Computational Fluid Dynamics Using Finite Volume Method
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Lecture - 37
Finite Volume Method for Fluid Flow
Calculations: SIMPLE algorithm – Part II

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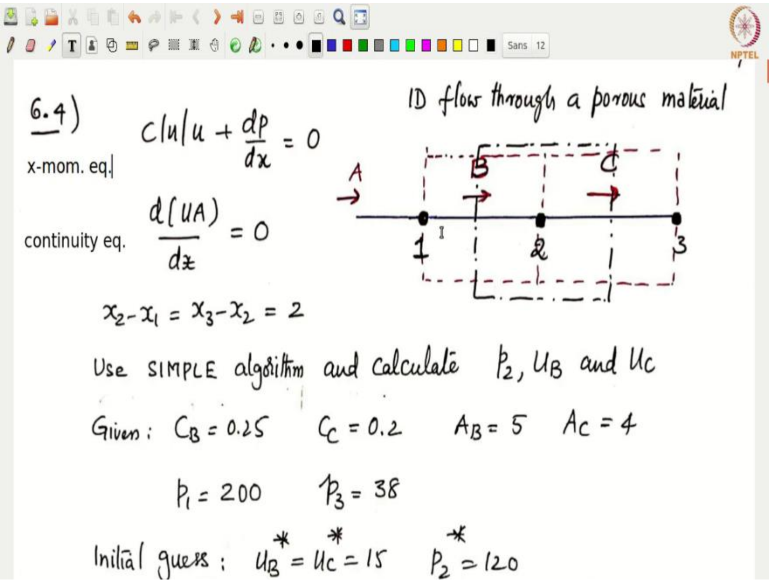

Hello everyone, welcome to another lecture as part of our ME 6151 Computational Heat and Fluid Flow course, ok. So, in the last lecture, we looked at overall simple algorithm on a staggered mesh and we also considered the effect of under relaxation for pressure and the momentum equations.

And finally, we looked at the velocity and the pressure boundary conditions when solving any problem using simple, right. So, in today's lecture, we are going to solve essentially formulate three problems from Patankar's book in chapter 6, that is the fluid flow equation.

So, we look at the each of these problems and formulate them using simple algorithm. So, the first problem we will look at is the problem 6.4 that is shown here; basically this is a 1 dimensional flow through a porous medium and the black circles here indicate essentially locations where the pressure is stored and the red arrows here in the x direction denote the A, B, C denote the staggered locations of the velocity vectors denoted by A, B, C, alright.

I have kind of also shown the corresponding control volumes; of course these extends in the y direction do not make sense, because this is a 1 D problem, ok. So, essentially the area vectors are all 1, ok. But you can see that, there is one control volume; this is basically for control volume for B and this a dashed line this will be control volume for C and this black one this is slightly taller is basically the one control volume for cell 2, that is the primary control volume for the pressure cell, ok.

So, the governing equations are given as $C|u|u + \frac{\partial P}{\partial x} = 0$ and $\frac{d(uA)}{dx} = 0$, ok. So, essentially this equation is basically your, what equation is this? This is your continuity equation, ok. And, what about this guy? This is the momentum or the x momentum equation, ok.

So, alright; now what about, this is the pressure gradient term, area is the cross sectional area that is basically given at what will be the effective area at different location. So, A_B is 5 units, A_C is 4 units and also where C is basically the porosity constant that is basically given C_B equals 0.25, C_C equals 0.2, P_1 and P_3 are given.

So, basically these two pressures; that means we are given a pressure boundary condition; P_1 is 200, P_3 is 38 that is given. And we were also asked to take initial guess as for the velocities as u_B star, u_C star as 15 and the pressure for the cell 2 as 120, ok.

So, essentially this is given and then you are asked to calculate use simple algorithm and calculate what is the final pressure P_2 and the velocities u_B and u_C, ok. The grid values that is basically the Δx between 2 and 1 is equals 2. And so, Δx for 2, 3 cell is also equal to 2, ok. So, basically this is a uniform, this is basically uniform mesh, alright.

So, what about the momentum equation? So, this is the pressure gradient term; what kind of a term do you call this guy as $C|u|u$? What kind of a term would this be? This would be would this be convection would this be a convection term? No, it is not a convection term; because if this is a convection term, it is not just that you should have u u, you should also have a nabla operator, right.

You should have some del dot operating on this thing which is not available. So, this is not a convection, is this is of course, not a unsteady term. This is also, is this a diffusion term? No, this is not a diffusion term; there is no nabla operator here. Then what kind of a term this would be? This would be essentially your, the only thing remaining is your source term. So, this would be a source term, ok

And is it linear or non-linear the source term? Source term is non-linear, because you have mod u times u. So, this is a non-linear term, ok. So, you will see how to categorize this particular thing in the formulation, ok. Then let us gets started. So, if you remember the, you remember the simple algorithm; what we do is basically write the momentum equations for the velocity control volume.

So, that means, you have to discretize at the momentum equation that is basically this guy at B and at C. Then we essentially write an equation for velocity corrections in terms of pressure corrections from these momentum equations for these staggered control volumes B and C. Then we write the continuity equation, that is for cell 2; that is for the primary cell or for the main cell right, the pressure cell.

Then using the continuity equation, we substitute for the flow rates from in terms of the flow stars and the flow primes. And then we substitute for the flow primes; and the velocity primes in terms of the pressure primes right and then there are a pressure correction equation.

Then we solve for the pressure correction equation and eventually correct the velocities and pressures and kind of keep iterating until the obtained value satisfy both the continuity and the momentum equations, alright.

Then let us gets started with discretizing the momentum equation on the cells B and C, ok. And thereafter we will look at discretizing the continuity equation on cell 2, fine. So, the momentum equation is basically $C|u|u + \frac{\partial P}{\partial x} = 0$.

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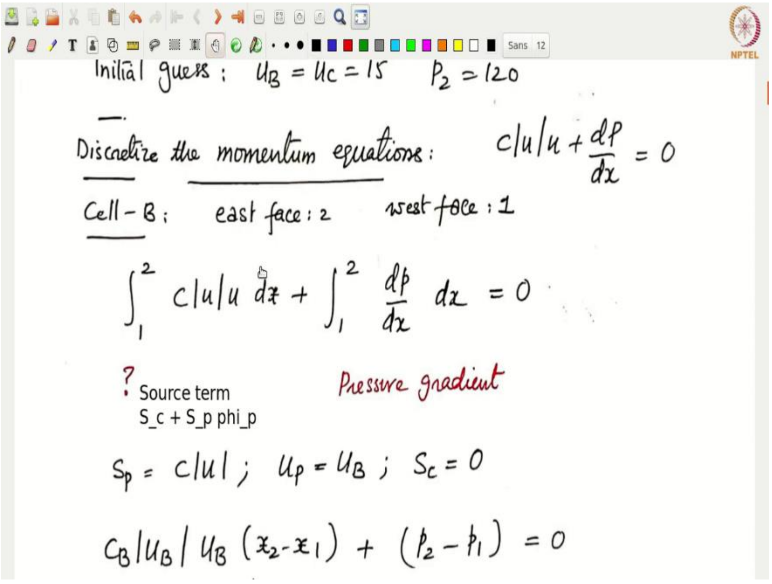

So, if we discretize on cell B, cell B has faces as cell B has faces 1 and 2 ok; that means the integration has to go from 1 to 2. We have $C|u|u$ plus integral 1 to 2 $\frac{\partial P}{\partial x}$ equal 0; $\frac{\partial P}{\partial x}$ equal 0, this is basically integration on the finite volume. So, we know that this is a pressure gradient term and we just categorize this as a, this as a source term, right. And we know that we can write the source as $S_C + S_P \phi_P$.

So, if you consider u as u p means u 2; then this is basically comes as your S_P right, basically S_C equal 0 and S_P equal $C|u|$, right. And S_P of course, also needs to be evaluated at the cell centroid. So, the cell centroid for this particular 1 to 2 is basically 2 is basically B right; essentially everything has to be evaluated B and u_P would be equal to u_B right, and S_C equal to 0, alright.

So, we know that this is basically a source term. Then if you write this in terms of the $S_P \phi_P$ and then $S_P \phi_P$, ϕ_P would be evaluate the cell centroid and that would be a constant. So, constant times integral dx would give you x and you have x 2 minus x 1.

So, that means, essentially what you have is C_B mod u B u B times x_2 minus x_1 right that is what we have; that is basically integration of this particular first term. And the second term would be dP dx, dx integration would be dP right, integration of dP would be P, that would be minus P_2minus P_1 is what you get here, ok.

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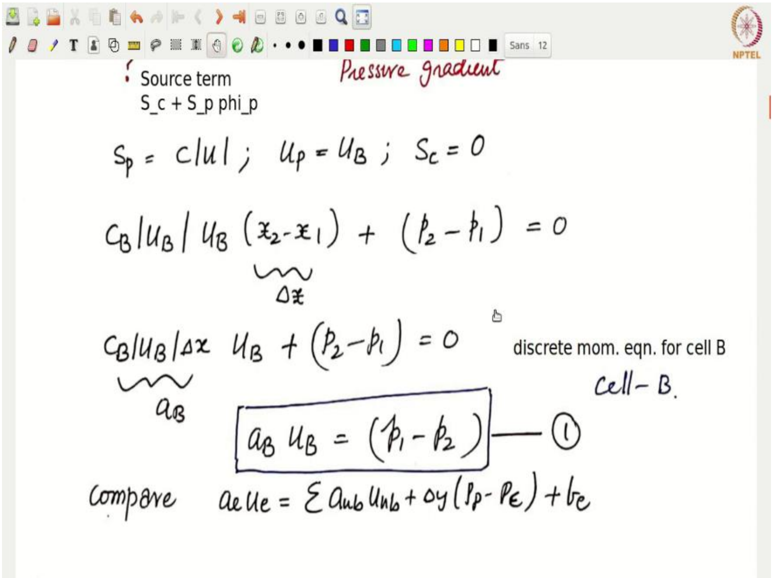

So, for cell B, now we pretty much have the discrete momentum equation; that means C_B mod u_B u_B times $x_2 - x_1$. So, if I write whatever is being multiplied with u_B, that is C_B mod $u_B \Delta x$ as a coefficient a_B; then the equation we have is a_B times u_B plus $P_2 - P$ equal 0, right. That means, this is the; this is the discrete momentum equation for cell B right; $a_B u_B = P_1 - P_2$.

Now, a_B here is the coefficient it is more like your, a_P term, right. So, if you compare with your original equation; if you remember we wrote $a_e u_e = \sum \mathrm{a_{nb}} \mathrm{u}_{nb} + \Delta y(P_P - P_E) + b_e$, right. So, in this particular case if you compare, you do not have b_e; you do not have contribution coming from $\sum \mathrm{a_{nb}}$ here, because otherwise you should have got a term which is like u_C times something that is not there. So, only the central coefficient that is $a_e u_e$ is basically $a_B u_B$.

Now, a_B is basically the coefficient for u_B, alright. So, that is the discrete equation for cell B. Now, if you move on we will do a similar thing; integration of the momentum equation for cell C, that is basically for this cell centroid. So, the limits of integration would be for faces that is 2 to 3.

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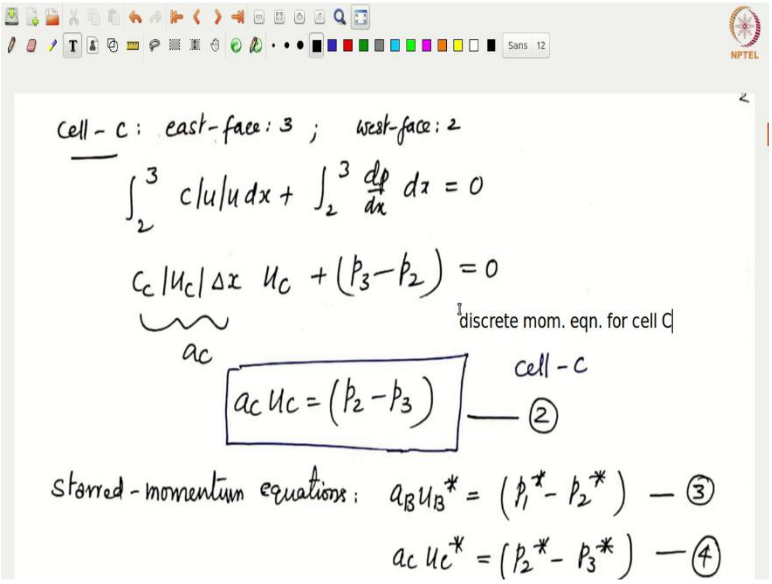

And then integrate in a similar way and uptime essentially for cell 3 we have west face is 2, east face is 3. So, we have to go from 2 to 3 C mod u u d x plus integral 2 to 3 $\frac{\partial P}{\partial x}$. Then again this is basically your $S_C + S_P \phi_P$. So, this would come out to be C_C mod $u_C \Delta x$ times u_C plus $\frac{\partial P}{\partial x}$ integration would be pressure; that means we will apply the limits we have P_3 minus P_2 and equal 0, ok.

And this coefficient whatever is multiplying u_C, if we take it as a constant; that is the basically the coefficient, because we have to linearize the system right, this is what is the linearized source term. And S_P is known while we are doing the inner calculations, right. So, that is why C_C mod u_C Δx would be the constant if we call it as a_C. What we have is basically $a_C u_C$ equals P_3 minus P_3. So, this is the discrete momentum equation for cell C, ok. So, we have the both the momentum equations.

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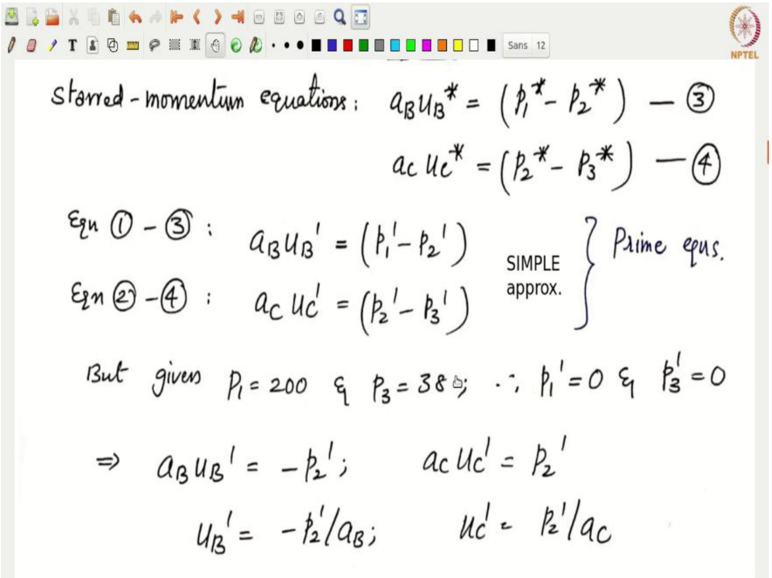

So, if you were to write in terms of the star quantity. So, essentially we just have to apply for a given pressure guess and a velocity guess; what would be your velocities that you converge to? So, in this particular case, we do not have a system to solve; because what we have is u_B^* expressed as pressure star. So, the moment you have a guess value for pressure, you can calculate what is the value for converged value for u_B^*, alright.

So, that means, your start momentum equations are $a_B u_B^* = P_1^* - P_2^*$; that is basically let us call it as equation 3. Then $a_C u_C^* = P_2^* - P_3^*$, ok. So, this is basically the star momentum equation for the cell C, ok. Now, what do we have to do?

We have to subtract the star momentum equations from the original equation right; that means we take equation 1 and subtract equation 3, similarly we take equation 2 and subtract equation 4. That means, what we get is, we get $a_B u_B$ minus $a_B u_B^*$ which will basically give you a_B times u_B minus u_B^* that is basically your u_B'.

So, equation we get is $a_B u_B'$ equals; similarly on the right hand side the non star values and the star values subtract of leading to the prime values that will be $P_1' - P_2'$. And similarly we have another equation that is $a_C u_C'$ equals $P_2' - P_3'$, ok. So, these are the, these are the velocity corrections expressed in terms of pressure corrections.

Now, what you see here is that, this is we do not have to apply the simple approximation here right, which was neglecting the contribution of $\mathrm{a_{nb}} \mathrm{u}_{nb}'$ primes and u_{nb}' here; $\mathrm{a_{nb}} \mathrm{u}_{nb}'$, because those are already 0 here. So, as a result, the simple algorithm need not be, the simple algorithm approximation need not be directly applied here; because the equations that we obtained already satisfy this condition that, the neighboring contribution is already 0 to the velocity corrections, alright.

But we were given what is the pressure at the locations 1 and 3. So, P_1 equals 200 and P_3 equals 38 this is what is already given; that means because the pressure is given, what will be the pressure corrections at these locations? They should be 0 that is why; that means P_1' equal 0 and P_1' equals 0, alright.

So, those are 0, that means if you substitute here; P_1' is 0 and P_1' is 0. So, the equation we get is $a_B u_B'$ equals $-P_2^{'}$; then we can write u_B' as $-P_2^{'}/a_B$. Similarly, $a_C u_C'$ equals P_2' right; because $-P_3^{'}$ is 0, then u_C' equals P_2'/a_C, ok.

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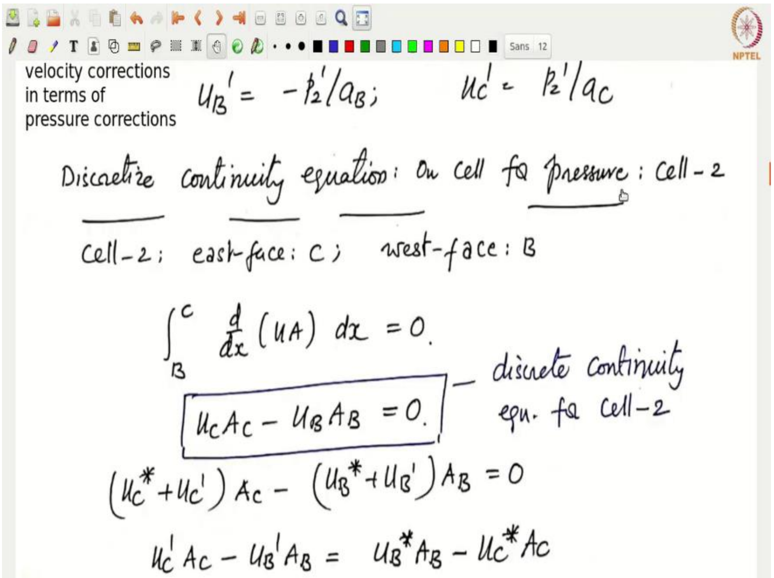

So, these are basically your velocity corrections in terms of pressure corrections, right. Essentially we just have only three cells, two velocity cells B and C and one pressure cell that is P_2. We just have to calculate what is the converged values for u_B, u_C and P_2, alright. Now, we are done with essentially discretizing the momentum equations. So, we will go and look at discretizing the continuity equation on the primary cell, on the cell for pressure that is basically cell 2, ok.

So, if you look at cell 2, the faces that we have are B is there on the left hand side and C is there on the right hand side; so that means if we go back. So, we have for pressure cell 2, that is this black cell we have the faces are B to C, ok. So, essentially we will integrate the continuity equation for B to C and the continuity equation is given as $\frac{d(uA)}{dx} = 0$.

So, that is what is given, that means if you integrate this you are going to get u A and if you substitute the face values what we get is, $u_C A_C$ minus $u_B A_B$ equal 0. So, this is your discrete continuity equation for pressure cell 2, alright. Now, we know that of course, because these are the corrected values as such u_B and u_B that if we represent them, then these are equal to 0; but if these were starred values u_C^* and u_B^*, they would not satisfy the continuity equation, right.

As a result we can of course, now decompose this u_C into star and prime, similarly u_B into star and prime; then what you get is u_C^* plus u_C' times A_C minus u_C^* plus u_B' times A_B equal 0, right. So, that means, we have decomposed this into star and prime values; then we can send the star values to the right hand side, because those are known at this particular time, right.

Essentially we can send u_B^* and u_C^* times A_C and u_B^* times A_B to the right hand side. So, what you get is, you get minus $u_B^* A_B$ going to the right hand side makes it $u_B^* A_B$ and you have plus $u_C^* A_C$ become minus $u_C^* A_C$ on the right hand side.

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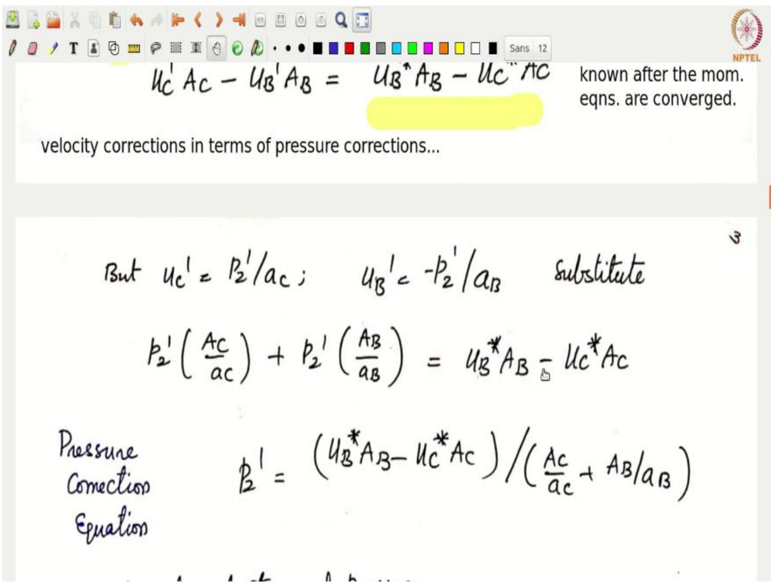

So, this is basically what is already known after the momentum equations are converged, right alright. That means, what we have left with this, you have you are left with $u_C' A_C$ minus $u_B' A_B$ equals such and such on the right hand side. Now, why do we do essentially?

We substitute for the velocity corrections right; we substitute for the velocity corrections in terms of pressure corrections right from the equations we have derived. So, that means, substitute for u_C' in terms of pressure correction that is basically u_C' equals $P_2' A_C$; similarly u_B' equals $-P_2^{'} A_B$.

So, substitute for these two; that means plug in these two into this equation. So, what we get is P_2' times A_C/a_C. So, capital A_C is the cross sectional area that is given, little a_C is the coefficient of u_C right in the momentum equation. Similarly, you get a u_B' has got a minus, u_B' has got a $-P_2^{'}/A_B$, as a result this becomes plus.

So, what you get is P_2' times A_B/a_B equals, on the right hand side as usual we have $u_B^* A_B$ minus $u_C^* A_C$. So, essentially we got an equation here; this is basically P_2' times A_C/a_C plus A_B/a_B equals $u_B^* A_B$ minus $u_C^* A_C$, ok.

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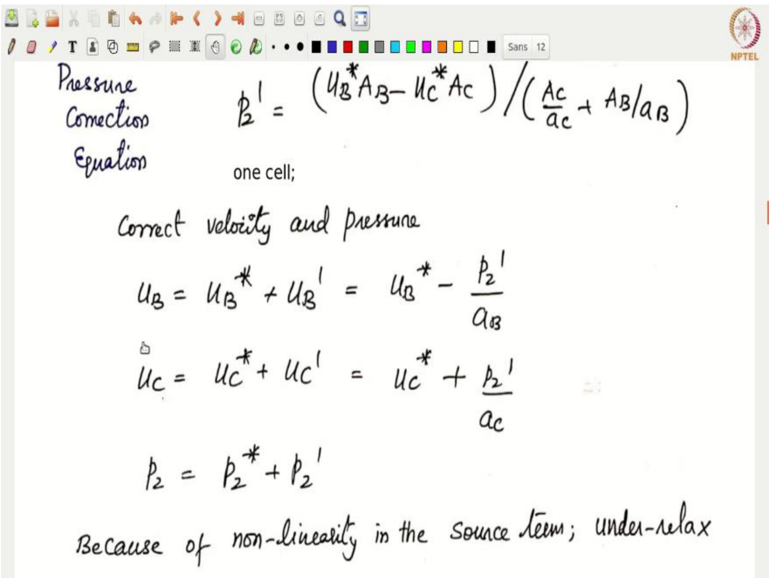

So, we can rewrite this, basically this is now your pressure correction equation in terms of the right hand side B right the star values and the coefficients, ok. So, this is your pressure correction equation. Now, in this particular case, because we have only one cell; we do not have to solve a Gauss Seidel, right. But in general, this would be similar to $a_P P_P'$ equals $\sum \mathrm{a_{nb}} \mathrm{P}_{nb}' + b$, ok.

So, you should be able to make these comparisons at any stage. But once we calculate what is the right hand side value here; we can calculate what is P_2', that means we will know what is the pressure correction. So, once you know the pressure correction, you can substitute into the velocity corrections, calculate the velocity corrections; then pressure and velocity can be corrected and the algorithm is complete.

You can go back and to the step, you know the previous step where we have guess the velocity and pressure and then flow down from there again, right ok. That means, once you know the pressure correction and velocity correction; you can correct the pressure that is P_2 equals P_2^* plus P_2'.

Similarly, velocities are $u_B = u_B^* + u_B'$, where u_B^* is the converged value from the momentum equation and u_B' equals in terms of pressure corrections that is $-P_2^{'}/A_B$ and $u_C = u_C^* + u_C'$ that is u_C^* plus u_C' equals P_2'/A_C, ok.

So, we have we now got u_B, u_C and P_2 these u_B and u_C of course, now satisfy continuity equation; but they do not satisfy the original momentum equation, because the momentum equation itself is non-linear, right. We have C mod u u, where C mod u itself we was taken as u A_B, A_B contains u_B^* values right, which are now got updated. So, they do not satisfy the momentum equations, the velocity.

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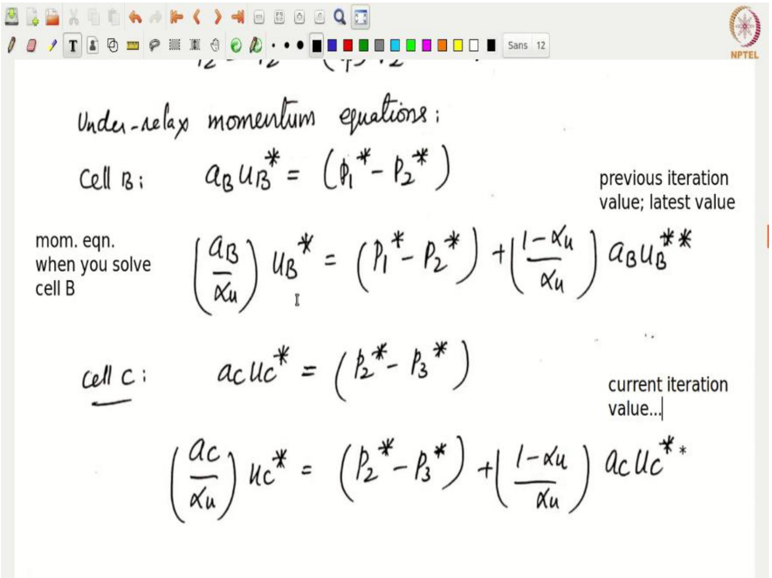

So, because of the non-linearity of course, they do not satisfy. And we also have to under relax the equations, because of the nonlinearity in the source term, ok. So, let us under relax the pressure; that means whatever is the P_2', you got you only obtain multiply that with some alpha p and add it to the star value to obtain the original value.

So, $P_2 = P_2^* + \alpha_P P_2'$. And of course, we know that we will not. So, basically do not under relax velocity in this way right; because if you do it this way, because then it will not satisfy the continuity equation. So, we go back and use the, in the context of momentum equations itself whenever we are trying to solve it; we have to under relax and use the under relaxed equations.

That means, our original equation for cell B is $a_B u_B^* = P_1^* - P_2^*$. So, if you under relax this equation, basically this is your a_P, a_P value right; this is like $a_B \phi_P$ right.

That means, you get a_B/α_u times u_B^* equals $P_1^* - P_2^*$ plus; for this contribution you would have $(1 - \alpha_u)/\alpha_u$ times $a_B u_B^*$. I have u^* star here, basically to indicate that this is the previous iteration value or the latest value that we have, that is that we can use here.

And essentially to make sure that this is different from what we have here, ok. So, you would use this momentum equation when you solve for the cell B ok; because this is the under relaxed equation and you can take some value for α_u as 0.8 or something.

And this under relaxation is necessary, because of the non-linearity in the source term, ok. That means you will use this equation instead of the equation we have written here in and you try to solve. So, when you try to solve the star momentum equations instead of 3 and 4, you would use the under relaxed equation.

Now, that means, if you look at cell C; what we have is $a_C u_C^*$ equals $P_2^* - P_3^*$. If you also under relax this equation with the same factor α_u, what you get is a_C/α_u times u_C^* right equals $P_2^* - P_3^*$ plus on the right hand side you have to add this extra term that is $(1 - \alpha_u)/\alpha_u$ times $a_C u_C^*$ star, this also has to be essentially star, right.

Basically I forget to write it here, this is also star star; indicating that this is the current iteration value that is available right, that is available, excuse me, right ok. So, essentially you would use these two equations in the solution of the star momentum equations. So, when you after you have guess the pressure and we guess the velocities, you would use this equation to solve for convergence of these values to solve using the Gauss Seidel or something.

In this particular case you do not have to; because you know all these things on the right hand side, so it can be computed, ok. Of course, now because we have modified the this above equation to use under relaxation; we not only need pressure guess, but we also need the velocity guess in order to calculate the velocity values, alright.

So, now the algorithm is actually complete; we have to of course write a code for this and run it, so that we would obtain the values for u_B, u_C and P_2, ok. So, that we will do it in the next class; but for today we will do the remaining two problems as well and formulate them as such.

Now, one question you may have is ok, what about. So, instead of using this equation, now what we are saying is that we will use this equation with under relaxation. But do not we have to go back and re do everything; because if we have change the momentum equation, do not I have to go back and do the these correction equations again.

You are right for example, now that. So, instead of 3 and 4 we are going to use the corresponding under relaxed equations; then do not I have to redo all these calculations of u_B' and use the correct equations, is not it the under relaxed equations or not? So, that is the question.

The question is, why am I doing 3 4? Why are we obtaining these from this equation and then we are saying ok, we cannot solve this equation; we want to do under relaxation and change these equations to something else right, which are the corresponding under relaxed equations.

Now, the idea is, even if you use the under relaxed equations here; in the simple approximation you will have those terms also will be neglected, just like the $\sum a_{nb}u'_{nb}$, the contribution coming from this $\sum a_{nb}$, $\sum a_B u'_B$ also will be neglected, ok. So, essentially you will, they will not be considered; that is simple approximation, excuse me.

So, as a result it does not matter whether you had use the under relaxed equation for deriving the velocity correction pressure correction equation or the regular equation, fine. So, that finishes the setting up of the first problem that is the problem 6.4.

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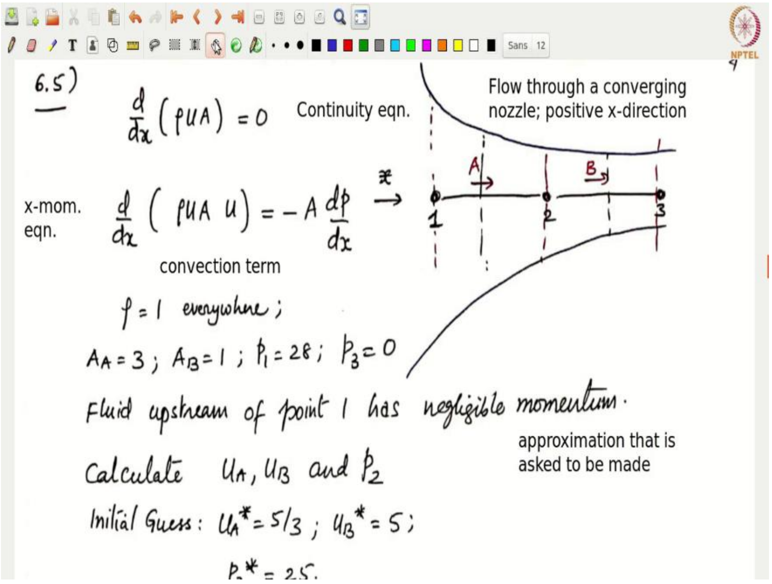

Let us move on to the second problem that is 6.5 from the, from chapter 6 of Patankar's book, ok. So, what this is basically, this is a again a this is a flow through a converging nozzle, ok. And the flow is going in the positive x direction. So, it is going from essentially left to right. So, there is a converging nozzle and then again the values 1, 2, 3 here locations denote the places where the pressure is stored and A and B denote the locations where the velocities are stored, alright.

Then two equations are given, the first one is $\frac{d(\rho uA)}{dx} = 0$. What is this equation? This is basically your continuity equation, right. And, what about this guy? This is $\frac{d(\rho uA\, u)}{dx} = -A\frac{dP}{dx}$. what is this equation? This is your x momentum equation. Now, what about the terms here? Of course, this is the pressure gradient. And what is this term? This is now; this is now the convection term right; basically ρuA u this is basically the convection term, ok.

So, we have and we do not have any source term. So, source term is 0. Also the effective areas at these locations are given at A and B, ok. And the density can be taken to be 1 everywhere that is what is given; then because this is a nozzle, so beyond 1 the area kind of grows very large, as a result the velocity is quite small here, ok. And the area is very large and the velocity is very small ahead of upstream of 1, ok.

So, the effective areas are given, area cross sectional area at A is 3, cross sectional area at B is 1 and the pressures are given. Again this is similar to the previous problem, with all the boundary conditions given are the pressure P_1 is 28 and P_3 is 0 that is what is given.

And it is also given that fluid upstream of 0.1, basically here has negligible momentum, right. That means upstream of 1 has negligible momentum; that means the flow rate upstream of this can be taken to be 0, because if we denote flow rate as ρuA, the momentum is mass times velocity.

So, mass is your ρA times u , right. So, upstream of 1 here velocity is so small and the cross sectional area is so large and essentially the momentum can be taken to be very small, alright. That is this is an approximation that is given, ok. So, this is an approximation that is asked to be made while solving problem, fine.

Then what else essentially you are asked to calculate what is the velocities at a locations A and B, that is the A and B is the velocity control volumes and at the pressure control volume that is P_2. So, you have to calculate what is u_A, u_B and P_2 and it is basically given that u is initial guess five thirds for u_A, 5 for u_B and 25 for pressure ok.

That is basically when you write the code, you can use this as the initial guess and solve the equations. As of now we will try to set up the complete algorithm using simple, ok. So, again what is the first step? First step is to write the momentum equations for the velocity control volume that is for A and B, write the momentum equations; then derive the prime equations a star equations and prime equations for A and B.

Then write the continuity equation discretization for cell 2 and relate the velocity corrections that you got from A and B in terms of pressure corrections and substitute for pressure corrections, essentially get a pressure correction equation, right. So, that is the idea.

So, we start off with momentum equation that is this equation discretize for A; then discretize for B, followed by discretize this for cell 2, ok. So, for cell A we have 1 and 2 as the faces, for cell B we have 2 and 3 as the faces, and for cell 2 the pressure cell we have A and B as the faces that is kind of good to remember, alright.

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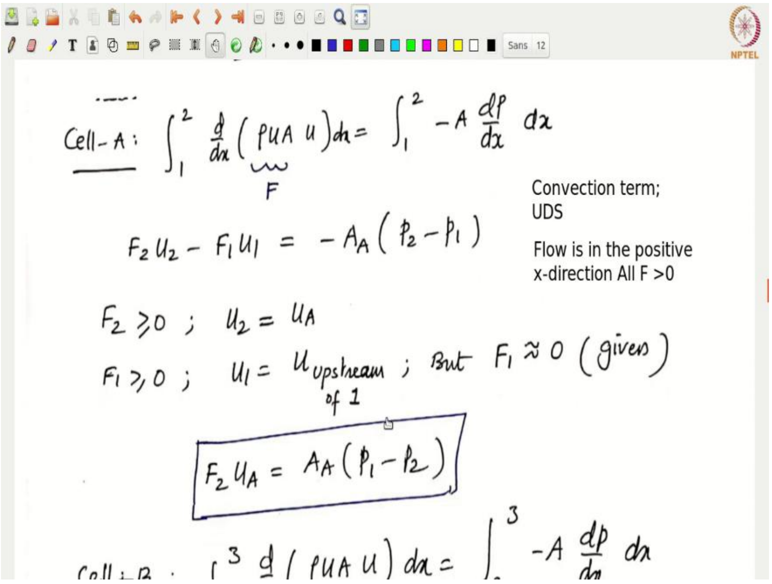

So, cell A is basically going from 1 to 2. So, what we have is integral 1 to 2 $\frac{d(\rho u A\, u)}{dx}\, dx$ equals integral 1 to 2 on the right hand side we have $-A\frac{dP}{dx}\, dx$, ok. This being the convection term, we can write this as the flow rate that is basically ρuA equals F times u. So, if you apply gauss divergence theorem and so on what you basically get is, you get Fe east, right. Or you can even integrate it here, because this is 1 D, you do not need to involve gauss divergence theorem.

So, this basically gives you ρuAu that is F times u; you calculate at both the limits that is $F_2 u_2 - F_1 u_1$, right. And on the right hand side, we have minus A; because this is a coefficient this, this is taken to be at the cell centroid value. So, for cell A, this is basically A_A, this is $-A_A$ times integral $\frac{dP}{dx}\,dx$ would give you integral dP which will give you pressure; that means if you apply the limits this is basically $P_2 - P_1$.

Now, of course, because we are given the convection term ok; we will make an approximation, we will use the upwind difference scheme. Although it is not specified in the problem, we will apply upwind difference scheme; we will use upwind difference scheme essentially. What that means is that, if F_e is greater than or equal to 0, then u_e equals u_P or if F_e is less than 0, then u_e equals u_E.

But fortunately in this case, you do not have to look for less than 0 cases; because all the flow rates are positive right, because the flow is going from in the essentially flow is in the positive x direction. As a result all F values are greater than or equal to 0. In fact, all are greater than 0; because there is some flow that is happening.

That means, F_2 is, because F_2 is greater than or equal to 0; what will be the value of u_2? u_2 is basically, if you go here F_2 is positive. So, u_2 can be taken to be it is up steam value that is u_A, right.

Similarly, if F_1 is positive, what will be the value of here? The value here would be F_1 is positive, u_1 can be taken whatever u upstream of 1 right; that means F_2 is because it is positive, u_2 can be taken as the upstream value and u_1 can be taken as u upstream of 1, ok.

But it is given that the momentum up stream of 1 can be can be neglected; that means F_1 can be taken to be 0, as a result this term is 0; that means what we are left with is we are left with $F_2 u_A$ equals $-A_A$. Or I, if I observe the minus inside, we can write this as A_A times $P_1 - P_2$, alright.

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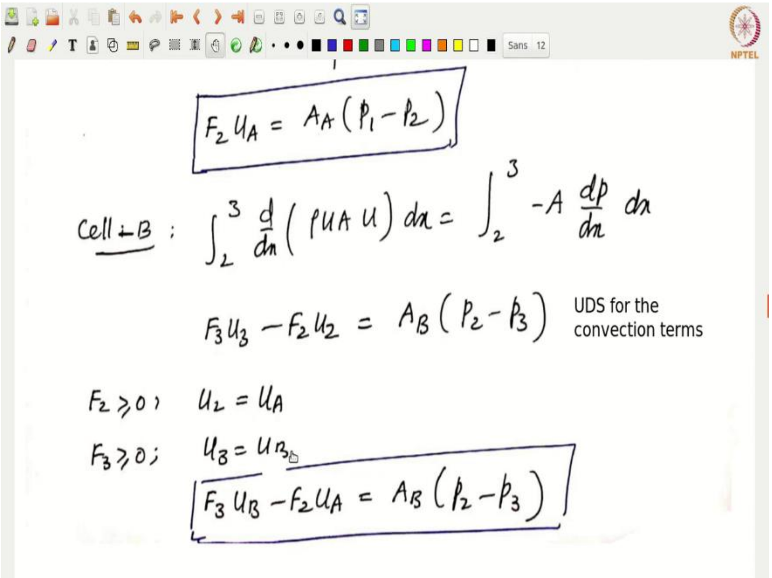

So, we have $F_2 u_A$ equals A_A times $P_1 - P_2$. And if we do the same exercise for cell B; what we get is integral 2 to 3, because for cell B we have the faces is 2, 3, right. So, that means, we can write this as integral 2 to 3 $\frac{d(\rho uA\,u)}{dx}\;dx$ equals 2 to 3 $-A\frac{dP}{dx}\;dx$ alright; that means again this is F times u.

So, is the integration value apply that at the both the limits, you get $F_3 u_3 - F_2 u_2$ equals cross sectional area for cell B is A_B, that is evaluate the cell centroid times integral $\frac{dP}{dx}\;dx$ would give you dP.

So, this is basically pressure, integral dP would be pressure; that means $P_3 - P_2$. Or if you observe the minus inside, you get A_B times $P_2 - P_3$, that is understood. Now, of course, you have to apply again upwind difference scheme for the convection terms right; that means F_3 because F_2 is positive, so u_2 equals u_A and u_3 equals u_B, right.

So, because F_3 is positive, u_3 equals u_B, u_2 equals u_A right; this is the upstream values for these flow rates, alright. That means, what we get is, for cell B, we get F_3 can be written u_3 can be written as u B, $F_2 u_2$ can be written as u A. Now, remember that, we are not replacing the F_3 with F_B here or F_2 with F_A; you are only replacing, because upwind difference scheme only tells you to replace phi sub e with the upstream values, not the flow rates, right.

So, these F's will remain the same; that means $F_3\ u_B$ minus $F_2\ u_A$ equals A_B times $P_2 - P_3$, right. Similar to what we had here; but here we did not, we did not have the other term, because the upstream value of the mass flow rate was given that you can take it as negligible.

Now, one question that might pop in your head is, basically we do not have storage for velocities at the points 2 and 3; what do I do for the flow rates? That is where we are coming down; essentially we will go and see if we can go something for these flow rates from the continuity equation.

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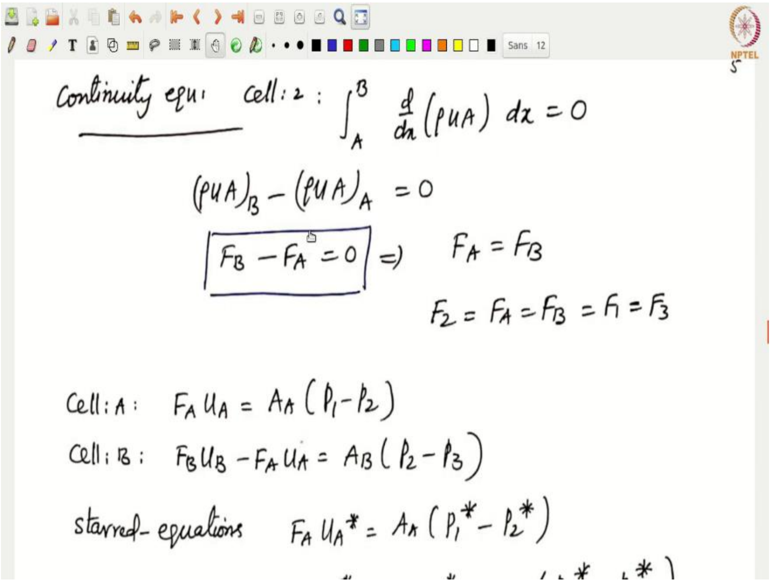

So, we are now done with deriving the, discretizing the momentum equations for cells A and B, right. Now, we look at the continuity equation for cell 2; that means cell 2 has got essentially limits as A and B as it is faces. So, that means, integral A to B $\frac{d(\rho uA)}{dx}dx$ equals 0 right, this is basically integration would be rho u A applied at B minus A.

So, basically what we have is $(\rho uA)_B - (\rho uA)_A = 0$; this is nothing, but your flow rate at B minus flow rate at A equals 0, that means $F_A = F_B$. So, $F_A = F_B$; what is that mean? That means, the flow rate of course, going through mass flow rate going through here is same as the mass flow rate going through here.

In fact, that means, that the mass flow rate going through 2 and 3 and 1 should also be the same; because of the from the principle of conservation of mass, right. That means, not only is your $F_A = F_B$; you can also take $F_2 = F_A = F_B = F_1 = F_3$ right, because this is conservation of mass. Or even if you take F_2 as linear average of F_A and F_B; you will still get both of them equal being you get a $F_2 = F_A = F_B$, right.

Now, this is where we can replace the F_2, F_3 that we have here with either F_A or F_B, right. So, I can now write this as $F_B u_B - F_A u_A = A_B(P_2 - P_3)$, right.

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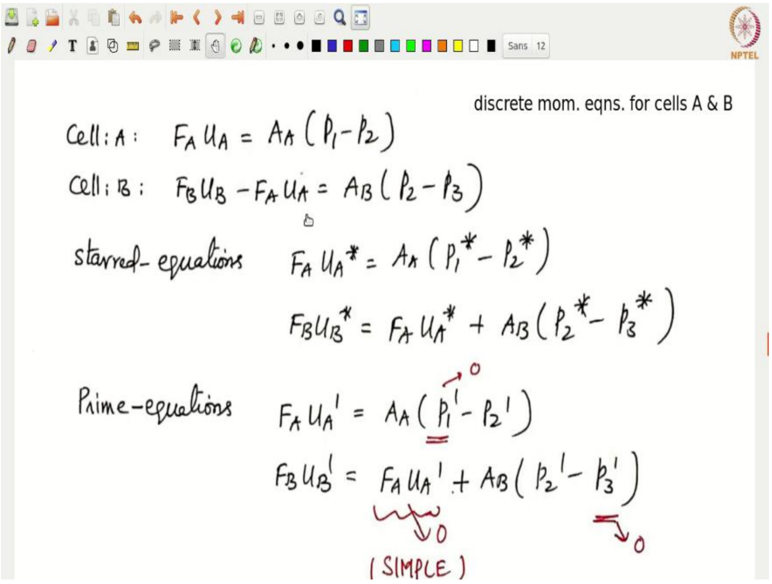

That can be written; that means our momentum equations. So, these are the discrete momentum equations for cells A and B, ok. What we have is $F_A u_A = A_A(P_1 - P_2)$ and for cell B, we have $F_B u_B - F_A u_A = A_B(P_2 - P_3)$, ok. So, how do we construct the star equations here? Starred equations are basically use a guess value for pressure; then the velocity value should be the star values that these equations have to be converge too, alright.

That means, the starred equations for cell A is $F_A u_A^* = A_A(P_1^* - P_2^*)$ and $F_B u_B^* = F_A u_A^* + A_B(P_2^* - P_3^*)$, alright that is given. Then how do we consider the prime equations? Prime equations is basically subtract the star equations from the original equation.

That means you get $F_A u_A'$ prime, because u_A minus u_A^* gives u_A' equals $A_A(P_1' - P_2')$ that is the prime equation for cell A. Prime equation for cell B would be $F_B u_B' = F_A u_A' + A_B(P_2' - P_3')$, right.

But in this we make the, because of the simple algorithm; the contribution coming from the $\sum a_{nb}$ prime, that is this term would be taken to be 0, ok. So, essentially this approximation is coming from simple algorithm, right. This is basically coming from simple algorithm that your $F_A u'_A$ equal 0, ok.

So, basically this is equal 0 and we also know that P_1 and P_3 are given P_1 is given as 200, P_3 is given as 38 I suppose, right; sorry P_1 is given as 28 and P_3 is given as 0, that means P_1 the pressures are given. So, the pressure corrections are 0. So, P_1 prime equal 0 and P_3 prime equal 0, right. That means, this is 0 and this is 0 and this is taken to be 0, because of the simple approximation, alright.

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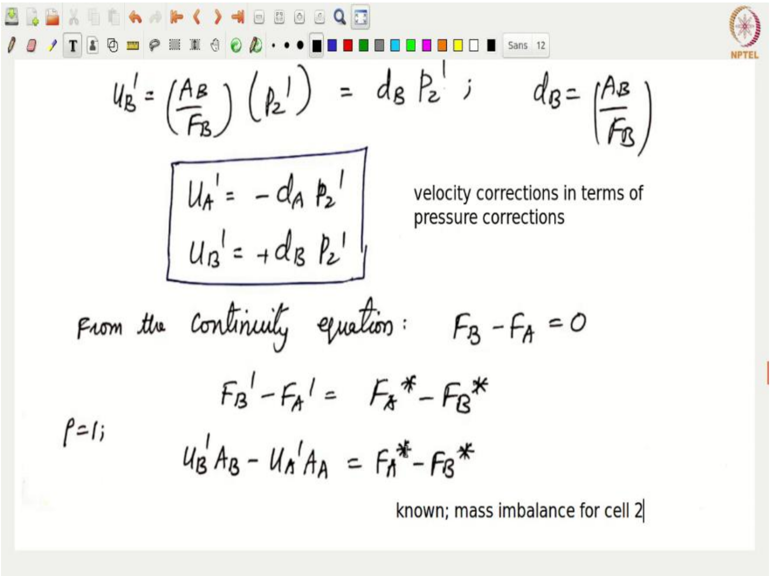

So, this is 0 as such; that means we have these things, then we can write this as u'_A as A_A/F_A, right. This is what we are calling it as some d right; remember we had Δy upon the central coefficient a east that we are written as d, d_e. Similarly, we have a prime equals capital A_A/F_A times $-P'_2$.

So, we can write this as $-d_A P'_2$, where d_A is your A_A/F_A this. Similarly, u'_B equals A_B/F_B that is A_B/F_B times P'_2. So, this can be written as $d_B P'_2$, where d_B equals A_B/F_B, ok. That means, we go now velocity corrections for cells A and B in terms of the pressure corrections that is P'_2.

So, this is your velocity corrections in terms of pressure corrections. Now, these are important, because this is what we need to substitute in the continuity equation. So, but from the continuity equation what we have is, we have $F_B - F_A = 0$. So, which of course, can be decomposed into star and prime values and the if the star values are sent to the right hand side; what we get is, we get $F_B' - F_A' = F_A^* - F_B^*$ right, this is basically sent to the right hand side.

Then a densities given as 1. So, we can write F_B as ρu_B right ρ u A; that means rho is 1. So, we can write this as $u_B' A_B - u_A' A_A = F_A^* - F_B^*$, right. So, the right hand side, this term is already known; this is the mass imbalance for cell 2, right. The amount by which the velocities do not satisfy the conservation of mass for cell 2 is the $F_A^* - F_B^*$ star.

Now, this is not equal to 0; if this is equal to 0, then we have reached convergence, right for momentum equations also right, that is why this is not equal to 0 at the moment. This we have could have computed ok, before coming to this step. Now, we substitute for A B prime u A, u_A' and u_B' from these equations. So, substitute for u_B' as $d_B P_2'$, and u_A' as $-d_A P_2'$.

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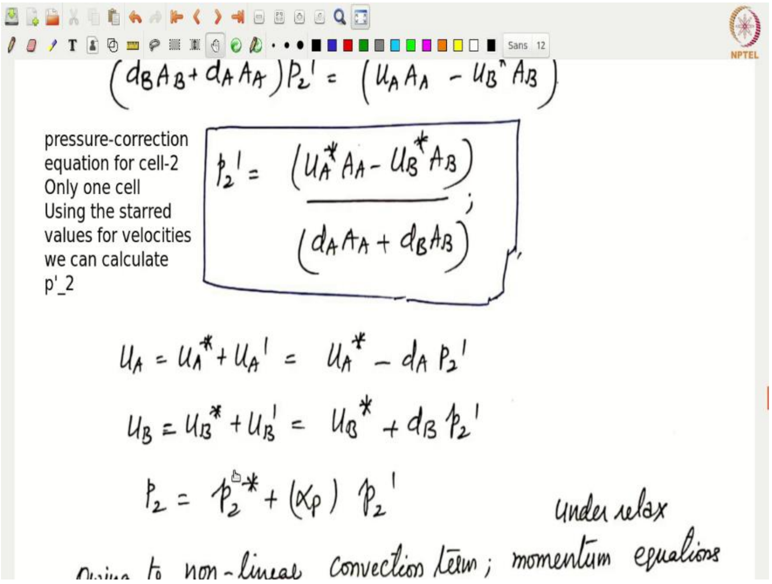

And then write an equation for P_2'. So, if I substitute that, then what we get is u_B' equals $d_B P_2'$. So, this is basically $d_B A_B$ and we have this is basically$-d_A P_2'$, that becomes a plus $d_A A_A$ times P_2', right that is P_2' equals on the right hand side we have F_A^* is rho equals 1. So, this is $u_A^* A_A$ minus $u_B^* A_B$ right; because we only know the star values and the cross sectional areas.

That means, we can we know everything on the right hand side; we know the coefficients here, the cross sectional areas are known and the d values are known, that means we can write an equation for pressure correction for cell 2.

So, this is basically the pressure correction equation for cell 2, again we have only, we have only one cell. So, we do not need to solve for Gauss Seidel, rather using the starred values for velocities right; we can calculate P_2' that can be calculated, fine. So, this is the pressure correction equation.

Now, once you know, once you obtain the pressure correction; you can now correct the velocities using $u_A = u_A^* + u_A'$, u_A' equals$-d_A P_2'$. So, this is $u_A^* - d_A P_2'$. Similarly, $u_B = u_B^* + u_B'$; that means, $u_B^* + u_B'$ is your $d_B P_2'$ and P_2 equals P_2^* plus again I use some under relaxation because of the non-linearity of the convection term this time, right.

So, this is basically $P_2 = P_2^* + \alpha_P P_2'$. So, owing to the non-linear convection term, we need to also under relax the momentum equations and under relax the pressure while we are correcting it.

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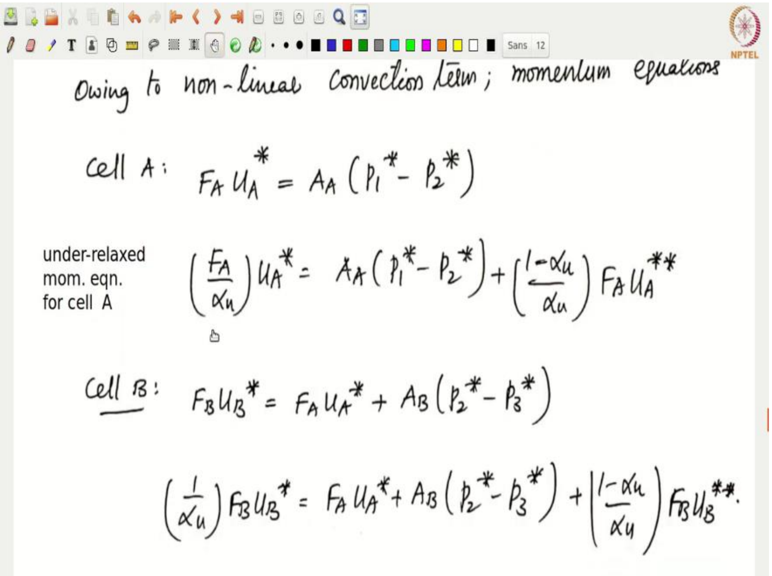

So, now how do we under relax the momentum equations? The original momentum equation, the star equation is $F_A u_A^* = A_A(P_1^* - P_2^*)$. So, if have to under relax them, you divide this by alpha and the extra term on the right hand side; thus this becomes F_A/α_u

times u_A^* equals $A_A(P_1^* - P_2^*)$ you get $(1-\alpha_u)/\alpha_u$ times this quantity that is $F_A u_A^*$ star ok, basically indicating these terms are already known.

So, that is your under relaxed, under relaxed momentum equation for cell A ok, which you will use when you solve for the guess velocities and guess pressures to calculate the converged velocities at the cells, ok. This is what you will use. Again the momentum equation for cell B was given as $F_B u_B^*$ star equals $F_A u_A^*$ plus $A_B(P_2^* - P_3^*)$ right; remember this is what the original equations we had written, right.

So, we had these two equations right and we wrote the star equations here. Now, we are trying to under relax them and if you were to under relax the cell B; then what you get is basically this is $F_B u_B^*$. So, you will get $1/\alpha_u$ times $F_B u_B^*$ star equals $F_A u_A^*$ star plus $A_B(P_2^* - P_3^*)$ plus you get $(1-\alpha_u)/\alpha_u$ times , ok.

So, this is basically your under relaxed equation for cell B, fine. So, again because of the non-linearity of the convection term, we have to do multiple iterations here, so that we converge to a final solution that is driven through continuity satisfying velocity fields by the simple algorithm, alright.

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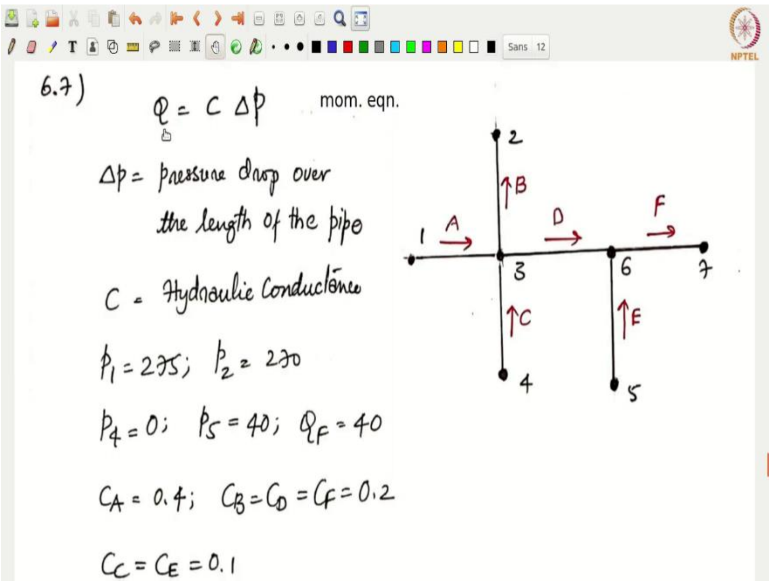

Let us look at the final problem from the book that is basically problem 6.7, which is a pipe network; that is let say it distributes water to a residential area, ok. Essentially we have different locations here and the flow through the pipe, the flow rate Q is given as some hydraulic conductance C times the ΔP, where ΔP is the pressure drop over the length of the pipe, ok.

This is basically the P upstream minus P downstream, that is the pressure drop over the length the L and C is the hydraulic conductance and Q is the flow rate ok. And the pipe network is shown here; so the locations dark circles here 1, 2, 3 all the way to 7 are the locations where the pressure is stored. And the arrows here denote the direction of the flow and there also the locations where the velocities are stored. So, for example, the velocities stored as at A, B all the way to F.

Now, few pressure values are given that is P_1, P_2, P_4, P_5 is given. So, all these things are given; 3, 6 and 7 are not given, and the flow rate also through F is given. So, Q_F is given this is 40 and the hydraulic conduction is given for all of them; that is C_A is given as 0.4, C_B D F that is B D F all these three are 0.2, C_C and C_E are given as 0.1 all the hydraulic conductance given.

So, we were asked to calculate what is the pressure p 3, p 6 and what is the value for Q_A, Q_B, Q C, Q_D and Q_E. We need to calculate what is A, B, C, D and E these flow rates is what you need to calculate. And the given equation is only the Q equals C times the delta p. Now, what equation is this?

This is momentum equation or this is now what kind of an equation this is the? What do we take this as? We should take this as basically momentum equation is not it; because you have kind of an integrated momentum equation ok, where the flow rate equals C times ΔP.

So, $\frac{dP}{dx}$ is not given similarly the $\frac{dP}{dx}$ of this guy is not given. So, this is the momentum equation; that means we have to solve for this equation at locations A, B, C all the way to F right that is what we have to do, alright. So, we have to calculate these two pressures and the flow rates for A to E alright; that means we know that this is momentum equation.

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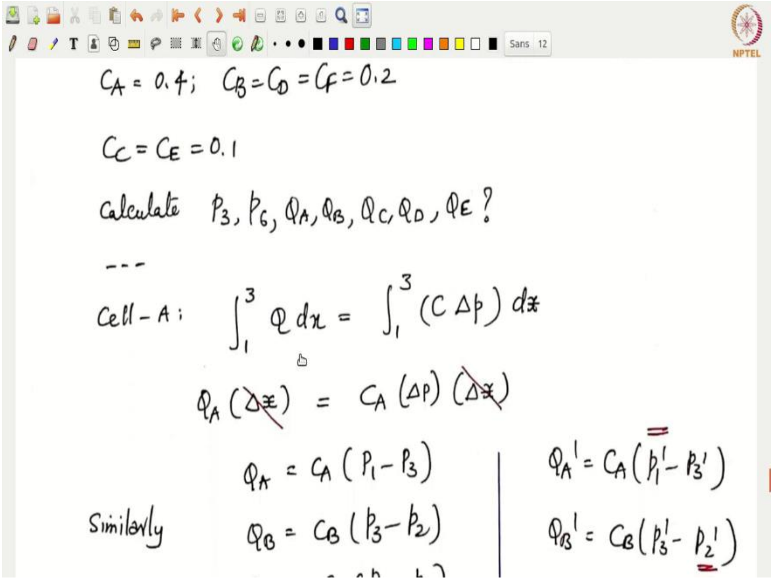

So, if you were to write for cell A, cell A the limits are 1 and 3. So, you have to integrate 1 to 3, Q dx equals 1 to 3, C ΔP dx, right. Now, what is Q? Q is the flow rate what in; under what term do you want to call this as, what type of term is this? This is basically like a source term right; because Q is kind of constant for the entire cell and the cell centroid value is what you want to take it as a representative, right.

So, Q would be Q_A and will be constant, so integral d x will be delta x. So, this will be delta x equals C you can evaluate the cell centroid that is C_A. What about ΔP? ΔP is also you know constant. So, C_A ΔP times integral dx would be Δx right, basically x_3 minus x_1, x_3 minus x_1. So, these two Δx get cancel. So, what you get is Q_A equals C_A times delta p. And what is ΔP? ΔP is the pressure drop across the length and the length is for 1, 2, 3, right.

So, along this much length right; that means P_1 minus P_3 would be ΔP for length a right for the vector A. So, that means, this is basically Q_A equals C_A times P_1 minus P_3; similarly we can write the equations. So, what will be Q_B? QB would be equal to C_B times P_3 minus P_2 right, because the flow is going in that way.

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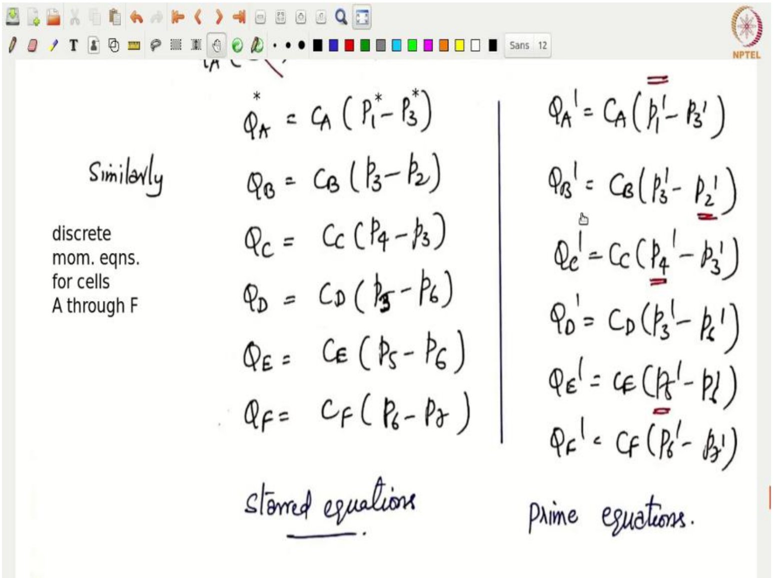

Similarly, what would be Q_C? Q_C equal to C_C times P_4 minus P_3. So, that is Q_C equals C_C times P_4 minus P_3. And what would be Q_5, sorry Q_D? Q_D would be C_D times P_3 minus P_6. So, that is your Q_D right; Q_D equal C_D times P_3 minus P_6. And Q_E would be C_E times P_5 minus P_6. So, that is Q_E is C_E times P_5 minus P_6. And similarly you can write what is Q_F equals C_F time P_6 minus P_7.

So, these are the now the discrete momentum equations for cells A through F, right ok. Now, once you know these basically these are the starred equations right, these are the starred equation; if I have to put star here for P_1^* and P_3^* and then what I obtain here is your Q_A^* right, this is your Q_A^*.

Now, if you subtract of the starred equations from the non-starred equations; what is get is the prime equation. So, we can write the prime equations on the right hand side; those are nothing, but Q_A' equals C_A times P_1' minus P_3'. So, it is kind of easy to see from these equations how to write them and Q_B' equals C_B times P_3' minus P_2'.

Similarly, Q_C' equals C_C times P_4' minus P_3', and Q_D' equals C_D times P_3' minus P_6', and Q_E' equals C_E times P_5' minus P_6', and Q_F' equals C_F times P_6' minus P_7', ok. So, we got all the pressure correction essentially prime equations in terms of; we do not have velocities now, but we have flow rates right, essentially Q is like our velocity. So, we kind of related the velocity corrections or flow rate corrections in terms of pressure corrections, alright.

So, now what is the step that follows? Basically you have to look with the continuity equation; but we were not given continuity equation right, because the only equation we are given is Q equals C times ΔP. But what are the locations where do we have to apply continuity equations?

We have to apply continuity equation at the pressure cells or the primary cell. That means, we have to apply this at 1, 2, 4, 3, 6, 5, 7, right; but again we do not have to apply at 1, 2, 4, 5, because 1, 2, 4, 5 the values are already given. So, we need to apply only at 3 and 6 right and at 7.

So, but at 7 we do not have to apply; because the flow rate is already given, so that is flowing in. So, we need to apply continuity equation at 3 and 6. What would be the continuity equation at 3? I mean continuity equation is basically at 3 is whatever that is flowing in minus whatever that is flowing out should be equal to 0.

That means, for cell 3, the conservation of mass is Q_A plus Q_C minus Q_B minus Q_D equal to 0 that is conservation of mass for cell 3; that is Q_A plus Q_C minus Q_B minus Q_D equal to 0. And the continuity equation for 6 would be Q_D plus Q_E minus Q_F equal to 0, right. So, those are the continuity equations.

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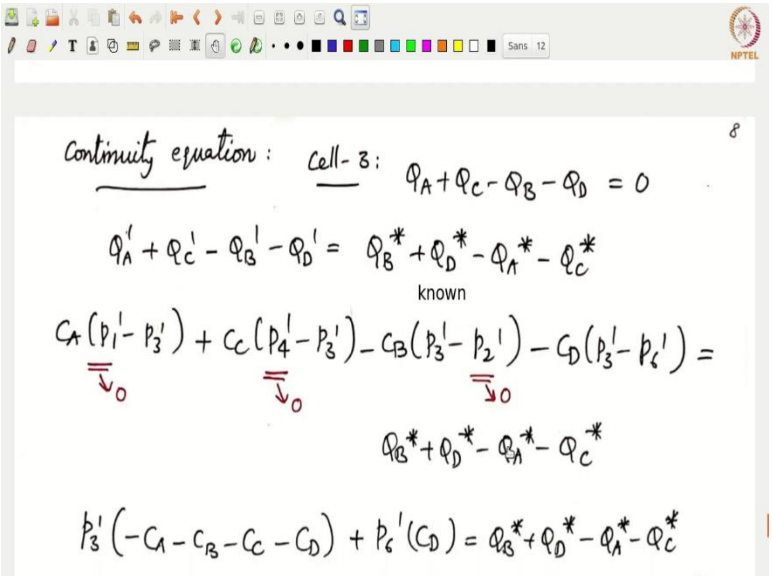

Now, that means, we can write the continuity equations for pressure cells, that is for cell 3 is Q_A plus Q_C minus Q_B minus Q_D equal to 0. Now, this can be decomposed into star and prime quantities and the star values can be sent to the right hand side.

So, what you get is Q_B^* plus Q_D^* minus Q_A^* minus Q_C^* on the right hand side, which is basically known right from the converged star values. And on the left hand side, we are left with Q_A' plus Q_C' minus Q_B' minus Q_D', right alright. So, we are left with this.

Now, we can substitute for Q_A' Q_C' Q_B' and Q_D' from the equations we have derived here, right. So, we will substitute for them; that means we substitute Q_A' equals C_A times P_1' minus P_3', Q_C' as C_C times P_4' minus P_3' minus Q_B' as C_B times P_3' minus P_2', and Q_D' as C_D times P_3' minus P_6' equals everything on the right hand side.

Now, we also realize that, because the pressure values are given at locations 1, 2, 4 and 5; the p primes at these locations are 0 right, we do not need to do any pressure correction there. So, the pressure correction is 0. So, as a result P_1' is 0; P_4' is also 0, P_2' is also 0 and these two are not 0, ok.

That means, we can write collect P_3' terms. So, P_3' equals minus C_A minus C_B minus C_C and minus C_D that is what we get. And we have plus p 6 prime that is coming from here, which is P_6' C_D equals something on the right hand side that is basically Q_B^* plus Q_D^* minus Q_A^* minus Q_C^*, ok

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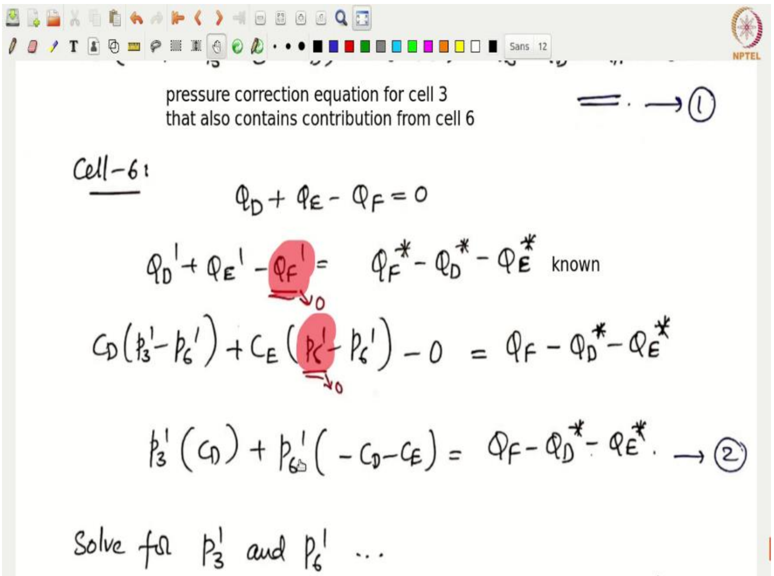

So, this is now your pressure correction equation right; this is very different from what we had till now, till now we had only one pressure cell, now we have two pressure cells that is C 3 and 6. So, this is pressure correction equation for cell 3 that also contains contribution from cell 6 right, that is P_6' is also there, ok.

So, we have two unknowns here, but this is one of the equations; the other equation will come from cell 6. So, for cell 6 what we have is, we have Q_D plus Q_E minus Q_F equal to 0 that is the continuity equation for cell 6, Q_D plus Q_E minus Q_F equal to 0, right. So, if you were to again write that equation, then we have decompose this into star and primes.

The equation we get is Q_D' plus Q_E' minus Q_F' equal to Q_F' star minus Q_D^* sorry Q_F^* minus Q_D^* minus Q_E^*. So, the right hand side thing here is already known. And because Q_F' is basically given; Q_F is given as 40, Q_F' would be equal to 0 right, because this is given this is equal to 0.

And we can substitute for Q_D' in terms of pressure corrections that is C_D times P_3' minus P_6' plus C_E times P_5' minus P_6' minus this is 0 equals; Q_F^* is nothing, but Q_F, right. So, this is Q_F minus Q_D^* minus Q_E^* which is already known and P_5 is given. So, as a result if p 5s prime is this is 0, ok.

So, this is 0; that means we again have equation in terms of P_3' and P_6'. So, P_3' times C_D plus P_6' times minus C_D minus C_E equals Q_F minus Q_D^* minus Q_E^*. So, this is our other pressure correction equation for P_3' and P_6'. So, we have this is a second equation and the unknowns are P_3' and P_6'.

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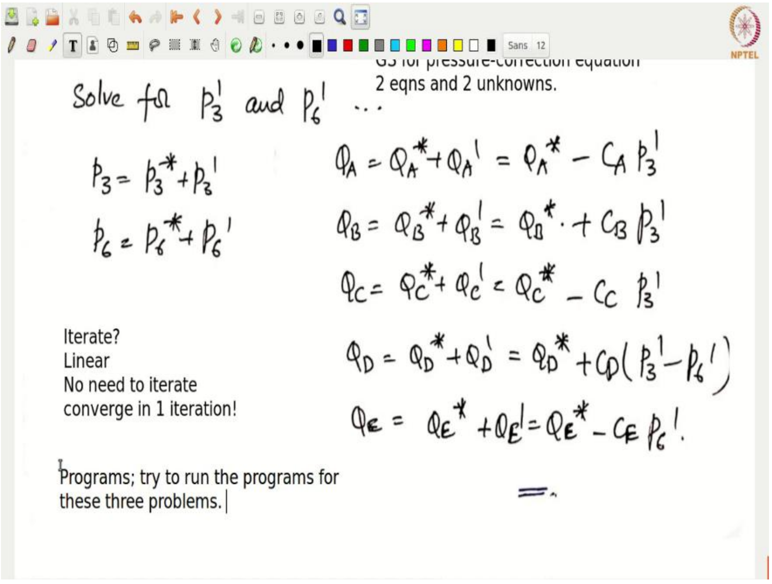

So, we have two equations and two unknowns. Of course, in a traditional sense, we would have solved for Gauss Seidel here right for solving the pressure correction equation; but here we have 2 equations and 2 unknowns, right. So, we do not, we do not have to solve for Gauss Seidel; we can directly solve for 2 equations 2 unknowns.

So, solve for P_3' and P_6', ok. Then once you know the pressure corrections, you can update the pressures using $P_3 = P_3^* + P_3'$. And $P_6 = P_6^* + P_6'$ and then we correct the flow rates right; essentially Q_A equals Q_A^* plus Q_A', where Q_A' equals minus C_A times P_3', because the other contribution of P_1' is 0.

Similarly, Q_B equals Q_B^* plus C_B times P_3', Q_C equals Q_C^* minus C_C times P_3', and Q_D equals Q_D^* plus C_D times P_3' minus P_6'. And finally, Q_E equals Q_E^* plus Q_E', for which we can write minus C_E times P_6'; these are basically the same equations we have derived before.

Only thing is that we have now set these underlined quantities to 0; because those are, those corrections are all 0, because their boundary conditions for pressure are given. Now, what you see; you got one, you got a you corrected your flow rates using the flow rate corrections.

Now, what about how many times? So, do we need to iterate here? Do we need to iterate? Because at this point our solution satisfies the flow rates are satisfied; but what about the momentum equation, is momentum equation linear or not linear? Momentum equation is linear; that means we do not have to iterate, because whatever in one iteration; it will basically converge, right.

Because, the momentum equation is linear right, the pressure corrections only will affect the flow rate. So, because this is linear, we do not need to iterate right even; it will just converge in one iteration, ok, so, that we will see when we try to solve these problems using a code, ok. So, basically that is all for today. So, in the next lecture, so we will look at, basically we will look at the corresponding programs. And we will try to run the programs for these three problems.

So, it is, it will be good if you can keep these notes handy, ok, so that we can refer to these equations and then look at the program, try to run the program and see and obtain the results for each of these problems, alright. So, I am going to stop here; if you have any questions, send them to me through email ok, alright.

Thank you, talk to you in the next lecture.