Computational Fluid Dynamics Using Finite Volume Method

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Lecture – 03
Review of governing equations: Conservation of energy

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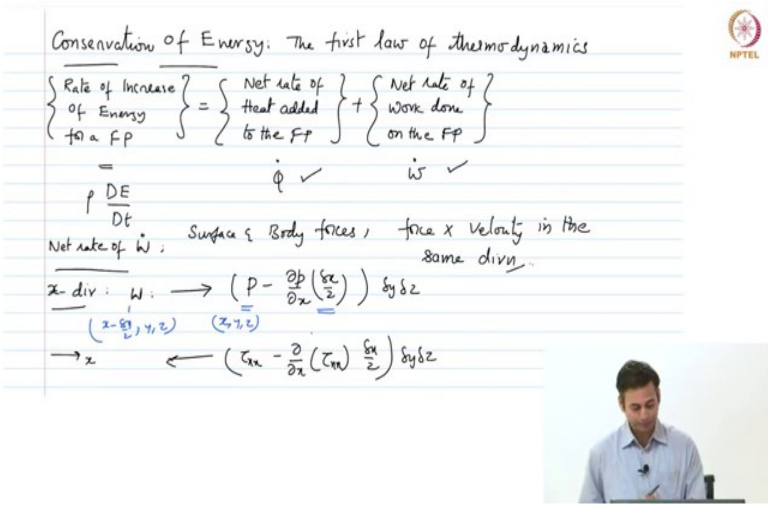

We are going to look at Conservation of energy, which is nothing, but the first law of thermodynamics, right ok. So, the conservation of energy states that the, it is also a rate equation, like the conservation of mass and the conservation of momentum and it says that the rate of increase of energy, right; for a fluid particle, we just say F P here equals the net rate of heat added to the fluid particle plus the net rate of work done on the fluid particle, ok.

Just like before we can write the rate of increase of energy on a unit volume basis, right. So, then this gives us $\frac{\rho DE}{Dt}$ where, E is the energy of the fluid particle, right. That is the stored energy of the fluid particle and D E, D t is the rate and multiplying by density we have the rate of increase of energy on a volume basis, ok.

And, coming to the two components we have here the net rate of heat added, we represent with Q dot and the net rate of work done we can write it as W dot ok. So, we will first look at the net rate of work done on the fluid particle by the several forces the are acting on it and thereafter we look at the net rate of heat added to the fluid particle ok.

So, coming to the net rate of work done on the fluid particle. We see that we have already known that the surface forces that act on it do the work on the fluid particle, right and also the body forces as well. So, first, like a look at the work done by the surface and body forces, that contribute to the work on the fluid particle.

So, essentially we, I am not going to redraw the picture that we have drawn in the last class. We have noted that for a fluid particle or for a fluid element we have several forces acting on it, right. If I consider the x direction, the forces acting in the x direction, so what we can say is the rate of work done is nothing but the force acting on the fluid particle, multiplied by the velocity in the direction of the force, right. That is the work done by a particular force right. So, that is we agree upon.

So, essentially force multiplied by the velocity in the same direction would give us the; would give us the net rate of work done, contributed by the forces in particular direction ok. So, essentially we do not want to rewrite, but rather we want to consider the forces that are acting in the x direction first ok.

So, if we consider the x direction, if you recall the fluid element from the last lecture, we had 6 faces, east, west, north, south and front and back. If I consider the west face, what are the forces that are acting on it, only in the x direction? We had a pressure force and we had a normal stress, right; that is τ_{xx}.

Now, if I use arrows to indicate the direction, on the west face we had a pressure that is acting on it in the positive x direction, right. By an amount that is $p-\frac{\partial p}{\partial x}\frac{\delta x}{2}$ right. We are getting this $\frac{\delta x}{2}$, because the p is defined at x y z and the west face is located at $\left(x-\frac{\delta x}{2}, y, z\right)$.

And then so this is the pressure that is acting on the west face. When we multiply this by the area, right; that was $\delta y\, \delta z$ that was the force that is acting on the west face, right; corresponding to pressure. And then we had another force that is acting on the west face, that was the τ_{xx} right and it was acting in the negative x direction, right. If I consider x to be positive in this direction and this was $\tau_{xx}-\frac{\partial \tau_{xx}}{\partial x}\frac{\delta x}{2}$, multiplying this with $\delta y\, \delta z$ would give me the force acting in the x direction on the west face right.

Now, we are interested in of course, calculating the rate of work done. So, we have to calculate the force multiplied by the velocity in the particular direction. So, I have to essentially multiply these two forces with u, right. That will give me the rate of work done for this.

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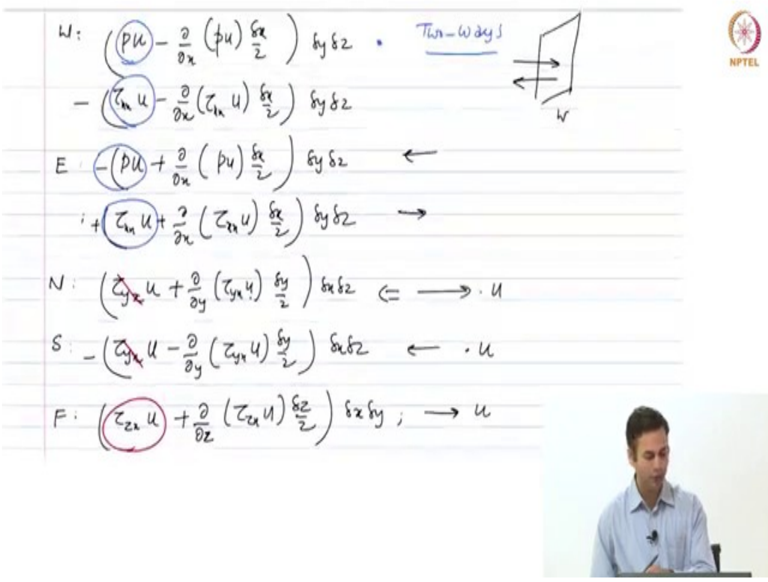

So, what I would do is, I would just like before I am going to construct for the west face. I am going to say that the work done; rate of work done for the west face on the west face by the pressure force would be p times u ok. That is the rate of work done at the centroid of this fluid element and then we want to calculate the rate of work done on the west face. So, it will be $\left(pu-\frac{\partial}{\partial x}(pu)\frac{\delta x}{2}\right)\delta y\delta z$ right; that would be the rate of work done.

Now, of course, you can do this in couple of way. One is to proceed the way we have done. Essentially, multiply p times u calculate the rate of work done at the centroid, and then use Taylor series expansion and calculated the face or you could even start off with u and p separately at the centroid, right. And then reconstruct them on the faces and the multiply them together, right.

In which case you would have this $(p-\frac{\partial p}{\partial x}\frac{\delta x}{2}$¿$\delta y\,\delta z$p minus partial p partial x times delta x by 2, delta y delta z you would have u minus right partial partial x u delta x by 2 as well, right. We had this before.

So, you could even multiply this and get the same answer as we get here ok. So, that would be the same that you can verify, alright. So, this is related to the pressure. Now, if we go back to the τ_{xx}, what will be the rate of work done by τ_{xx}? We just have to again multiply that with the direction of right. The velocity in that particular direction that will be $\tau_{xx}u-\frac{\partial}{\partial x}(\tau_{xx}u)\frac{\delta x}{2}¿\delta y\delta z$. So, this is the net rate of work done by p and τ_{xx}.

Questions still here, this part? Ok, alright. So, then I am not going to write the forces again. So, what we will do is we will try to kind of look up directly from your notebook and write the net rate of work done on the other faces, ok. So, we have now constructed this for the west face.

If I do it for the east face, what would be the net rate of work done? This time this will be (

$pu+\frac{\partial}{\partial x}(pu)\delta x/2¿\delta y\delta z$ right. This was acting in which direction?

In the negative x direction, right. So, this was acting in the negative direction. So, the work done would be with a minus right, because this was minus ok. And then what will be the other force? τ_{xx} right. So, that would be$(\tau¿¿xxu+\frac{\partial}{\partial x}(\tau_{xx}u)\delta x/2)\delta y\delta z¿$ would be a plus or a minus?

This will be a plus, because it is acting in the positive x direction. So, it will be plus ok. So, these are the on the east face, right. And then we have two more forces; one on the north face and one on the south face, right.

Similarly, we had two more forces, one on the front and one on the back as well ok. So, essentially that would be, if I if we write those together that would give us on the north face what was the force we had?

Student: Tau.

τ_{yx} right, that is essentially acting on the positive north face in the direction of x times which velocity should I multiply this with?

Student: (Refer Time: 09:02).

I should multiply this with v, because right; because why?

Student: (Refer Time: 09:08).

But we are still looking at the forces in the x direction, right.

Student: (Refer Time: 09:12).

So, I should still multiply this with?

Student: u.

u, right; we are looking at forces. This is acting in which direction?

Student: (Refer Time: 09:17).

In the x direction, this will only do a work if you multiply this with?

Student: (Refer Time: 09:21).

u, right. So, I multiply this with u and plus $\left(\tau_{yx}u+\frac{\partial}{\partial y}\left(\tau_{yx}u\right)\frac{\delta y}{2}\right)\delta x\delta z$partial partial x tau y x u times delta x by?

Student: (Refer Time: 09:30).

This should be $\frac{\delta y}{2}$ right, we are constructing on the north face and then multiply this with $\delta x\,\delta z$to get the net rate of work done, ok. Similarly are there any other forces acting on the north face, in the x direction?

No right. So, in the south direction, we have the reconstruction of this guy with a minus right.

So, that would be$\left(\tau_{yx}u-\frac{\partial}{\partial y}\left(\tau_{yx}u\right)\frac{\delta y}{2}\right)\delta x\delta z$ right. Which direction is the second one acting?

Student: South.

On the south, in the negative x direction right. So, this would this would come with a with a minus, ok. This will come with a minus because it is acting in this direction and then we still multiply with the velocity ok.

So, we have north and south, then what about the front face? Front face we have tau z x right. I multiply this with u only, right. Because this is still the force acting in the x direction and then, because it is front face it would be a plus partial partial z, I think I made a mistake here these two should be.

Student: y.

Partial y right. Is not it?

Student: (Refer Time: 10:51).

Yeah question?

Student: (Refer Time: 10:54).

With what?

Student: North and south.

North and south I multiplied with u right.

Student: (Refer Time: 11:02) (Refer Time: 11:03) how will be (Refer Time: 11:05).

We will right I have multiplied with u.

Student: (Refer Time: 11:08).

Which side?

Student: (Refer Time: 11:12) get an error.

Here yeah.

Student: Yeah, I think (Refer Time: 11:16).

So, I have taken the force here. So, the question is do we multiply with u or not? So, this arrow indicates the force, I am multiplying with u right. That is what I mean; I do not mean that you multiply this again with u right. So, this is already multiplied with u right. Pressure times the area itself is your force, I multiply that with velocity, right. Does it clear your question? So, do not worry about this part here ok.

So, what is $\tau_{yx}\delta x\,\delta z$? That is the force right. I multiply that with u right, only to get an impression about whether it is positive or negative I am drawing these arrows here, right; ok.

So, this is already, so this effect is basically what we have on the left hand side right ok.

Now, what about the front face? It will be $(\tau_{zx}u+\frac{\partial}{\partial z}(\tau_{zx}u)\frac{\delta z}{2}¿\delta x\delta y$right. This is acting in the positive direction, we multiplied with u. When it comes to the back face, we have right. This will come with a negative; right, because this was act force acting in the negative x direction we multiply with u ok.

Now, what is all these terms that what we have written? We have we have written down all the forces right or all the net essentially the net rate of work done because of all the forces that act in the x direction, ok.

Now, all these should be summed up, right; to get the net rate of work done by all the forces together, ok. So, that is what we are going to do. So, I am going to sum up all these things, which will give us the w dot in the x direction of the forces, ok.

Now, if I want to sum it, sum it up I want to kind of, want you to ask you to look at it before I sum it up ok. So, what are the terms that will remain in this thing, if I sum all these things? So, start with the west face.

Student: Sir.

Yeah.

Student: Next one same delta (Refer Time: 13:27).

This one, because if you take the face here, right. So, this was the west face, we had pressure acting in this direction and then the tau acting in this direction.

Student: Tau (Refer Time: 13:40).

So, the pressure, so which is negative here? This one is negative here, right. Is that correct? Ok. What about the east face, east face is correct?

Negative acting in the negative x tau x is acting in the positive ok. So, this is a mistake ok, thank you. What about the north and south, are they correct? Front and bottom front and back. Fine ok, then what terms will remind and what will go away if I club these things? What about p u and minus p u? Right. When I club all these things, they will cancel ok.

What about tau x x times u and minus tau x x times u? So, essentially this these goes away right, this and this goes away and this and this goes away right. So, essentially the first terms of the Taylor series expansion, right; will all get cancelled. Is not it?

So, essentially if you look at here, this goes away with this and then similarly this term goes away with this term, right ok. So, what we end up is essentially, we end up with only the second terms and the second terms are what? And partial partial x, right with a minus here and there is a plus times minus here. So, this is also a minus right; so we have one. So, these are two halves sum up to one right.

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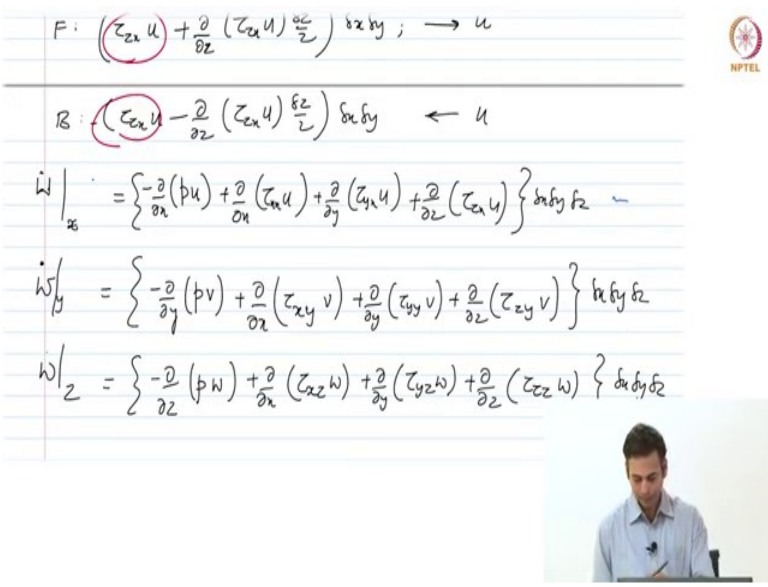

So, that will give me minus that will give me. So, essentially if I look at w dot because of the forces in the x direction right that will give me$\{-\frac{\partial}{\partial x}(pu)$ right, because of the summation of this guy and this guy, right ok. Times we have delta x delta y delta z that I have not written down, I will write it down later. What else we have? We have these 2 terms right that is essentially tau x x u and here, right.

So, this is a plus right we have to minus here and then there is a plus here ok. So, we have these two terms, also summing up to one with a positive sign. So, this will be how much?

This will be $\left\{\frac{-\partial}{\partial x}(pu)+\frac{\partial}{\partial x}(\tau_{xx}u)+\frac{\partial}{\partial y}(\tau_{yx}u)+\frac{\partial}{\partial z}(\tau_{zx}u)\right\}\delta x\delta y\delta z$ plus partial partial x tau x x times u multiplied with delta x delta y delta z which I have not written down.

Similarly, we have plus right partial partial y tau y x times u fine. So, this will be plus partial partial y tau y x times u multiplied with delta x delta y delta z and then we have plus partial partial z tau z x times u, all of these guys are multiplied with the elemental volume, ok; fine.

So, we have now written down the rate of work done, expression, ok. Now, instead of going through this again for each of the faces, can we guess the rate of work done by the force in the y and z directions?

We can easily guess right. So, that would be what would be the then, what will be the total rate? Ok. Let me write down the rate of work done in the y direction, then we will write down the total rate ok.

So, this will be how much by analogy by looking at the, this equation? Right. We can write down here this would be this should come out to be $\left\{\frac{-\partial}{\partial y}(pv)+\frac{\partial}{\partial x}(\tau_{xy}v)+\frac{\partial}{\partial y}(\tau_{yy}v)+\frac{\partial}{\partial z}(\tau_{zy}v)\right\}\delta x\delta y\delta z$ minus partial partial y p times v,.

Student: x and y.

x y right, this would be x y multiplied with v right, ok; multiplied with v plus partial partial y tau double y times v plus partial partial z tau z y times v, right. Essentially, the second index should be the same for all these, right. They are all should be y, because it is the forces acting in the y direction, ok. All these guys are multiplied with what? Delta x delta y delta z ok. Fine, ok alright then we can of course, write down what is the rate of work done in the z direction again. So, that would be $\left\{\frac{-\partial}{\partial z}(pw)+\frac{\partial}{\partial x}(\tau xz\, w)+\frac{\partial}{\partial y}(\tau yz\, w)+\frac{\partial}{\partial z}(\tau zz\, w)\right\}\delta x\delta y\delta z$ minus partial partial y p times what?

Student: w.

W ok, yeah;

ok. We have all these three, now we can club all of them.

Student: Delta.

Into a total rate of work done, term ok.

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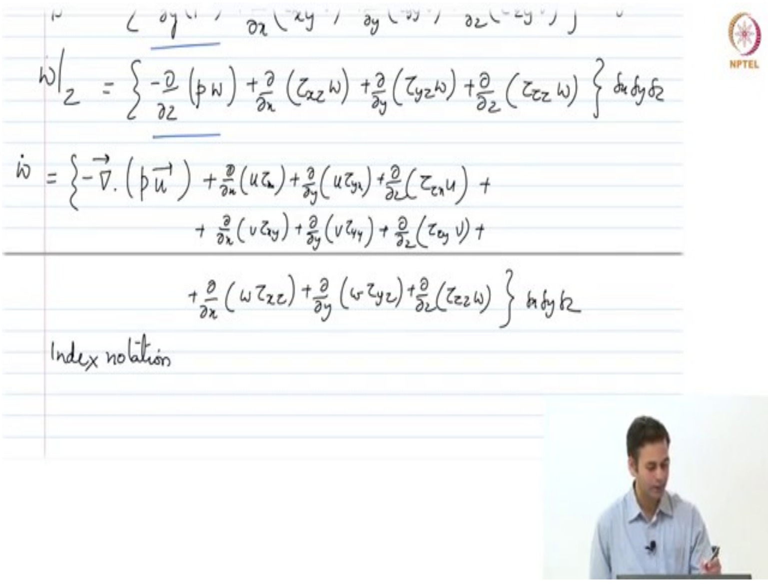

So, that would be what? w dot right. The net rate of work done because of all the surface forces is what? So, we have these three terms, partial partial x partial partial y partial partial z with pressure multiplied with u. Can we write using a nabla notation? If I were to write that would be minus grad right.

Student: (Refer Time: 19:22).

Which we multiplied with what?

Student: (Refer Time: 19:25) bar.

Dot what?

Student: (Refer Time: 19:27).

$\{-\vec{\nabla}.$ ¿, right. That would give me all these three terms ok. Again of course, this is multiplied with the volume which I have not written plus we have several terms here which of course, I can repeat them here again. So, that would be $\{-\vec{\nabla}.$ ¿ partial partial x u tau x x partial partial y.

I am just writing down here again$\{-\vec{\nabla}.$ ¿wehave$\frac{\partial}{\partial x}(w\tau_{zz})+\frac{\partial}{\partial y}(w\tau_{yz})+\frac{\partial}{\partial z}(\tau_{zz}w)\}\delta x\delta y\delta z$ w tau x z w tau y z tau z z times w ok. So, all these guys multiplied with delta x delta y delta z right.

Is that correct? Or any mistakes in the summing up. We can of course, write this in a compact notation right using, using the index notation right. In which we can club all these terms that we have, all these terms we have here ok.

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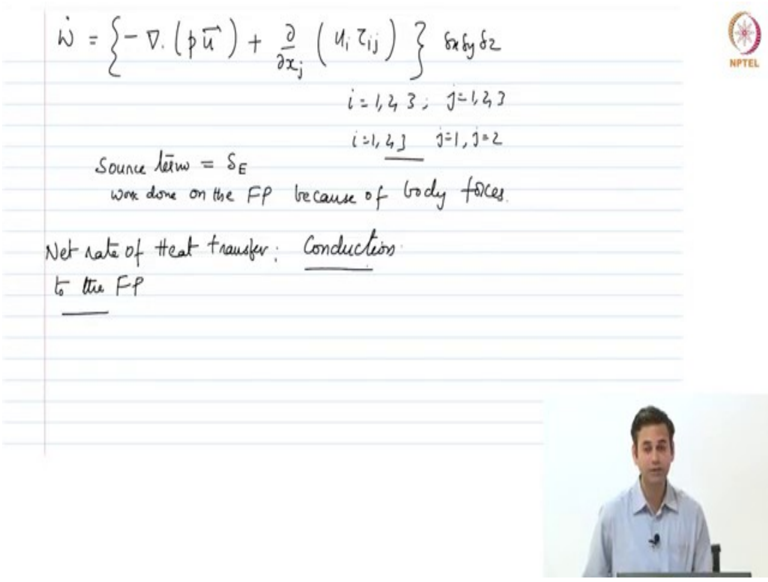

Because you have it on your notes, this was what? This was the w dot equals ok minus del dot p u bar that is what we have, plus we had these nine terms right. So, I can use index notation and then club them all of them up into what? How do I write it? Partial partial x j ok, times u I tau i j, right. Would that give me all the nine terms?

It will give me right, it will give me all the nine terms times delta x delta y delta z right. So, this you have to verify, because there is a double subscript, there is a repetition of the subscript that will add to summation, ok.

So, essentially what you have to do is you have to substitute i equals 1 2 3 and j equals 1 2 3 at a time. So, essentially choose j equals 1 and plug in i equals 1 2 3 repeat and then plug in j equals 2 and then repeat again for all the i's and so on. And, because there is a repeated index they are all sum to they will all sum up ok. So, you can write of kind of write this in a index notation like this very good.

So, essentially, we found the work done by the surface forces. Now, there is also amount of work done, because of the gravity right. Essentially that one, so that we will integrate using a source term ok. So, let us call this source term as some S E ok, this is because of the work done on the fluid particle. Let say because of body forces, ok; that is what we have.

Now, let us look at the net rate of heat transfer term, right. So, the net rate of heat transfer, right to the fluid particle is what we want to look at now. So, we will consider net rate of heat transfer because of conduction, ok. We are not consider me effects of radiation, ok.

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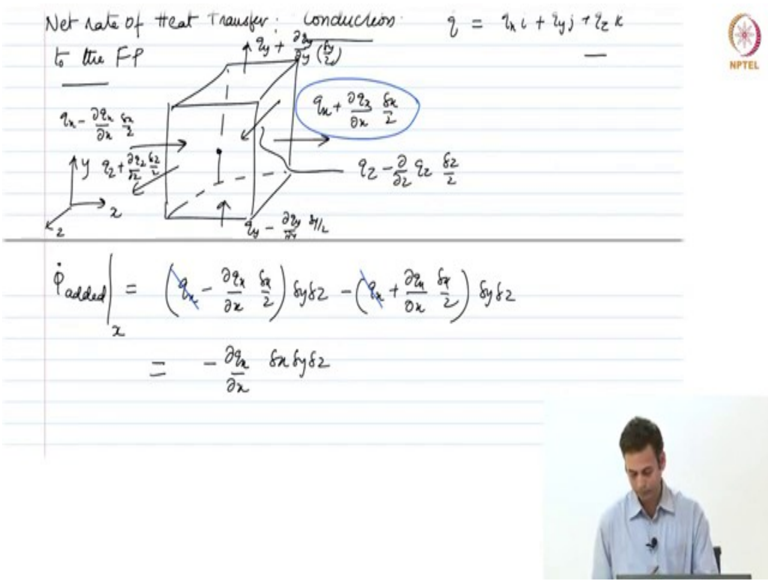

So, what do we have? We have we will consider again the fluid element which is x y z and the heat flux vector that we have q bar is a basically q x I hat q y j hat plus q z k hat, right.

Essentially, we have three components of the heat flux vector, ok. Again, if we define this q bar at the centroid that is at x y z, we can reconstruct this on the faces on the faces, what would that be? That would be essentially the heat flux that is coming in at heat flux rate,

would be $\left(q_x - \frac{\partial q_x}{\partial x}\frac{\delta x}{2}\right)\delta y \delta z - (q ¿¿ x + \frac{\partial q_x}{\partial x}\frac{\delta x}{2})\delta y \delta z ¿$ Similarly, we have the heat flux coming

in here, thi$q_y - \frac{\partial q_y}{\partial y}\frac{\delta y}{2} \wedge q_y + \frac{\partial q_y}{\partial y}\frac{\delta y}{2}$

q y minus partial q y by partial y delta y by 2 and through here we have q y plus partial q y by partial y delta y by 2 similarly in the z direction. So, this would be

$$q_z - \frac{\partial q_z}{\partial z} \delta z / 2 ¿ \wedge q_z + \frac{\partial q_z}{\partial z} \delta z / 2 ¿$$ ok.

So, we have all these heat flux rates that are coming in and leaving ok. So, the heat flux is coming through the west face and leaving through the east face ok. So, we have to look at what is the net rate of heat transfer added to the fluid particle.

So, Q dot that is added would be what? Would be essentially whatever is being added from the west face would consider as positive. Whatever is leaving on the east face would be negative ok. So, this would come out to be q x minus partial q x by partial x delta x by 2 ok. I will first consider the x direction ok.

So, this is being multiplied with delta y delta z would give me the heat transfer rate, because this is the flux ok. So, I multiply with delta y delta c, this is being added and then what is leaving the east face? This quantity is leaving the east face right. So, this is minus q this is minus q x plus partial q x by partial x delta x by 2 times delta y delta z that is being leaving.

So, what terms will remain here again q x gets cancelled and then we have 2 halves of the partial derivative term here. So, they will add up to 1. So, that will be minus partial q x by partial x times delta x delta y delta z would be the term that retains in the heat transfer in the x direction ok.

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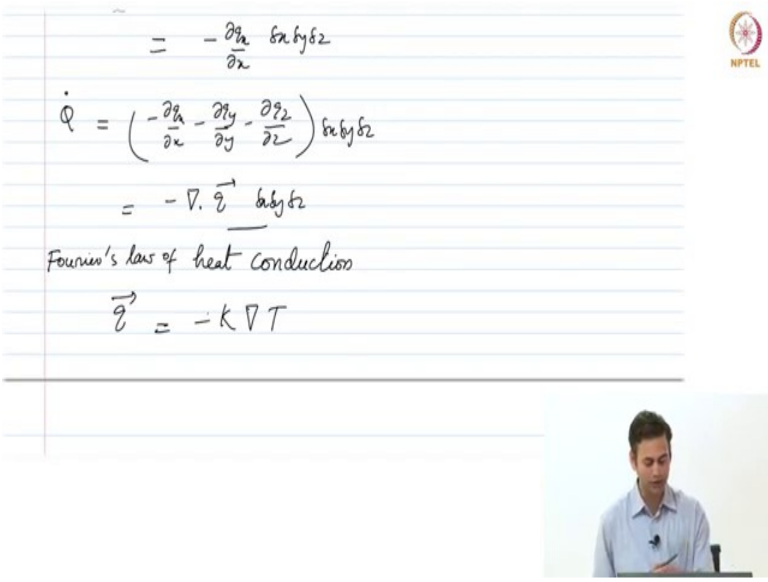


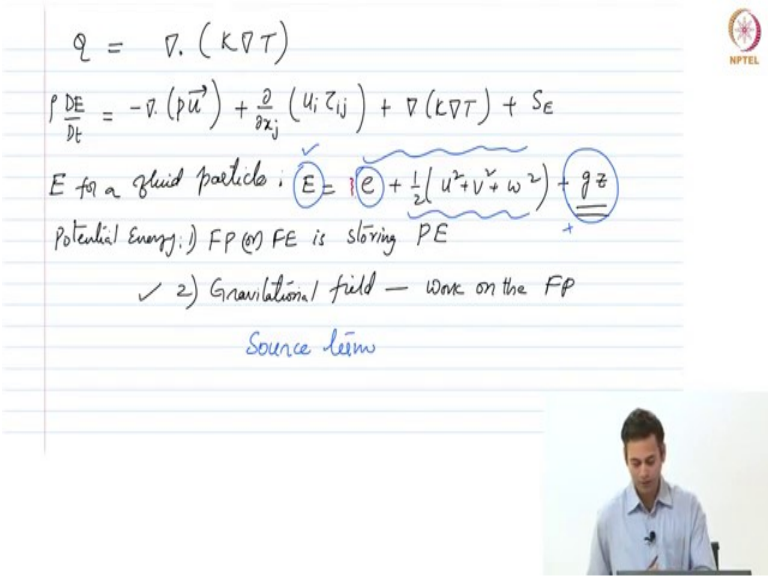
That means, what would be the total heat transfer that is being added to the fluid particle? This would be a summation of the x y and z components, ok. That will be $\left(-\frac{\partial q_x}{\partial x}-\frac{\partial q_y}{\partial y}-\frac{\partial q_z}{\partial z}\right)\delta x\delta y\delta z$ So, what is this? This is nothing, but we have the q bar vector right and then we have nabla operator how can I write this? This should be minus, is it a divergence or a gradient? It will be a divergence, right; it will be a divergence $-\nabla.\vec{q}\ \delta x\delta y\delta z$ So, we can simply write like this is the total heat added alright. Now, we are in a position to write down our total equation ok, the energy equation that is rho times d.

Before we do that, I want to simplify this to let us say in terms of temperature. So, what is, what does a Fourier's law of heat conduction tell us about heat flux vector and the temperature? What is Fourier's law of heat conduction for q bar? $\vec{q}=-k\nabla T$ q bar equals?

Student: (Refer time: 27:50).

Minus k grad T right there is a minus here ok, where does the minus come from? Because heat flows in the direction of?

Student: (Refer Time: 28:01). (Refer Slide Time: 28:09)



Decreasing temperature gradient ok. So, essentially that is why you have a minus grad T. Now we can plug back this into the above equation that we have, so which gives us q equals. So, there is a minus here in this equation and there is a minus here if I plug in substitute for q bar as minus k grad T we are going to get$Q=\nabla .(k\nabla T)$ del dot k grad T right this is the heat added to the fluid particle all right. So, we are going to look at now the total, essentially the energy equation, ok. That is given by rho times D E, D t equals, what are the terms we have?

We have the net rate of work done on the fluid particle that is how much? That was

$\frac{\rho De}{Dt}=-\nabla .(p\vec{u})+\frac{\partial}{\partial xj}(u_i\tau_{ij})+\nabla .(k\nabla T)+SE$ minus del dot p u bar right isn't? correct plus we have this nine expressions right this gigantic expressions which we kind of wrote down in a compact form ok, I will write it in a compact form here and then later on if we needed we will expand it.

That was dou partial partial x j u i tau i j right, is that correct? Yeah would that give us all the nine terms plus what else we have? k del dot k grad T coming from the heat plus we have a source term for the gravity S e ok.

So, all these terms this is our energy equation alright. Now, what does this energy that we have for a fluid particle? What does it contain? e the total energy that you have for a fluid particle is a summation of internal energy right which we usually represent in thermodynamics with u, but here we already have u as well as u bar ok. So, we will represent with another letter e ok.

So, this is the little e is the internal energy for the fluid particle plus you have kinetic energy that is half of u square plus v square plus w square right. Is this u v w same as the u v w we wrote before?

Student: Yes.

Or different?

Student: Same.

It is the same, right it, should be the same plus what else we have?

Student: Potential.

We have the potential energy ok. So, we have the potential energy as well which is given by g times.

Student: (Refer Time: 30:33).

Some elevation. Now, this potential energy that we have, can be thought of in two different ways ok; one is the way we write it here in which the fluid particle or fluid element is storing potential energy, that is one way to look at it, the other way is to look at in terms of some work being done on the fluid particle by this gravitational field ok. So, essentially we have a gravitational field and this is doing work on the fluid particle ok.

So, we would like to take the this approach, in which case we do not have to write the potential energy term explicitly here, rather we can have a source term that takes care of doing work on the fluid particle, ok. As a result we do not have this term here, but we would add a corresponding source term on the right hand side of the equation, ok which is basically the gravitational field is doing work on the fluid particle ok.

Now, essentially, we end up with only then two terms this e and half u square plus v square plus w square. Now, we are doing all this because it is a common practice to write an energy equation for the internal energy e rather than the total energy capital E ok. So, essentially what we want to do is, we initially want to have an equation for internal energy and thereafter depending on the fluid we would like to have an equation for temperature ok, we have an incompressible fluid.

So, for that matter we are trying to get derive what does e contain in terms of the little e and the kinetic energies ok. Now, what do we have to do if we want to get an equation for little e? We have already an equation for capital E. We have to write an equation for the kinetic energies and subtract off that from the equation for capital E right, then we can get an equation for the little e that we have ok. So, that is what we are going to do.

So, now how do I get an equation for the kinetic energies? How do I get an equation for kinetic energy? Essentially it is straightforward you take the momentum equations right, you already know the momentum equation. For example, if you take the x momentum equation what we have is? What is the term on the left hand side we have rho times D u D t right. So, what I can do is I can multiply this with u. I can write it as rho times u times D u D t which is nothing, but rho times half of D u square by D t right. So, we got one term of this equation right like this.

So, essentially what we have to do is we have to multiply the x momentum equation with u, y momentum equation with v and the z momentum equation with w and then add all of them together. Then we are going to get an equation for D by D t of this quantity that is half u square plus v square plus w square right.

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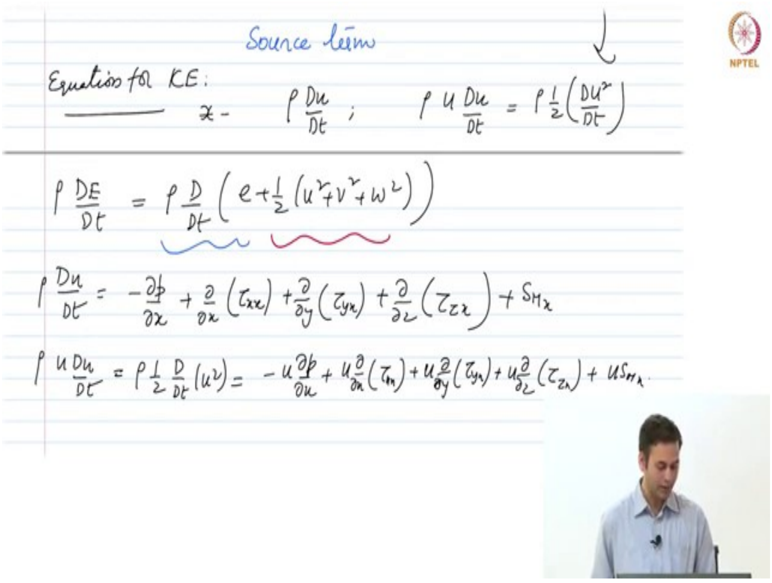

Essentially what we have is we have an equation for. So, our energy equation is $\frac{\rho De}{Dt} = \rho \frac{D}{Dt}$¿ rho times D E D t right. This is nothing, but rho times D E D t times e plus half u square plus v square plus w square.

Now, eventually we want to write an equation for rho D E D t right. So, we are trying to come up with an equation for this quantity; that is rho times D by D t of the kinetic energy ok. So, which we just found out that we can easily simply multiply with the momentum equations on both sides with the corresponding velocity and then add them together ok.

So, if I write the momentum equation which you would help me write here. So, what would be rho times D u D t? What the momentum equation we had? You have to kind of flip your pages back and then tell me what is rho D u D t. $\frac{\rho Du}{Dt} = \frac{-\partial p}{\partial x} + \frac{\partial}{\partial x}(\tau_{xx}) + \frac{\partial}{\partial y}(\tau_{yx}) + \frac{\partial}{\partial z}(\tau_{zx}) + SMx$ That is minus partial p partial x right.

Student: (Refer Time: 35:02).

Z x right plus we had some source term that we wrote it as S M x ok. So, this is our momentum equation in the x direction ok. So, we would multiply this with u that would give me$\rho \frac{Du}{Dt} = \rho \frac{1}{2} \frac{D}{Dt}(u^2) = -u \frac{\partial p}{\partial x} + u \frac{\partial}{\partial x}(\tau_{xx}) + u \frac{\partial}{\partial y}(\tau_{yx}) + u \frac{\partial}{\partial z}(\tau_{zx}) + u\, SMx$ rho u D u D t which is nothing, but rho times half D by D t of u square equals minus u times partial p partial x plus u times partial partial x tau x x u times partial partial y tau y x plus u times partial partial z tau z z x plus u times S M x right. Straightforward we just multiplied that. Now, let us go back ok. I am try to write an equation also for v then we can end up club it together.

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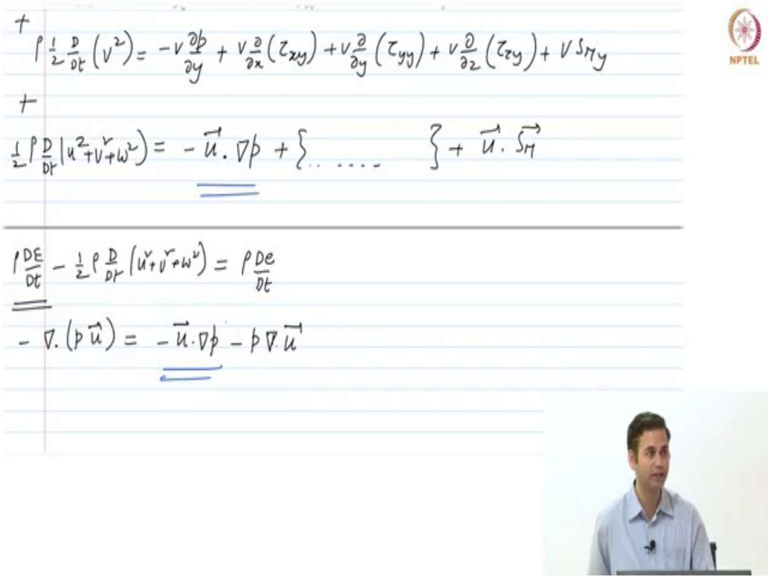

So, that will be $\rho \frac{1}{2} \frac{D}{Dt} \left(v^2 \right) = -v \frac{\partial p}{\partial y} + v \frac{\partial}{\partial x} \left(\tau_{xy} \right) + v \frac{\partial}{\partial y} \left(\tau_{yy} \right) + v \frac{\partial}{\partial z} \left(\tau_{zy} \right) + v\, SMy$ rho half d. Now, we do not have to write the third expression, but what does these two equations have to do? We have to kind of club them together right to get half u square plus half u square we have to sum these two equations together plus we have to add the w equation as well ok, we add all of that.

And then we realize that d rho D by D t of with a half u square plus v square plus w square would be what? Would be a really long expression right. What will be the first term of that? You have minus u partial p partial x right and you have minus p partial p partial y and you have minus w partial p partial z. What is that? Can we write in a vector form?

Student: Minus u bar.

Minus u bar dot.

Student: Grad.

Grad p right plus we have again these terms that is u partial x all these things right. We have these nine terms right; three from each of these equations which I am not repeating here. So, we have so many of these terms plus what does the last term we have? u times S M x plus v times S M y; that is nothing, but u bar dot S M bar right if I represent my source as a vector then it will be u bar dot S M bar.

Now, what we will do is, we will kind of subtract it off in our mind and then write try to write the total equation that we have here which I am going to take your help. So, essentially what will be the equation for them? $\rho\frac{DE}{Dt}-\rho\frac{1}{2}\frac{D}{Dt}\left(u^2+v^2+w^2\right)=\rho\frac{De}{Dt}$ Rho D E D t minus half of rho D by D t of u square plus v square plus w square that is nothing, but your rho D little e D t. Isn't? Because this capital E contains this plus little e plus this half kinetic energy square.

So, this is the equation we want to write. Now, you have to go back and see what are the terms that retain get retained here when we write this equation. What is the pressure term in the in this case? The pressure term was del dot minus. Was there minus? Minus del dot p.

Student: u bar.

$$-\nabla.\left(p\vec{u}\right)=-\vec{u}.\nabla p-p\nabla\vec{u}$$

U bar. So, if I expand this what will this be? This will be minus u bar dot grad p right minus p times del dot u bar right. So, when I am subtracting this off I have this term same as this term right. So, what term would retain? Only this minus p times del dot u gets retained ok.

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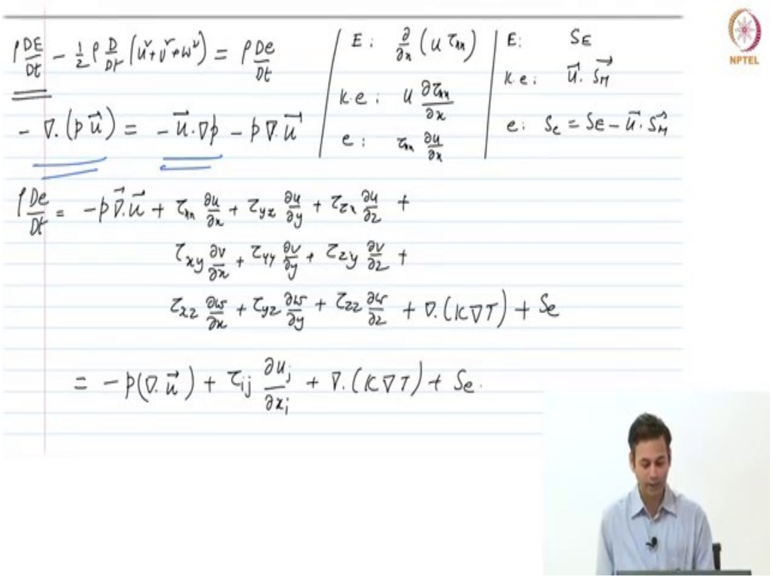

So, that would be; that means, my equation for rho D e D t would contain minus p times del dot u right, because the equation for rho D capital E D t contains this term which can be expanded into these two components and one of the component already exist in the half kinetic energy equation alright. So, we have this first term.

Now, what does the equation for rho D E D t contain in terms of the tau x x tau y j stress tensor and u v w? What we had was we had a really long 9 terms just like here. Only thing was these terms the u was inside the partial derivative right. If you go back what we had was we had partial partial x u tau x x u tau y x and so on right. So, if you expand that in a product rule this is one of the terms right. So, only the other term remains. Is that correct?

Student: Yes.

Yes ok. So, only the other term remains in which case what will be the term look like? That looks like the partial derivative should operate on the velocity instead of the shear stresses that is the only term that retains ok. Then we can write it as plus ok. Can you help me write this what term gets retained? So, if I write it here your E derivative equation contained $\frac{\partial}{\partial x}(u\tau_{xx})$ partial partial x u tau x x right. Whereas, your k equqation contains terms like $u\frac{\partial \tau_{xx}}{\partial x}$ right.

So, in the equation for little e we will have only a term that is $\tau_{xx}\frac{\partial u}{\partial x}+\tau_{yx}\frac{\partial u}{\partial y}+\tau_{zx}\frac{\partial u}{\partial z}+\tau_{xy}\frac{\partial v}{\partial x}+\tau_{yy}\frac{\partial v}{\partial y}+\tau_{zy}\frac{\partial v}{\partial z}+\tau_{xz}\frac{\partial w}{\partial x}+\tau_{yz}\frac{\partial w}{\partial y}+\tau_{zz}\frac{\partial w}{\partial z}$ and so on ok. Is that correct? Any mistakes?

So, the second index should be y and x y z and the first equation should go as the first terms that should be compatible with the denominator of the partial derivative right fine, this is perfect this is correct ok. So, we now just have simplified these equations plus what is the remaining terms we have coming from the capital E equation?

Source term; so, that is essentially we have this del dot k grad T as well right. So, that was del dot k grad T plus coming to the source term, what was the source term we had for the capital E equation? We wrote something like S e to account for all the gravity and all the body forces and in the equation for kinetic energy we had u bar dot S M bar right remember here ok.

So, now I define another new source term that source term for little e would be I would call it S little e which is nothing, but S capital E minus u bar dot S M bar ok. I would just simply write it as S little e. Of course, this also can be simplified minus p del dot u plus I can write this as tau i j what partial u j partial x i plus del dot k grad T plus S little e, that is a very simple equation ok.

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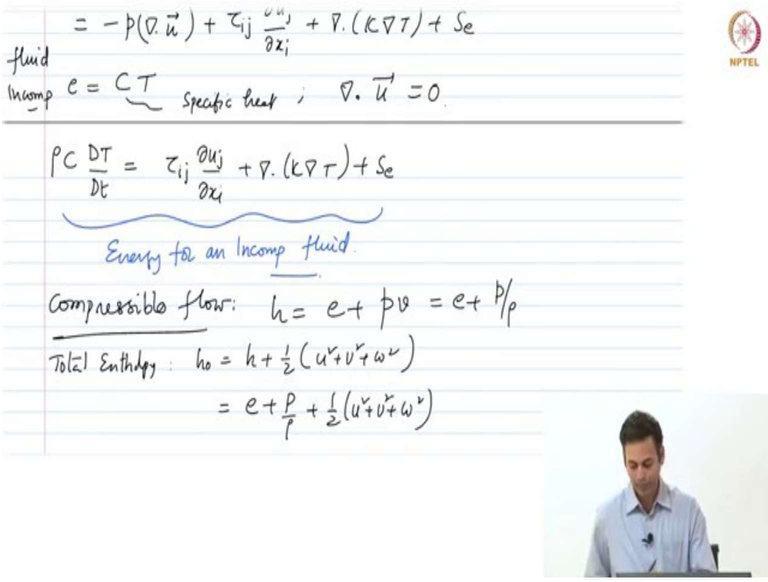

Now, we got an equation for internal energy. Let us take a particular case of an incompressible fluid or an incompressible flow in which case your internal energy e for a fluid which is incompressible. What is little e? The e is nothing, but your internal energy. Internal energy can be expressed in terms of specific heat and temperature, for a fluid you have only one specific heat right which is C, we call it C times T ok, where C is the specific heat and T is your temperature ok. What about for an incompressible fluid this quantity del dot u bar? 0 right. This is 0 for an incompressible fluid which we have established in the context of mass conservation ok.

So; that means, if we go to this equation, we can plug in e equals C T in this equation and this term goes to 0. So, the only things we are left with is these 9 gigantic terms and then the heat conduction and so on ok. So, we can now write down an equation for temperature that is the energy equation for an incompressible fluid; that is rho times C times D capital T by D little t equals this expression tau i j partial u j by partial x i plus del dot k grad T plus S little e ok. So, that is your energy equation your; energy equation for an incompressible fluid.

Of course, we have not yet closed how to calculate the shear stresses right? We do not have a model about the shear stresses, that we will do and that is what defines a fluid to be either

Newtonian or non Newtonian that we have to see. Questions till now? So, you would essentially solve this energy equation in terms of temperature for an incompressible fluid ok.

So, this is nice and good, but for compressible flows you would usually work with enthalpy ok. If you are dealing with compressible flows which we are actually not discussing the solution algorithm say as part of this course, but for the sake of completeness I would write down here. So, if you have a compressible flow you would get an equation, you will write equations in terms of enthalpy because that is much more easier to work with ok. So, enthalpy would be summation of what? Your internal energy and?

Student: (Refer Time: 46:34).

And the and the flow work right, the product of the pressure times, times the what?

Student: (Refer Time: 46:41).

Specific volume. So, this is little v, we will work with this equation. So, this is nothing, but what. So, I can write this as e plus. So, this is nothing, but e plus p upon rho 1 upon density is your specific volume. So, your e is what? So, if I write your. So, I can write h equals e plus. So, my e is C p ok.

So, I can also calculate what is my total enthalpy right or stagnation enthalpy ok that would be what? That would be h naught equals h plus the kinetic energy right, essentially plus half u square plus v square plus w square ok. So; that means, I can plug in $h=e+\frac{p}{\rho}+\frac{1}{2}\left(u^2+v^2+w^2\right)$ h equals e plus p upon rho plus half u square plus v square plus w square.

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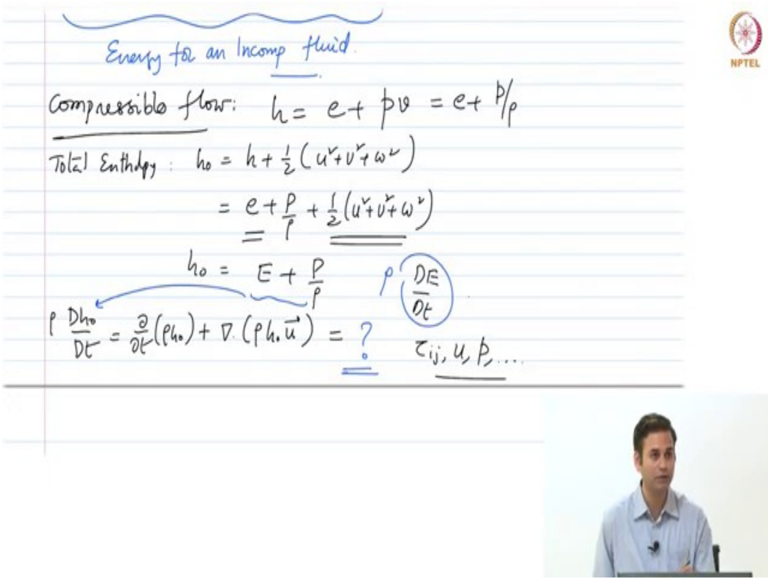

Now, we can rewrite this expression as h naught equals. What is little e plus half u square plus v square plus w square? That is my capital E right without the gravity terms in there. So, this is capital E plus P upon rho ok. Now, what I can do is for compressible flow I would write an equation for h naught ok, the total enthalpy that would be $\rho \frac{Dh_0}{Dt} = \frac{\partial}{\partial t}(\rho h_0) + \nabla.(\rho h_0 \vec{u})$ right. We just introduce the transformation equals something on the right-hand side right.

Now, how do we get the something on the righ- hand side? You plug in what is h naught h naught is your e plus P by rho plug in e plus P by rho into this equation right which will give you D E by D t plus D by D t of P right with a minus plug into that. So, essentially get an equation for D E D t get an equation for D E D t substitute that back into this equation for D E d t. So, you will only get an extra term that is D P D t right that is what you will get.

And do you have an equation for D E D t or rho D E D t in that sense? We have already derived an equation for rho D E D t ok. So, substitute that then you would get an equation for this enthalpy and then you can finish an equation for total stagnation enthalpy, here in terms of the tau i j u and pressure and so on that you can easily obtain by substituting for d rho D E D t from the previous equation we have already derived ok.