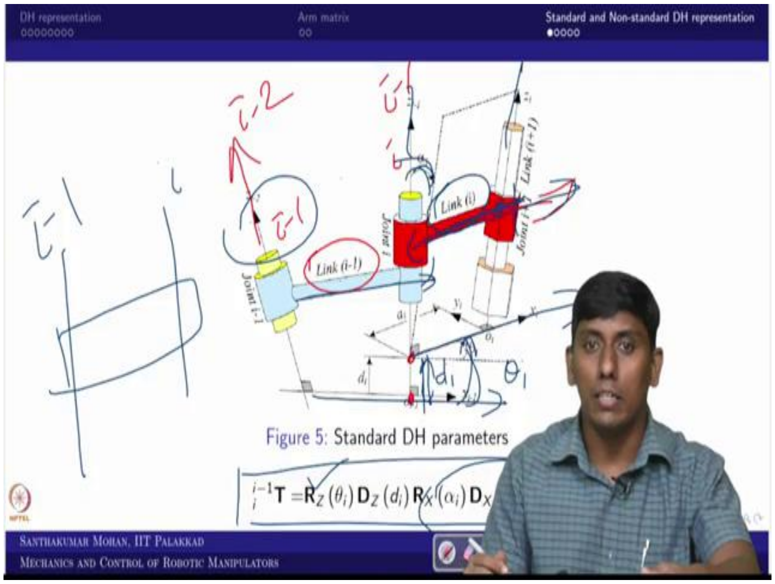

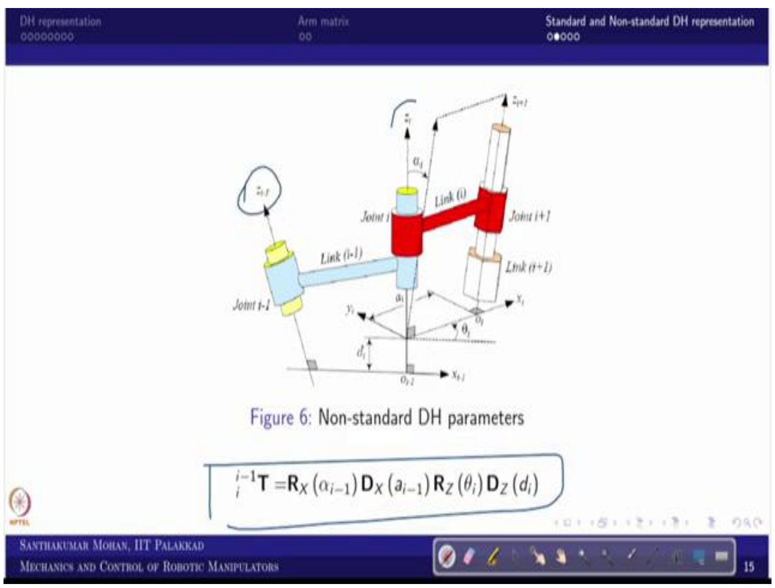

So, one we call standard, so standard in the sense what we see. So, you have link i minus 1 that would be having link i minus 1 would be having axis would be i minus 2 to i minus 1 that way we can take, this is what we take in the other way round, this is i and this i minus 1. So, what we are trying to realize the standard form you take this point and this you take how this link progress and how this link is coming. So, now this is coming like this and this is progressing this. So, this angle you can call theta i and this distance you call di.

Further what we are seeing that the Zi to Zi minus 1 what angle after this in the sense you are looking at link i, so, what angle twisted and what angle, sorry what distance is travelled along the

X. So, if you are using that way, first you have rotated this one, then you are translated, then you have rotated and then translated in the sense you will get a different way.

But one benefit is this is giving the information of next to joint that is what this alpha and a but benefit what here is so everything would be ai, alpha i, di and theta i. But what we did in the previous, this is mixed stuff i minus 1 and i. So, that is why we call this as nonstandard form, but this is very much beneficial when you think about understanding.

So, you can see like here, so, we take i minus 1 and i, but if you look at the previous one, although you call i minus 1 link, but the axis you will denote one before, so, this is always confused. So, when you think about actuator, you want to control the actuator 1, but on this paper, you have to control the actuator of 0, because you are actually using the axis 1 down. So, you can see like our convention if the link i minus 1 would be having 2 joints, which is i minus 1 and i but if you take a standard form, so, that would be i minus 2 and i minus 1, that is the basic difference.

So, but we are all the time, we are favoring the nonstandard form, because that will give you more intuition and whenever you are getting tau which is torque for certain motor or the force of certain linear motor or the angular displacement of any motor that would be directly denoted to the axis which you are referred to, but the standard is not that way. So, now you can see this is the nonstandard and the previous one is standard.

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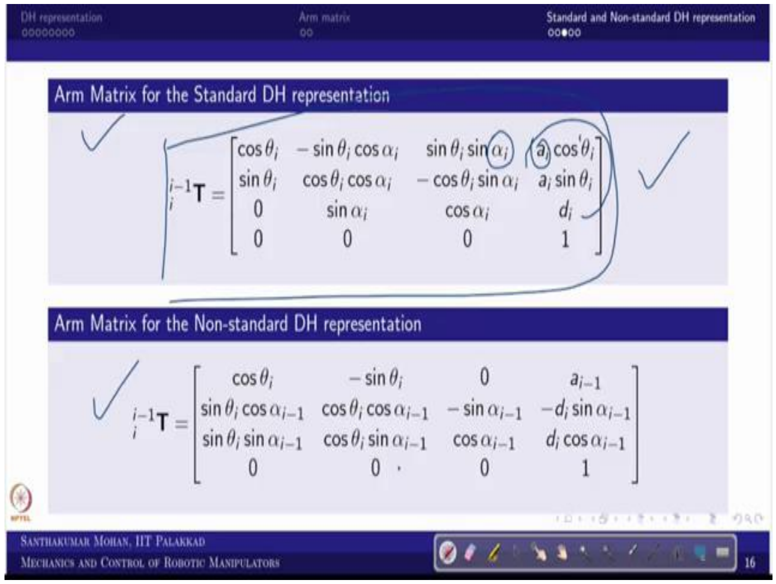

So, these are the two matrices will come. So, you can see very, very similar. So, this is the joint information is clear, but the link information is ai and alpha i is proceeding joint, the link parameter is relative information of 2 links, 2 joints so that would be in this case the proceeding link, but in this case, the current joint to the proceeding link and the joint angle also would be that way, the link to link and joint to joint which is exact correspondence.

So, that is why we use the non-standard, but some of the textbooks which you are going to refer may be giving the standard form. So, one should know. So, this is what we have given. So, the other way round the people used to say so this nonstandard is something like you are starting from the base to the end, the other way round is you are standing in one particular joint. So, you see the joint parameter with respect to previous and you see the link parameter with respect to you and what is proceeding.

So, in the sense you will be standing here and you will call the information before and you will give the information to the front, but that non-standard one is slightly different, you will travel along the path. So that is why I favor this non-standard one.

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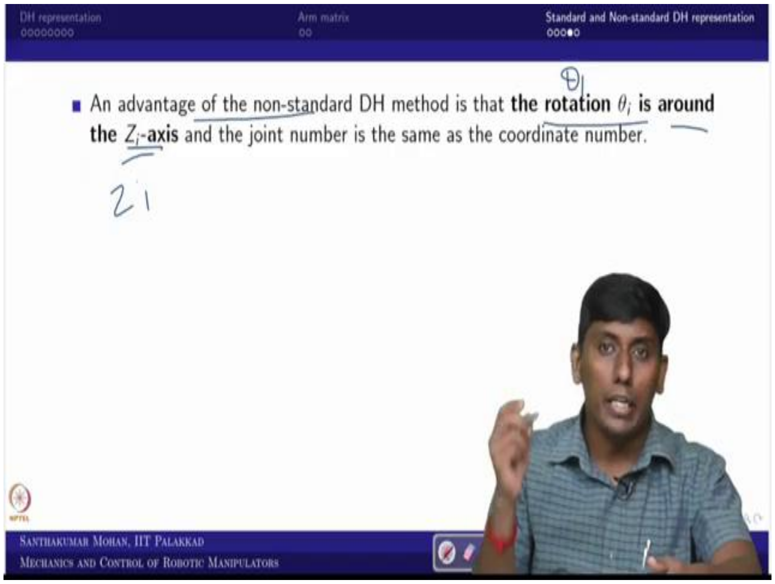

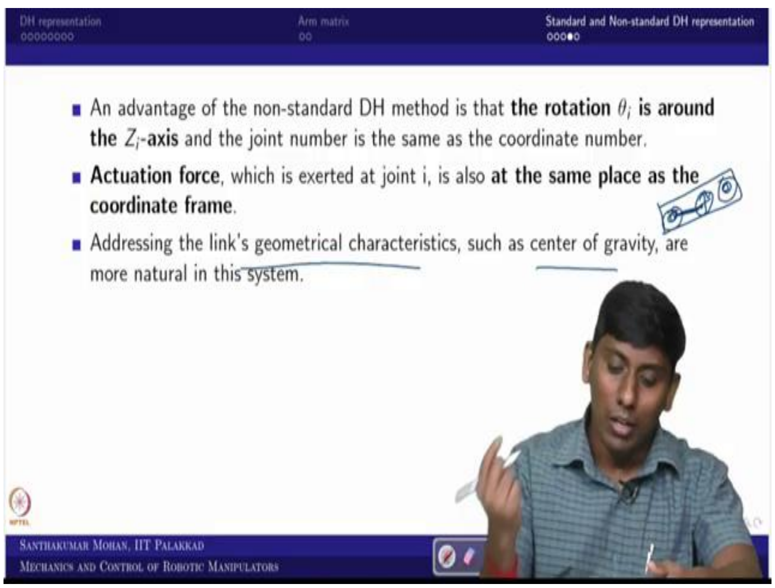

So, I will just end up with this to further slide. So, what is that, one is the advantage of having nonstandard is the rotation of theta i always around the Zi. So, in the sense you denote Zi as 1 and the theta 1 is corresponding to that, in the sense your actuator would be belongs to that, the joint number and the coordinate number coincide.

And because of that your activation force is directly the same place not like previous one. So, this is one of the main benefits that is why we are using nonstandard. In addition to that what happened the link geometrical characteristics always beneficial because this would be directly can be denote the center of gravity.

When you have your body and this is the joint and this is the centroid and this is another joint, you can always denote this with respect to this frame or this frame, but if you use the standard you have to recall all the time before.

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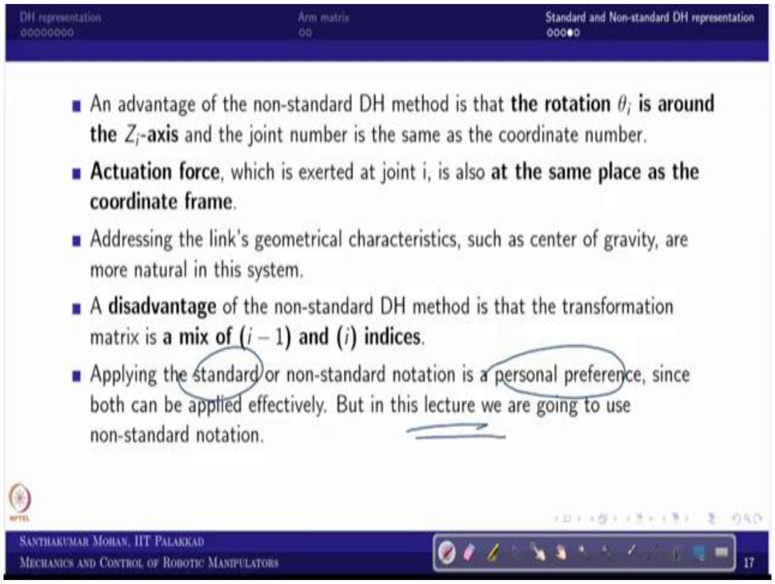

And the disadvantage always there. So, the disadvantage is mixed of the indices, what indices, so, i minus 1 and i would be inside arm matrix, where alpha i minus 1 and ai minus 1 would be exists, theta i and di would be other set. So, this is the disadvantage, but it is not going to create much confusion once you are getting familiar.

But as I already said applying which method you want to do it is a purely personal preference, I am preferring this and my course entire thing is going to follow the non-standard one, but some of the textbook would be talk about the standard one. That is what the one of the easiest thing as I already mentioned this lecture we are going to use the non-standard representation.

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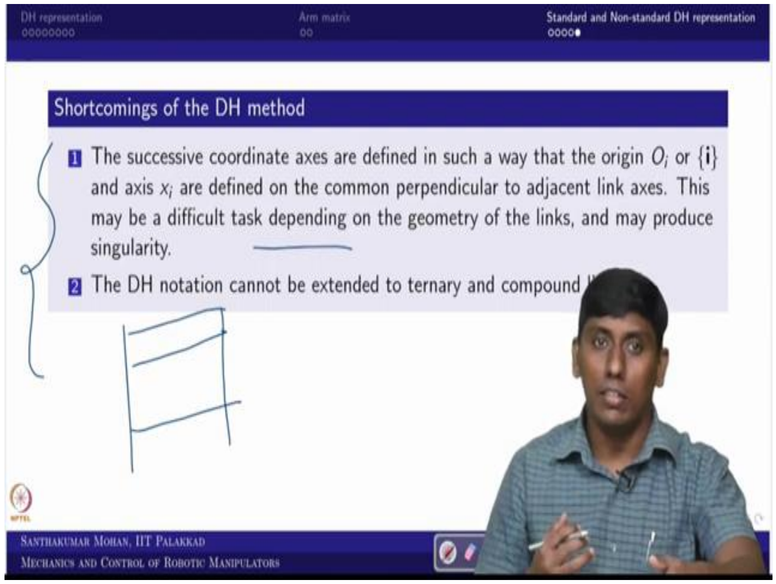

So, now, we will talk about general, general in the sense what is the benefit of DH we have seen but what are the shortcoming. So, the shortcoming already we have discussed in the beginning itself, every link should be having considered as a binary link. In that case if you use a ternary link or quaternary link, it is not straightforward.

Second thing is every joint should be 1 DoF system. But if you have a spherical joint or universal joint then you are to consider as virtual mass or virtual body and then you have to get it. In the sense you have to what you call, you have to decompose those joints into number of 1 DoF joints. So, that is the thing.

And third thing is if you have a parallel axis so, the common normal can come at infinite location. So, how will you find that? So, this is another case, it gives some benefit and the same time it gives a non-trivial solution also. So, that is what the idea here. So, in that sense, you can see, these are the cases. So, now, you know like what is DH representation, how we obtained the arm matrix. And now you know, what is the shortcoming of DH parameter or DH method and what is the basic difference between the standard and non-standard.

The next lecture, we will see some of the examples, we will see some of the physical robot and we will configure into a line diagram, then that line diagram we will mark it as frames, and then we will find it whether this Denavit-Hartenberg representation is useful or not. So with that, I am ending this particular lecture. See you then, bye. Take care.