Computational Fluid Dynamics Using Finite Volume Method

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Lecture – 20
Finite Volume Method for Diffusion Equation: Unsteady diffusion time-stepping schemes

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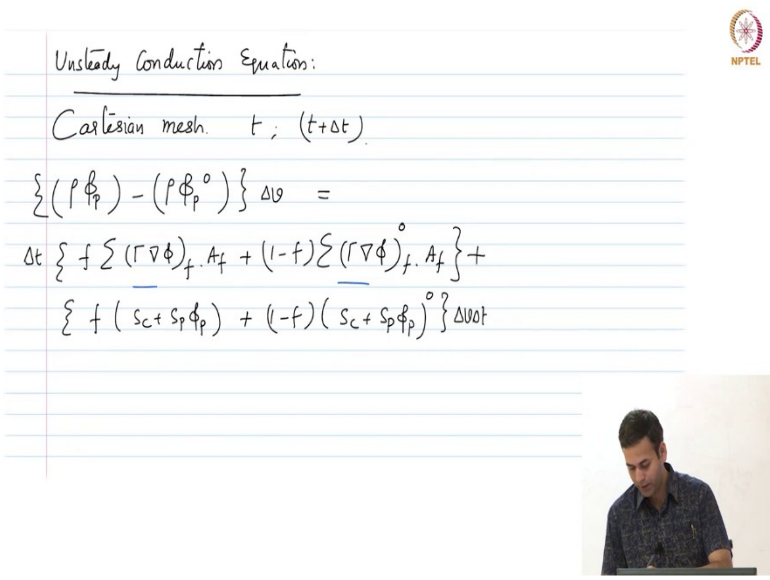

Good morning; let us get started. So, we were discussing about the; in the last class about the unsteady diffusion right, uncertainty conduction that is unsteady conduction equation. We have also considered a Cartesian mesh alright and discretized the unsteady diffusion equation. And we found one kind of an equation which is has an interpolation right between 0 and 1 levels right or t and $t+\Delta t$ levels, we arrived at an equation that looks like the following.

$$\{(\rho \phi_P)-(\rho \phi_P^0)\}\Delta V=\{f\sum (\Gamma \nabla \phi)^1{}_f \cdot A_f+(1-f)\sum (\Gamma \nabla \phi)^0{}_f \cdot A_f\}\Delta t+\{f\,(S_C+S_P\phi_P)^1+(1-f)(S_C+S_P\phi_P)^0\}\Delta V\,\Delta$$

So, that was $\rho \phi_P$ right; I am dropping the superscript 1 here minus $\rho \phi_P^0$ right times ΔV right equals; on the right hand side we have, what? It was $f\sum$ alright. How much was it? It was $(\Gamma \nabla \phi)^1{}_f \cdot A_f$, where again the superscript 1 is dropped plus we have $(1-f)\sum (\Gamma \nabla \phi)^0{}_f \cdot A_f$ right.

This entire term is multiplied with; multiplied by what? Δt and then we have plus, we have a source term right that is also has some $f\left(S_C+S_P\phi_P\right)^1$ right plus $(1-f)\left(S_C+S_P\phi_P\right)^0$ and these two terms are multiplied by what?

Student: ΔV.

ΔV; Δt right, that is the equation we have. And then we also kind of looked at how does the discretization of the diffusion fluxes kind of helps us rearrange this for the simplify; so that is what we looked at. Now, can you help me write down the final discrete form from here so that we can continue.

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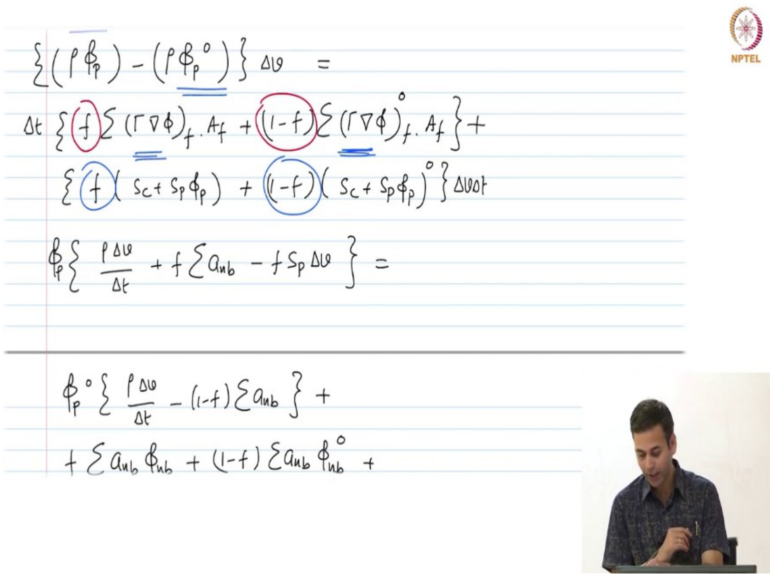

So, that is basically; if we rewrite this as, if we collect the coefficients for ϕ_P right; so the coefficients for ϕ_P would be and also you divide the entire equation with Δt right and collect the coefficients for ϕ_P.

So, that would be what? You have one contribution coming from this term right; that would be $\frac{\rho\Delta V}{\Delta t}$ right. And then we have a contribution coming from ϕ ah, this term right $(\Gamma\nabla\phi)$ at 1, there will be a contribution to ϕ_P right; all of that will have a minus sign. So, when they come to the left hand side, they it will all become plus.

So, we have plus; what? $\sum a_{nb}$ times ϕ_P; that is what we will have and what other contributions come to ϕ_P, from the source term that is S_P term right. So, I would call this as S_P; S_P and then this is S_P; S_C^0 and S_P^0. So, this will be minus, this will be a $-f\,S_P\,\Delta V$ times.

Student: ΔV.

ΔV and there is no Δt because we have divided with Δt. So, this is the term that we have equals; what will be on the right hand side now? Of course, let us collect the terms for ϕ_P^0 right. So, the first time is ϕ_P^0 would have is the $\frac{\rho \Delta V}{\Delta t}$ right; that is the left over term on the left hand side, this comes to the right hand and it will become $\frac{\rho \Delta V}{\Delta t}$ and what other terms contribute to ϕ_P^0?

Student: $(1-f)$.

$(1-f)\sum (\Gamma \nabla \phi)^0{}_f$; so essentially this term right. Now, out of that term; you have central coefficient which is ϕ_P^0 and the other coefficients are ϕ_{nb} which goes to something else right; they go to ϕ_{nb}^0. So, that would be what? That would be plus or minus?

Student: Minus; plus.

Plus right, that will plus or minus?

Student: Plus, plus.

For ϕ_P^0?

Student: Minus.

It will be minus right because you will write $\phi_E - \phi_P$; so you will have a minus; so this will be $-(1-f)\sum a_{nb}\phi_P^0$ right. Of course, you have one more contribution coming to ϕ_P^0; what is that? S_P term which I am not writing here, I would write it in the b term fine. So, we have this term plus what else will be there? What are the terms do you get? You get of course.

Student: ϕ_{nb}.

Again, from the first term from here; you would get ϕ_{nb}right, that would be how much? That would be $f\sum a_{nb}\phi_{nb}$ right; that is what we would get and then plus you have another term coming from here, that is what? $(1-f)\sum a_{nb}\phi^{0}{}_{nb}$ right, that is what we will have plus; I have all the remaining source terms right. So, we are done with all these terms; is not it? The deficient fluxes we are done; $(\Gamma \nabla \phi)$ terms right; that is all accounted for.

Now, what is remaining is the source terms; out of the source terms, we already send the $S_P \phi_P$ term to the left hand side. So, what reminds is $f\,S_P+(1-f)(S_C^0+S_P^0\phi_P^0)$ times what?

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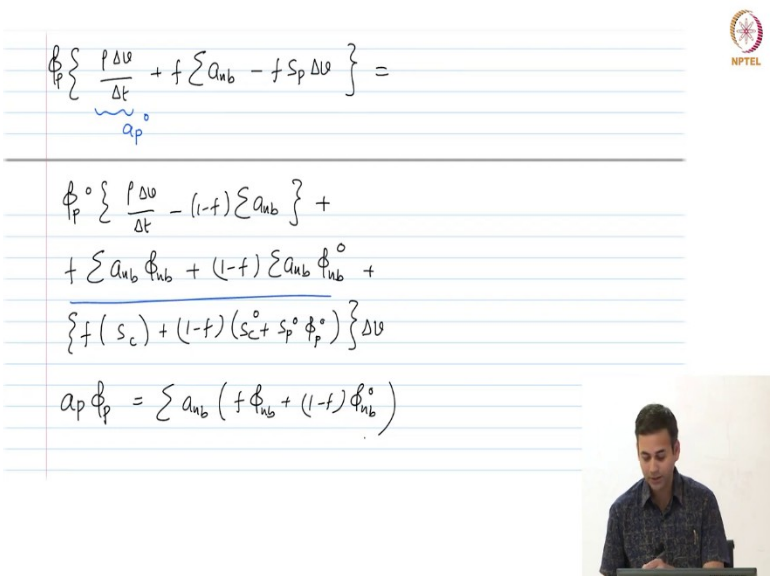

Student: ΔV.

ΔV. Now, there was one question yesterday; S_C and S_C^0, they are the same right; can we just cancel them and then write S_C^0 and things like that. So, one thing you have to kind of think about this thing is; is $S_C$and S_C^0 the same? Are they the same? Ok, now that is where you have to kind of think. So, we are talking about here an unsteady equation right and if you end up with solving iterations per every time step; then would S_C and S_C^0 be the same?

Student: (Refer Time: 06:50).

It will not be right because you have now two iterations; one is the outer iteration which is the time stepping thing and you have the inner iterations which is to solve; a linear system. In that context, S_C^0 would be different from S_C^1, is not it?

Because you would be using the star values as you update your S_C^0's right and S_C^1 will remain the same right you see that. So, you have to understand the difference between S_C and S_C^0 in the context of time stepping schemes. Think of it, we will come back to that later again when we write down the equations.

So, as a result it is not the same of course, somebody I mean that there was a question yesterday; the f we have that we have used here in these two and the f we have used in these two could they be different?

Yes, they could be different right; in which case the kind of interpolation you would use or the scheme you would use for the diffusion fluxes would be different from the scheme you would use for the source terms that is allowed. But as far as the; at least the present discussion is concerned, we are assuming that you use the same scheme for discretizing diffusion and the source terms, but this is very much possible in; in which case you will use some f_1 and f_2 and so on.

If it is not the case; then I am going to kind of tell that diffusion fluxes are discretized with so and so such and such scheme and with source terms with such and such scheme. So, that we will kind of tell you fine; so this is; this looks good. Now, let me rewrite this thing. So, we would like to call this coefficient as some a_P^0; then I can rewrite this as, as what?.

This would be $a_P \phi_P$ on the left hand side equals, equals from the two terms that we have here; can I rewrite this as $a_P \phi_P = \sum a_{nb} \left(f \phi_{nb} + (1-f) \phi_{nb}^0 \right)$; is that? I just clubbed the two sigma's right.

Here again a_{nb} are the same right; somebody there was a question yesterday, would; can a_{nb} be different? They can be different if you have Γ's being different right. So, which we are not considering at the moment; we are assuming that the Γ we have is only a function of space, but not a time function of time or anything else; fine.

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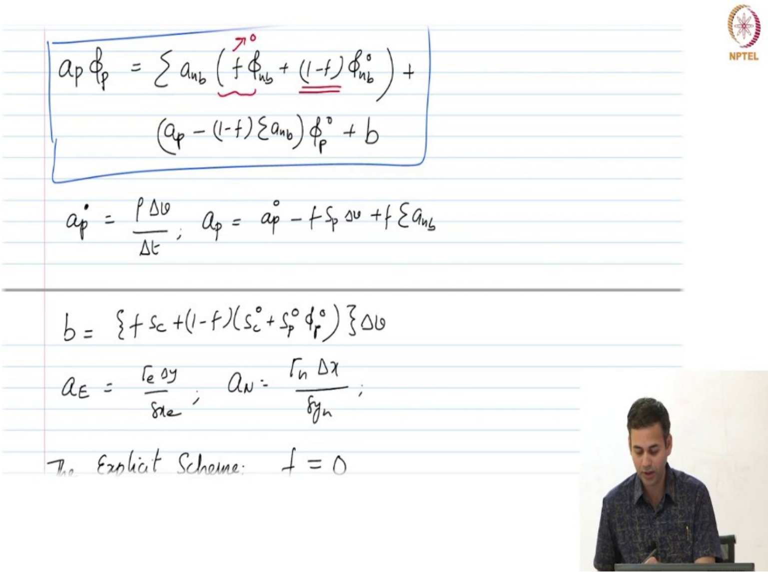

So, we can rewrite these two; then what else? What are the terms are remaining? Plus we have this first guy that is $\left(a_p - (1-f) \sum a_{nb} \phi_P^0 \right)$ right? That is our simplified equation. So, that is our general equation for unsteady diffusion equation right. Is it correct?.

Then what about what are these coefficients? What is a_P^0; $\frac{\rho \Delta V}{\Delta t}$. What is a_p? a_p is what? a_p is this entire term right, that is basically this entire term right; that is what? $a_p = a_P^0 - f S_P \Delta V + f \sum a_{nb}$, what else b right.

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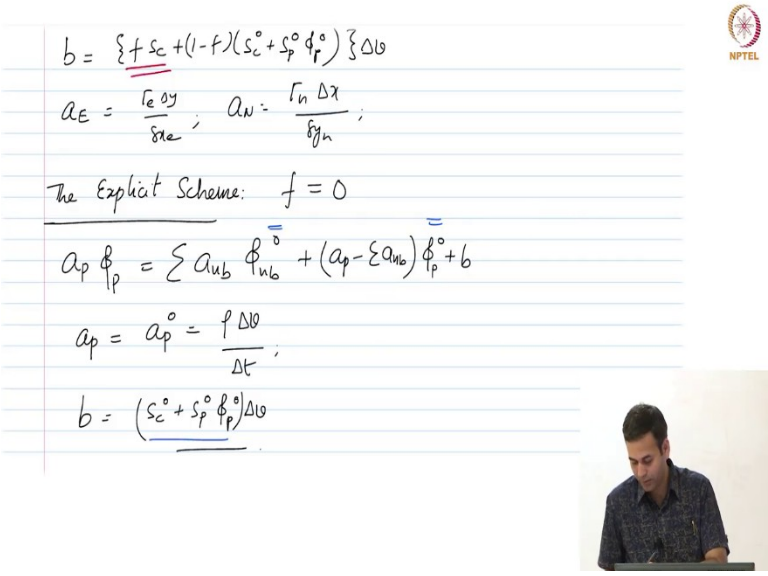

What would be b? b is the source term that is $\{f S_C + (1-f)(S_C^0 + S_P \phi_P^0)\}\Delta V$. Now, what about the coefficients? a_E, a_W; all those things, they are the same as what we had in the study diffusion equation right; for a Cartesian mesh.

So, this was a_E equals ah; how much was a_E? $\frac{\Gamma_e \Delta y}{\delta x_e}$ and a North would be $\frac{\Gamma_n \Delta x}{\delta y_n}$ and of course, you can complete a West and a South as well. So, those are your a_{nb} values fine; questions till now? This is basically kind of recap of what we learned yesterday.

So, learn let us now look at different schemes. So, you can get three; we are going to discuss kind of three different schemes one is the explicit scheme which is basically you set the interpolation factor $f=0$ and that is going to give you an explicit scheme. Now, if you set $f=0$; can you tell me what will be the resulting equation? Resulting equation will be $a_P \phi_P$ equals, what will be on the right hand side? $f=0$; how much? $\sum a_{nb}$.

Student: ϕ_{nb}^0.

ϕ_{nb}^{0}; because the first term would drop out right, so this will; this will basically go to 0 because f is 0 right.

So, only term that remains is this guy ϕ_{nb}^{0}; what else? Plus what else? f equals 0; $\left(a_p-\sum a_{nb}\right)$; would the term there or not?

Student: It will be there.

It will be there right. So, that will be what? Plus $\left(a_p-\sum a_{nb}\right)$; times how much?

Student: ϕ_P^0.

ϕ_P^0 plus some b right, now what is a_p? a p is how much? a_p is.

Student: (Refer Time: 13:20).

a_P^0 because if you look at a_p, this term is there right oh no; not, this term; this term right. So, you have a_p; contains a_P^0 plus this is now 0, this is 0 because f is 0 right.

So, $a_p=a_P^0$; so $a_p=a_P^0=\frac{\rho \Delta V}{\Delta t}$. Of course, a_{nb}'s are the same, what about b? b is $f\,S_C$; this is 0 right, what about the remaining term? This will be; this will survive, so this is $\left(S_C^0+S_P\phi_P^0\right)\Delta V$ right, that is b correct, everybody with this? Ok, no mistakes alright.

So, now let us look at this equation and make some comments; what is the first thing that you see in this equation? What is on the right hand side? Is everything on the right hand side known or unknown?

Student: Known.

Or is it a mix.

Student: Known.

Everything is known right because you have all of them evaluated at what time level?

Student: 0.

At 0; so this is ϕ_{nb}^{0}; ϕ_{P}^{0} and b contains all the 0 terms right. So, everything on the right hand side is known right and what is the unknown here? Only on the left hand side; ϕ_P is the unknown, that is at the current time level right that is at $t+\Delta t$.

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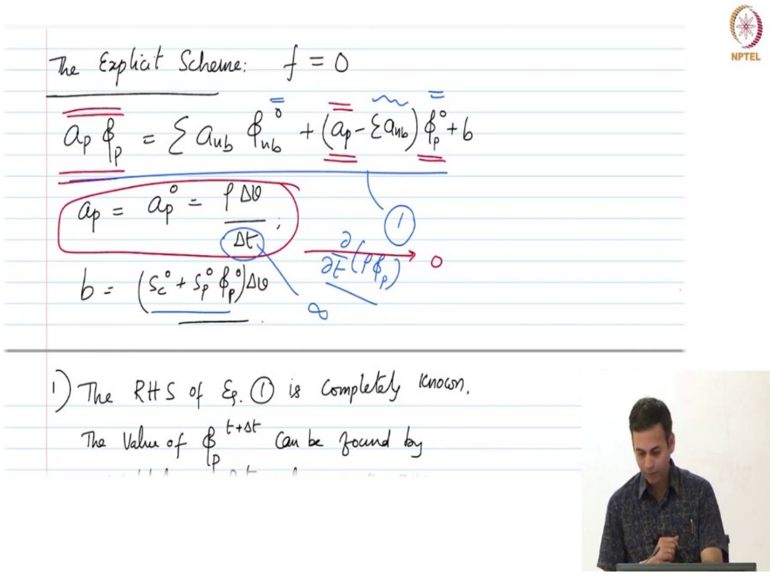

So that means, if you were to make some comments; first comment is the right hand side of; let us call this as some equation, let us call this as some equation 1. So, right hand side of equation 1 is completely known right.

So, that means the value of $\phi_{P}^{t+\Delta t}$ can be found by substituting ϕ_{P}^{t} values right in the RHS alright, that is what we can do correct; that means, it is a simple substitution. You know what is ϕ_{P}^{0}; you just plug it in, calculate what is ϕ_P right. Do we need to solve a system of linear equations here? No, because we are going cell by cell and every cell you just substitute what is there, in that cell in the previous time cell; this is much easier, is not it? Now, everybody we can go very easily right; so this is there.

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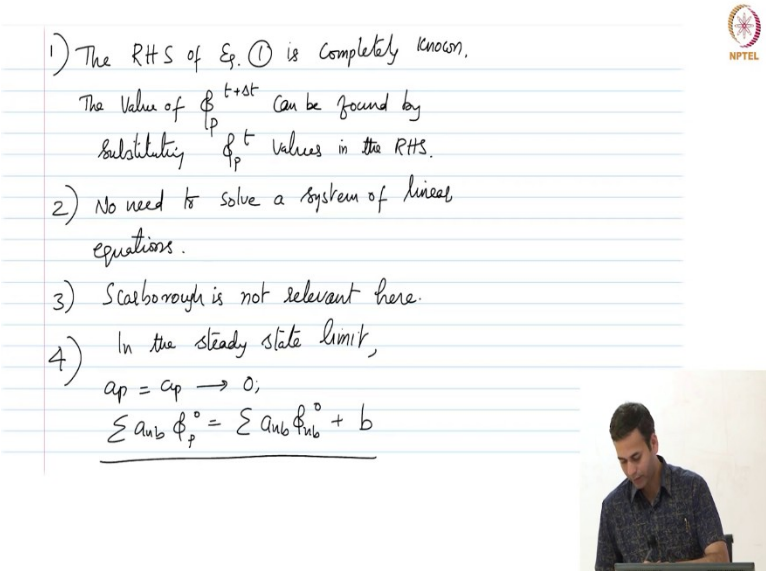

That means, we do not have to solve for; no need to solve a system of linear equations right; that is good, we do not have to solve for system of linear equations. What about Scarborough criteria? Does it satisfy Scarborough criteria or not; we do not care if it satisfies or not right because.

Student: (Refer Time: 16:50).

We are not solving for a system anymore right. So, we do not really care whether it is all satisfies Scarborough or not because we do not have to solve for a x equal to b right. Of course, we will be worried if the iterations do not converge right; so there will be something that is equivalent of Scarborough criteria that we will see. So, Scarborough is not relevant here, fine.

What about the behavior as we approach; let us say we are trying to solve a, a problem for which there is a steady state solution exists. So, we know that there is a steady state solution, but we started off with this unsteady diffusion equation. You have started off with this thing, but you know that there is a steady state solution that exists.

So, in that case; what will be the equation in the limit of steady state? Let us say as we in the steady state limit, now what will happen to the discrete equation; which coefficients would go to 0?

Student: a_P^0.

a_P^0 and what about a_p?

Student: (Refer Time: 18:10).

a_p also goes to 0 right; a_p is gone, a_P^0 is gone. So, what are we solving? Ok, so essentially these two would go to 0 because it is kind of a steady state, then what will happen to the remaining parts of the equation? That means, this term is gone right and this term is gone, what is the resulting equation?

You have $-\sum a_{nb}\phi_P^0$, this goes to the left hand side right. So, essentially you get $\sum a_{nb}\phi_P^0 = \sum a_{nb}\phi_{nb}^0 + b$, does it look familiar? This is nothing, but you are.

Student: Steady.

Discretized steady state diffusion equation right; so essentially we are recovering our steady state equation in the limit of steady state, from the equation; that means, in the limit of steady state; that means, a_p and a_P^0; both tend to 0. As a result, the equation we get by rearranging these terms is nothing, but send this guy to the left hand side right. So, that would be $\sum a_{nb}\phi_P^0 = \sum a_{nb}\phi_{nb}^0 + b$ alright, this is our original equation in the first place for a; if you had started off with the study diffusion, fine.

Now, what about; this is from the point of view of the equation. Now, let us say your iterations converge or your time steps converge and you obtain a steady state solution.

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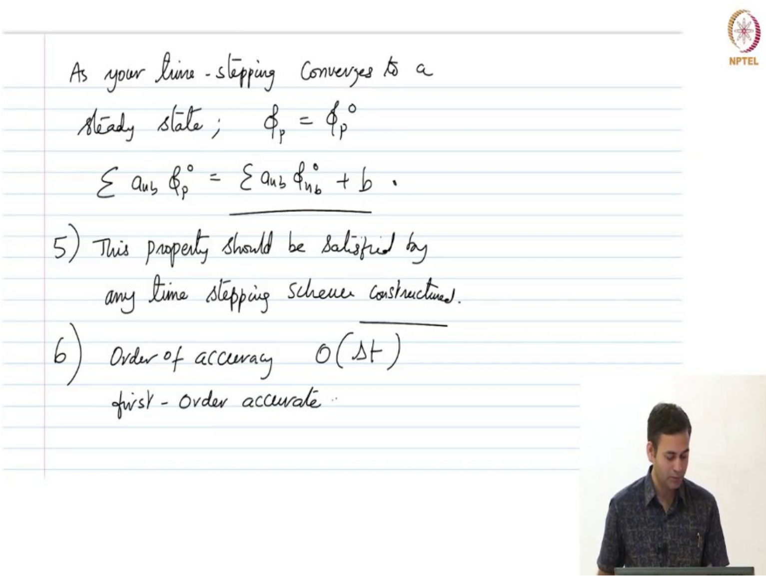

As you are time stepping converges to a steady state, what is that you would obtain; what is, which quantity becomes unchanging? ϕ_P and ϕ_P^0, would they become the same; if you have reached a steady state?

Student: (Refer Time: 20:04).

Right, essentially; that means, ϕ_P equals ϕ_P^0; then what will happen to this equation? $\phi_P = \phi_P^0$, so which terms get cancelled?

Student: a p (Refer Time: 20:14).

$a_P \phi_P$ would be the same as $a_P \phi_P^0$ right? Essentially, this term goes away; then what remains? We get the steady state equation right; that means, as this equation approaches, as the problem approaches a steady state through iterations, through time stepping; then you are recovering the original steady diffusion equation right; that property should be valid for any time stepping scheme that you construct.

Do you see that everybody? Right, as $a_p = a_P^0$; $a_P \phi_P$ would be the same as $a_P \phi_P^0$ on the right hand side. As a result, the only term that survives are the terms that survive are the $\sum a_{nb} \phi_{nb}^0 - \sum a_{nb} \phi_P^0 + b$, right? Which, you can rearrange to read it as; the same thing as before right.

So, essentially this means $\phi_P = \phi_P^0$ right, then you would recover the same equation that is $\sum a_{nb} \phi_P^0 = \sum a_{nb} \phi_{nb}^0 + b$; this would be the same equation, as we got before right. So, essentially; if there is a steady state to your problem, then your unsteady solution or the discretized equation would also converge to that steady state by satisfying the original equation, yes.

Student: Sir, how does a_P in the case of steady state?

In the case of second one or first one?

Student: First one.

First one?

Student: a_P (Refer Time: 21:43).

Now, essentially you do not have terms from coming from a_P and a_P^0 because what was the; how does these terms are coming about? These are coming about because you have this term which is $\frac{\partial}{\partial t}(\rho\phi_P)$. Essentially, you are you have a steady state; so there is no change with respect to time.

Student: (Refer Time: 22:06).

As a result, this goes to 0; so the two terms that; I out, got out of this thing are the $\rho\phi_P \Delta V$ and $\rho\phi_P^0 \Delta V$ right which manifested into a_P and a_P^0 into our equations right. Of course, another way to look at is you take in the context of steady state, your delta t kind of goes to infinity right; you are looking at a steady state. So, you; you are essentially independent of the delta t at a very long time, as a result a_P and a_P^0 kind of go to 0; both of them right.

Student: So, delta is t times scale.

Delta t is times; so that is why you should not look at it that way, but that is the only timescale you have. So, in the limit of a steady state you the; you do not have any time stepping that is coming into play right.

As a result, you do not have these coefficients a_P and a_P^0 which came about because of this unsteady term that is $\frac{\partial}{\partial t}(\rho\phi_P)$ right; because if this was this goes to 0 then you do not have these terms coming into play at alright, this is a_P and a_P^0 would not be there.

Other questions; fine; so let us make some more comments, so we see that it kind of converges to the same solution; so this property has to be satisfied by should be satisfied by any time stepping scheme, that is constructed or used fine. So, all the time stepping schemes that you would come up with should satisfy this criteria.

Now, we will see that we have not done it, the order of accuracy of the scheme is only first order; this is only order Δt, this is only first order accurate. So, that means as you refine the Δt; your temporal error only goes down by order Δt. So, that is not very good; so you have to take like smaller and smaller delta t.

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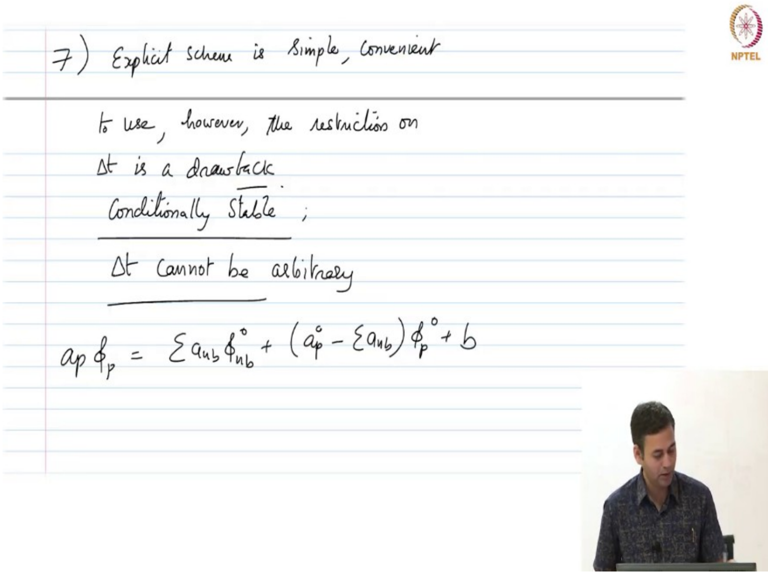

So, this method; the explicit scheme is simple right and it is very convenient to use. So, that is why explicit schemes are kind of used in most of the computations, but; however, there is a drawback. However, the drawback is the; restriction on Δt is a drawback. So, that means you cannot take any Δt that you wish; so you have to follow one particular condition.

So, there is basically only conditionally stable; that means, delta t cannot be arbitrary; Δt cannot be arbitrary. So, we will come to this point to; through two different ways, one is when we do the von Neumann stability analysis. The other one is by looking at the equation; it is kind of a just by looking at the equations, we can heuristically say what will happen to this thing.

So, if you look at the equation shall I; I will just write it down again, can you help me write it? It is become messy. So, $a_P \phi_P = \sum a_{nb} \phi_{nb}^0$ plus what do we have¿; is that correct? $a_P \phi_P = \sum a_{nb} \phi_{nb}^0 +$¿¿, that is right. So, that is the equation we have for explicit scheme.

Now, what about the coefficient for ϕ_P^0; that is $a_P^0 - \sum a_{nb}$. Is there a possibility that this coefficient would become negative? Can it ever become negative? We know that $\sum a_{nb}$ are always positive, they are all positive; what about a_P^0? Is it always positive or not?

Student: Positive.

It is positive, but can this become negative? It can. So, if under what conditions can this become negative?

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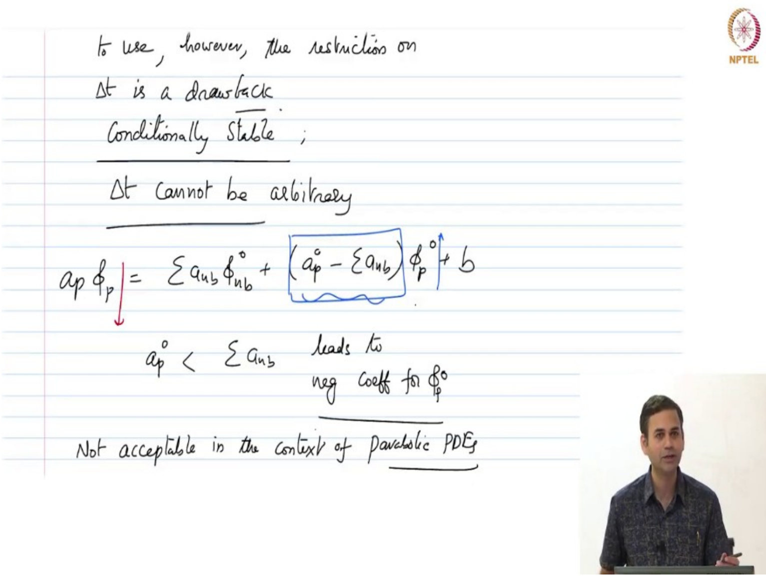

If $a_P^0 < \sum a_{nb}$, this would become negative right. So, this will become a lead to; this leads to negative coefficient for ϕ_P^0. So, it becomes negative; now, why do we care? We care because if this coefficient becomes negative, then what will happen to phi p?

Let us say, we have a minus n here; let us say in one particular Δt at; at time t, ϕ_P^0 is kind of a large value, it increases because you have a negative. What will happen to the ϕ_P at the current time level? This would go down right, as this ϕ_P^0 is increasing with time steps; this will go down right.

So, this kind of behavior is not acceptable in the context of parabolic PDEs. So, if you have the negative coefficients and increase in value of ϕ at one time level would lead to a decrease in value of t at the next level in the same cell. So, this kind of behavior is not acceptable; so this is not acceptable in the context of parabolic PDEs.

As a result, we have to make sure we kind of meet with this condition and the condition would be what? Would be a_P^0 has to be either greater than or equal to some of the a_{nb}; so that means, a_P^0 has to be greater than or equal to $\sum a_{nb}$.

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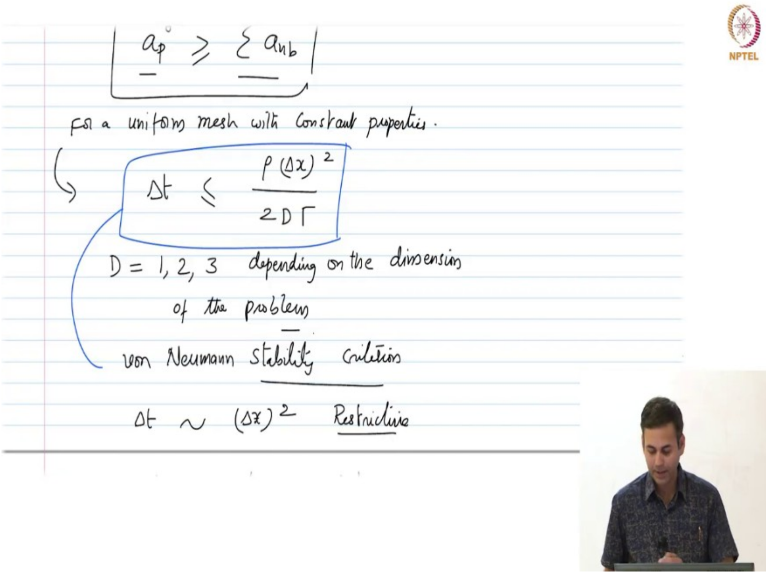

So, that is the condition we have to satisfy. So, we can show that this condition boils down for a uniform mesh with constant properties. It boils down to something that looks like this; Δt that we have should be less than or equal to $\frac{\rho \Delta x^2}{2 D \Gamma}$, where D is either 1, 2 or 3 depending on the dimension of the problem.

So, if you are looking at a 1 D problem; this is $\frac{\rho \Delta x^2}{2 D \Gamma}$. Now, we will prove this little later; using von Neumann stability analysis; we can of course, also prove this by just writing what is a_{nb} and what is a_P; you can substitute the definitions, $\frac{\rho \Delta V}{\Delta t}$ and $\frac{\Gamma_e \Delta y}{\delta x_e}$ and so on and then prove that it will; this condition is; is satisfied. So that means, you cannot take any arbitrary Δt; it has to be dictated or should be smaller than this condition.

So, this in the literature is known as the von Neumann stability criterion; which has to be satisfied by the explicit scheme, in order to get converged solution; in order to get physically possible and converged solution.

Now, alright; so this condition is too restrictive because your Δt now goes as Δx^2, that is too restrictive. Because if you; if you keep refining your mesh, you have to take it much smaller than the Δx^2. So, that is very restrictive because you can only go with a very small Δt, you can march ahead.

So, it is very restrictive; that is why the explicit schemes are not used in to a great extent because of the limit on the Δt.

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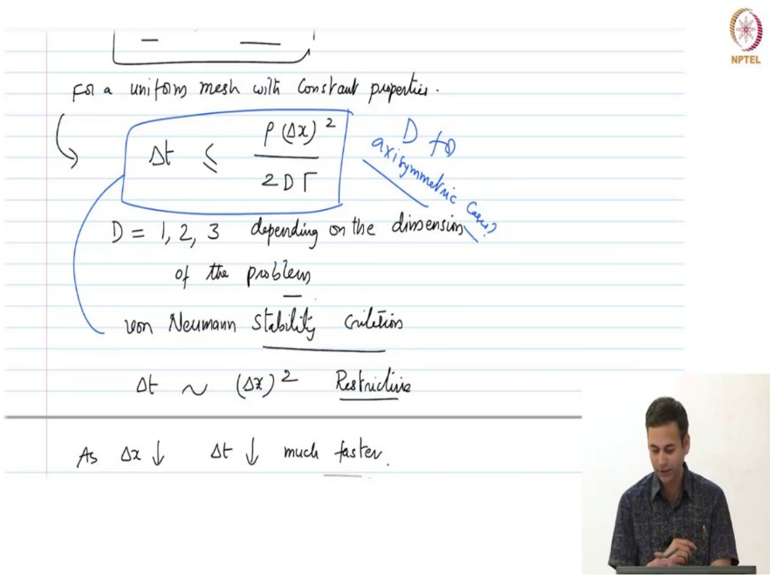

And also as Δx goes down; Δt goes down much faster right. So, this is kind of a big restriction for the explicit schemes. We will do this again in the context of von Neumann stability analysis that will be coming up in the next few lectures; fine, questions till now?

Student: Can Δt be negative?

Can Δt be negative? Can Δt be negative? Well, can it be negative? Cannot be, right; yes.

Student: (Refer Time: 31:50).

Which is?

Student: a_p is greater than.

a_p; is it a_p or a_P^0? a_P^0right? This one.

Student: a_p.

In the actual equation, it is a_p; is it which is the actual equation, you mean the one; I have written here, this is a mistake; this should be a_P^0 right. Is not it? That should also be a_P^0. Anyway, for the context of explicit schemes; $a_p = a_P^0$ right, a_p is same as a_P^0 for the context of explicit schemes.

Other questions, why is that essentially an increase? Essentially, these kind of solutions are not supported by parabolic PDEs. You cannot have; for example, if you in the context of diffusion problems, we saw that everything has to be bounded in the absence of source terms right.

Similarly, when you have a parabolic PDE; you cannot have an increase in value at one time step causing, a decrease in value at the next time step; as you for the same cell. Now, why is it a problem? This actually leads to; eventually this leads to what? Leads to oscillations which will lead to divergence right. Essentially, your solution will kind of a oscillate will start oscillating and then it will eventually diverge; that is what actually is dictated by this.

We will try this with an example in the assignment; we will have where Δt will be given to be certain value, if you start taking it to be larger than the von Neumann stability condition; you will see that see that the solutions you would get are kind of display oscillatory nature and eventually they will diverge.

Essentially, you would get a very large values, but this behavior is perfectly fine; if you have let us say a hyperbolic PDE right or if you have let us say a source term or a (Refer Time: 33:38) term and things like that; other questions; yes.

Student: (Refer Time: 33:46) run a simulation backward in time?

So, you want to run a simulation backward in time; so we have this principle of causality right. What is principle of causality? What is principle of causality? You have a cause and effect right; it is not effect and cause right. For example, only today can influence tomorrow, today cannot influence yesterday right. So, if you want to run a simulation backwards; I do not know what does it mean, it is as simple as going back to yesterday right.

Student: (Refer Time: 34:21).

And if you can put it physically, I would be happy to answer.

Student: (Refer Time: 34:26).

Ok.

Student: (Refer Time: 34:28).

Ok.

Student: (Refer Time: 34:31) in a small time step.

Ok.

Student: (Refer Time: 34:33).

You want to go back by a small time step for the simulation.

Student: (Refer Time: 34:39).

I do not know that is essentially same as you start off with the old condition and proceed in the future right. Instead of going back, you start off with whatever is the time and then you proceed forward right; that is the main difference between time stepping and spatial differences, is not it? You do not have.

So, spatial schemes if you look at have; you can go both ways right, you have dependency on $i+1$, $i-1$ 1, $i+2$, $i-2$ and so on, but Δt; you have only dependency on either t; $t+1$, $t+2$ right; not on the previous. You can have on the previous, but they should be known right, but you do not go in the backward direction right; other questions, no? Clear?

Fine, so you can now code the explicit scheme right; essentially it is calculation of ϕ_P from the old values right, very straightforward.

Student: (Refer Time: 35:33) value of D equal axisymmetry.

What will be value of D, for an axisymmetric case? That we have to see; probably it will be slightly maybe, it will come out to be 1; I do not know, I have not; I have not solved it, maybe that is a good question. What is the value of D for axisymmetric? Maybe, you can calculate and tell me; what is D for axisymmetry?

Now, I have one question. So, let us say we have no iterations right which is kind of a good news; so why do not we solve every steady state problem using unsteady with this explicit scheme?

Why should we bother ourselves to write this TDMA; line by line TDMA, all these things and or Gauss Seidel or cannot we just pose or pretend that all the steady state problems that we want to solve; like till, now we would say that they are all unsteady and then we will just add this extra term and then start using this explicit scheme.

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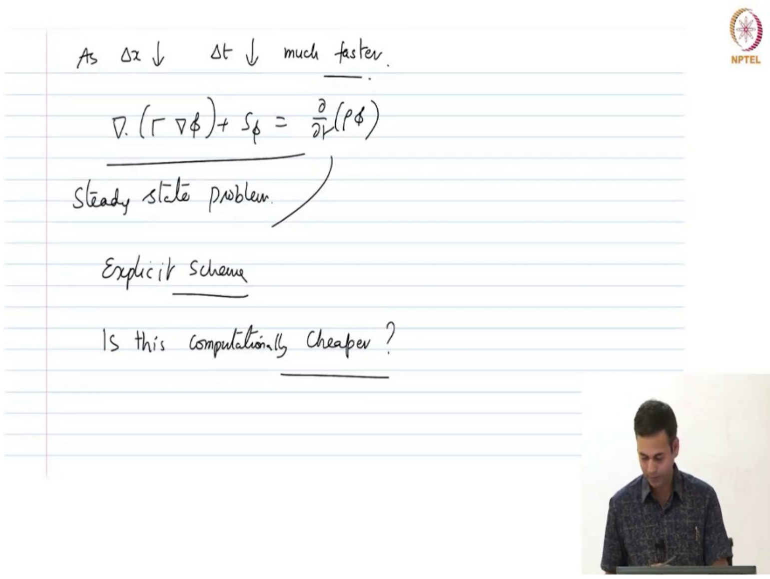

Would that; would not that be easier? Would not that be faster? Is the question clear? Right, we wanted to solve for $\nabla\cdot(\Gamma\,\nabla\phi)+S_\phi=0$, so essentially we want to solve for a steady state problem right; for which we know that the problem itself is steady state.

So, what I am saying is; we add this, we say that we pretend that this is not steady state; we start off with the $\frac{\partial}{\partial t}(\rho\phi)$ right and then we construct an explicit method using explicit scheme. I would come down and then use these equations and just keeps substituting with some guest initial condition; I would just keep updating the values from here, would not that be faster and easier than solving for a steady state problem in the first place?

Student: We stopped at $\phi_P=\phi_P^0$. We stop at $\phi_P=\phi_P^0$; is it not easier? That is something for you to think; is this easy or is this computationally cheap right or is it correct? So, that is what; question is clear right? Ok, fine; this is for you to think. Alright, let us move on to the next method that is the implicit method, fully implicit method; you would obtain a fully implicit method, if you have if you set f equals 1.

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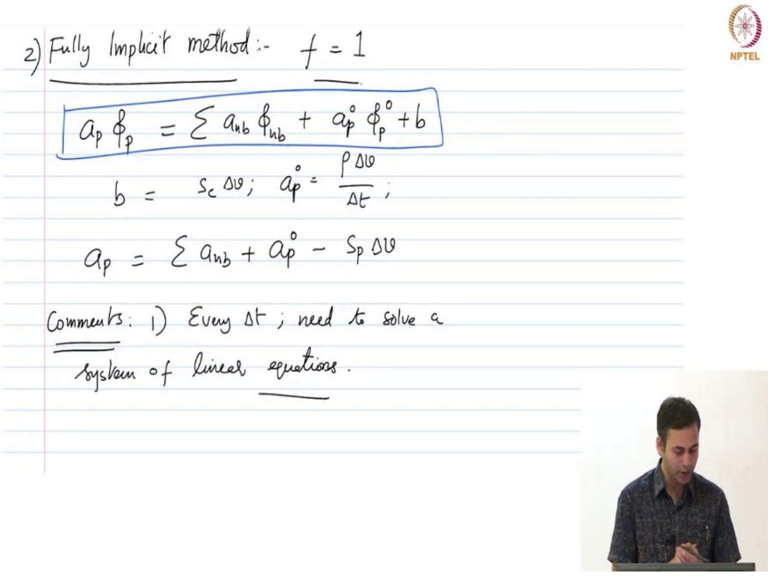

So, in the previous case we kind of assumed that f equal to 0; that means, the ϕ_P^0 values; 0 leveled time values prevailed over the entire time step right. Here we are assuming that f equals 1; that means, over the entire Δt, the ϕ values at $t+\Delta t$ prevail over the entire Δt right; that is what we are assuming.

Now, essentially substitute f equals 1 and tell me what would be the equation. So, we have to go back to the original equation; substitute f equals 1, again this would be some $a_P \phi_P$ equals; f equals 1. So, second term drops out on the right hand side; so the first time will remind; no, I think; so that is, what would be on the right hand side? Sigma.

Student: a_{nb}.

a_{nb}; ϕ_{nb}.

Student: Plus a_P.

Plus.

Student: a_P^0.

a_P^0; a_P^0times ϕ_P^0; that is all because the second term has f times something; so that is 0, so $a_P^0 \phi_P^0$.

Student: Plus b.

Plus b and what would be b?

Student: $S_C \Delta V$.

$S_C \Delta V$, that is all and what about a_P? Is the same as a_P^0? No, it is not, what is a_P? You will have $a_P = \sum a_{nb} + a_P^0 - S_P \Delta V$; is that correct? And a_P^0 is how much? Same as before; that is $a_P^0 = \frac{\rho \Delta V}{\Delta t}$ fine. Now, what do you, what can you say about this equation? So, we are; this equation you would solve at a particular time step right, for a particular Δt.

So, what kind of; if you want to make some comments, what can you talk about say about this equation? Is it, can you substitute for ϕ_P values and get it, similar to the previous case? You cannot right because the ϕ and b are unknown. So, what do you; what would you need here? For every Δt, you need to iterate or essentially you need to solve for a system right.

So, every Δt requires; every Δt need to solve a system of linear equations; that means, you have to use Gauss Seidel or a TDMA, if it is 1 D or a line by line TDMA; for every Δt.

Now, do you see the two loops; if you are let us say using Gauss Seidel, the inner loop is where you would solve for a particular Δt and the outer loop is where is your time stepping loop; right your Δt is going with by amounts of Δt right.

So, there are two loops; now do you see the difference between S_C^0 and S_C^1; in this context? Right, you would keep some things the same for the entire Δt, but certain things, you would update as you go within the inner iterations right. So, that is where this difference of S_C^1 and S_C^{01} comes into play right; fine.

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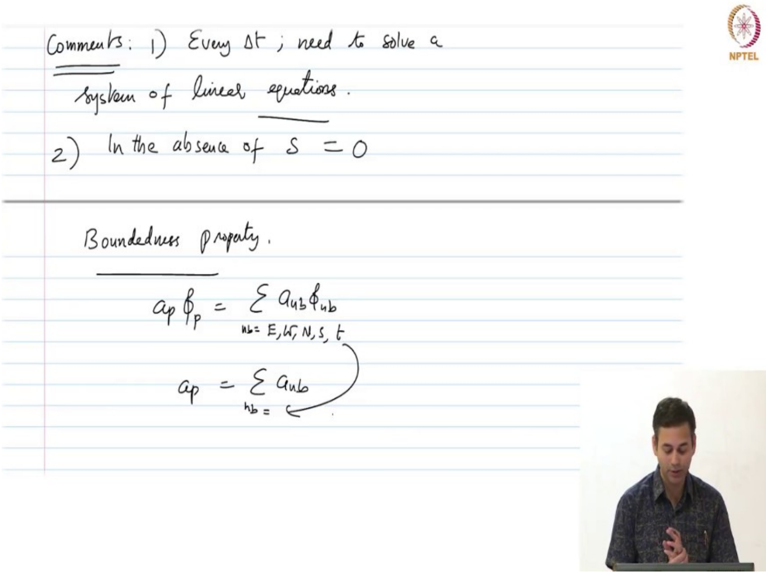

So, every Δt; you need to solve a system of linear equations, this is lot more work right of course. Then, what else? Then, the second thing is; if you look at this equation, a_P; let us say in the absence of source terms, in the absence of source terms; how does this equation look like? This equation looks like; $a_P \phi_P = \sum a_{nb} \phi_{nb} + a_P^0 \phi_P^0$ right, b is 0.

So, essentially this is nothing, but $\sum a_{nb} \phi_{nb}$ where the neighbors are East, West, North, South and also the ϕ_P^0 right. So, the previous time level can also be thought of as a time neighbor right. You have four space neighbors and one time neighbor; that means, we are now back to this original concept of boundedness right. Because a_P is now equal to $\sum a_{nb} + a_P^0$ and $a_P \phi_P$ equals $\sum a_{nb} \phi_{nb} + a_P^0 \phi_P^0$.

We are back to this boundedness, what do you say the problem right or the boundedness; what would you say in terms of the property right. So, we have a boundedness property right we are back to that. So, the boundedness property is now satisfied by this method right because now we are back to $a_P \phi_P = \sum a_{nb} \phi_{nb}$ where nb goes from East, West. North, South and time and $a_P = \sum a_{nb}$ where nb goes the same way; as it is right.

So that means, the coefficients are all will come out to be fractions of a_P right; $\frac{a_{nb}}{a_P}$ would be what? It comes out to be; that means ϕ_P is bounded by ϕ_{nb} and ϕ_P^0 right.

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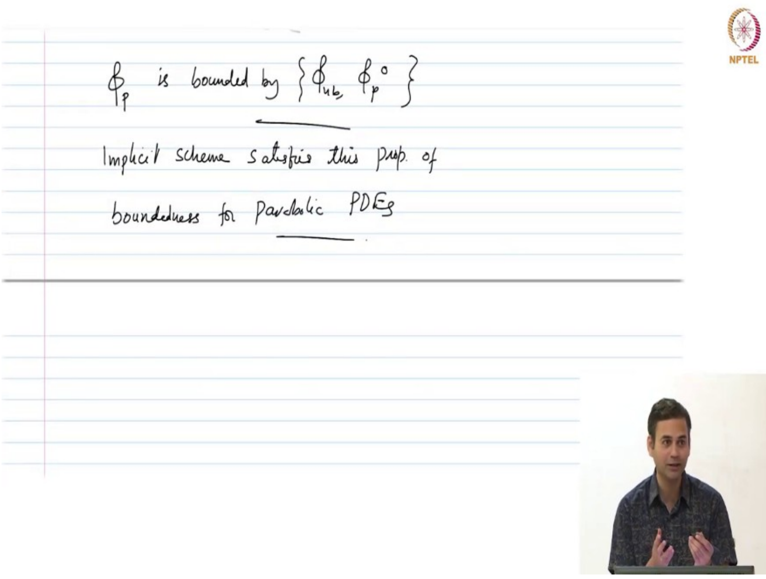

Now, will it be not bounded in any context? Let says source term is still 0, would there be a condition where it will not be bounded? Let us say, you take any Δt; would there be an effect of Δt on this thing?.

No, right because it does not matter any Δt you take, accordingly a_P^0 will adjust and then a_P will still be some of $\sum a_{nb}$ and then $a_P \phi_P$ would be still some of $a_{nb} \phi_{nb}$ right; that means, you would never get this problem of negative coefficients right. And always it is bounded which is the property that has to be satisfied by parabolic PDEs; as a result implicit scheme satisfies this property, very good.

Now, what is the advantage of the implicit method? We just saw that it has to solve Gauss Seidel or something like that, now where is the advantage coming into play? We are putting in more effort, what is the advantage? We just discussed.

Student: Δt.

Δt, essentially you can take any Δt that you like right and you still get a solution. Now, that does not sound quite right because you may get a solution which is kind of not infinite; but it may not be accurate, if you take a very large Δt.

So, the Δt you take from implicit schemes is of course, much larger than the Δt that can be taken by the explicit methods; whereas, this Δt cannot be very large that the solution is inaccurate.

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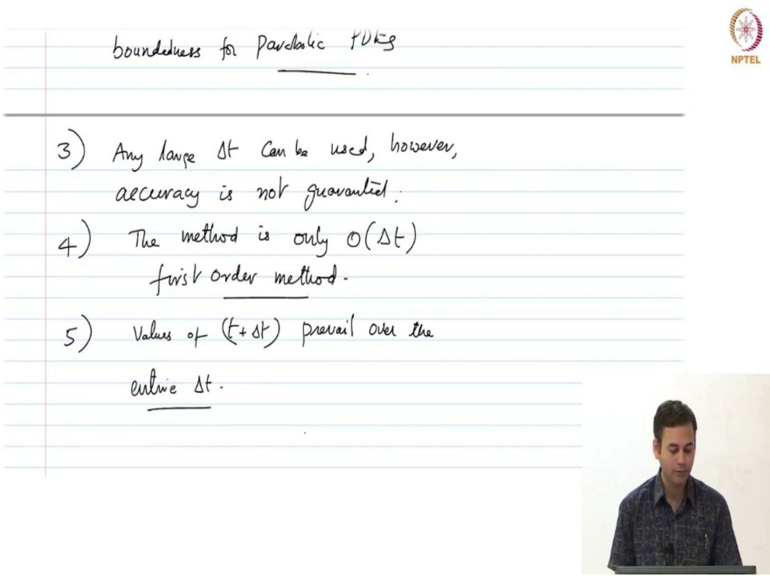

So, essentially one of the comments is any large Δt can be used; however; if it is too large, accuracy is not guaranteed, but still you would get a bounded solution; you will not get an infinite solution; however, accuracy is not guaranteed.

But convergence is guaranteed; it will converge to some answer, but it will not be accurate because the Δt is very large ok. Of course, we also we will prove later that the method is; is only order Δt. So, if you reduce the Δt by half, they are only goes down by half; it is only a first order method, its only a first order method right.

So, we have; so as a result; you kind of use implicit method, but not to a large extent because we have another method that we will discuss tomorrow that is the (Refer Time: 47:26) method which is the combination of these two.

So, essentially in the implicit method, the values of $t+\Delta t$ prevail over the entire Δt right, that is ϕ_P^1 values or ϕ_{nb}^1 values, prevailed over the entire Δt; that is what we are considering fine. So, we will kind of stop here; we will meet in the next class and discuss about the final method that is the (Refer Time: 48:01) method.

Thank you.