Computational Fluid Dynamics Using Finite Volume Method
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Lecture - 38
Finite Volume Method for Fluid Flow Calculations: SIMPLE algorithm – Part III

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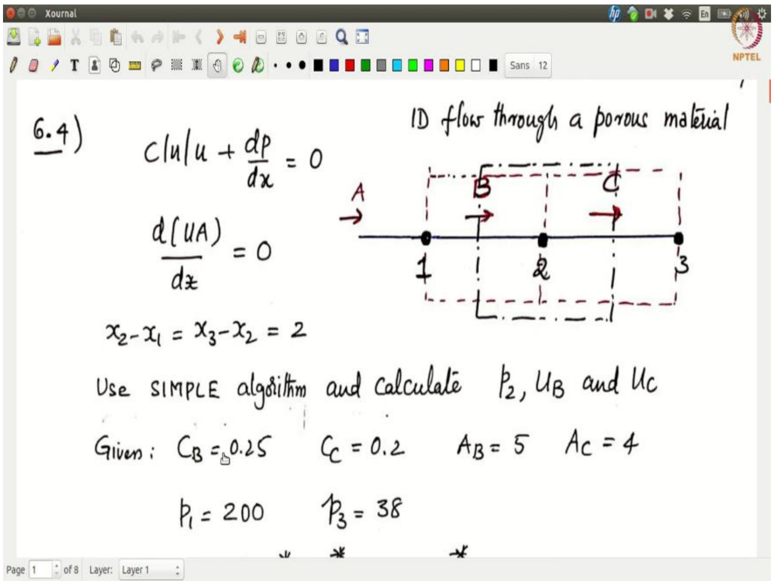

Hello everyone. Welcome to another lecture as part of our ME6151 Computational Heat and Fluid Flow course. So, in the last lecture, we looked at 3 problems from Patankar's book, right. From chapter 6, we looked at the 6.4, 6.5 and 6.7, these 3 problems we percolated them, kind of set the simple algorithm loop. So, in today's lecture we are going to see the corresponding programs for these problems and kind of run and obtain the answers, ok.

So, the first problem, we will tackle today is the problem number 6.4 that is the flow through a porous a 1-dimensional flow through a porous material. Essentially, this is governed by the equation $C|u|u + \frac{\partial P}{\partial x} = 0$, right. And we were given the pressure points which are 1, 2, 3 and the velocity points which are B and C, right.

And, the continuity equation was given as $\frac{d(uA)}{dx} = 0$, Δx that is x_2 minus x_1 or x_3 minus x_2 was given as 2. And we were also given the constants the porosity coefficients, right, C_B, C_C as 0.25, 0.2.

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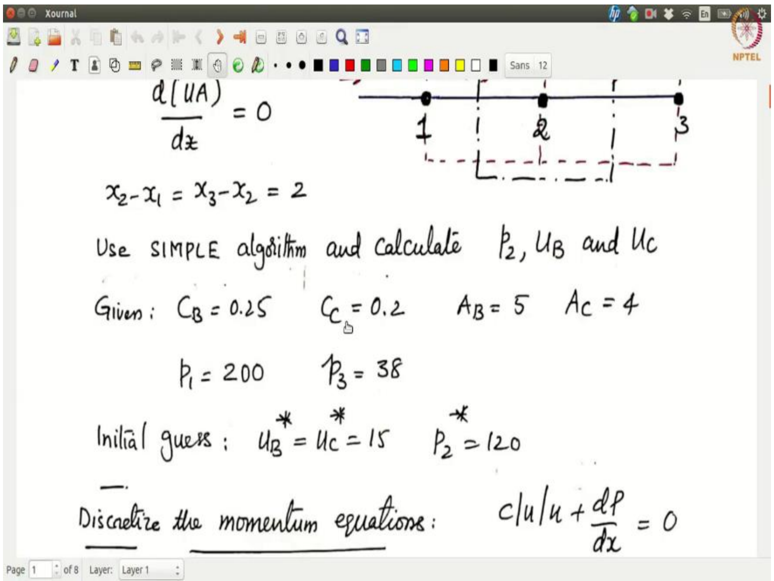

And the cross sectional areas A_B, A_C are given as 5 and 4. And the pressure boundary condition is given at the points 1 and 3 that is 238, right. And we were given an initial guess that is $u_B^* = u_C^*$ equals 15 and P_2^* equals 120, right. And we have to find what is the converged value for the velocities u_B and u_C, and also the pressure 1, P_2, right. That is what we have to find. So, the corresponding code the program with they have is in FORTRAN.

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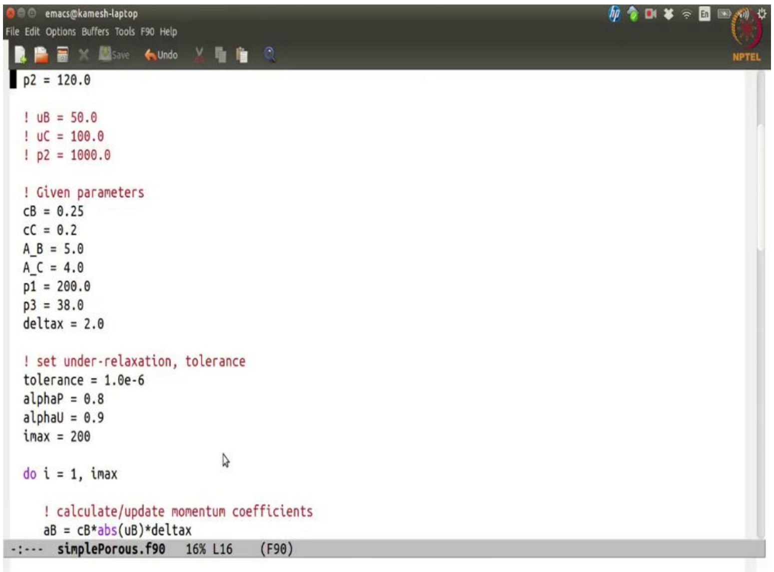

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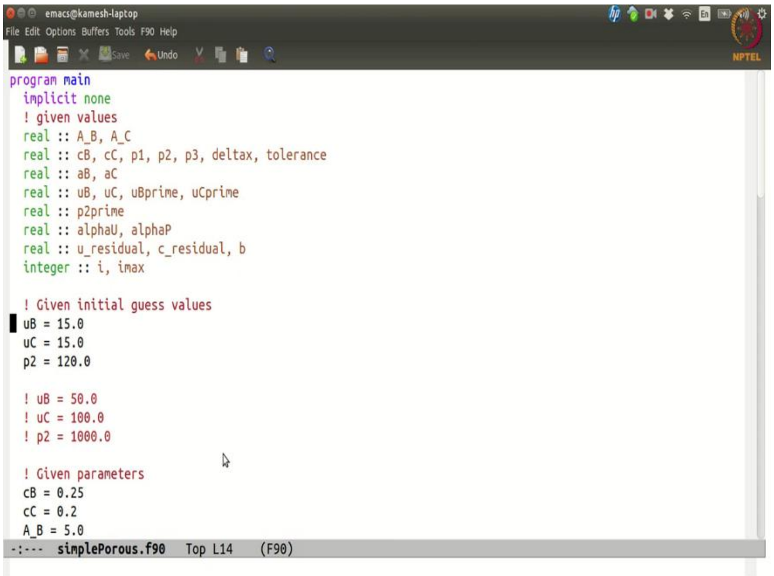

All the programs that I have that I am going to show you today are in FORTRAN. So, I do not have the C counterparts of these, but if you want you can easily write them by looking at by looking at the code that this code that I will share with you. So, if you want you can write a C counterpart of it or you can even run use the FORTRAN programs to kind of play with them and see how they are written and how whether it will work for different values or not, and then kind of learn from it, ok.

So, essentially what we are looking at this code for simple porous that is problem 6.4. So, we have we kind of, so essentially this is the declaration part, so essentially we are declaring these variables that is the cross sectional areas A_B, A_C and the porosity cB, cC and the pressures P_1, P_2, P_3; Δx is basically your x_2 minus x_1 and so on and your tolerance and a_B, a_C here with little a these kind of correspond to the coefficients, right, in terms of the a_B is the coefficient for the u_B, right whenever we write.

And then u_B, u_C are the velocities at B and C locations. u'_B u'_C are the corrections for velocities for u_B, u_C, and P'_2 is the correction for pressure at the location 2. Then, we have the under relaxation that is α_u, α_P the coefficient.

And then we also have something known as residual which is both for u as well as for continuity, ok, for the u momentum equation and for the continuity equation. And then we have this b, b corresponds to the b term on your in your continuity equation, right, ok.

And then we have we kind of make use of couple of integers i and imax, ok. So, what is given to us? The initial guess was given as u_B equals 15, u_C equals 15 and pressure equals 120, right that is what was given, essentially u_B, u_C is equal to 15 and pressure equals 120. So, we initialize to that we have another set here which have we will try later, and anything in an exclamation series a comment, ok, so you do not have to worry about that.

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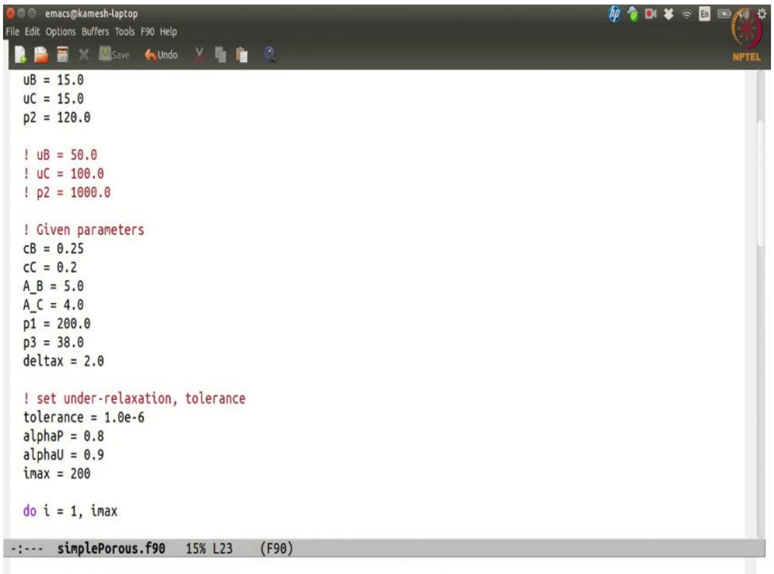

Then, the given parameters are the porosities cB, cC is 0.25 and 0.2, that is basically given here. And the cross sectional areas are 5 and 4, that is basically A_B is 5, A_C is 4 and pressures the Dirichlet boundary condition for pressure that is P_1 is 200 and P_3 is 38, ok. So, that is basically taken here. And the Δx is equal to 2 that is your x_2 minus x_1 or x_3 minus x_2 equals 2, ok. So, these are all the data that is already given in the problem. So, this data is given.

Next, what we do is we kind of set these values the tolerance values, so what is the smallest value we want to converge this code 2, that is 1 into 10 power minus 6. Then, the under relaxation values for the pressure we kind of set it as 0.8, for the velocity we set it set it as 0.9, ok.

Then, we would also need to do some iterations. Here I put the iteration limitation as 200, and hopefully we will kind of break out of the loop before we reach the 200 iterations, ok; that we need to see. If we do not break out of all these, then we have to increase this number to something like 500 or 1000, and then run them.

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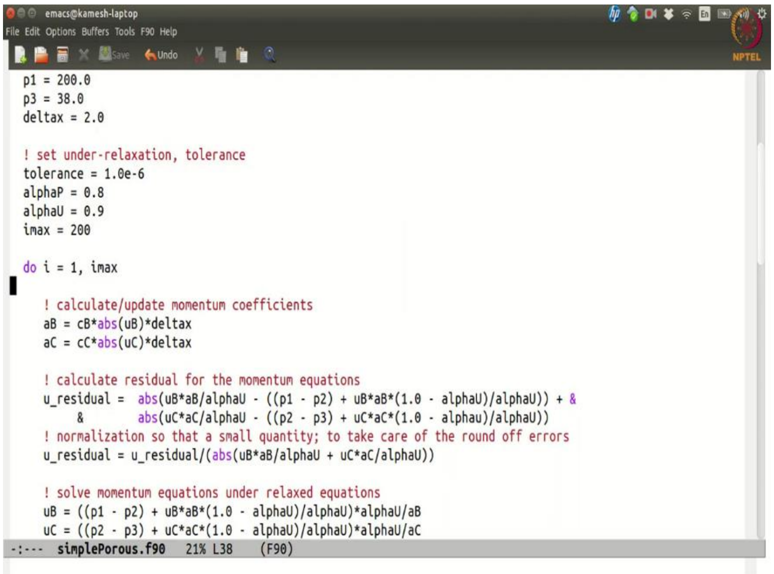

So, here we have a do loop this is basically similar to your for loop in C, ok, in C programming language or C plus plus, fine. So, this is similar to a for loop. Essentially we are going from i equals 1 to imax in steps of 1, ok. So, that is what we have. And then, we need to calculate what is this coefficients for the momentum equations, right.

So, you remember the momentum equation was aB, u_B equals something, right, where the coefficient aB was cB times mod cB times Δx, right. So, if you go down I think when you discretize the equation.

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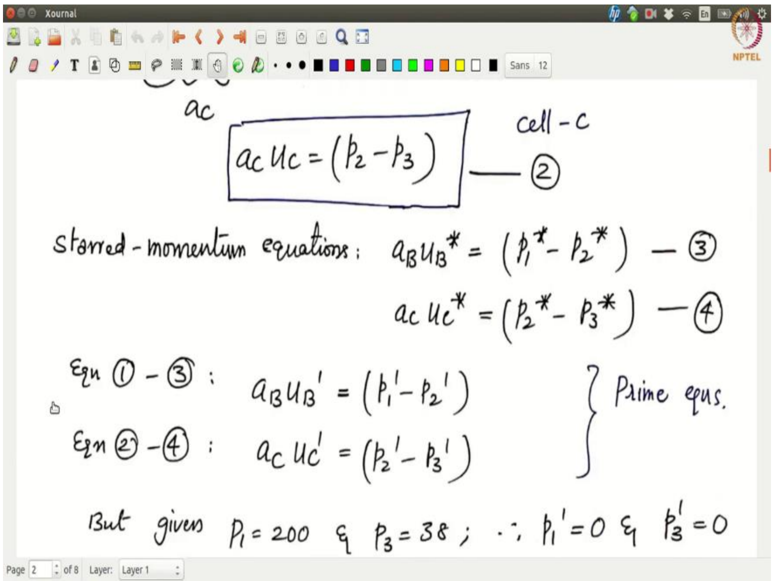

So, these are the equations we got, right. Essentially, the coefficient here aB is basically how much was it? aB was C_B mod u_B Δx, right. So, that is what we have written here.

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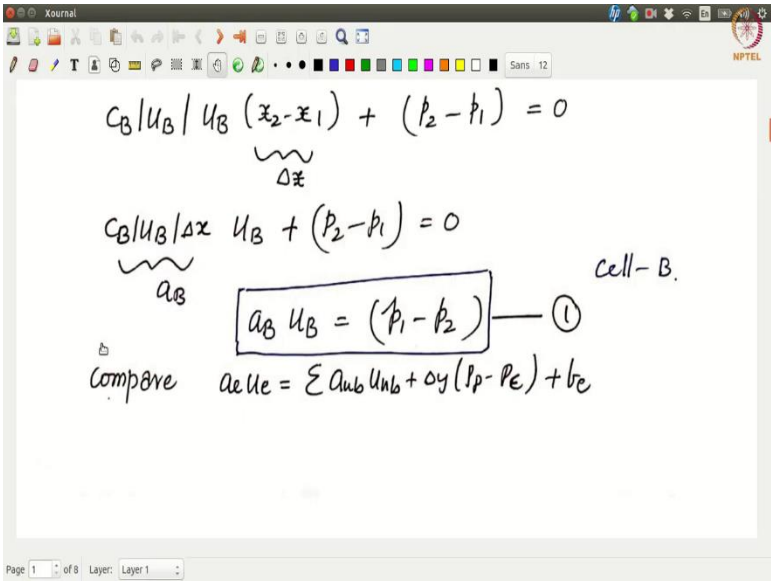

So, cB mod u_B Δx is your coefficient aB. Similarly, aC, cC mod u_C times Δx, also these are the coefficients for the velocities at the staggered locations, for uB and u_C. Now, we will not worry about this at the moment. So, these this is basically the residual I will come back to this in little while. So, before we do that, so let us not worry about this part here. So, let us look at the solution of the momentum equation. So, how do we solve for the momentum equation?

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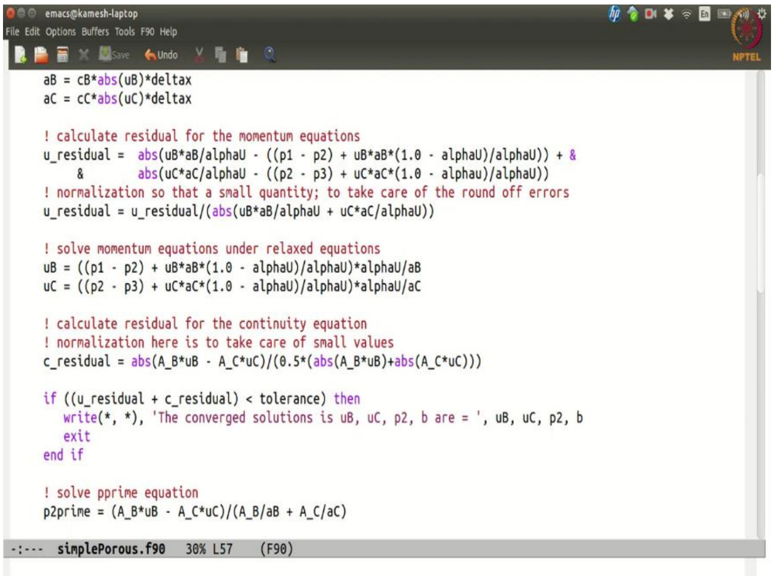

Momentum equation is basically you have aB u_B equals P_1 minus P_2. So, you calculate u_B as P_1 minus P_2 by a_B, right. But of course, this does not look like that because these are under relaxed equations, ok. So, we have to look for the under relaxed equations that we have written towards the end of the formulation, so that is basically this one, ok.

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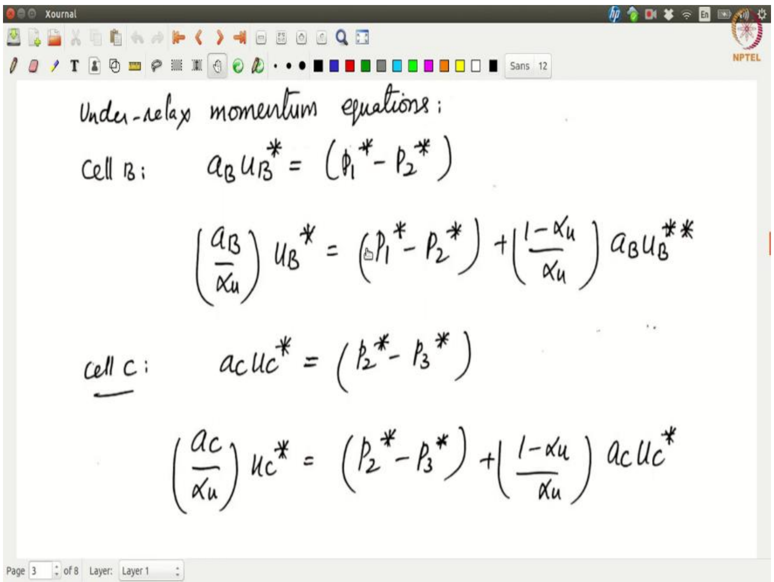

So, we are talking about a_B/α_u, u_B^* equals P_1^* minus P_2^* plus $(1-\alpha_u)/\alpha_u$ times a_B u_B^*. So, if you want to calculate u_B^*, then you take this entire thing and then multiply that with α_u by a B, right.

That is going to give you what is u_B^* for a guess value of pressure and velocity, ok. So, that means, if you look at here, so what we have is u_B^* equals you have P_1 minus P_2 same as what we had here plus we have 1 minus alpha by alpha times aB u_B^* that is this scan it as is $(1-\alpha_u)/\alpha_u$ times $a_B u_B^*$, ok. So, this is $a_B u_B$.

Now, this entire thing of from here to here, right has to be multiplied with α_u/a_B because this is coming from the left hand side, right. So, essentially you multiply with α_u/a_B on the right hand side to get u_B^*. That is what we have. Essentially, you multiply with α_u/a_B, ok.

Similarly, we write the equation for cell C that is P_2 minus P_3 plus $(1-\alpha_u)/\alpha_u$ times a_C u_C^* and the entire thing has to be multiplied with α_u and divided by a_C, ok. So, u_C equals P_2 minus P_3 these are the guess values. So, we do not have this star notation here because we know that eventually the star is basically what we have is there as the current iterate value and that is what will be used here, ok.

So, we have P_2 minus P_3, and then plus we have u_C times aC times u_C into 1 minus alpha by alpha and this entire thing will be now multiplied with alphaU by aC to get what is u_C value, ok. So, this is the solution of a momentum equations, right. So, once we obtain these thing we got a new values for u_B^* and u_C^*, ok, alright.

Then, let us calculate what is the value of the continuity equation. So, the continuity equation is nothing, but A_B, u_B minus A_C u_C, ok. So, let us also not worry about this residual at the moment. And we know that somehow we will kind of exit this loop if the sum of the residuals is less than the tolerance we have specified, ok.

Then, we say that the solution is converge. Now, we will come back to what is this residual, why we are using this as a check we will come back to that in little while, ok, ok. Then, once you have the star values for u_B and u_C, then you essentially go down and solve the pressure prime equation, right. What was the pressure correction equation? The pressure correction equation was P_2' equals u_B^* A_B minus u_C^* A_C.

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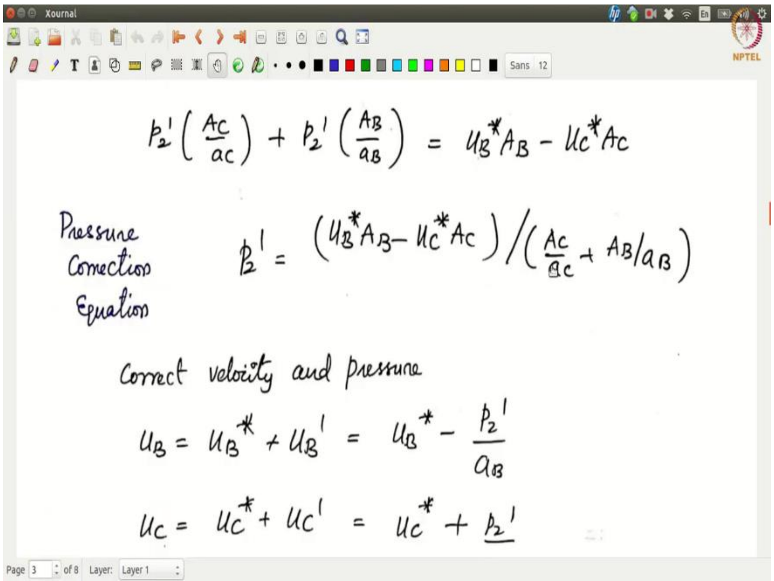

So, that is P_2' equals A_B uB minus A_C u_C divided by A_B by A_B plus A_C by A_C, ok. So, A, B, C is A_B by A_B plus A_C by A_C. So, that is what we have. So, we are solving for pressure correction equation because we have only one cell, we essentially do not have this solve for a system rather whatever we have would be fine, right. Essentially, whatever essentially we can just write it as an explicit equation, right. We do not have to solve for a system here.

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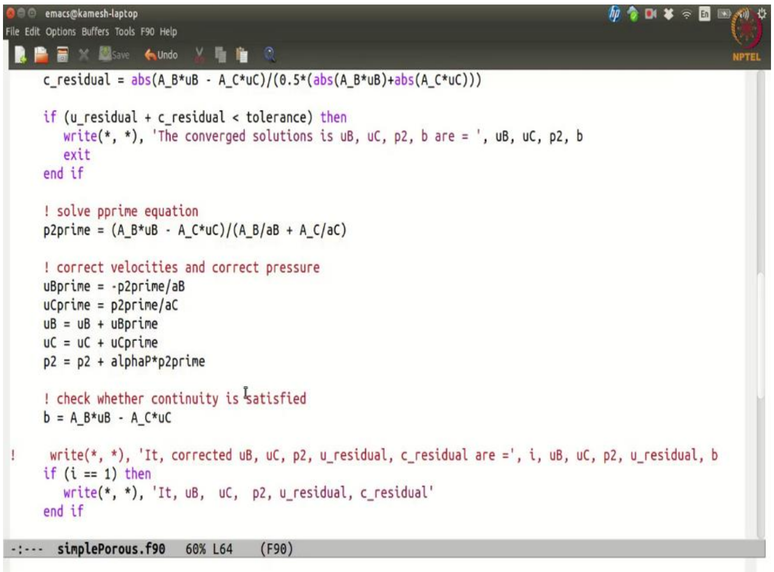

But once we know the pressure correction, then we can correct the velocities using whatever we know, right.

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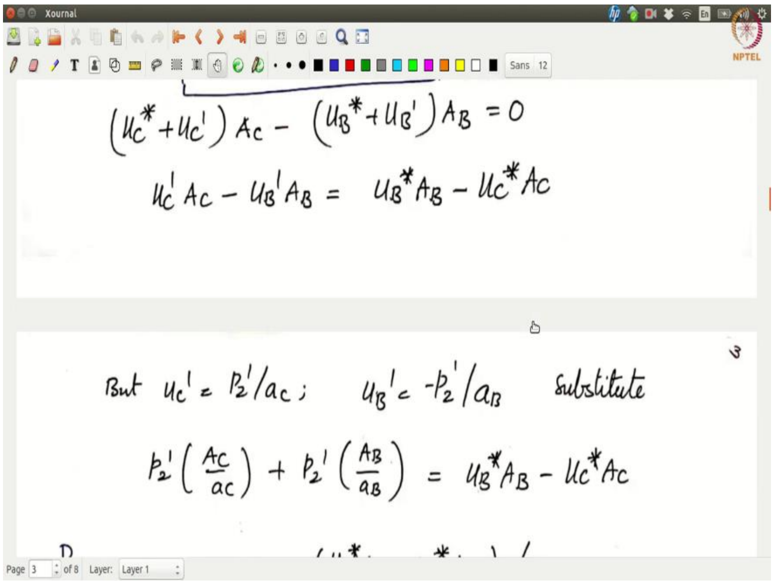

So, what is the relation between pressure correction and velocity correction? Once you know P_2' your u_B' would be minus P_2'/a_B that is written here and this is minus P_2'/a_B, that is u_B' and u_C' equals P_2'/a_C, right this is basically your P_2'/a_C, ok.

Then, u_B equals u_B^* plus u_B'. So, here by the time I come here u_B^* and u_B are the same, right. So, here what I am using is this is u_B^*, when you add it to u_B' the new value you would get I am still I am again storing it in u_B, ok. So, u_C gets all written. So, u_C^* plus u_C' would give me u_C, fine. Then p 2 equals p 2 plus α_P times P_2', ok. So, this is the pressure correction equation and these are the velocity corrections and pressure correction, fine; so far so good.

Then, what do we have? Then we have to check, so basically we have done now one iteration, after one iteration we need to check whether we satisfy continuity or not. What is the continuity equation that we have? Continuity equation is basically if you go back to the problem; where is the continuity equation? Continuity equation is this one this is basically A C u_C minus A_B u_B, right. That is basically your A_B u_B minus A C u_C.

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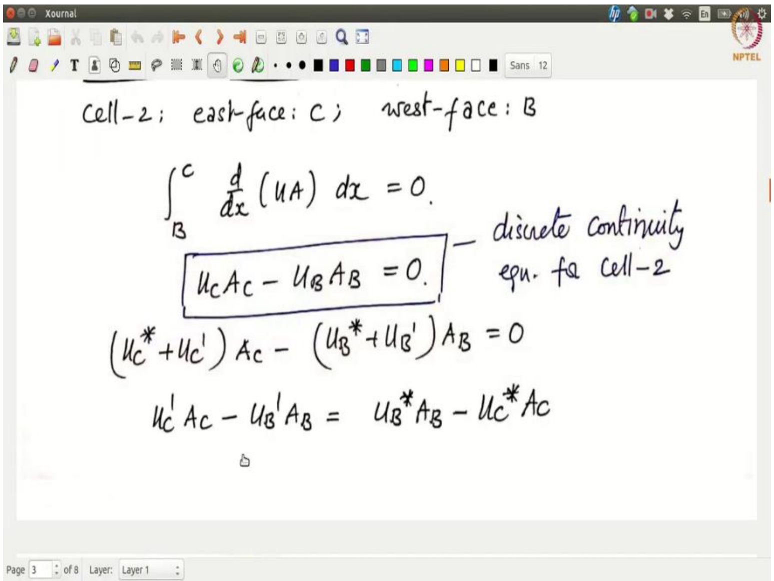

So, if this b satisfied by the new velocity is that is u_B and u_C that we just calculated here then that means, we got a continuity satisfying flow field, ok. So, that is the idea, ok.

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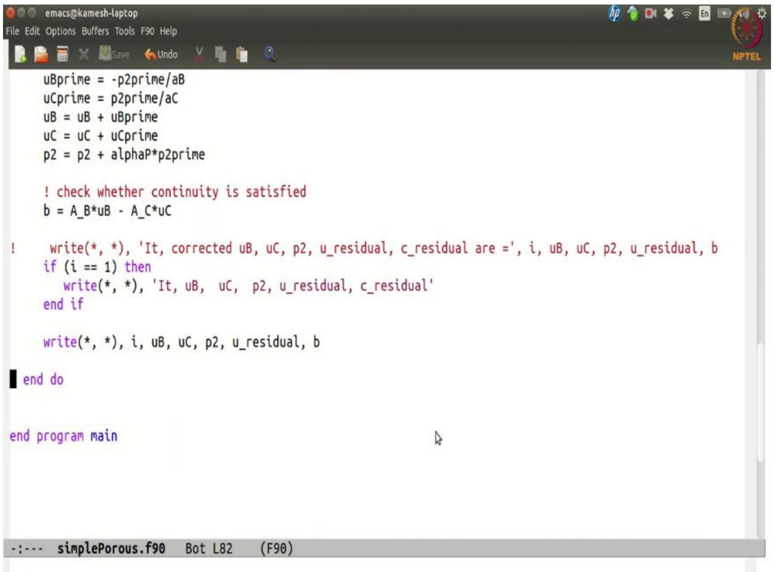

Then, we kind of print off the values after every iteration. So, i is our iteration count and u_B and u_C are the velocities, p2 is the pressure. So, essentially the program the problem asked us to calculate all these 3 values. Then, we print something known as the u residual that is basically the residual coming from the momentum equation and b is nothing but is the amount by which the flow field satisfies the continuity equation or not, ok. So, that is what we are printing after every iteration.

Now, let us get back to the u residual and c residual, ok. So, this is nothing but the u residual is nothing but, if we define something as residual if you have an equation let us say ax equal to B then the amount by which ax is not equal to B is known as residual. So, if you take all the terms to one side that means, the residual for ax equal to B would be B minus ax. So, B minus ax if it is not equal to 0 that means, the x that you got is not satisfying the solution.

So, that means, we have to still improve the x such that B minus ax equal to is equal to 0, ok. So, that is what is residual which should be used to see whether the system is satisfying the solution is satisfying the system or not, ok. So, that means, u residual is nothing, but the entire equation that you have taken to one particular site so that means, we have aB u_B, aB u_B^* by alphaU. This is basically your left hand side.

So, we are taking everything in to the left hand side. So, we have on the right hand side essentially P_1 minus P_2 plus this is all the under relaxation part. So, this entire thing, right, from here to here is the entire momentum equation which is basically taken to the left hand side. And similarly, we have we are looking at momentum what is the; what is the residual for the other cell. So, this is the residual for cell B and this is the residual for cell C, ok.

So, we are taking an absolute value of these residuals because we do not want the residual of this to be subtracted by this one, ok. So, we want to see how much is the amount by which this is not satisfied and how much is the amount by which the C cell, values are not satisfied then we calculate these two, ok.

That is the sum. So, u residual gives you the residual for cells B and C. Then, we are also updating u residual by it is absolute value of the central coefficient, that means a p, u p, a p, right, that is u_B a B by alphaU. So, we are kind of normalizing this.

This is only kind of a normalization, so that like a small quantity that is coming up from these terms would not be over seen, ok. So, as a result, we are just taking the first two terms, essentially the left hand side values and we are trying to normalize whatever u residual we have computed here such that even if this comes out to be small it is with respect to what is computed here and we would know that the value that you got here is not really small compared to this, ok. Such that we still have to kind of converge.

This is basically to take care of this is to take care of the numerical errors or the round off errors, right. So, we do not want very small values to be seen as, this is 0 or something, ok. So, this is basically. But by the time u residual goes to let us say less than the tolerance that means, we are satisfying the momentum equation, that means, the calculated u_B and u_C now satisfy the momentum equations to the up to the tolerance, ok. That is what it tells us, right.

So, that means, if we if u residual comes out to be 0 or the tolerance value that means, the u_B u_C that we have calculated now satisfy the momentum equation, right. That is what it is, ok, ok, alright, ok. And you also realize that before when I do this thing I have already used the aB as the updated values, ok. So, you may have a question here in terms of well we talked about the discrete momentum equation and the momentum equation.

So, we are talking about the discrete momentum equation in this context, but I have just updated the aB values here, ok. That means, what I use here in this equation is the latest value. So, this is the non-linear value whatever we have, right. The non-linear value would be computed here because this is the same u_B that is used to check for this, ok. So, that means, it will satisfy the same non-linear equation as such, ok. So, we have just plugged in these values, alright.

Now, let us also look at the continuity residual. So, we want to see whether the continuity value that is A_B u_B minus A C u_C whether this is 0 or not, ok. Again, we are kind of taking a average of A_B u_B and A C u_C and multiplying with half essentially this is to take care of the relative value of A_B u_B minus A C u_C, right.

This is basically the normalization here is to take care of a small values that may come which may really look small, but they are actually not small in comparison to what is the each of the flow rates, ok.

Further, we are doing this thing. Otherwise you can even check with A_B u_B minus A C u_C, ok, fine; essentially, these two the continuity residual and the momentum residual. This is basically what tells us that if these two together are less than the tolerance that we have specified which was some 1 E minus 6 or something, right which was a 1 E minus 6 here.

So, that means, if that is if both of them are less than the tolerance then the then that means, we have kind of converged to a particular solution, ok.

And then we are going to print what is the value of u_B u_C and pressure at the cell and we also going to print the b, b is nothing but the value of the continuity equation, ok. So, that is what we are going to do, alright. So, that is as far as the program is concerned.

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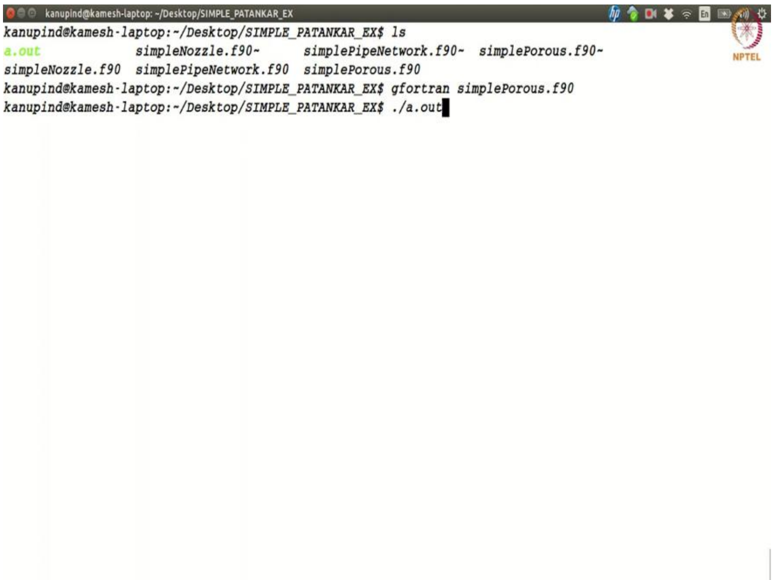

Now, let us look at running this program. This is simple for us. So, I have these programs here. So, I use gfortran, gfortran simple porous dot f90 and then I would run it, ok.

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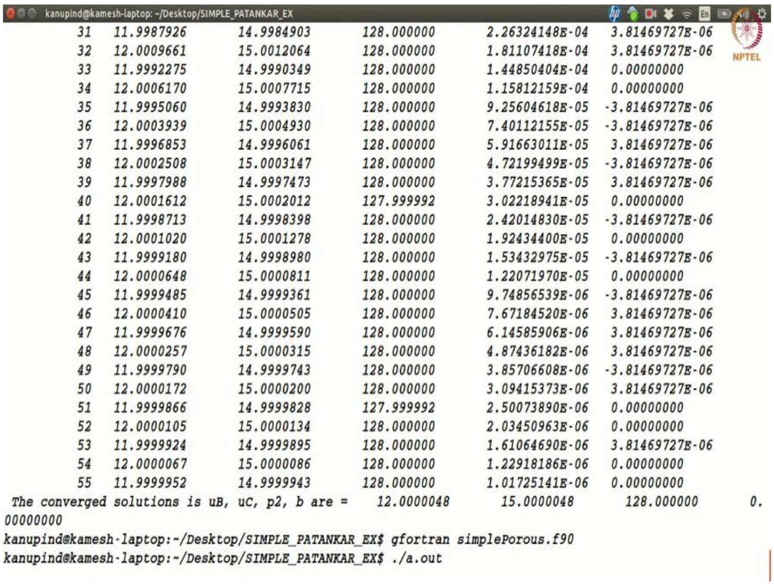

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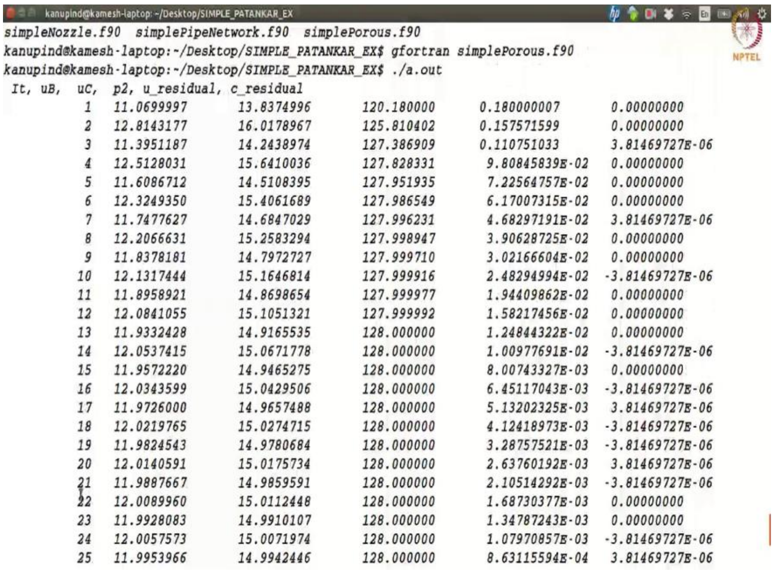
kanupind@kamesh-laptop: ~/Desktop/SIMPLE_PATANKAR_EX

```
simpleNozzle.f90   simplePipeNetwork.f90   simplePorous.f90
kanupind@kamesh-laptop:~/Desktop/SIMPLE_PATANKAR_EX$ gfortran simplePorous.f90
kanupind@kamesh-laptop:~/Desktop/SIMPLE_PATANKAR_EX$ ./a.out
```

It,	uB,	uC,	p2,	u_residual,	c_residual
1	11.0699997	13.8374996	120.180000	0.180000007	0.00000000
2	12.8143177	16.0178967	125.810402	0.157571599	0.00000000
3	11.3951187	14.2438974	127.386909	0.110751033	3.81469727E-06
4	12.5128031	15.6410036	127.828331	9.80845839E-02	0.00000000
5	11.6086712	14.5108395	127.951935	7.22564757E-02	0.00000000
6	12.3249350	15.4061689	127.986549	6.17007315E-02	0.00000000
7	11.7477627	14.6847029	127.996231	4.68297191E-02	3.81469727E-06
8	12.2066631	15.2583294	127.998947	3.90628725E-02	0.00000000
9	11.8378181	14.7972727	127.999710	3.02166604E-02	0.00000000
10	12.1317444	15.1646814	127.999916	2.48294994E-02	-3.81469727E-06
11	11.8958921	14.8698654	127.999977	1.94409862E-02	0.00000000
12	12.0841055	15.1051321	127.999992	1.58217456E-02	0.00000000
13	11.9332428	14.9165535	128.000000	1.24844322E-02	0.00000000
14	12.0537415	15.0671778	128.000000	1.00977691E-02	-3.81469727E-06
15	11.9572220	14.9465275	128.000000	8.00743327E-03	0.00000000
16	12.0343599	15.0429506	128.000000	6.45117043E-03	-3.81469727E-06
17	11.9726000	14.9657488	128.000000	5.13202325E-03	3.81469727E-06
18	12.0219765	15.0274715	128.000000	4.12418973E-03	-3.81469727E-06
19	11.9824543	14.9780684	128.000000	3.28757521E-03	-3.81469727E-06
20	12.0140591	15.0175734	128.000000	2.63760192E-03	3.81469727E-06
21	11.9887667	14.9859591	128.000000	2.10514292E-03	-3.81469727E-06
22	12.0089960	15.0112448	128.000000	1.68730377E-03	0.00000000
23	11.9928083	14.9910107	128.000000	1.34787243E-03	0.00000000
24	12.0057573	15.0071974	128.000000	1.07970857E-03	-3.81469727E-06
25	11.9953966	14.9942446	128.000000	8.63115594E-04	3.81469727E-06

It kind of took 55 iterations. So, the columns here are the first column is iteration the second one is u_B this is u velocity, u_C is the third column, then pressure is the 4th column, then we have the u residual and c residual, c residual is 0.

So, one thing you note is that for every iteration you see that the c residual that is the continuity equation that is the b term is always 0 or less than the tolerance, right; 0 0 1 E minus 6 0 that means, we are or simple algorithm is running driving these u_B, u_C fields through continuity satisfying field.

So, at every location that is 0, right, it is always 0. And finally, the solution is converged to u_B of 12. So, velocity is at B cell is 12 and velocity at a C cell is 15 and the pressure is 128, ok, alright. So, that is what we have. And what about the; what about the momentum equation? So, the if you look at the momentum residual initially this is somewhat large value that is the large in the sense this is 0.18 and you can see that it continuously decreases.

So, as it converges down as the u and B where p get u_B and u_C and p get updated it kind of starts decreasing and the momentum residual comes down to 1 E minus 6, ok. So, but the momentum equation eventually satisfies at this condition, but you can see that the continuity equation is always satisfied by the velocity fields that we got; ok.

And you can also see that the initial guess that we have given is kind of comes down from 15 and 12 or something and then it kind of comes down to these values, right. So, what was the initial guess again? Initial guess given was 15, 15 and 120.

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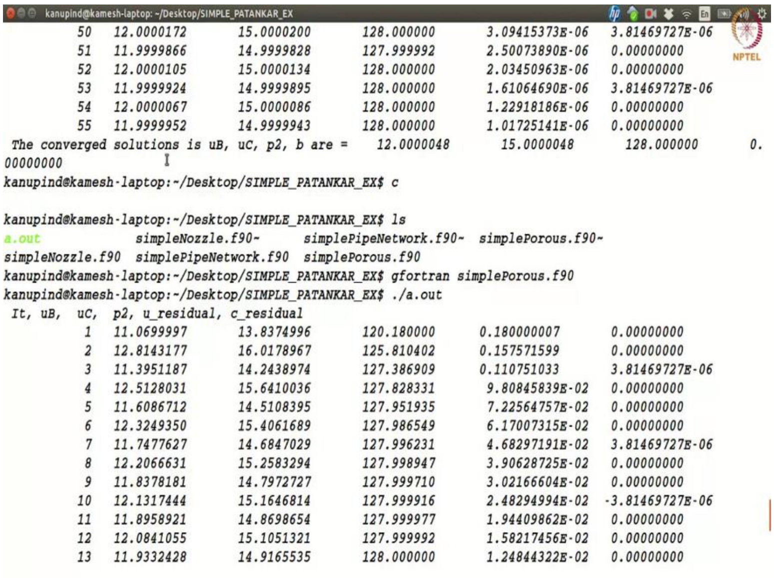

So, initially it kind of little bit oscillates here between 11 12 and then it kind of eventually reaches a value of 12, and u_C was given a value of 15 that starts off with from 15 it comes down to 14, 13.8, then 16 and then it kind of oscillates and the pressure was given as 120, so it starts with 120 and eventually it reaches a value of 128, ok. So, these are the final values for u_B, u_C and the pressure, ok.

Now, let us see, let us not use what is given by the problem that is in the book that is basically 15, 15 120. Let us use these new values that is basically u_B is 50, u_C is 100, pressure is 1000, ok. So, remember that we have converged in 55 iterations. So, we are going to rerun this problem. So, this is gfortran simplePorous and then run it, ok, boom. So, it kind of converts this in just 61 iterations and it converts the same values that is a 12, 15 and 128. Did it start with what we have given?

(Refer Slide Time: 21:58)

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It,	uB,	uC,	p2,	u_residual,	c_residual
1	8.94399834	11.1800022	345.120056	0.872228622	-1.52587891E-05
2	15.3845577	19.2306976	188.793625	4.42267513	0.00000000
3	9.96248722	12.4531078	145.022217	0.371808380	3.81469727E-06
4	14.0050488	17.5063114	132.766220	0.405778408	0.00000000
5	10.6543102	13.3178883	129.334534	0.239252210	0.00000000
6	13.2295208	16.5368996	128.373672	0.241705969	7.62939453E-06
7	11.1192265	13.8990326	128.104630	0.159514040	0.00000000
8	12.7674093	15.9592628	128.029297	0.148228243	-3.81469727E-06
9	11.4275856	14.2844820	128.008209	0.104940914	0.00000000
10	12.4837360	15.6046705	128.002304	9.24212411E-02	-3.81469727E-06
11	11.6298809	14.5373507	128.000641	6.83973953E-02	3.81469727E-06
12	12.3066969	15.3833714	128.000183	5.81962951E-02	0.00000000
13	11.7615213	14.7019024	128.000046	4.42990400E-02	-3.81469727E-06
14	12.1951351	15.2439184	128.000015	3.68670560E-02	3.81469727E-06
15	11.8467026	14.8083773	128.000000	2.85714399E-02	3.81469727E-06
16	12.1244240	15.1555300	128.000000	2.34429650E-02	0.00000000
17	11.9016104	14.8770123	128.000000	1.83773115E-02	3.81469727E-06
18	12.0794439	15.0993042	128.000000	1.49419606E-02	3.81469727E-06
19	11.9369144	14.9211435	128.000000	1.17993187E-02	-3.81469727E-06
20	12.0507679	15.0634604	128.000000	9.53796227E-03	-3.81469727E-06
21	11.9595776	14.9494715	128.000000	7.56717985E-03	3.81469727E-06
22	12.0324612	15.0405760	128.000000	6.09414931E-03	3.81469727E-06
23	11.9741106	14.9676380	128.000000	4.84946184E-03	0.00000000
24	12.0207624	15.0259533	128.000000	3.89604340E-03	0.00000000
25	11.9834232	14.9792786	128.000000	3.10631539E-03	0.00000000
26	12.0132818	15.0166025	128.000000	2.49168812E-03	0.00000000
27	11.9893875	14.9867353	128.000000	1.98894367E-03	-3.81469727E-06
28	12.0084972	15.0106220	128.000000	1.59392809E-03	0.00000000

Yes, it does it kind of starts, but then you can see that very immediately it kind of comes down to the velocities come down to 8, 9 and 11 or something, right.

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12	12.3066969	15.3833714	128.000183	5.81962951E-02	0.00000000
13	11.7615213	14.7019024	128.000046	4.42990400E-02	-3.81469727E-06
14	12.1951351	15.2439184	128.000015	3.68670560E-02	3.81469727E-06
15	11.8467026	14.8083773	128.000000	2.85714399E-02	3.81469727E-06
16	12.1244240	15.1555300	128.000000	2.34429650E-02	0.00000000
17	11.9016104	14.8770123	128.000000	1.83773115E-02	3.81469727E-06
18	12.0794439	15.0993042	128.000000	1.49419606E-02	3.81469727E-06
19	11.9369144	14.9211435	128.000000	1.17993187E-02	-3.81469727E-06
20	12.0507679	15.0634604	128.000000	9.53796227E-03	-3.81469727E-06
21	11.9595776	14.9494715	128.000000	7.56717985E-03	3.81469727E-06
22	12.0324612	15.0405760	128.000000	6.09414931E-03	3.81469727E-06
23	11.9741106	14.9676380	128.000000	4.84946184E-03	0.00000000
24	12.0207624	15.0259533	128.000000	3.89604340E-03	0.00000000
25	11.9834232	14.9792786	128.000000	3.10631539E-03	0.00000000
26	12.0132818	15.0166025	128.000000	2.49168812E-03	0.00000000
27	11.9893875	14.9867353	128.000000	1.98894367E-03	-3.81469727E-06
28	12.0084972	15.0106220	128.000000	1.59392809E-03	0.00000000
29	11.9932070	14.9915094	128.000000	1.27327978E-03	-3.81469727E-06
30	12.0054379	15.0067968	128.000000	1.01976295E-03	3.81469727E-06
31	11.9956522	14.9945641	128.000000	8.15097650E-04	3.81469727E-06
32	12.0034809	15.0043516	128.000000	6.52574061E-04	-3.81469727E-06
33	11.9972172	14.9965200	128.000000	5.21971495E-04	3.81469727E-06
34	12.0022268	15.0027838	128.000000	4.17648786E-04	0.00000000
35	11.9982195	14.9977741	128.000000	3.33958713E-04	0.00000000
36	12.0014248	15.0017805	128.000000	2.67150463E-04	3.81469727E-06
37	11.9988604	14.9985752	128.000000	2.13657069E-04	0.00000000
38	12.0009117	15.0011396	128.000000	1.70973290E-04	0.00000000
39	11.9992714	14.9990892	128.000000	1.36757299E-04	0.00000000
40	12.0005827	15.0007286	128.000000	1.09283152E-04	0.00000000

And the pressure is very high, this is basically the pressure correction has come down from 1000 to 345 in like one step, right. And then you can again see that the momentum residual comes down and the continuity is always satisfied and so on, ok.

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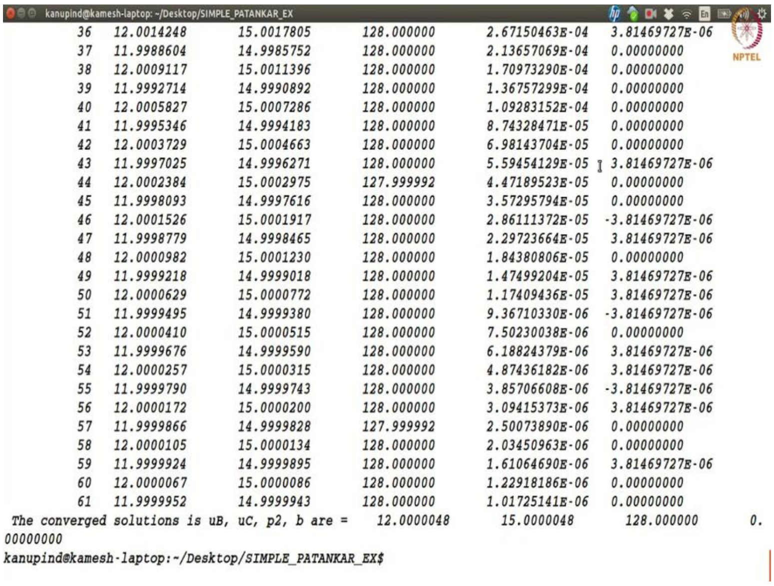

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36	12.0014248	15.0017805	128.000000	2.67150463E-04	3.81469727E-06
37	11.9988604	14.9985752	128.000000	2.13657069E-04	0.00000000
38	12.0009117	15.0011396	128.000000	1.70973290E-04	0.00000000
39	11.9992714	14.9990892	128.000000	1.36757299E-04	0.00000000
40	12.0005827	15.0007286	128.000000	1.09283152E-04	0.00000000
41	11.9995346	14.9994183	128.000000	8.74328471E-05	0.00000000
42	12.0003729	15.0004663	128.000000	6.98143704E-05	0.00000000
43	11.9997025	14.9996271	128.000000	5.59454129E-05	3.81469727E-06
44	12.0002384	15.0002975	127.999992	4.47189523E-05	0.00000000
45	11.9998093	14.9997616	128.000000	3.57295794E-05	0.00000000
46	12.0001526	15.0001917	128.000000	2.86111372E-05	-3.81469727E-06
47	11.9998779	14.9998465	128.000000	2.29723664E-05	3.81469727E-06
48	12.0000982	15.0001230	128.000000	1.84380806E-05	0.00000000
49	11.9999218	14.9999018	128.000000	1.47499204E-05	3.81469727E-06
50	12.0000629	15.0000772	128.000000	1.17409436E-05	3.81469727E-06
51	11.9999495	14.9999380	128.000000	9.36710330E-06	-3.81469727E-06
52	12.0000410	15.0000515	128.000000	7.50230038E-06	0.00000000
53	11.9999676	14.9999590	128.000000	6.18824379E-06	3.81469727E-06
54	12.0000257	15.0000315	128.000000	4.87436182E-06	3.81469727E-06
55	11.9999790	14.9999743	128.000000	3.85706608E-06	-3.81469727E-06
56	12.0000172	15.0000200	128.000000	3.09415373E-06	3.81469727E-06
57	11.9999866	14.9999828	127.999992	2.50073890E-06	0.00000000
58	12.0000105	15.0000134	128.000000	2.03450963E-06	0.00000000
59	11.9999924	14.9999895	128.000000	1.61064690E-06	3.81469727E-06
60	12.0000067	15.0000086	128.000000	1.22918186E-06	0.00000000
61	11.9999952	14.9999943	128.000000	1.01725141E-06	0.00000000

```
 The converged solutions is uB, uC, p2, b are =    12.0000048      15.0000048      128.000000       0.
00000000
kanupind@kamesh-laptop:~/Desktop/SIMPLE_PATANKAR_EX$
```

So, that is the as far as the first problem is concerned, ok, alright. So, let us see, let us now look at the next problem, alright that is basically the simple porous is done.

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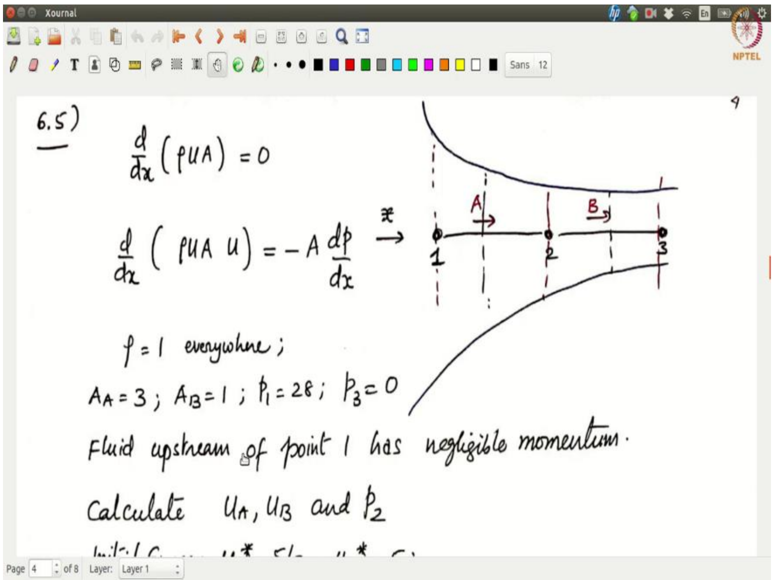

So, what is the second problem? The second problem was the nozzle problem, right. So, in the nozzle problem what we have is basically there is a nozzle that is given and whose continuity and the momentum equations are given. So, the continuity equation is d by dx of rho u A equals 0, and the momentum equation is d by dx of rho u A u equals minus A dp dx, right. So, essentially that is what is given. And we have discretize this equation and density is given as 1 everywhere A A equals, A_B equals, 3 and 1, respectively.

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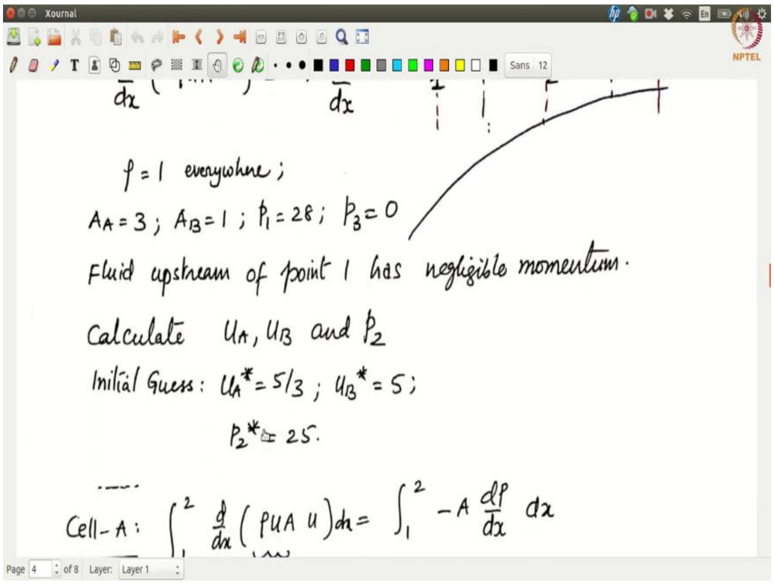

And the pressure boundary condition is given that is 28 and 0, right. And we formulated this and the initial guess is also given, ok.

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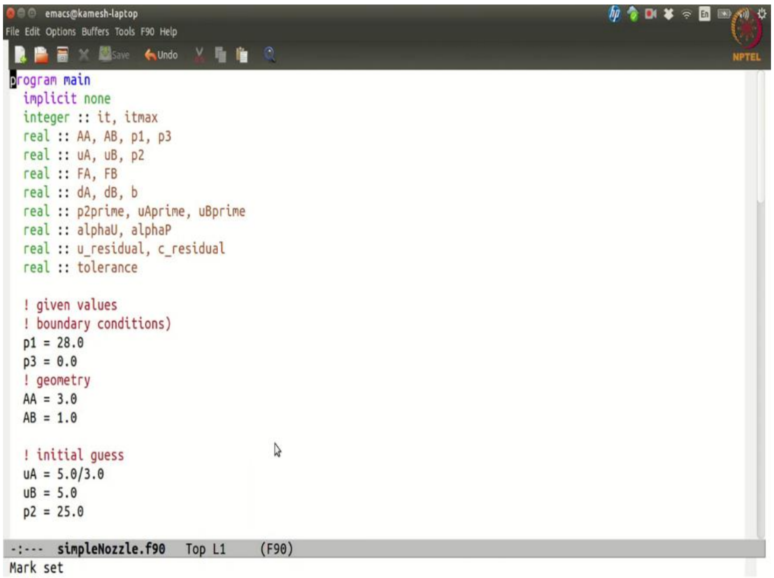

So, let us see what is the second problem looks like simple nozzle. So, again we have these are the definitions declaration of the variables that is we have AA, AB, these are the cross sectional areas and then in P_1 and P_3 are the pressures, and then uA, u_B are the velocities, right, P_2 is the pressure essentially we have to calculate what are these values, ok. And then FA are the flow rates, FA and FB; dA dB are the coefficients that we get in the prime equations, ok.

Similarly, we have p2prime, uAprime and u_B' these are the velocity and the pressure corrections and then we have alphaU and alphaP, ok, alright. So, if you see the given boundary conditions are P_1 equals 28, P_3 equals 0, right. These are already given. So, P_1 equals 28, P_3 equals 0 that is already given and the geometry is given as cross sectional areas let us 3 and 1, right. This is 3 and this is 1, A A is 3, A_B is 1 and the initial guess is given as five-thirds for u A, u_B equals 5.

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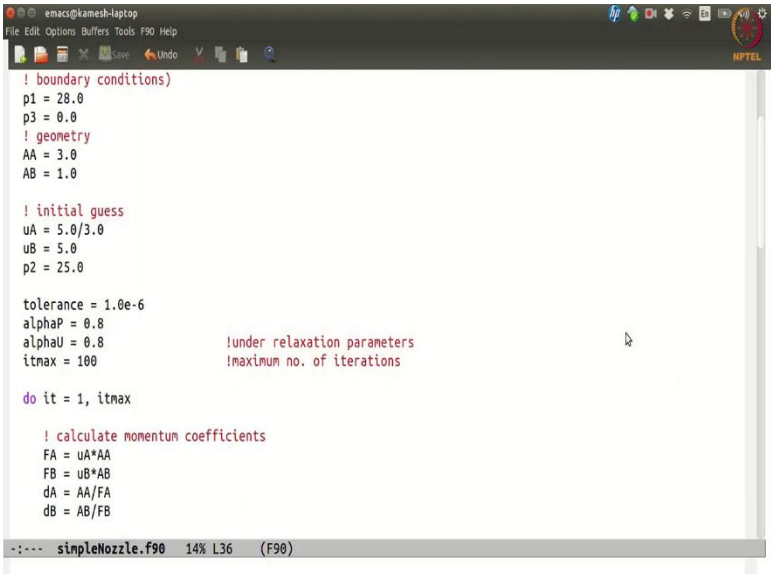

So, we take this as 5, and P_2 is 25, ok. So, this is 25, ok, alright. That is the initial guess for pressure, ok. The tolerance value is 1 E minus 6. Again, I set it to 1 E minus 6 bar equations to kind of converge. Then we have α_P equals 0.8, α_u equals 0.8, these are basically your under relaxation parameters, ok.

And then again I said these are the iteration max is maximum number of iterations before which we want to we hope to come out of the loop. So, this is the loop. So, do iteration equals 1 to it max. So, we calculate the momentum coefficients, ok.

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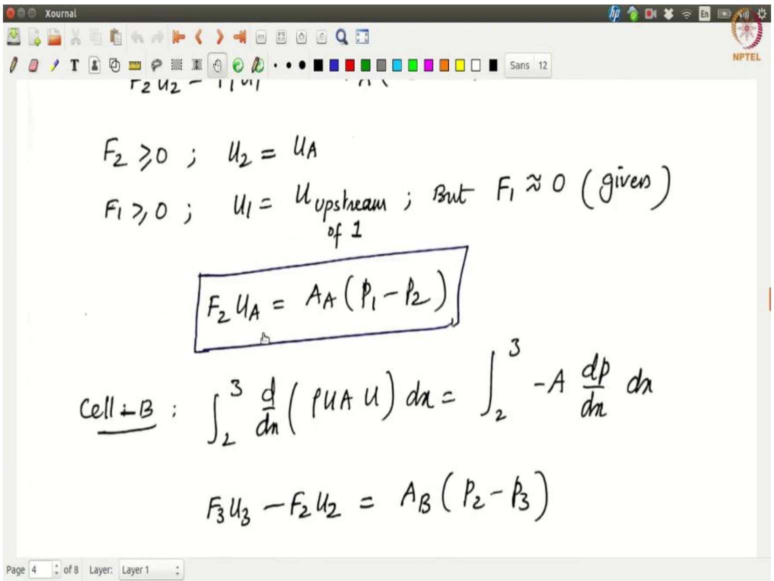

So, the momentum coefficients are basically we have if you go back to the problem what are the coefficients for momentum? Basically, your F 2 that is nothing, but your FA, right, so ok. For the cell A we have FA and FB, right.

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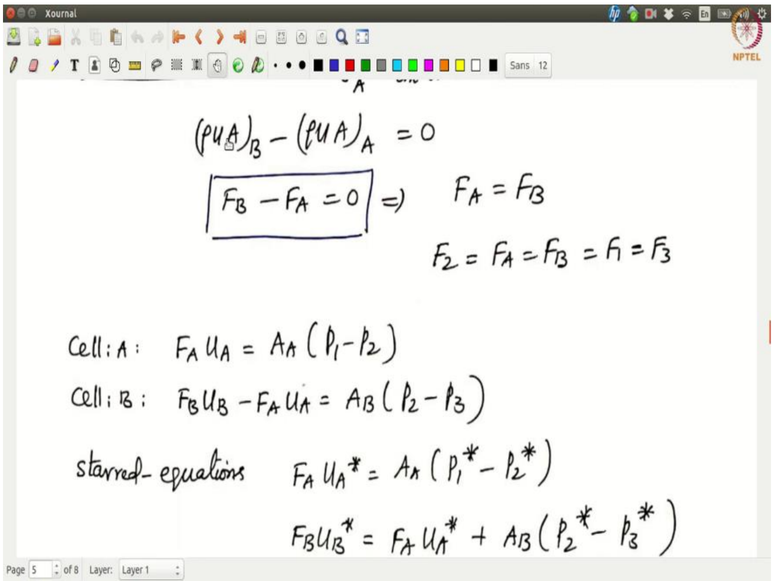

So, we look at what is F A. F A is your is basically u_A times AA, right velocity times area and then we have rho equals 1 similarly FB is u_B AB. So, that is the coefficients here.

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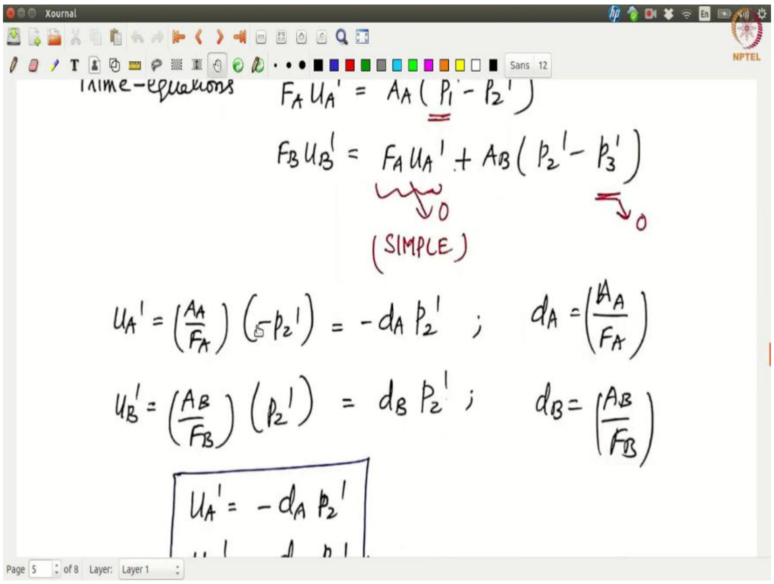

Then, d A which we get here, this we defined as d A that is AA by FA. So, we calculate we assign d A equals AA by F A and d B equals A_B by F B, right. So, we have those two values as well.

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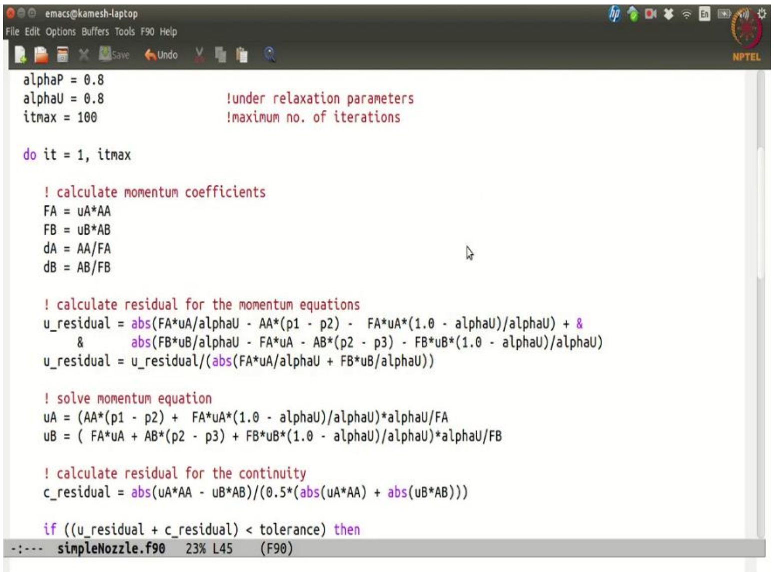

And then we use the this is again the residuals. So, basically taking the entire momentum equation to one side; this is what we have, ok. We will come back to this little later.

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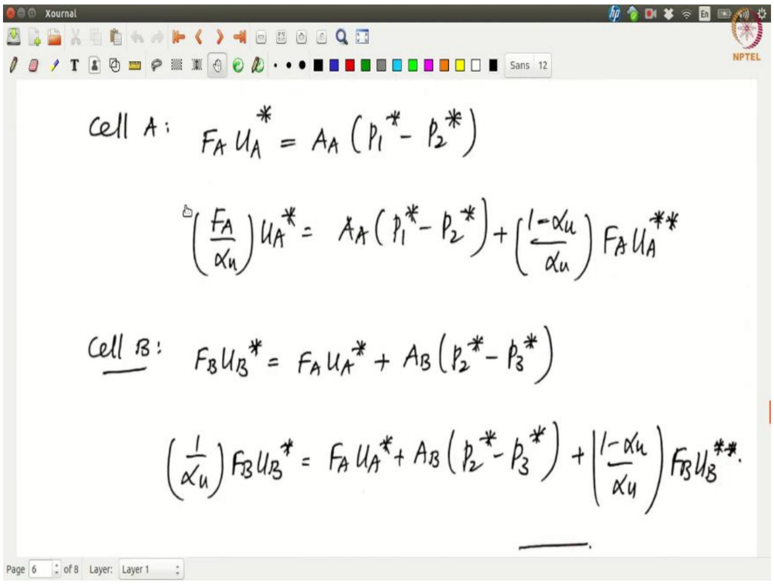

So, what is the momentum equation? Momentum equation is we have to look for the under relaxed equation that we have written that is basically this value, right that is your u_A^* equals AA times P_1^* minus P_2^* that is AA times P_1^* minus P_2^* plus this is basically your under relaxation component that is 1 minus alpha by alpha times FA u_A.

And then this entire thing has to be entire equation has to be multiplied with alphaU times F A, multiply with alphaU divided by F A to get what is u A star, ok. So, that is how you get the u A star. And u_B^* is also similar u_B^* has basically F A u A that is your F A u A here, right, here. Then the second term is a B times P_2 minus P_3. So, that is your AB times P_2 minus P_3 plus this thing is again coming from the under relaxation.

And what we have is a multiply this with this entire thing with alphaU by divided by F B to get value of F B star, ok. So, that is u_B equals FA u_A plus AB times P_2 minus P_3 plus the under relaxation component and then we have multiplication with alphaU times F B, ok, alright, ok. So, just got it here, ok; so, this is your u_B, fine.

Then, again the continuity equation. What was the continuity equation? Continuity equation was; continuity equation was F B minus FA, right that means, rho equals 1. So, this is u_B A_B minus u A A A, right. So, if your continuity residual would be u A A A minus u_B A_B, again I am dividing with whatever is the continuity value average value such that this becomes kind of normalized, ok.

Similarly, now we can understand how this residual is calculated. This is the absolute value for cell A, ok. So, this is basically the absolute value for cell A, right. So, this is F A u A by alphaU. This is the left hand side equation, left hand side part of the equation and this is the right hand side brought to the left hand side, right.

So, essentially we have this is for cell A and we take the absolute value similarly we take the other value that is for the B cell, we have F B u_B by alphaU minus FA u_A all these things basically this is your first cell B, ok.

And again we use, we normalize with the central coefficient that is FA u_A by alphaU and FB u_B by alphaU, to basically get to normalize this value, ok. So, that is what we are doing.

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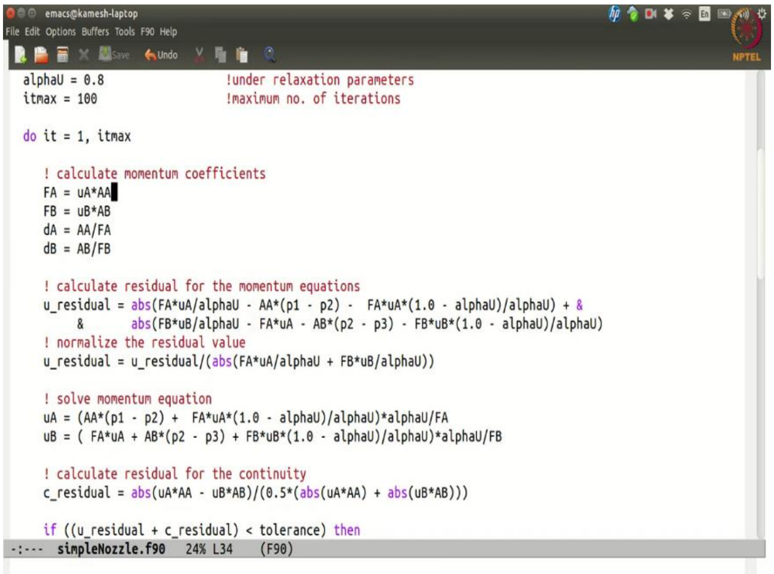

So, that means, basically this is to normalize the residual value, such that it is not going to be very small or very large, ok. So, that is the residual for velocity. So, that means, if we go back to the algorithm we started off with calculating the coefficients, then we computed this all the momentum equations then we have to solve for the pressure correction equation. So, P_2' is basically coming from our equation.

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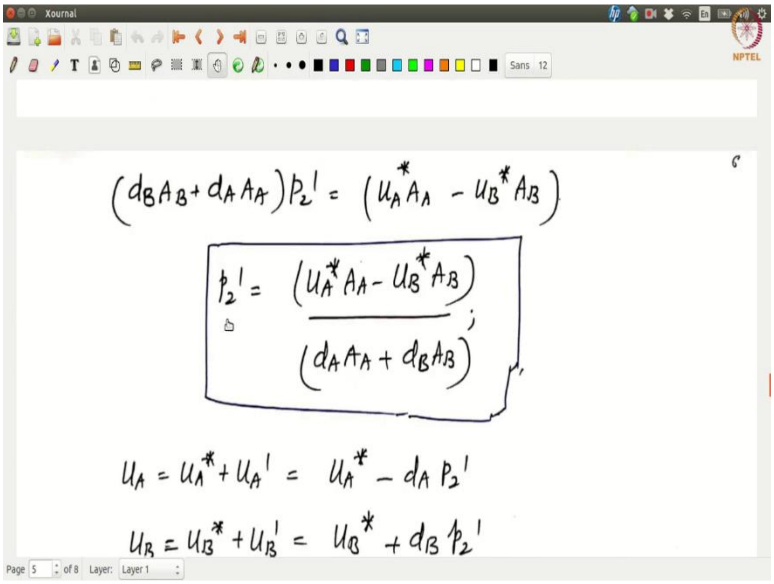

What was P_2'? P_2' was, this guy, right this is P_2' equals u star A A minus u_B^* A_B upon d A A A plus d B A_B, ok. So, that is what we have in the program. This is P_2' equals u_A AA minus u_B AB divided by dA AA plus dB AB, ok. So, that is what we have. Again, we have only one cell. So, we do not have to solve for a system. We just plug in what is the value of u_A^* and u_B^* and calculate what is P_2', alright, ok.

Then, once you have the pressure correction value, then you can use pressure correction to correct the velocities, right. So, what is the formula for correcting the velocity corrections? Velocity corrections are basically u A prime equals minus d A P_2', so that is uAprime equals minus d A p2prime and u_B' equals plus d B p2prime, right that is what we have from here, u_B' equals plus d B p2prime.

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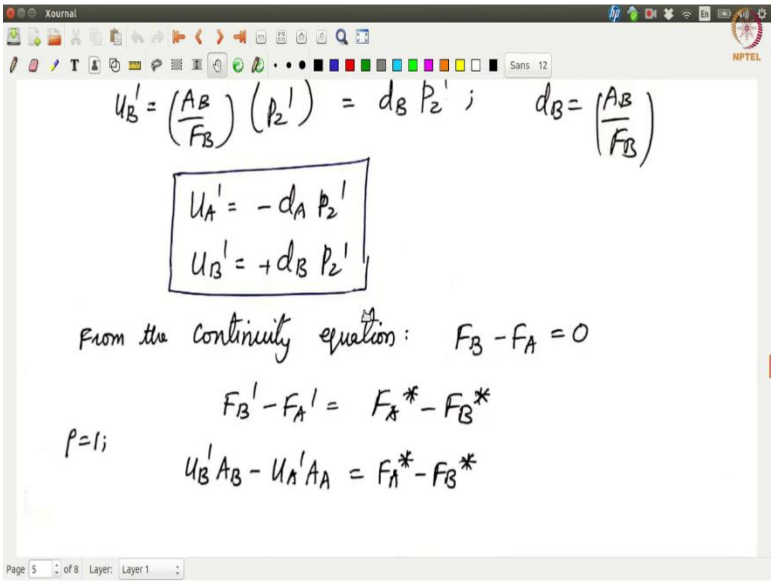

So, we correct the velocities, then we can update the velocity u_A as u_A^* plus uA prime and u_B as u_B^* plus u_B' and the pressure as P_2 equals P_2 plus $\alpha_P P_2'$, ok. Again, we check for convergence. So, this is basically whether continuity is satisfied or not.

So, we look for u_A^* AA minus u_B^* AB, this is the value of our continuity equation, right. So, with whatever corrected value, so this should satisfy continuity at each and every iteration, ok. Then, we print out the b value that is the continuity value u_A, u two let us print out this at the end, ok. So, that means, we print out what is u_A, u_B, and pressure, and b, ok.

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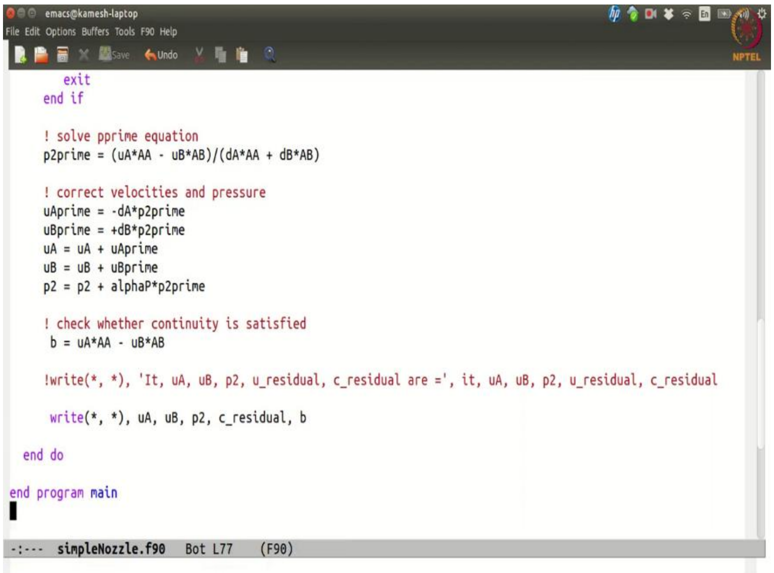

Now, if you want you can also print out what is the c residual, alright. So, the c residual can also be printed, ok. So, that is the overall algorithm, and then we hope to exit this before the iterations finish through this condition, right. So, essentially this is the condition, this is basically tells you that if your residual some of the residuals is less than the tolerance, then we say we have reached the convert solution and the solution is blah blah blah, that is u_A u_B and P_2 and then we exit, ok.

So, but we have to make sure that the we do not run out of the iterations as such, ok. Let us also print what is the iteration count here. So, that is i, right, ok. So, we should know that we have not reached more than 100, ok. Let us see if we can run this program. So, this is simple nozzle. So, gfortran is simple nozzle, so problem here has no implicit type, ok. So, this is there is no i here, this is basically iteration, ok, alright, ok.

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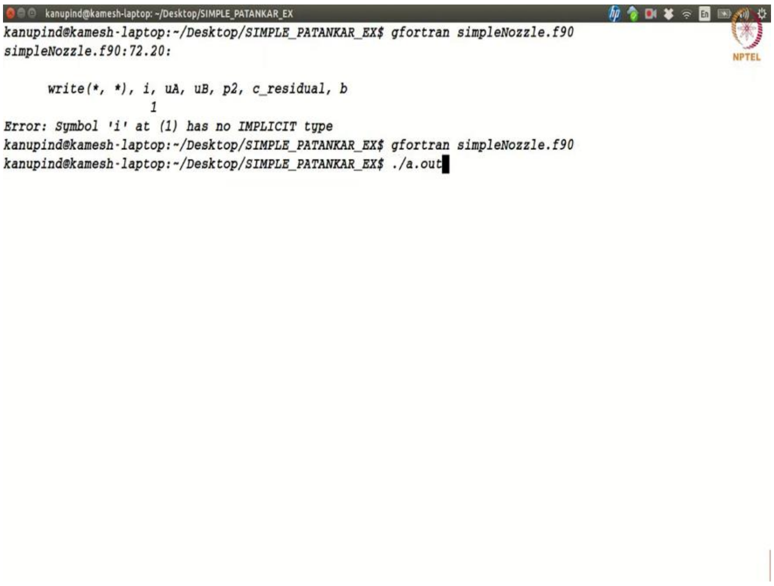

Successfully compiled, then let us run it this is dot slash a dot out, ok, very good.

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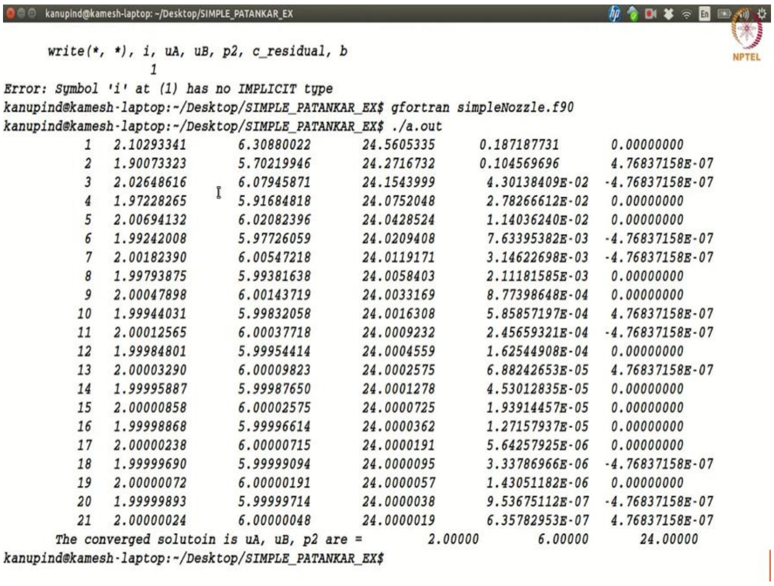

So, our program converged in 21 iterations and you can see this is the value of the velocity and at u A and this is the value of the velocity at u_B. So, we converged it to 2 and 6, and this is the value of pressure, this is 24 is the value of the pressure. And again, you can see that the continuity residuals are always 0 or somewhere small value, whereas, the momentum residuals, right; that is u; oh this should be u residual, ok. I will go to it again, ok.

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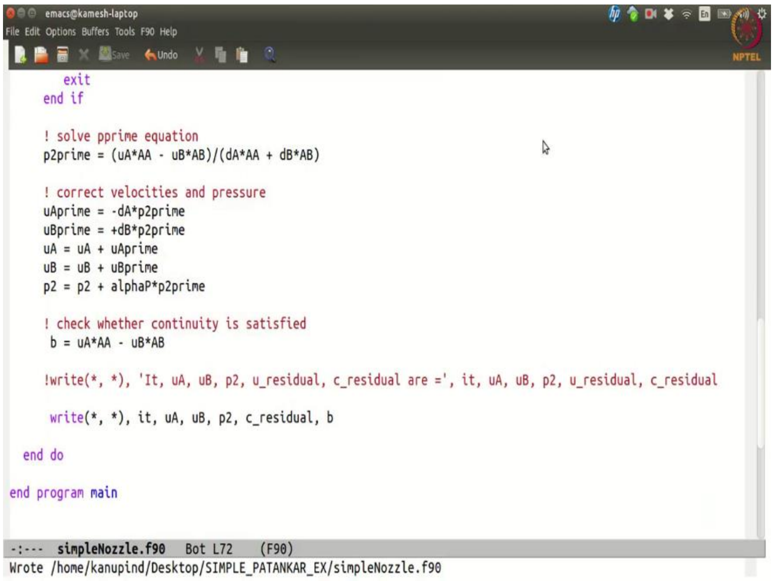

So, whereas, you can see that the u residuals are always coming down, right. Momentum is always coming down whereas, the continuity b equation is always equal to 0, ok, fine.

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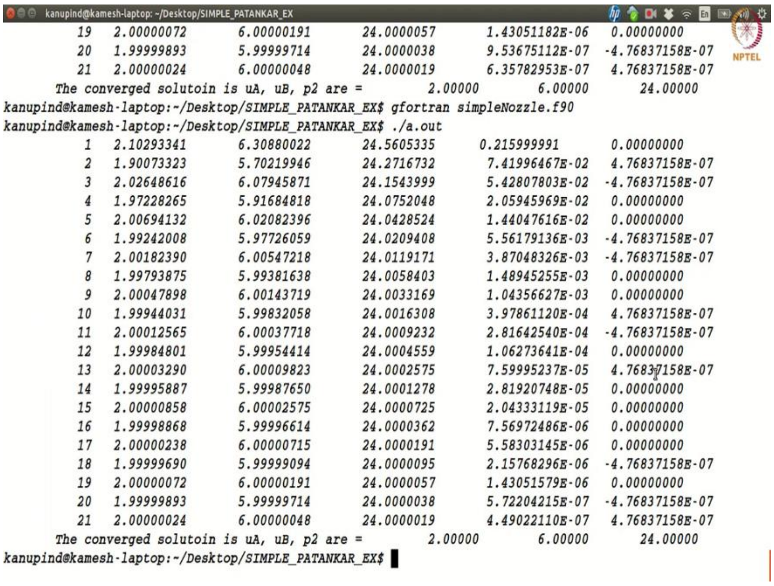

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kanupind@kamesh-laptop: ~/Desktop/SIMPLE_PATANKAR_EX
          19   2.00000072       6.00000191       24.0000057       1.43051182E-06   0.00000000
          20   1.99999893       5.99999714       24.0000038       9.53675112E-07  -4.76837158E-07
          21   2.00000024       6.00000048       24.0000019       6.35782953E-07   4.76837158E-07
       The converged solutoin is uA, uB, p2 are =       2.00000       6.00000       24.00000
kanupind@kamesh-laptop:~/Desktop/SIMPLE_PATANKAR_EX$ gfortran simpleNozzle.f90
kanupind@kamesh-laptop:~/Desktop/SIMPLE_PATANKAR_EX$ ./a.out
           1   2.10293341       6.30880022       24.5605335      0.215999991       0.00000000
           2   1.90073323       5.70219946       24.2716732       7.41996467E-02   4.76837158E-07
           3   2.02648616       6.07945871       24.1543999       5.42807803E-02  -4.76837158E-07
           4   1.97228265       5.91684818       24.0752048       2.05945969E-02   0.00000000
           5   2.00694132       6.02082396       24.0428524       1.44047616E-02   0.00000000
           6   1.99242008       5.97726059       24.0209408       5.56179136E-03  -4.76837158E-07
           7   2.00182390       6.00547218       24.0119171       3.87048326E-03  -4.76837158E-07
           8   1.99793875       5.99381638       24.0058403       1.48945255E-03   0.00000000
           9   2.00047898       6.00143719       24.0033169       1.04356627E-03   0.00000000
          10   1.99944031       5.99832058       24.0016308       3.97861120E-04   4.76837158E-07
          11   2.00012565       6.00037718       24.0009232       2.81642540E-04  -4.76837158E-07
          12   1.99984801       5.99954414       24.0004559       1.06273641E-04   0.00000000
          13   2.00003290       6.00009823       24.0002575       7.59995237E-05   4.76837158E-07
          14   1.99995887       5.99987650       24.0001278       2.81920748E-05   0.00000000
          15   2.00000858       6.00002575       24.0000725       2.04333119E-05   0.00000000
          16   1.99998868       5.99996614       24.0000362       7.56972486E-06   0.00000000
          17   2.00000238       6.00000715       24.0000191       5.58303145E-06   0.00000000
          18   1.99999690       5.99999094       24.0000095       2.15768296E-06  -4.76837158E-07
          19   2.00000072       6.00000191       24.0000057       1.43051579E-06   0.00000000
          20   1.99999893       5.99999714       24.0000038       5.72204215E-07  -4.76837158E-07
          21   2.00000024       6.00000048       24.0000019       4.49022110E-07   4.76837158E-07
       The converged solutoin is uA, uB, p2 are =       2.00000       6.00000       24.00000
kanupind@kamesh-laptop:~/Desktop/SIMPLE_PATANKAR_EX$
```

So, we can of course, again play with these values the initial guess values and see whether the program runs or not, ok. I leave that for you to do, ok. So, that is about the second problem that is in flow through a nozzle, ok, alright. Now, let us look at the third problem that we formulated in the last class that is basically now in these both problems we have non-linearity, right.

In the first problem, in the porosity case we have nonlinearity in the source term. In the second case, we have non-linearity through the convection term, right. So, that is why we had to do so many iterations like some 20 iterations or so, fine.

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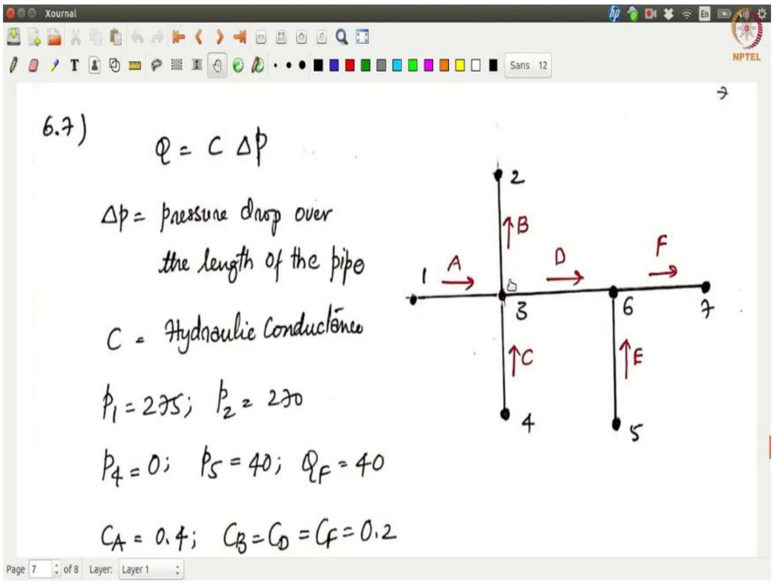

Now, let us move on to the third problem. Third problem was a flow through a residential pipe network, right. So, essentially we have Q is given as C times delta p, where C is the hydraulic conductance, delta p is the pressure drop over the length of the pipe. And what we were given is we were given this pipe network with the flow rates and the pressure stored at the 1, 2, 3, 4, all the way to 7 and the velocity stored at A_B all the way to F, right.

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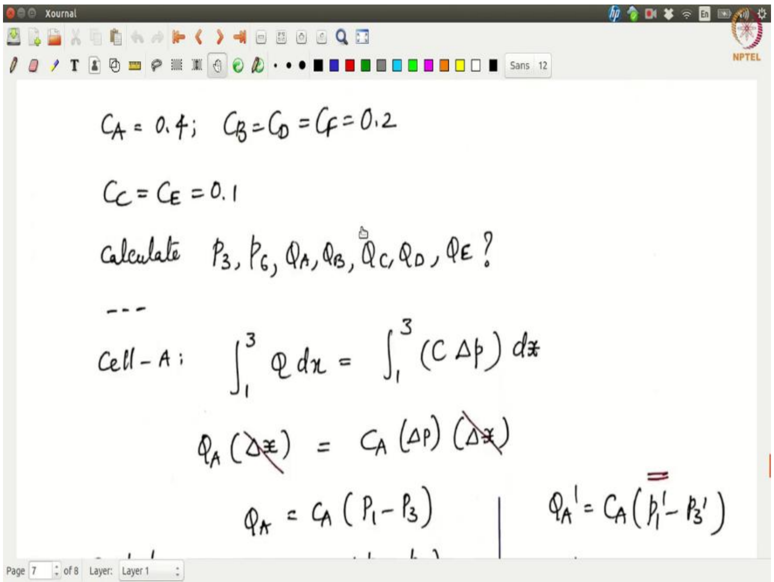

And we were also given some of these pressures that is P_1 is given as 275, P_2 is given as 70, P_4 as 0, P_5 as 40 and so on and Q_F as 40, ok, alright. So, and also the of course, the values of all the hydraulic conductance are given C_A to C_F, and we were ask to calculate what is the pressure at this P_3 and P_6; and what is the flow rate through Q_A, Q_B all the way to Q_E, ok. So, we have to calculate what are all the flow rates.

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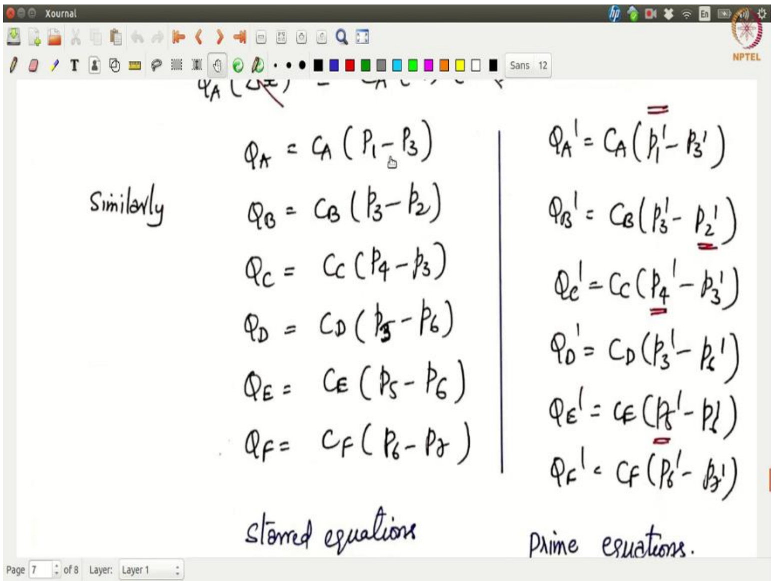

We have formulated the problem using finite volume method, and we said these are the momentum equations; these are the corrections, pressure corrections in terms of the flow corrections.

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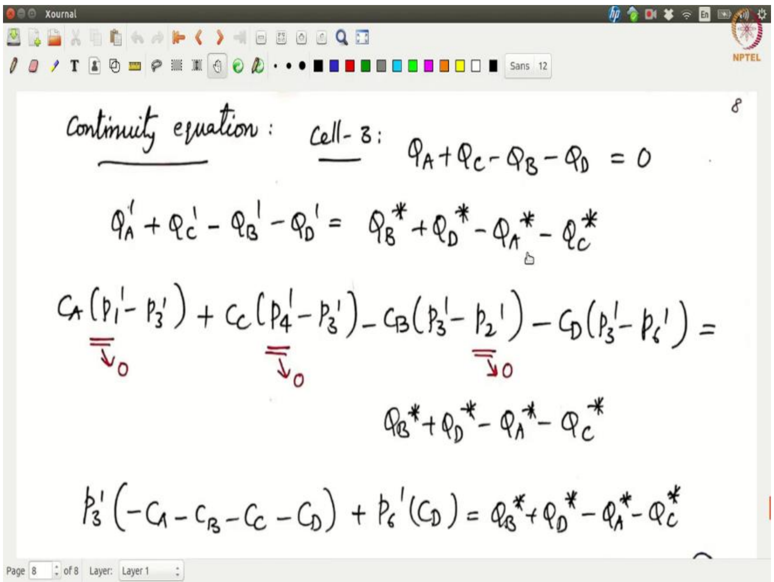

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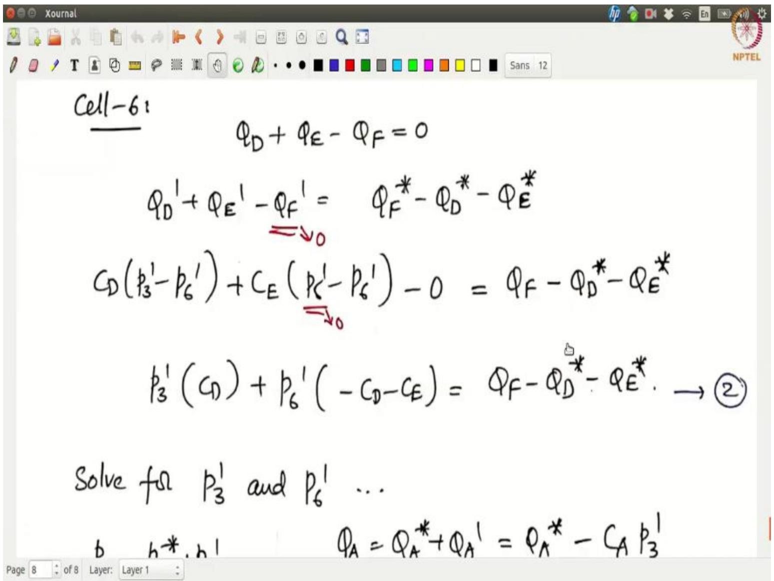

Then, we wrote the continuity equation or the mass conservation equation, and we said that the continuity equation will give you one equation in terms of P_3' and P_6' and another equation at junction 6 as another equation in terms of P_3' and P_6', with the right hand side as known values, ok. Now, we have two equations and two unknowns, right P_3' and P_6', alright.

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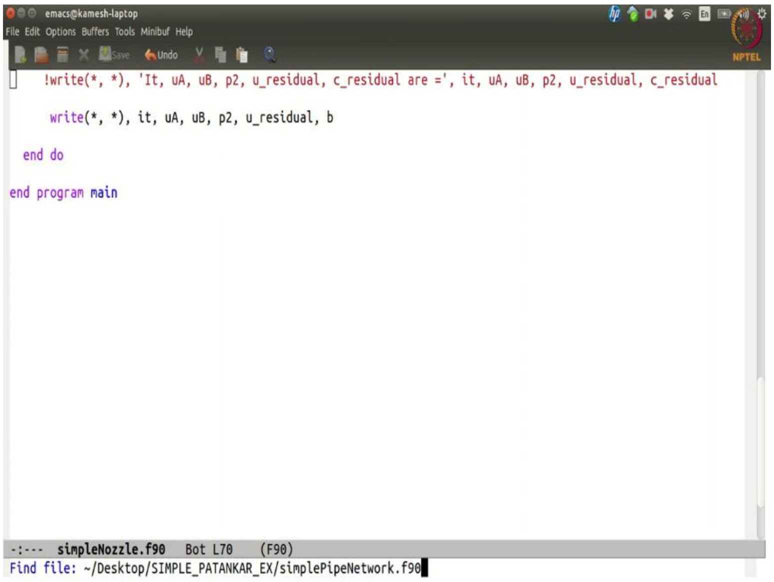

Let us look at the corresponding code. So, the corresponding code is a simple pipe network, ok.

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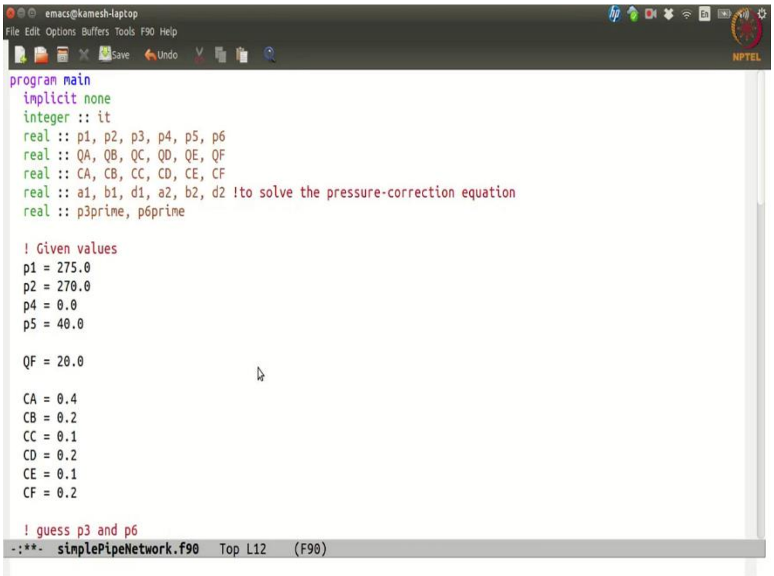

So, page up, ok. So, again this is the main program. So, these are all the variables that we define, P_1 to P_6 the pressures, Q_A to Q_F the flow rates, C_A to C_F the hydraulic conductance values, right. And then a 1, b 1, d 1, a 2, b 2, d 2 are basically the coefficients that are written.

These are basically these are to solve the we had two equations and two unknowns, right, so to solve the pressure correction, ok, pressure correction equation, right. So, these are the coefficients we will see them later. And then of course, these are the P_3' and P_6' are the pressure, correction values at 3 and 6 locations, ok, fine.

Then, what was the initial; what was the given values? Given values for P_1 P_2 are given as 275 and 270. So, if you look at the values P_1 is 275, P_2 is 270, P_4 is 0, all right, P_4 is 0, P_5 is 40, and then Q_F is 20, right, Q_F is 20. Then, C_A to C_A to C_F all these values I have written out here, C_A, C_B, all the way to C_F, C_F is 0.2, ok. So, these are the hydraulic conductance values.

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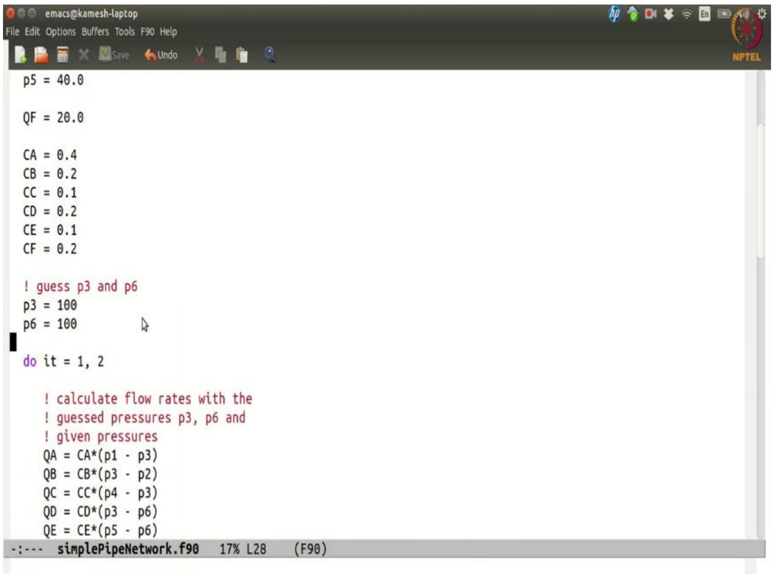

Now, initial guess for P_3 and P_6 probably may not be given in the problem, so I have taken it as 100 and 100, ok, of course, you can now see if you want to have a different value as the initial guess, ok. Now, ok, here I have only I will let me put I will then put like two iterations, ok, but you can put more iterations, but they are not required. We will see why we do not need so many iterations in this particular problem, ok.

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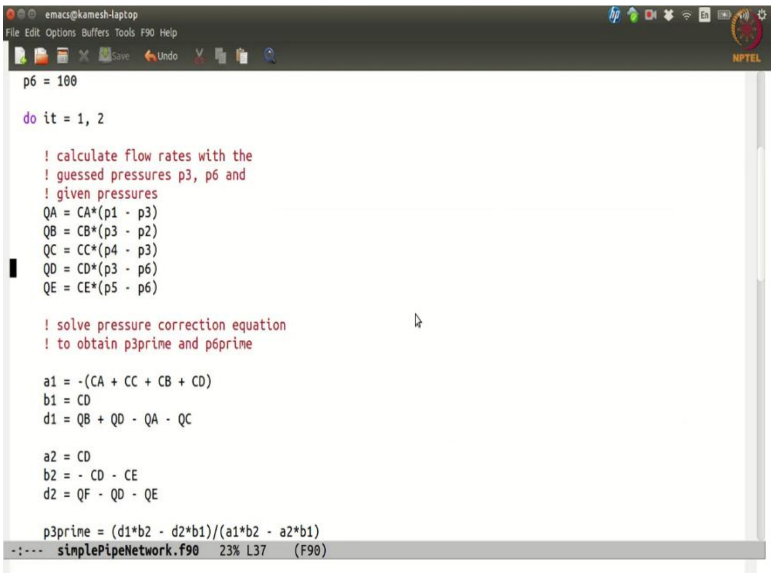

So, if we look at the equations what we had was, ok; so, the first step is write the solve the momentum equation. Momentum equations are Q_A equals C_A times P_1 minus P_3. So, that is what we have. This is Q_A equals C_A times P_1 minus P_3. And similarly, Q_B equals C_B times P_3 minus P_2 that is the other equation. Similarly, Q_C equals C_C times P_4 minus P_3 that is what we have here; Q_C is C_C times P_4 minus P_3 and Q_D and Q_E, ok.

So, with the initial guess values for pressure or the using the given values for pressure we calculate what is the flow rates, ok. Then, what we do is basically we got these flow rates, then we have to solve the pressure correction equation, ok. In order to do the pressure correction equation, we realize that we had two equations set two unknowns, ok. So, what we do is we basically write out the coefficients. So, I am writing these coefficients as basically a1, b1 and d1 and sum a2 a2, b2 and d2, right.

So, that I can solve two equations and two unknowns; so, a1 is basically you are C_A, C_A plus C_B plus C_C plus C_D with all minus. So, that is what I have here, right. a1 is minus of a C_A plus C_C plus C_B plus C_D. Similarly, b1 is your C_D, ok. And d1 is your right hand side that is Q_B^* Q_D^* minus Q_A^* minus Q_C^*, ok. So, these values are given. These are now taken from here.

Then, for the second equation we have this coefficient we are writing it as a2. So, a2 is C_D, b2 is minus C_D minus C_A, and d2 is Q_F minus Q_D^* minus Q_E^*, ok. So, that is the value.

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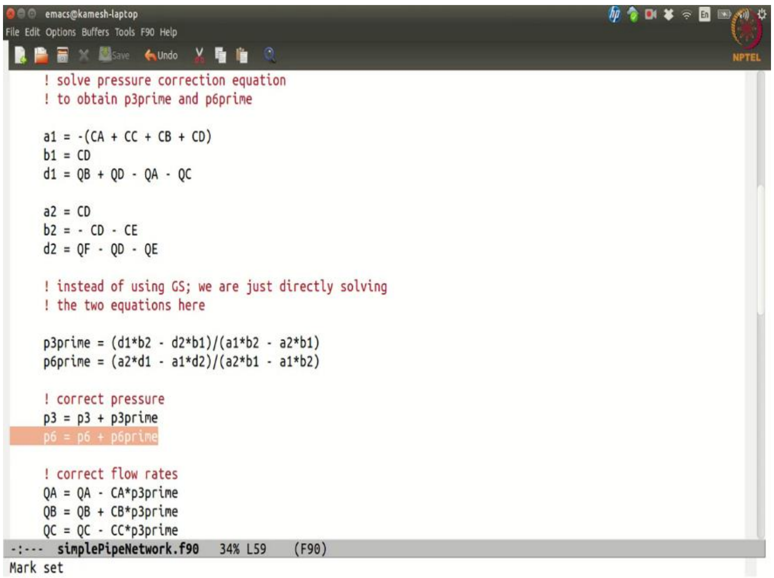

Then, of course, I can basically I have two equations two unknowns I can write what is P_3' in terms of these coefficients. So, I would just eliminate the variables and calculate this as d1 b 2 minus basically a proper proportion of these d 1 b 2 minus d 2 b 1 by a 1 b 2 minus a 2 b 1 that gives me what is P_3'. Similarly, I can write down what is P_6' as a 2 d 1 minus a 1 d 2 upon a 2 b 1 minus a 1 b 2, ok.

So, basically we are just solving, instead of using Gauss-Seidel we are just directly solving the two equations here, ok, alright. So, that is what we do. Then, once you obtain what is P_3' and P_6' we can just correct them, right. So, basically correct P_3 equals P_3 P_3^* plus P_3', similarly P_6 equals P_6^* plus P_6', ok. So, we have done that.

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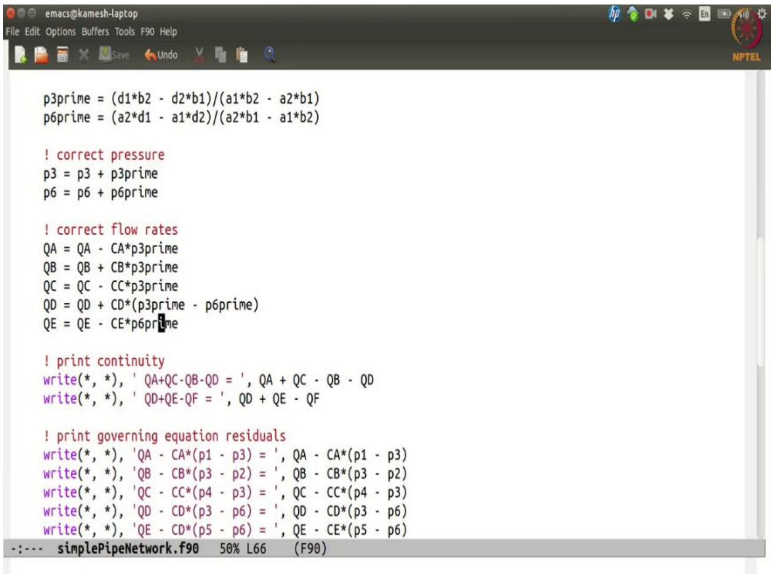

Then, what do we do? Then we correct the flow rates that is Q_A equals Q_A^* plus Q_A', but we know that what is Q_A'. Q_A' is nothing but, Q_A' is nothing, but C_A times this is 0 minus P_3'. So, this is minus C_A times P_3'.

Similarly, we know we can correct what is Q_B; Q_C that is Q_C minus C_C times P_3' because P_4' is 0, because P_4 is given as a pressure boundary condition and Q_D equals Q_D^* plus Q_D' that is C_D times P_3' minus P_6' that is this value. Then, Q_E equals Q_E plus Q_E^* plus Q_E' Q_E' is C_A times P_5' is 0.

So, this is minus C_A P_6', ok. So, this is minus C_A P_6', fine. Then, we print the continuity equation. Continuity equation is two equations, one for each cell junctions 3 and 6. So, this is Q_A plus Q_C minus Q_B minus Q_D and the second junction at 6 is Q_D plus Q_E minus Q_F, ok. Then, we print the residuals for the momentum equations, right. These are the residuals for the momentum equation.

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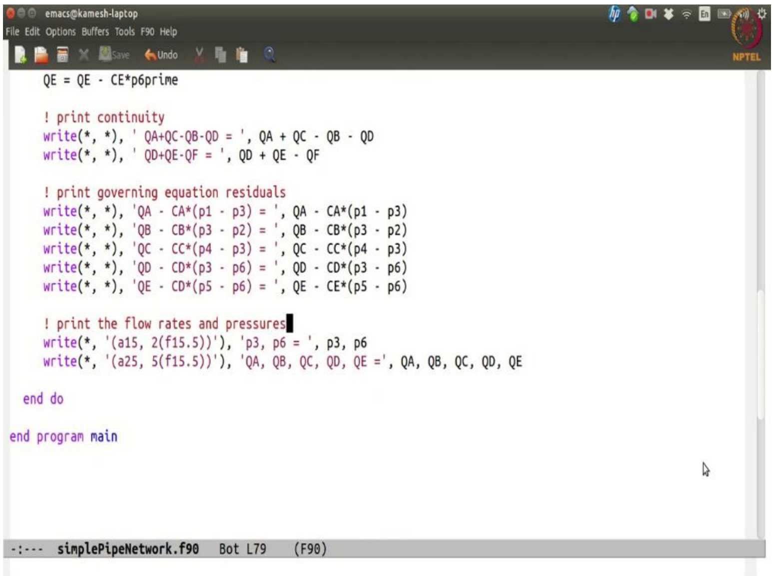

Now, if these one also satisfied together with the continuity equations that means, if you satisfy all of these, that means we have converged our solution, ok. If that is the case we print what is the pressures P_3 and P_6 and the flow rates these are basically what is asked in the problem to calculate, right.

Remember, if you go back to the problem we were asked to calculate all these pressures and the flow rate. So, that is these P_3, P_6, Q_A, Q_B, Q_C, Q_D, and Q_E, ok. So, that is the program.

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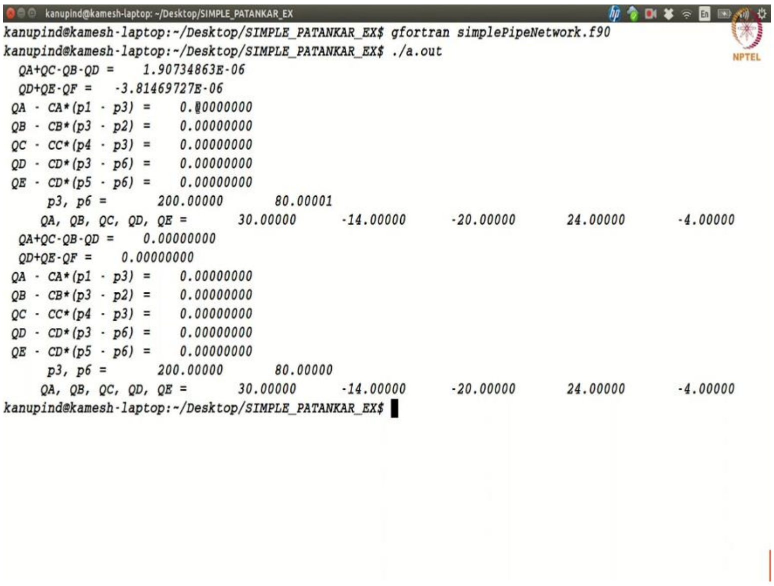

So, let us try to run this. This is simple pipe network. So, gfortran simplePipeNetwork, dot slash a dot, ok. So, what do we see here? We see that in just one iteration the everything got converged, right because it satisfies continuity after the corrections, this is basically the continuity values. Both are of the order of 1 minus 6 and the momentum residuals are also satisfied.

So, that means, everything is satisfied after one iteration, right. And the pressure came out as 280, and the flow rates as 30 minus 14 minus 20, 24 minus 4, right. And although we have done another iteration this is not required because the values actually do not change, right. These values are the same as these values where we get here. This is because only one correction is required because the problem is linear, right. There is no non-linearity.

As a result, the velocity corrections and pressure corrections that we have satisfy both the continuity and momentum equations exactly after just one iteration, ok. So, that is why without any problems without any having more iterations we were able to converge these in just one iteration, ok.

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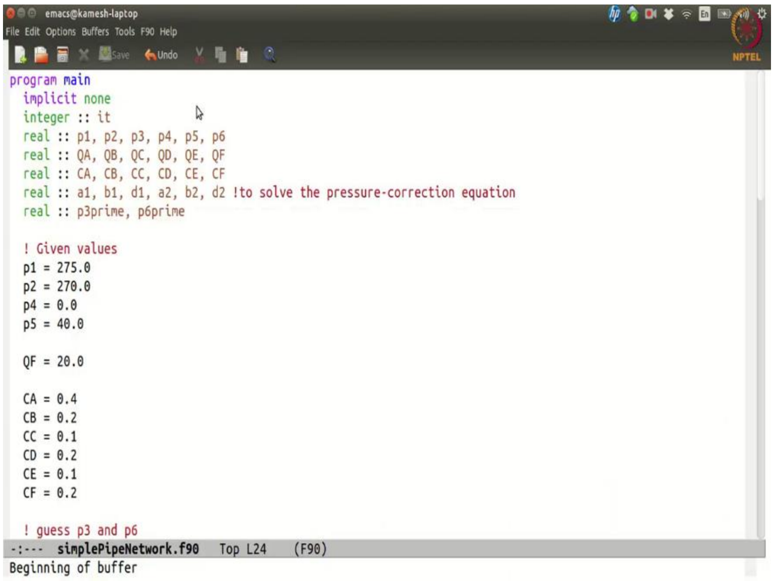

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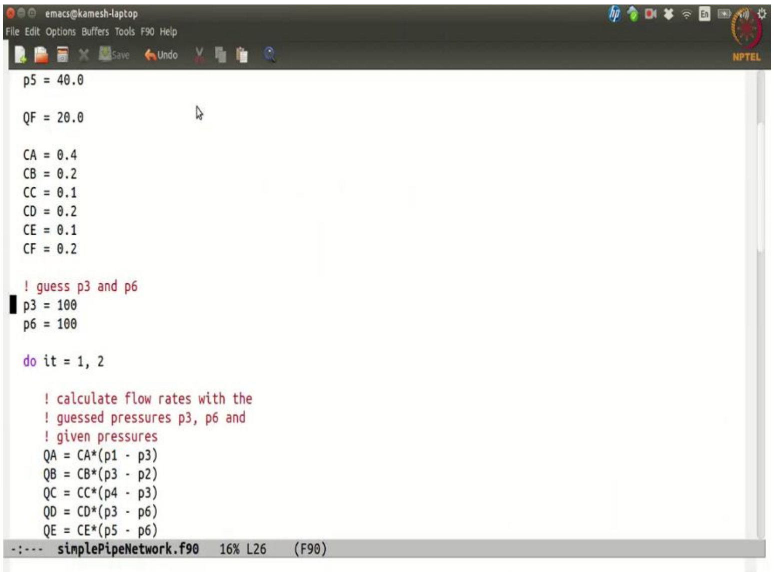

So, you can have like 5 iterations, but it does not make sense because it will just converge to the same value, right. This so does not make sense.

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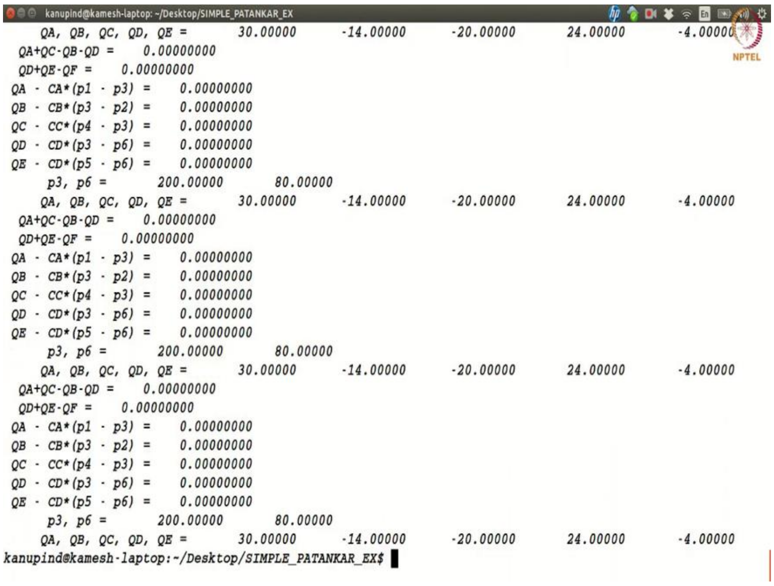

Of course, you can have a different value for these. So, you can have pressure as 200. Let us see if it converges or not.

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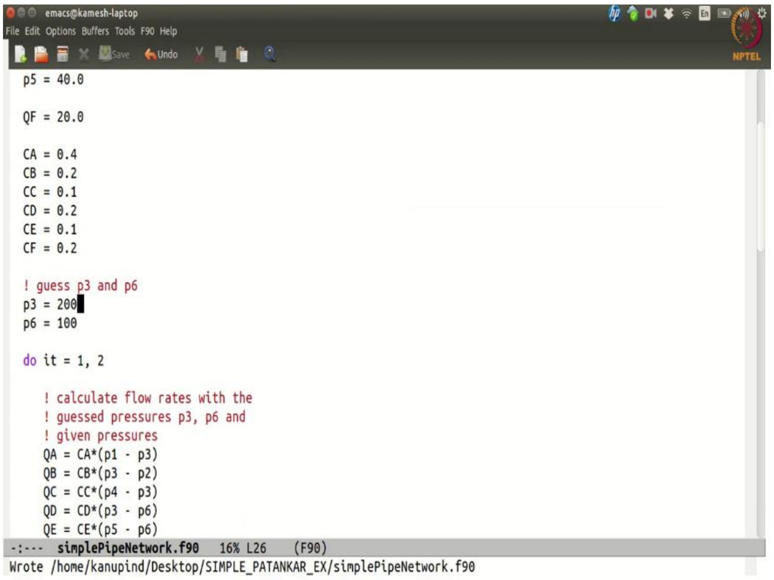

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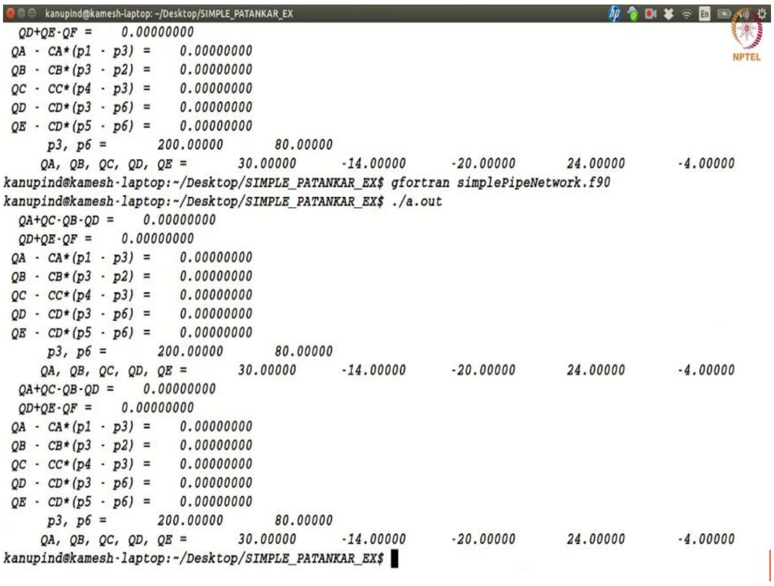

Yes, it converges to the same value 280, ok. But then 200 is a poor guess because the exact value is 200, so let us say 500, ok.

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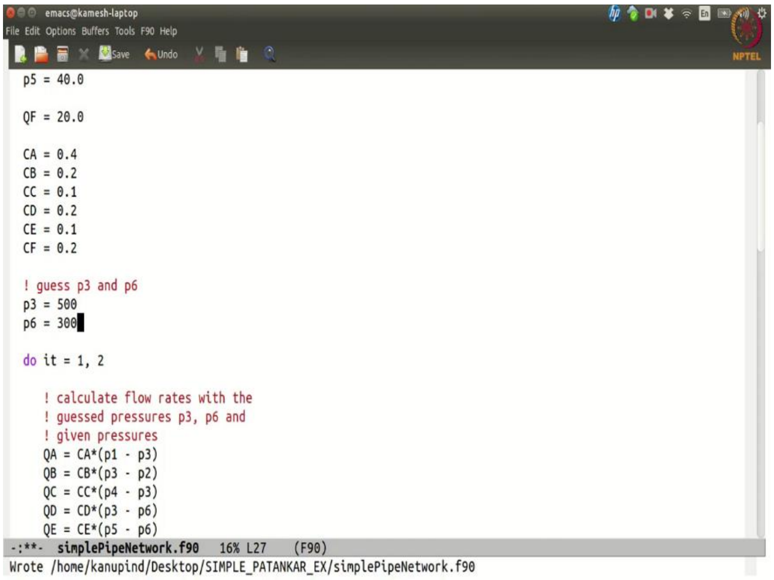

And then say this as 300, and see if it works. Yes, it works. So, it should work because there is nothing wrong, ok, alright. So, that means, this is done, ok.

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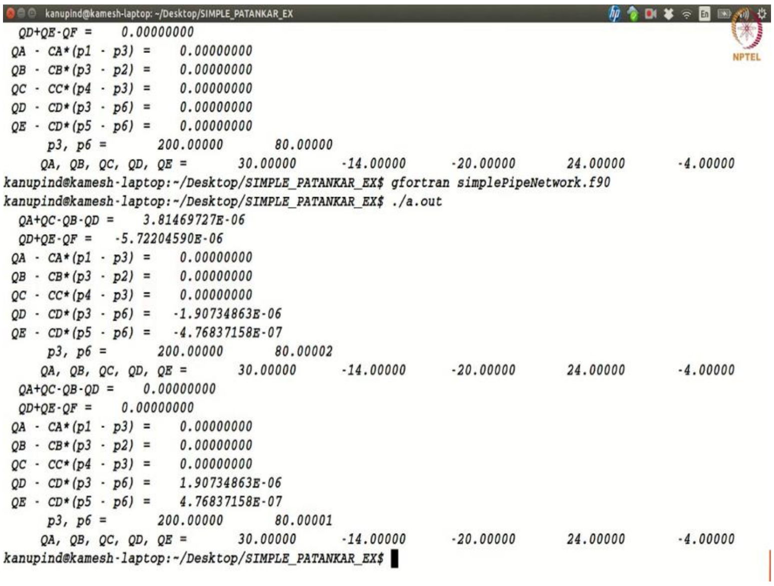

So, what I do is, what I will do is I will share these programs with you on Moodle. So, you can download them, and then run them, kind of learn from them, and then they probably will be useful later on with an assignment or something like that, ok, alright.

Then, I will stop here. I will talk to you in the next lecture. If you have any questions send them through email to me, ok, alright.

Thank you.