Computational Fluid Dynamics Using Finite Volume Method

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Lecture - 08
Overview of Numerical Methods: Finite Volume Method

Alright. Good morning everyone let us gets started. So, in the last class we looked at overview of finite difference method today we are going to look at a finite volume method ok. So, that is overview of finite volume method ok.

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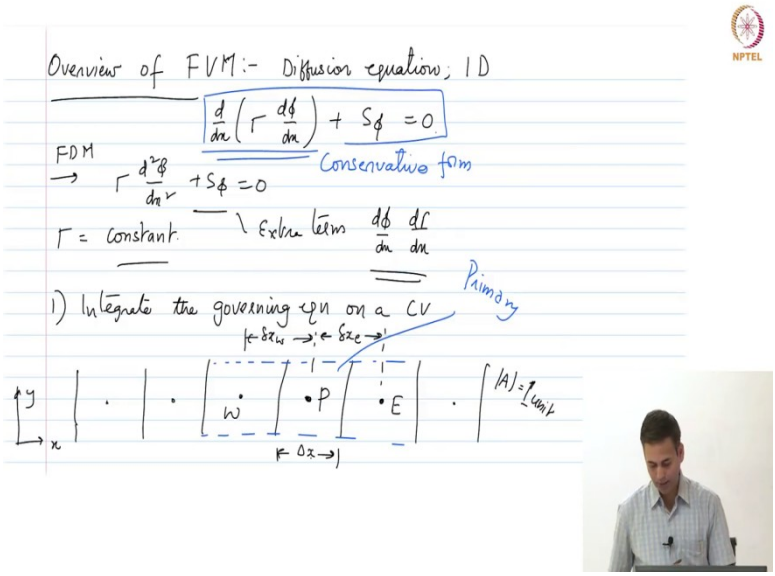

So, for diffusion equation ok. So, essentially we are looking at diffusion equation. Now again it is a 1 dimensional diffusion equation. So, I am writing the diffusion equation here as

$\frac{d}{dx}\left(\Gamma\frac{d\phi}{dx}\right)+s_\phi=0$ ok.

Now, one thing you kind of notice here is that yesterday when I wrote the finite difference method, I wrote the diffusion equation as $\Gamma\frac{d^2\phi}{dx^2}+s_\phi=0$whereas, today I am kind of purposefully writing in this form which is what form? Conservative form ok.

This is a conservative form now of course, these two equations are one and the same if you have gamma as constant ok. They are not the same if gamma is not constant and you would get another term. For example, if you were to write in a non conservative form you would get an extra term right extra term what would that term be? That would be d phi dx times d gamma dx will be there right that extra term will be there if you were to write in this form ok.

Now, having said that we will not; we will not work with the non conservative form in the finite volume, we will only work with the conservative form as it is written here. We will come back to the discussion on these differences a little later, when we kind of compared the finite difference and the finite volume metals together ok.

Now, let us kind of convince ourselves that we want to work with this conservative 1 D diffusion equation with a source term and we want to apply finite volume method ok. So, the first step is to basically integrate the governing equation on a control volume ok. So, we are going to integrate this, before that we need a kind of a control volume.

So, what I would do is I would do a 1 D domain. So, that is basically I have some discretization these are my cells this is 1 dimensions ok. So, these areas that we have that I am drawing here. So, this is a these indicate that the domain is only in the 1 D the dotted lines indicate that domain is only in 1 D and we will have lot of cells like this right. Just like what we had yesterday in the last lecture for the finite difference method like we have these vertices now we have some cells here I would like to call this as a primary cell P ok.

So, this is a primary cell which we always indicate with P and then we call this cell as a cell that is East to the primary cell we denote with E and then this cell we denote with W saying that this is kind of West to the primary cell ok. So, we have no three cells similar to the i i plus 1 I minus 1 that we had yesterday in the finite difference method.

Now, of course, you would have lot of cells as well, but we would develop the equations for these three cells essentially involving these three cells for the primary cell and see if we can apply the same for each and every cell in the domain ok.

Now we also assume that this area vector that we have here say if we say this is x axis this is y axis. These area vectors are the magnitude of these areas is 1 unit. So, this is a 1 unit that is what we assume because this is 1 D it need not be it could be some area a as well fine.

So, if I were to integrate this and we also say that the domain has a distance, the cell has a width of delta x and we will also indicate that the distance between the centroids of the primary and the East cell with a little del x e and little del x w.

So, these are the distance between the primary cell centroid and the East cell centroid and the primary cell centroid and the West cell centroid ok. So, we have del x e del x w and capital del X capital delta X these indicate the corresponding distances.

Now, of course, if you have a uniform mesh all these delta x delta x e delta x w they are all equal ok. If they are if you do not have a uniform mesh then these are all could be different fine. So, having said this, we will move on to integrate the equation the governing equation which is this guy right we will integrate this on the control volume.

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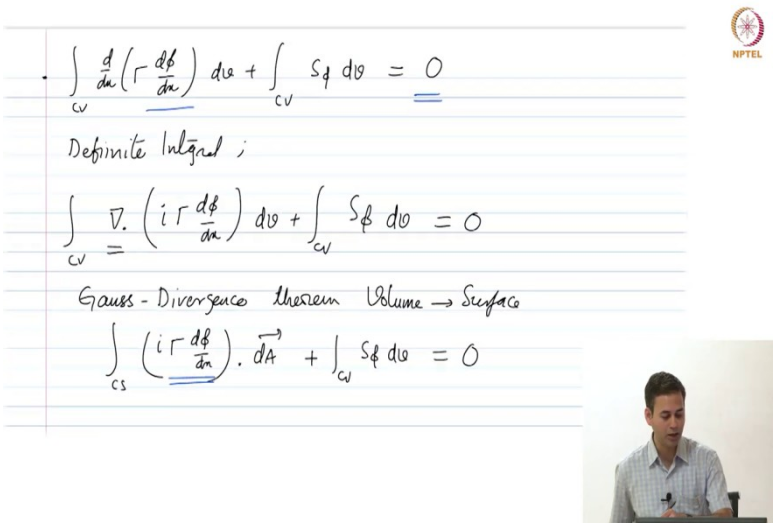

So, that would be integral control volume $\int_{cv} \frac{d}{dx}(\Gamma \frac{d\phi}{dx}) d_v + \int_{cv} s_\phi dv = 0$ ok. Why do not I integrate this 0 will not we get a constant here. If I integrate everything on the left hand side should not the right hand side become a constant?

Student: Yes sir.

It should become a constant is not it? But I still write it as 0 is that correct or no? 0 is correct not correct I have integrated the equation right, but I want to write it as 0 what is the reason?

Student: It is a definite integral (Refer Time: 05:52).

It is a definite integral right ok. So, essentially it is a definite integral. So, we do not have any constants of integration right. So, the constants of integration not there. So, because this is a definite integral as a result 0 remains 0 on the right hand side because we are integrating on the particular control volume with given extents ok.

Now, I have somehow convinced you and wrote this equation, but this equation is not quite nice right this is not what we want to work with. If you want to apply gauss divergence theorem, this is not how it should look like.

So, we will realize that and we will change it, we will change it to control volume we want to have the del operator right. So, that is del dot right and then this should have ideally been written as $\int_{cv} ¿ \nabla . (i \Gamma \frac{d\phi}{dx}) d_v + \int_{cv} s_\phi dv = 0$

Now, does it look in a better form right your del dot is i dou dou x, j dou dou y and k dou dou z and then dotted with i gamma d phi dx that will give you basically the same thing that we have before ok.

Now this is in a nicer form because we want to apply gauss divergence theorem now we also realize that for 1 D problems gauss divergence theorem kind of need not be applied we could have even get away without applying Gauss divergence theorem that we will see in little while ok.

First let me apply Gauss divergence theorem and convert the so, essentially use Gauss divergence theorem and convert the volume integrals to surface integrals alright. So, I would convert the first term that is into a control surface, what would be Gauss divergence theorem?

Del dot a vector dv equals the vector times the area vector write a dot product. So, essentially this will be $\int_{cs} ¿ \left(i \Gamma \frac{d\phi}{dx} \right) . \vec{dA} + \int_{cv} s_\phi dv = 0$ ok. So, this is in a nice form now.

Student: (Refer Time: 08:02).

Where?

Student: (Refer Time: 08:04).

In this equation. So, this equation is basically a kind of a scalar equation, I converted into a vector form right with a del dot and i gamma d phi dx right. This is same as the previous equation we have i dot i becoming this the same thing right I just wrote it in the divergence form right that we realize ok.

Now, we will move on to. Now what we do is we I kind of assumed that this gamma d phi dx term that I have remains the same or I can represent it with. So, we are essentially what we are doing is we have this control surface. So, what does this control surface? This control surface is nothing, but is nothing, but what?

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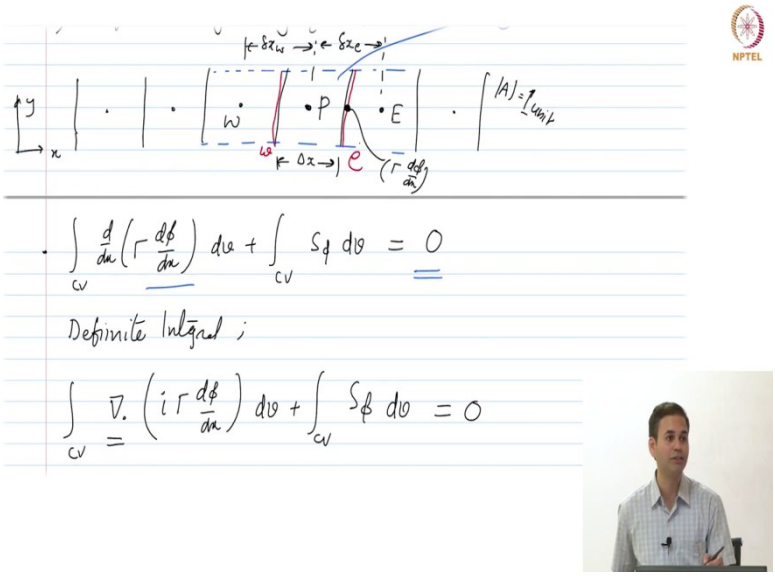

This surface and this surface right that is what I am integrating; that means, I have two surfaces we will call them this face as little e that is between the primary cell and the capital E and we will call this face as little w ok. So, these are the faces which we indicated little e and little w ok.

Now, we have two control surfaces, we have to evaluate the integral on this integral dA right that is basically on this entire face. So, what I assume is that, I assume that this value gamma d phi dx can be represented using the centroid value of the face ok. So, if I evaluate and it I assume that it kind of reminds the same.

So, I have a face centered value of gamma d phi dx and this can be assumed to be a constant which is a centroid value of the face and I am going to integrate with that constant value the

center integration. And we have two such integrations because this is a control surfaces and that those can be now replaced with a summation right because I am assuming that these integrations are now over with a constant value right.

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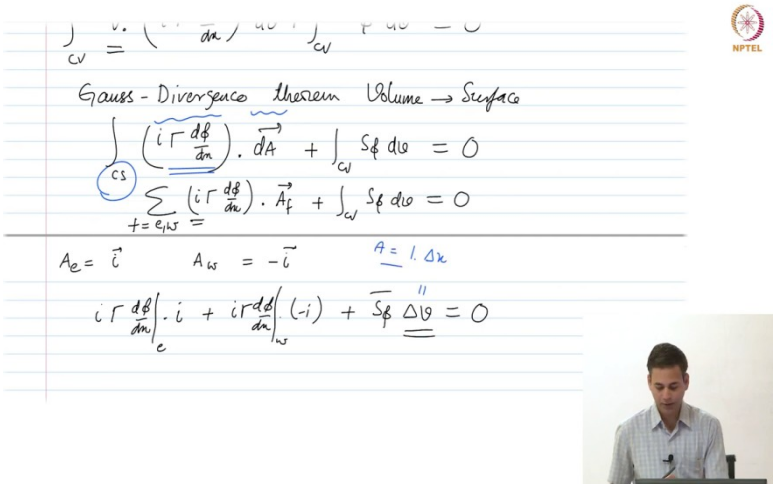

So, I can simply write this as this integral i gamma d phi dx dot d I can simply write it as sum over the faces that is east face and west face right and this integration is on a for a constant value. So, this will be $\int_{cs}\left(i\Gamma\frac{d\phi}{dx}\right).\vec{dA}+\int_{cv}s_{\phi}\,dv=0$ would be now replaced with A bar ok.

SomeΣ¿ A f bar we will see what A f bar is plus we have this c alright is everybody with this? So, this integration of this value over elemental dA is now replaced with a summation of the total value multiplied with the area right dotted with the area. So, this is fine right everybody agrees with this? Everybody agrees now what is Af bar? So, Af bar is nothing, but f goes from east and west Af bar would be A east and A west right.

What is A west or A east? 1 unit and in a vector form this would be i right this will be i bar and a west would be.

Student: Minus.

Minus i bar alright minus i cap or minus i bar ok. So, we have these two, now if I were to dot with these Af bar what will be the terms? We have i dot i in the when we apply east and we have i dot minus i when we apply the west face right.

If I plug in a east and a west into this equations what we have is, we have gamma d phi dx i dot i plus i gamma d phi dx dotted with minus i right. I have these two terms for the east and the west where of course, this is evaluated on the east this is evaluated on the west right this entire thing ok.

Plus now what I introduce another approximation for the calculating the source term, we say this is source term can also be calculated at the centroid of this cell ok. So, I somehow calculate an and average value for the source term at the centroid and that value prevails over the entire cell fine.

So, I have some average value for the source term which is a representative of the cell centroid value and that average value prevails over the entire cell; that means, that is equal to the integral S phi d v ok. So, I have some. So, I would replace this with S phi bar which is an average value for the entire cell times delta v equals 0 ok. I am introducing an approximation here fine ok.

Now, what is delta v? Delta volume of the cell that is nothing, but 1 times delta x right this is the area, area equals 1 times delta x fine.

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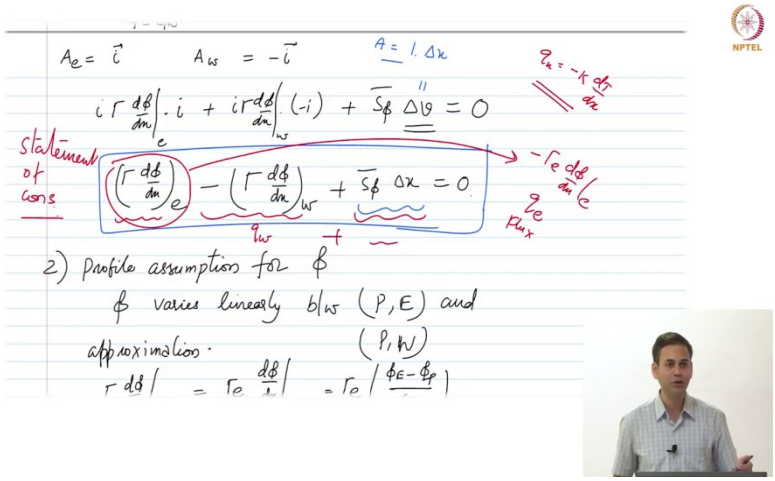


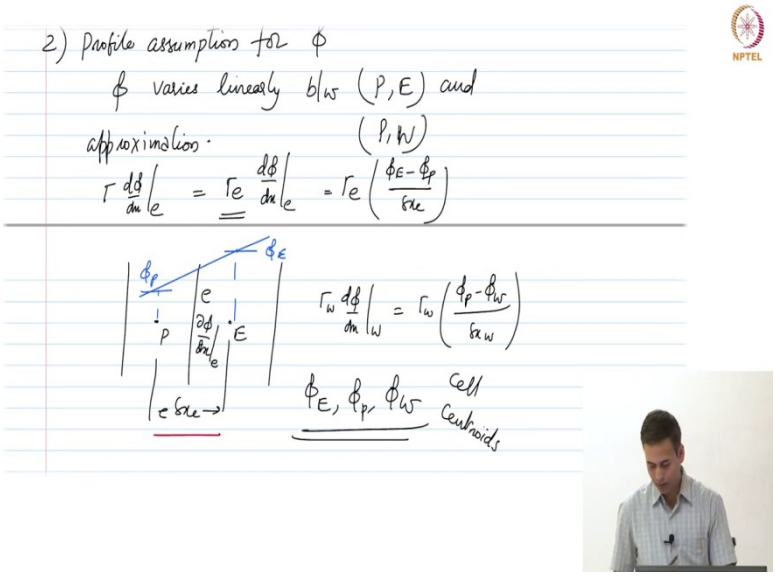
So, far so good now what we have is i dot i is 1. So, you have $\left(\frac{\Gamma d\phi}{dx}\right)_e - \left(\frac{\Gamma d\phi}{dx}\right)_w + \vec{s\phi}\,\Delta x = 0$.

So, now we got some statement here which we will come back to it now this now let us introduce. So, the integration is done then now we introduce a profile assumption for phi ok. Now we have these gradients d phi dx and d phi dx on east face and on west face these two

have to be somehow linked to the values of phi at the cell centroids ok. So, for that reason we are introducing a profile assumption for phi, what we say is that, we say that phi in the profile assumption phi varies linearly between P and E and P and W.

So, between this cells the variation of phi we are saying that it varies linearly ok. Now that is an approximation we are now introducing. Just like the approximation we have introduced in calculating the source term where we said an average value of the source can be found and which is the value at the centroid multiplied by the volume would give you this integration that is one assumption and this is the another assumption where phi is assumed to vary linearly between the cell centroids ok.

Now, you may have a question why will it vary linearly? It need not vary linearly, but we are going to assume that it varies saying that the local variation between these cells if these cells are fine enough could be represented using a linear variation ok. Now if these cells are large in a larger and if there is a non-linear variation for phi, this will not work correctly as a result you will get errors ok. So, fine.(Refer Slide Time: 15:09)



Now, with this assumption we can of course, calculate the derivatives that we got here that is gamma d phi dx on the east face this is nothing, but $\Gamma \frac{d\phi}{dx} = \Gamma_e \left.\frac{d\phi}{dx}\right|_e = \frac{\Gamma_e(\phi_e - \phi_p)}{\delta x}_e$ I can write the diffusion value evaluated on the east face times d phi dx on the east face. Now because I have introduced a linear profile assumption.

If you recall; if you recall what we had was we had these this is our P this is our East let us say our variation for phi is something like this alright. So, what we have is this distance between these two is del x e right and the values are this is phi East, this is phi p right. So, we are evaluating the value of the gradient on this East face right. So, this dou phi dou x on the East face would be phi East minus phi p upon the distance right that is del x e ok. So, that is what we have introduced.

Similarly, I can write $\Gamma_w \left.\frac{d\phi}{dx}\right|_w = \Gamma_w \frac{(\phi_p - \phi_w)}{\delta x_w}$ because my I points my x axis is in the positive right. So, that is why I get p phi p minus phi w by delta xe right because delta x is has to be measured in the same direction ok.

Now we have introduced a linear profile assumption for the gradients. So, what we have in a sense done is, we have brought back the value of phi at the cell centroids right. Now we have an equation which is which involves phi east phi p and phi w right these are all at the cell centroids alright very good.

So, then we will go back to our equation and substitute these things. So, our equation was gamma east d phi dx east minus gamma west d phi dx west plus St bar times delta x equals 0.

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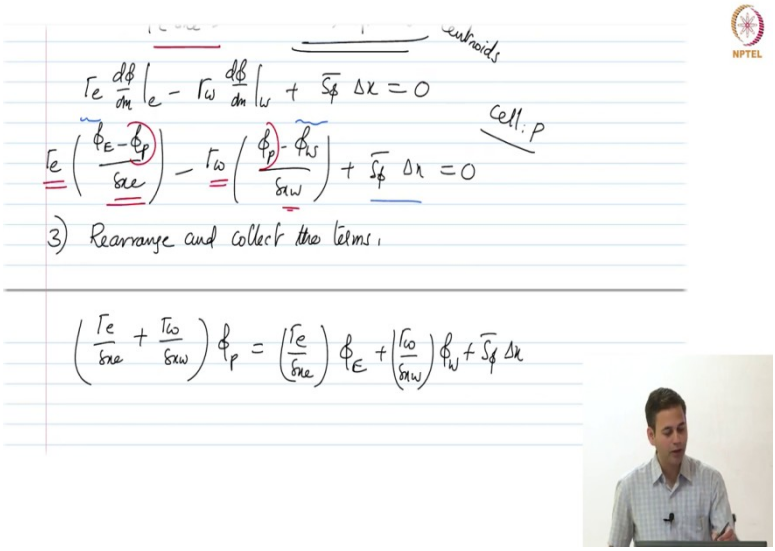

So, I am going to plug in gamma east d phi dx east as $\frac{\Gamma_e(\phi_E - \phi_p)}{\delta xe} - \frac{\Gamma_w(\phi_p - \phi_w)}{\delta xw} + \vec{s\phi}\,\Delta x = 0$.

We got one linear algebraic equation for the cell P ok. This is for cell P I am going to kind of another step would be to kind of rearrange and collect the terms or collect the terms and rearrange. So, what I am going to do is there are two coefficients for phi p. So, I am going to send them to the right hand side that will be $\left(\frac{\Gamma_e}{\delta_{xe}}+\frac{\Gamma_w}{\delta_{xw}}\right)\phi_p=\left(\frac{\Gamma_e}{\delta_{xe}}\right)\phi_E+\left(\frac{\Gamma_w}{\delta_{xw}}\right)\phi_w+\vec{s\phi}\,\Delta$

So, if I take this coefficient for phi p and this coefficient for phi p I can send these two terms to the right hand side that will give me with the positive multiplied with the phi p equals we have gamma east by .

Essentially, this term the phi w phi east will remain on the left hand side and the phi p terms go to the right hand side ok.

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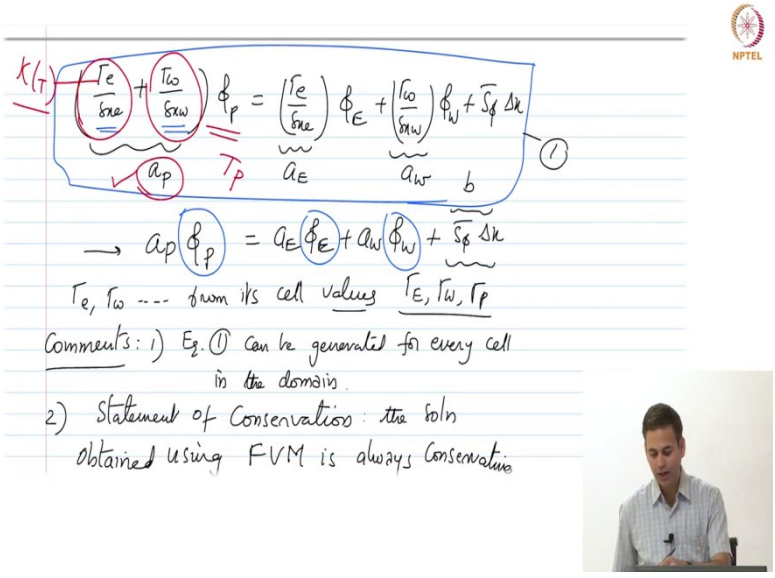

So, we have an equation here we will write in our in a particular form where we call this coefficient multiplying phi p with the value a ok. a is our coefficient because it is multiplying phi p for primary cell we call it ap ok. Similarly this coefficient we will call like to call it as a sub E and this coefficients are a sub w ok.

Now we will write this equation as a p phi p equals a East phi East plus a West phi West plus S phi bar delta x this we can call it as some constant term b fine. So, essentially now we got a linear algebraic equation in terms of phi p phi East and phi w of course, we have assumed that we somehow can calculate the gamma on the faces right.

The gamma diffusion coefficient on the faces can be somehow calculated from its a cell values right. If somebody gives us only description of diffusion coefficient at the cell centers that is gamma East, gamma West and gamma P, we somehow interpolate and then calculate the values on the faces because that is what is going into these coefficients of a East a West and a p that we have now implicitly assumed alright.

So, if you look at this equation, now we can write one such equation for every cell right ok. Let us look at some comments ok. So, essentially this is let us say equation 1, equation 1 type or equations equation one can be generated for every cell right in the domain ok.

Now the entire process has started off with a statement of conservation ok. So, we started off the entire process with a statement of conservation, which is how do we say its a statement of conservation? If I go back remember we had one equation where this one, I say that this is a statement of conservation because what is this term gamma d phi dx e? So, if we define our q bar as what or q q x if I write like to write this as q x would be what?

Student: Kdt.

K dt.

Student: dx.

K dt dx that is all.

Student: (Refer Time: 22:13).

With a minus that is very important right. So, this is q x, this is basically in which direction is this q x going? In the positive x direction right that is why it is minus K dT dx.

So, if I were to write this if I guy kind of send this guy to the right-hand side right this will be what? This will be a minus gamma east d phi dx on the east this is what? This will be q e right some flux that is in the leaving the east face right because in the positive x direction what is this? This will be q west that is also its kind of entering through the west face because its minus gamma d phi dx.

So, this is entering through the west face and. So, the flux entering through the west face plus we have the source something is generated is balancing the flux leaving the east face ok. This is a statement of conservation ok. So, this is a statement of conservation right of course, this

is not new we have done this before where in general we said if we integrate the governing equation we are going to get a statement of conservation, but we are kind of looking at in a 1 D perspective ok.

So, because we have integrated it on a control volume we ended up with the statement of conservation and this statement of conservation with profile assumptions, we have arrived at this linear equation right.

This equation is a manifestation of statement of conservation in a linear algebraic form right ok. So, that means, because it starts with the statement of conservation the solution obtained using finite volume method is always conservative always satisfies conservation.

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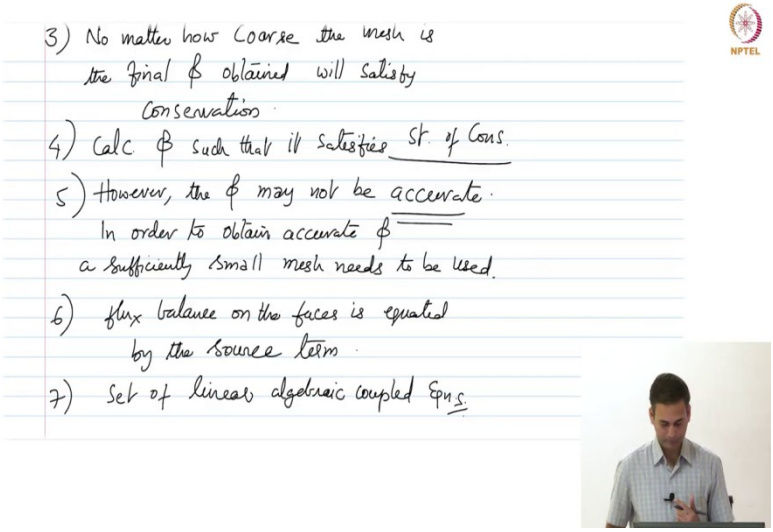

Now, we say this thing; that means, no matter how coarse the mesh is the final phi obtained will satisfy conservation right. For example, if I have a 1 D problem I can take 100 cells or I could take 5 cells or I could take 3 cells right I can take any number of cells, but every phi that I get for each of these meshes will satisfy conservation. Because I am finding phi essentially I am calculating phi such that it satisfies these statements right it satisfies statement of conservation. So, that way every phi that we get for any number of cells in the domain always satisfy conservation.

Now, are those correct are they accurate? They need not be accurate ok, but still what you get is a statement of conservation that is the kind of usefulness of the finite volume method. So, however, the phi may not be accurate essentially for it to be accurate you have to take a

sufficiently small size cells ok. So, that is what. So, in order to obtain accurate phi sufficiently a sufficiently small mesh needs to be used.

So, accuracy is not guaranteed, but phi is always satisfies conservation ok, but accuracy will be guaranteed if you go for a finer and finer meshes ok. Essentially we have the flux balance on the faces is equated by the source term right that is what we saw from that conserved equation alright.

So, now again because we write these things for every cell, we essentially ended up with a set of linear algebraic coupled equations right because these are coupled because whatever is phi east for one cell would be phi P for another cell and phi west for another neighbouring cell ok. So, these are all coupled. So, essentially we ended up with a system of linear equations right ok.

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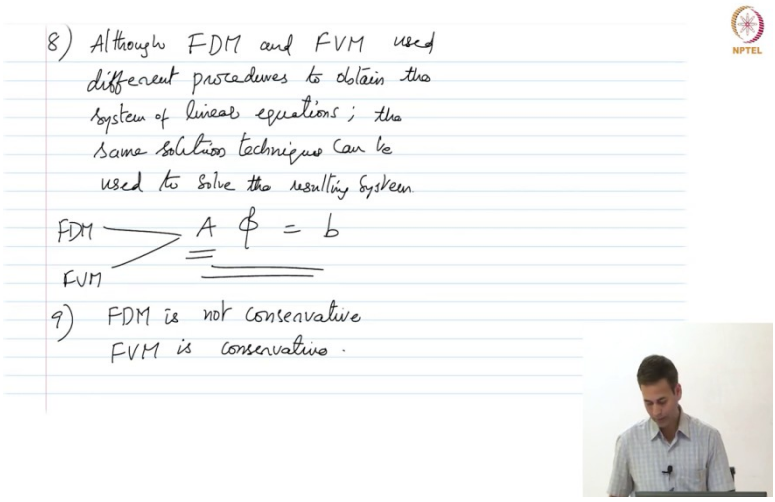

Few more comments are although finite difference method and finite volume method used different procedures to obtain the system of linear equations the same solution techniques can be used to solve the resulting system.

So, what we mean is essentially the ax equal to b that you got right or a phi equal to b that we got from finite difference. So, we got some A phi equals b right. Now this A is something for finite difference and something else for finite volume method or it could be the same we do not know depends on the particular problem of interest ok. Now this A could be different, but eventually we are casting both of them in A phi equal to B ok.

So, from here on if you want to solve we can have the same solution technique to solve for this linear system of equations ok. So, that means, if you write one solution technique that can be used for both finite difference methods and finite volume methods you do not need a separate technique as such to solve for this system.

Few more comments essentially the finite difference method is not conservative whereas, the finite volume method is conservative fine good ok.

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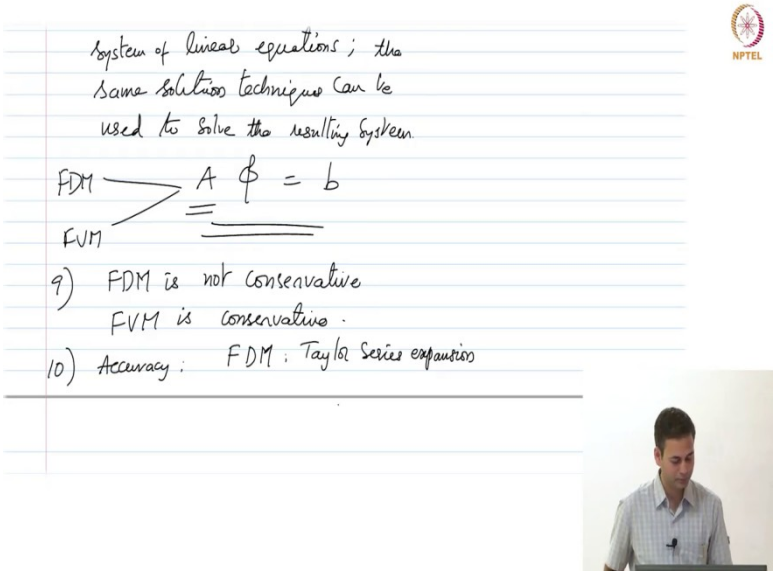

So, one more thing is corresponding to the accuracy, in finite difference method how do you increase the accuracy of finite difference methods? You have the of course, the mesh size is there you make it finer.

And finer, but does not matter how fine you make it you have this accuracy is kind of determined by the discretization scheme that you have used right. So, if we had used a let us say a second order truncation error right if we have neglected the order delta x square terms, then the finite difference method you have got will be second order accurate right. That means, the in finite difference method the accuracy is governed by the number of terms included in the Taylor series expansion right ok. So, essentially Taylor series expansion governs the accuracy.

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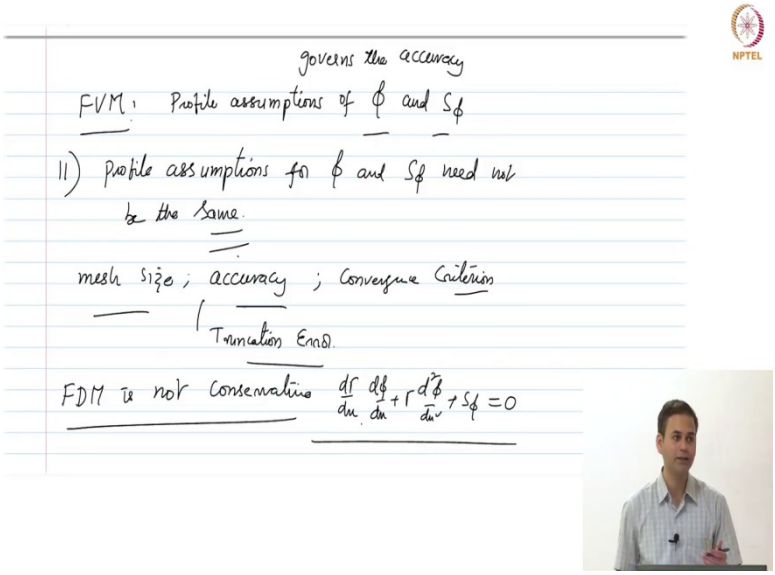

Now, how would you increase the accuracy in a finite volume method or which governs the accuracy? The profile assumption right mesh size is something different we will come back to that. So, one kind of dilemma you probably have is if I take a finer and finer mesh I am going to get some better solution that is true ok.

So, that is true, but irrespective of the mesh size that you take if you have the same mesh size, if you want to increase the accuracy you can also go for a more terms in your Taylor series expansion for the same mesh size.

So, what we are looking at is in a finite volume method if you want to change the accuracy, you have to change what will be the counterpart of Taylor series expansion and finite volume method. What we have just made right we made some assumptions profile assumptions right we have evaluated the variation of phi as well as variation of source terms right.

So, essentially if you want to change the accuracy for finite volume you have to change the profile assumptions of phi and the source terms these have to be changed. If you increase them you get a better accuracy higher accuracy.

Now, one more thing is the profile assumptions for phi and S phi need not be the same. So, we said phi varies linearly whereas, S phi we said the centroid value will be taken. So, this need not be the same you can go for a second order you know quadratic variation for phi and

a constant value for S phi and so on ok. So, these need not be the same ok. Questions till now? Yes.

Student: (Refer Time: 32:06).

Mesh size yes.

Student: (Refer Time: 32:10) because while calculating it cannot be (Refer Time: 32:14).

Right.

Student: (Refer Time: 32:18).

Right. So, ok.

Student: (Refer Time: 32:27).

So, the question is what is the difference between or the relation between mesh size accuracy and convergence criteria. So, essentially if you are you have a scheme the order of accuracy of the scheme is determined with the truncation error you have used ok. So, the accuracy of a scheme particularly depends on the truncation error or for example, let us take finite difference method you have a second order truncation error as a result your accuracy would be dictated by that by the second order accuracy.

So, now that does not mean that you get a very accurate result ok. Now you still have to use smaller and smaller meshes until you get a correct result ok. Now and then the convergence criteria is something which is basically tells you that your solution is not changing with time, you probably have obtained some kind of a steady state solution ok. Now these things I understand are kind of very confusing we will the next part of this lecture is basically looking at that ok.

So, the next discussion will be on accuracy, convergence stability and consistency that is what kind of continues after this because we have now looked at the solution methods. So, we will also look at these four topics, we will not look at the post processing as such at the moment ok. Because yesterday if you will; if you will go back we said this chapter is very short, we have we are getting basically an overview of the numerical methods. So, we looked at the finite difference finite volume how to integrate what are the differences.

Now, we will look at all these things such as the accuracy and things like that ok, but we will come back to these things again once we solve for 1 D and 2D diffusion equations then I think it will be little more clear fine.

Student: (Refer Time: 34:22).

No. So, the question is FDM is not conservative is that because we have not included the other term right essentially we took gamma as constant and took it out now this is a good question, it is not because we have taken gamma to be constant ok.

If you had included gamma also as a variable and got two terms right $\frac{d\Gamma}{dx}\frac{d\phi}{dx}+\Gamma\frac{d^2\phi}{dx^2}+s\phi=0.$ So, if you had if we had solved this one this will still come out to be non-conservative ok.

Now, this non conservativeness or conservativeness also depends on how gamma and s phi vary. If you have a linear problem most likely is that the solution are the system of linear equations you get that is A f phi equal to b that we get from finite difference and finite volume will come out to be the same. If you have a uniform mesh linear variations for gamma and s phi, then finite difference also will start satisfying because a phi equal to b would be the same for both ok.

But the moment you have non-linear variations for S phi and gamma and you have non uniform cells then finite volume method will continue to be a conservative solution will continue to give conservative solution whereas, finite difference will not ok.

Now, this will be there in your assignment ok. So, there I will set up a problem where you would work with both finite difference and a finite volume for a 1 D system and you would prove to yourself that it will not be conservative even if I include all these terms ok. Of course, if you look at look at the equations one thing you can see is that these terms here have only delta x in the denominator right whereas, define a difference methods we had a two gamma by delta x square right.

Why is that? Why is the difference? Taylor series because we have integrated these equations now right we have integrated the equation. So, the derivative order of the derivative has now decreased by one right the d square terms have become d by d right d by dx because of the integrations we got rid of 1 delta x there that is the reason ok.

Now, this does not matter even if you can always prove to me that if we have a uniform mesh with constant gamma and 0 source or a constant source, even with this things you would get the same constant values the coefficient values will be the same that can be proved ok. Only thing is and you have these non-linear source terms diffusion coefficients and non-uniform meshes, you would see that finite difference would not yield solution that is conservative ok.

Now, I am stressing this on the conservative part because that is the strength of the finite volume method ok, but it has nothing to do with a statement that finite difference is not correct ok. Final difference will also be correct only thing is that it will satisfy conservation as a whole in the entire domain ok.

Once you obtain a solution it will probably be it will be also conservative the solution final solution obtain, but at different stages it will not be conservative on a cell by cell basis fine. So, that I think will be clear when we do the assignment problem. Other questions?

No, let us move on. So, we have few more small things here which is these kind of definitions such as what is the accuracy.

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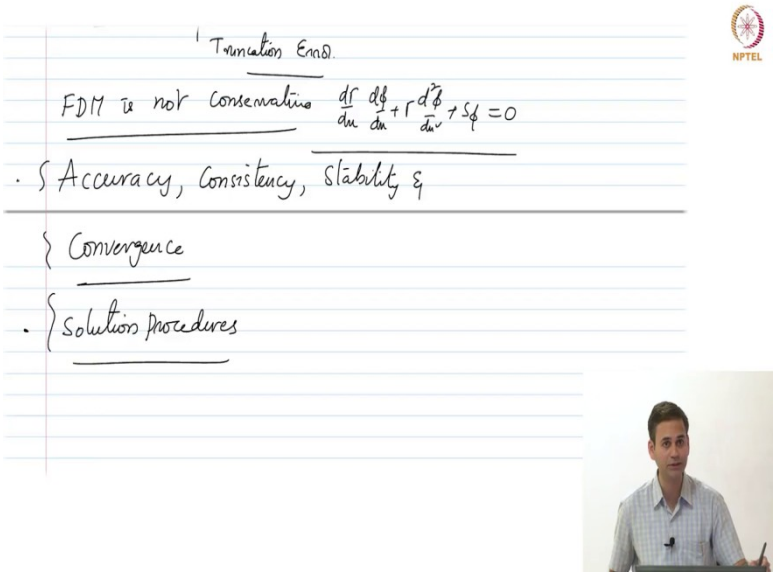

So, we are going to look at what is accuracy, consistency, stability and convergence ok. So, these things is what we are going to look at or we could even we should also be looking at the solution procedures because we have not discussed we obtained the f equal to b, but we did not discuss how do we solve for it ok.

So, I am going to discuss all of these because there is a question I thought we will go with this maybe we will go back to first solution procedures and then we will come back to the accuracy fine ok.

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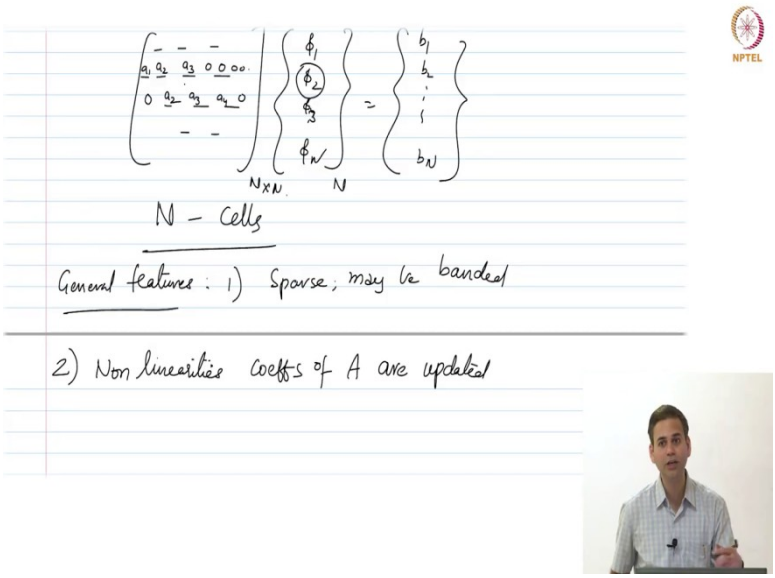

Coming to the solution processes what we what we got now, is basically A phi equals b right we have a system a matrix which has some coefficients right times phi 1 phi 2 and so on phi n equals something on the right hand side that is known that is b 1 b 2 and so, on b N where N could be the number of cells you have fine.

So, this is what we have, what would be a typical characteristics of this equation what will be general features of this equation? General features of this matrix may be.

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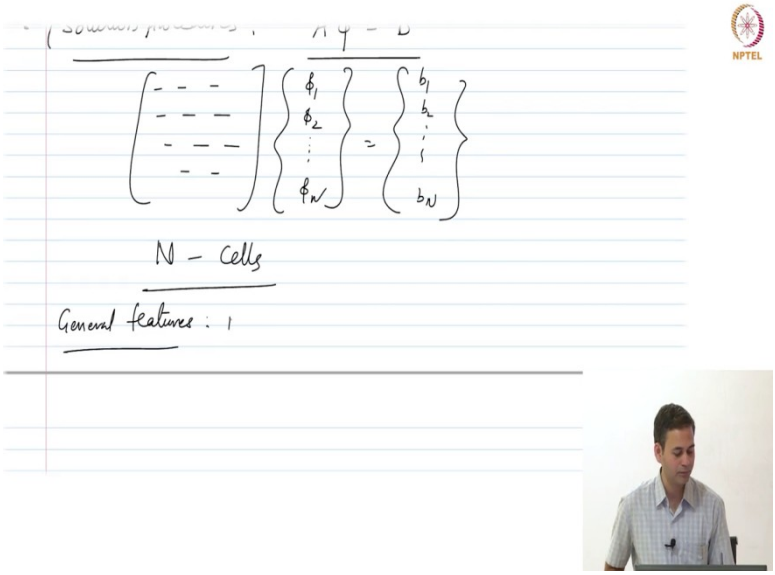

So, if you go back the equation only connects p 2 ok. So, we have this equation where the particular P value of the phi is connected to East and.

Student: West.

West right its only connected to East and West, it is not connected to East of East right if you have if I am at I am only connect i plus 1 and i minus 1, but not to i plus 2 and i minus 2 that is of course, to do with the linear profile assumption. If I had made a quadratic assumption I would have gone to East East to West West and so, on right we are not talking about that.

Let us say we use linear profile assumption for the entire course; that means, we are only connected to our neighbors, i plus 1 i minus 1 as a result if I were to put S an equation like this for example, when I write an equation for phi 2 which will correspond to the second row here, I would have; I would have some this is a 2 will be there some coefficient and I will have some a 3 some a 1 what will be the connection to phi 3 or phi 4?

Student: 0.

0 rights. So, essentially this will be 0 and to all other cells will be 0s right. Similarly if you are at phi 4 you would have some nonzero value or phi 3 you would have some nonzero value for here you will have something here you will have something a 4 what will be here?

Student: 0.

0 essentially you have essentially you have lot of 0s right. So, the matrix is quite.

Student: Sparse.

Sparse right. So, the matrix is quite sparse for example, let say N is there, this is an N by N matrix. So, one of the characteristics is it says sparse and maybe a banded matrix right because you have these diagonals. So, it could be a banded matrix as well and there are a lot of 0s this is a sparse matrix. So, this is one characteristic.

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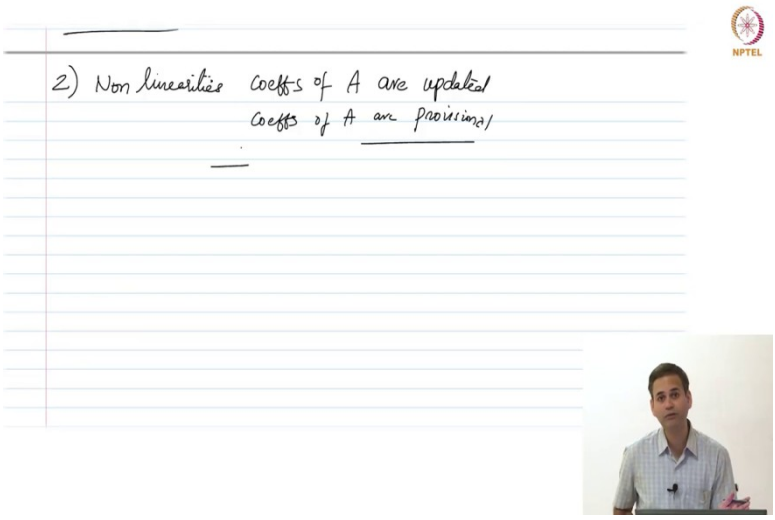

Another characteristic is that the coefficients themselves what we got these a 1, a 2 a 3 and so, on they themselves may not be fixed for the entire solution process ok. For example, if we have nonlinearities if we have nonlinearities then the coefficients of A are updated as you are solving.

For example, gamma let us say the diffusion coefficient k depends on temperature, you solve if a temperature k is a function of temperature with your guessed value of temperature you obtain some k right, but then as you keep getting better and better temperatures the k value gets updated right that k value goes into your coefficients a p a e a West ok; that means, these coefficients this matrix itself keeps getting updated if you have non linearities in the flow problem do you see that.

So, for example, go back what is the coefficient we have here ok? So, we have here this is the coefficient ap contains gamma e gamma West. If gamma were constant no problem everything is constant and you just have a fixed for the entire problem, but gamma let us say

gamma is K which is a function of temperature this instead of phi P I am solving for T P right.

So, as T P is getting updated in the cells the particular value of K T also needs to be updated right. If K T is getting updated your ap needs to be updated right. If ap is getting updated then your coefficient matrix a the coefficients are getting changed right; that means, we have to update; that means, these coefficients are only provisional if you have a non-linear problem coefficients are updated; that means, these are only coefficients are of A are only provisional.

So, these are the two characteristics that this particular system of linear equations that we obtained shows we better make use of those characteristics and tailor our numerical solution techniques to suit those needs ok.

Thank you.