Computational Fluid Dynamics Using Finite Volume Method
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Lecture – 17
Finite Volume Method for Diffusion Equation: Problem solving using line-by-line TDMA

So we were solving this problem, the 2 dimensional problem right of heat conduction for the line by line TDMA method right, or the line by line method.

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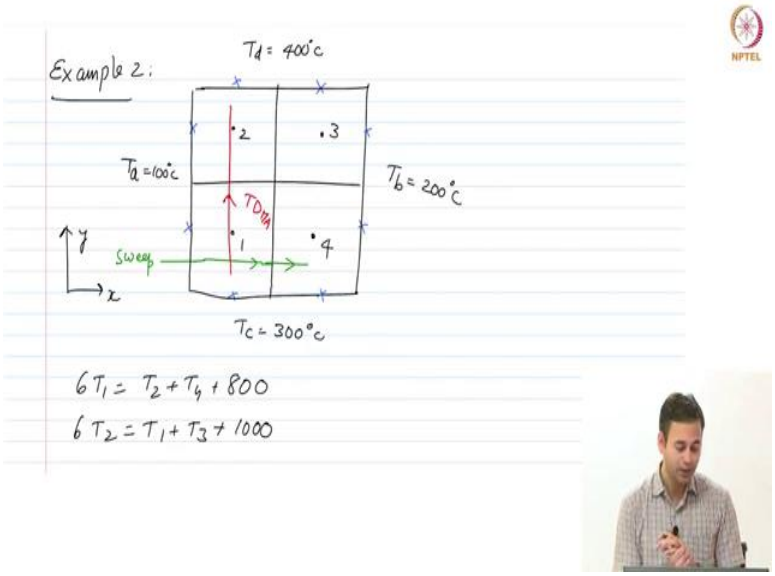

So, this is the example 2 right. Where we said, we have a domain with 4 cells right. We call this as 1, 2 and did I call this as 3 or 4?

Student: 4.

This is 4. So, we came in like clockwise right. So, this is 3 and T 1 or T a was 100, T b was 200.

Student: 200.

200 and T c is 300 and T d is 400 degree Celsius. And then we have these face centroids right, that get connected essential points pack ok. So, and then we have these face centroids here, right, is these correct? 1 2 3 4 right, in the clockwise, did we say like.

Student: (Refer Time: 01:30).

I wanted to choose two above, because we want to apply TDMA in that direction. It will be easy in terms of numbering right, ok.. So, we had this and then we have, in the yesterday's class we have seen, we have written down the linear algebraic equations, right. We have discretized and written down the equation. So, can you tell me what are the equations we have for this problem? This was $6T_1 = T_2 + T_4 + 800$

Student: 800.

800. And then $6T_2 = T_1 + T_3 + 1000$

Student: T 1.

Student: 1000.

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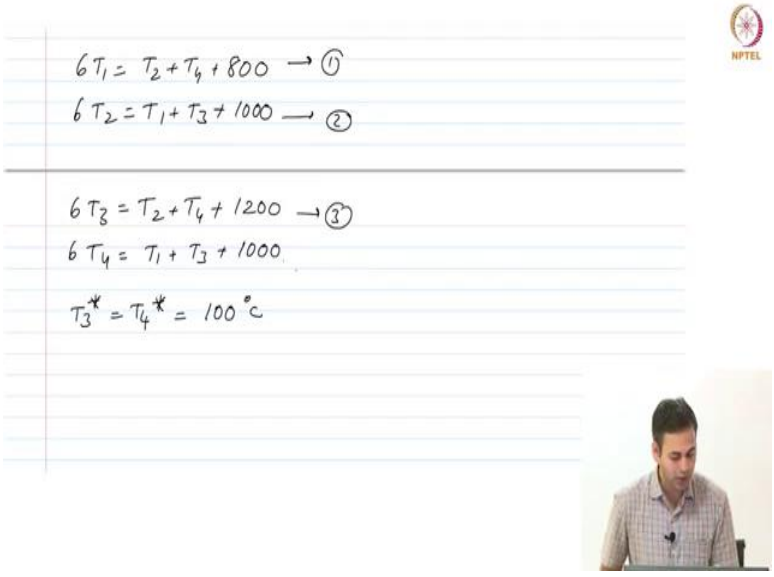

And then, what else we have? 6 T 3 equals T 2 plus T 4.

Student: (Refer Time: 02:12).

Plus 1000.

Student: 200.

200 ok, 1200 and then we have 6 T 4 equals T 1 plus T 3 plus.

Student: (Refer Time: 02:22).

1000 ok, 1000 that is what we have. So, essentially we have 4 equations. Now, we are deciding that we wanted to do, let us say TDMA along the y direction ok. So, this is our x and this is our y directions, and we would do, guess values or the sweep direction would be your x direction ok. So, this would be your sweep direction right, and the y direction would be your TDMA direction.

Now, we need to guess values for the cells 3 and 4 right. In fact, we have to guess values for all the, all of them right. Because we would be using the phi T 3 star and T 4 star.

So, let me start off with guess values for T 3 star equals T 4 star equals, I will start off with 100 ok, 100, T 3, T 4. I can also guess for T 1, T 2, but it does not make sense anyway, because we are using a direct solver in that direction. So, we do not need those values, because they will get anyway over written right, ok. Then fine then we have our first equation, that is this guy, this is our equation 1, 2, 3 and 4.

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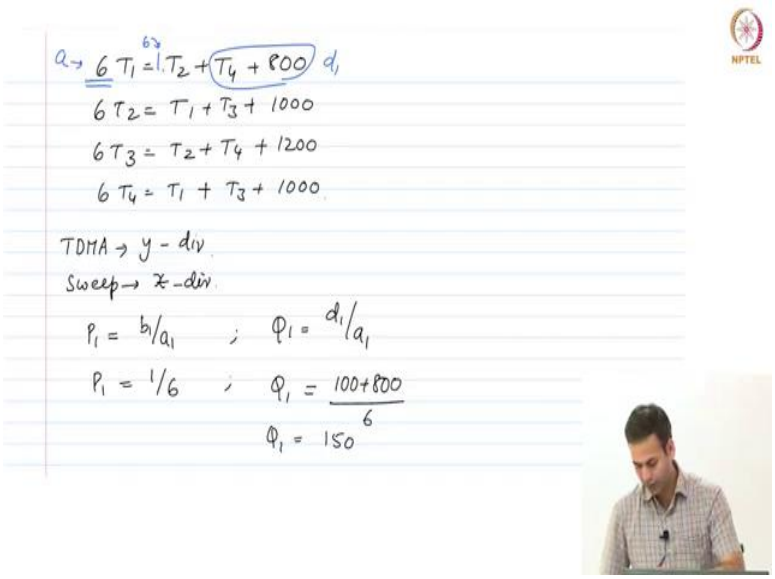

6 T 1 equals T 2 plus, how much? $6T_1 = T_2 + T_4 + 800$

Student: T 4.

. And $6T_2 = T_1 + T_3 + 1000$. And $6T_3 = T_2 + T_4 + 1200$

Student: 1200 and then $6T_4 = T_1 + T_3 + 1000$ So, we have these equations. Now, what we want to look at, is we want to apply TDMA in the y direction, right. TDMA in the y direction and sweep in the x direction ok. That is what we wanted to do.

So, then for TDMA what is that we have to do first? We have to calculate what are the coefficients P 1 and Q 1 right or P i and Q i. So, what are these coefficients for the cell 1, if we start off with 1 right. So, what is P 1?$P_1 = \frac{b_1}{a_1}$ I want the expression first, this. And then what is Q 1? $Q_1 = \frac{d_1}{a_1}$ Now, what is abcd in the expressions? a is 6 right, this is your a. What will be b? b is a 1, that is to the.

Student: i plus 1.

To the i plus 1. So, in this case it is actually j plus 1, right; we do not have. So, that is coefficient of T 2 right. So, this is 1, so this is 1 is your b. What about c? c is 0 right, c is 0, because you do not have a connection to T 0 right. So, c 1 is 0. So, what about d?

Student: (Refer Time: 05:26).

T 4 plus 800 right. Essentially, whatever is remaining this is your d 1, right; that is your d 1 ok. Now, how much would be; how much would the, these values come out to be? b 1 is 1 and a 1 is 6, that is your P 1. What about Q 1? $Q_1 = \frac{100+800}{6}$ so that would be 100 plus 800 divided by 6 right.

So, this is 900 by 6. That will be$Q_1 = 150$ 150 P 1 and Q 1, then what do we have? P 1 and Q 1 are then. Then what about P 2?

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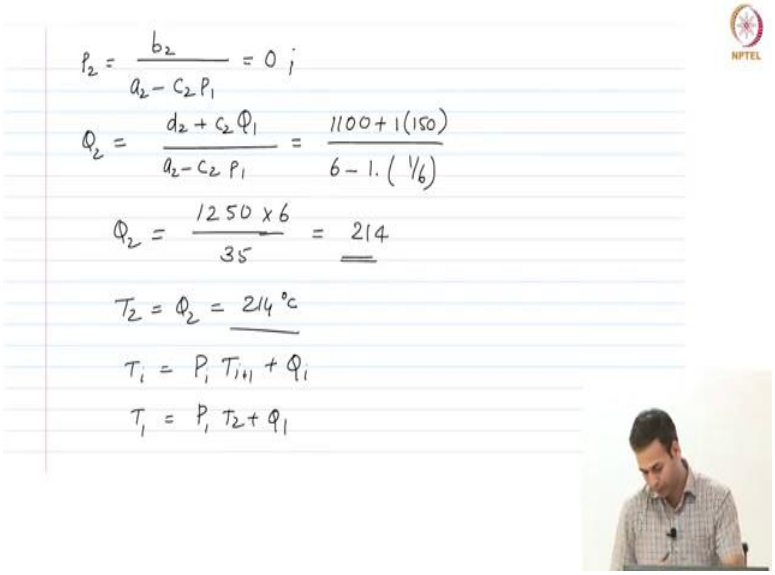

P 2 would be how much? $P_2 = \frac{b_2}{a_2 - c_2 P_1} = 0$

Student: (Refer Time: 06:13).

I want expression that is b 2 upon.

Student: (Refer Time: 06:16).

Student: c 2.

. So, we are looking at the cell 2. What is b 2, b 2 is?

Student: 0.

; because we have to look for?

Student: (Refer Time: 06:31).

We have to look for the cell which is in the traverse direction. So, we do not have 1 right, because here 1 is i minus 1 right. T 3 is a guessed value. So, we do not have another cell on the right other direction, right.

So, essentially I have kind of a half figure here, ok. Let me draw this anyway completely. So, I could make it better ok. So, essentially for T 2 cell, i plus 1 is not there here, right. We do not have a cell. So, essentially b 2 is how much? 0.

Is this expected? If you remember go back, we already had b n equal to 0 right. Which gave us b n equal to 0 right. So, essentially P 2 is now the last cell in the y direction, in which our TDMA. So, this is expected ok. We are not doing anything wrong, this is correct, this is no mistake.

Then what about Q 2? $Q_2 = \frac{d_2 + C_2 Q_1}{a_2 - C_2 P_1} = 1100 + \frac{1(150)}{6 - 1.\left(\frac{1}{6}\right)}$ Q 2 expression is d 2 right, plus c 2.

Then what is d 2? What the second cell, what will be d 2?

Student: (Refer Time: 07:54).

1000 plus?

Student: Plus.

T 3 star, right. So, that is 1100. So, this is 1100 that is d 2 plus, what is c 2? Is there c 2? c 2 is what coefficient of i minus 1 that is 1, so this is your c 2 right. So, this is your c 2 that is 1. So, we will come down. So, this is 1 times how much is Q 1? 150 upon a 2, how much is a 2? a 2 is coefficient of T 2, this is your a 2. So, this is how much?

Student: 6.

6, then minus c 2 is 1. What about P 1? One sixth right. So, this is one sixth. So, how much would be Q 2? This is 1250 times 6 by 35, is it? Ok. So, can you tell me how much this will be approximately?

Student: (Refer Time: 09:04).

257.

Student: (Refer Time: 09:06).

214 or something 54 something right? 214.0 something degrees 214.0 something ok. Let us not worry about degrees ok. So, this is Q 2, fine. Now, to have to solve for other variables,

no; we are done right. We are done with forward substitution. We just have 2 cells right 1 and 2. So, we already reached n this is Q n. Now what do we do?

Student: (Refer Time: 09:33).

Q n equals T n right. Q n equals T n. So, T 2 is how much?

Student: Q2.

Q 2 this is 214 degree Celsius ok.

Now, we will use the reconciliation that we already have, that is $T_i = P_i T_{i+1} + Q_1$. If I plug in i equals 2 in this, this is T 2 equals P 2 sorry, it should be P 1, I going to write it for P 1. So, this will be $T_1 = P_1 T_2 + Q_1$ So, fine.

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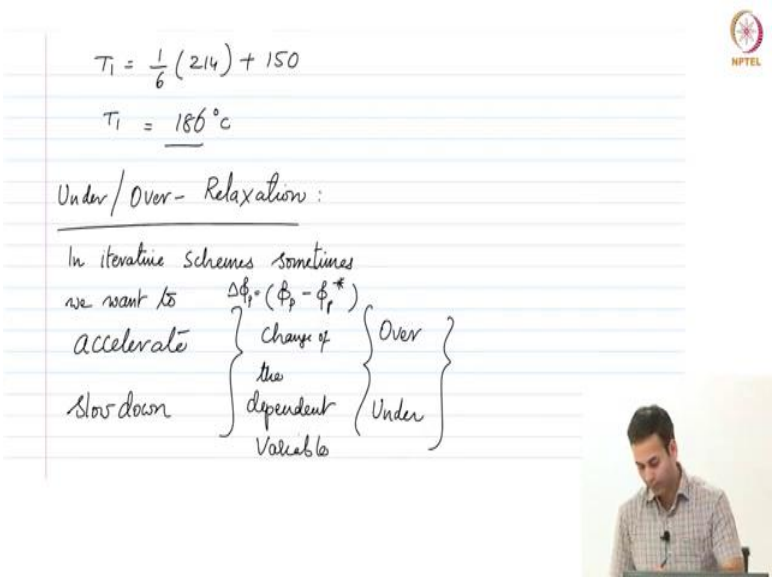

So, this is how much? T 1 is P 1 we got it as how much? One sixth and T 2 is

Student: 2.

214 plus Q 1 was 150 right, sorry not 150 how much was it?

Student: 150.

150 is correct?

Student: Yes.

150 is correct. So, how much would this be?

Student: (Refer Time: 10:33) 186 or something, this is 186 degree Celsius. So, this is by guessing the values of T 3 star and T 4 star as 100, right. Now can you do the same thing for 3 and 4 cells? Right. What you would use for T 1 and T 2? You would use these values, right. We are using the updated values.

Now, does these numbers make sense? You have got 156 for T 1 , do they, are they bounded and do they look similar.

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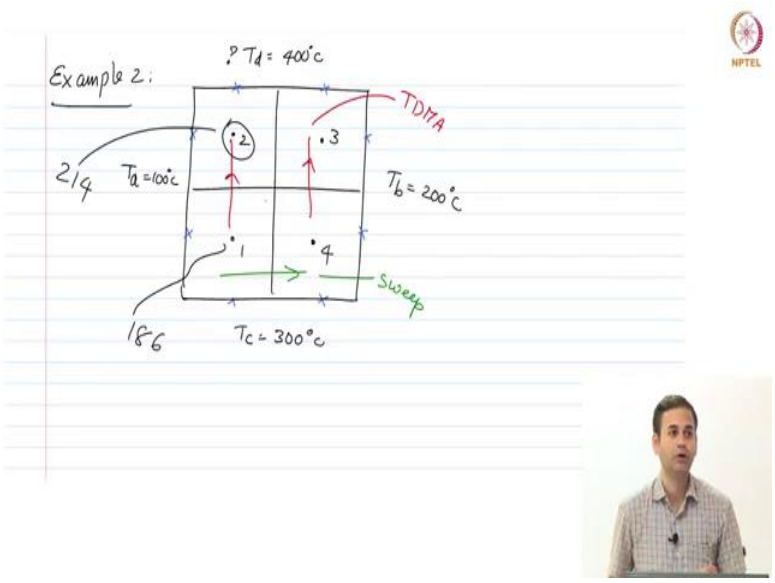

So, this we got as 1, 186 right. And the other one is 214. Do they look somewhat ok, right; it is kind of higher, because you have a higher value on the upper temperature and so on. You will use these values as the guesses instead of 100, 100 for T 1, T 2 and then do the same TDMA here right. And then what you do? What is a after this? You come back again, right. T 1 T 2 and then and then you keep iterating until you reach some kind of a converge solution, right.

Would you be able to do T 3, T 4 and kind of continue this process? Right. You can also write a sample code right, based on this algorithm fine. So, that is our sweep and this is our TDMA, ok. And then we kind of keep coming to these two, one after the other, right. In succession until you kind of converge fine.

Questions still now? Is it clear line by line, is clear?

Student: (Refer Time: 12:08).

Essentially, in some in some sense for example, you can looking in terms of a some kind of a norm, a L 2 norm or a infinity norm or essentially you can also look at when all the changes T 1, T 2, T 3, T 4 are less than some you know the difference between T 1, T 2, T 3, T 4 is not much between the previous and the current iterations ok. You can look it up in a, some kind of a norm as well, fine.

Other questions? 100 was just a guess, you could have, I could have started with 0 as well ok. Anything, you start off with 100, you can also start off with anything you like. Will that depend on the convergence right?

Student: (Refer Time: 12:53).

Will that depend, yes it will right. Essentially, you have guess something very close. For example, you have taken an average of all the values and guess that you would probably converge quickly, right. And you have taken, there is something very far of then, it will take longer iterations to converge right.

Now, all these things we are discussing you can play with it once you write your own codes, right. For the 1 d equation or for the 2 d equations we can start playing with all these things ok. Other questions?

Student: (Refer Time: 13:19).

So, essentially you want to start off with a guess value that is outside the bounds yes you can, right; for example, 0 is out of bounds, right; you can start off. Other questions? But your guess value also plays a role in kind of convergence, you may actually diverge as well.

That is also possible, right. So, its, so essentially the first solution that you would start off should also be kind of a good guess, ok.

Student: (Refer Time: 13:48) Scarborough.

Scarborough satisfied but I mean not only in this particular case, but in general if you take any algorithm, ok. No? Ok, clear fine. So, we have done the line by line method as well. Then

let us move on with our next concept. So, one more thing we did not talk about is the under relaxation, right. That is something we did not talk about essentially, that is relaxation. So, this is either under or over relaxation, fine.

So, in the iterative schemes that we have, sometimes we want to either accelerate or slow down, ok. The change of the dependent variable; that means, the from iteration to iteration. Let us say you have some phi P and you have some phi p star right.

So, this change that is your delta phi P is what you want to kind of control, ok. Either you want to increase it or you want to kind of slow it down from iteration to iteration. And that is what gives raise to either a over relaxation or an under relaxation ok. So, if you are trying to accelerate this change then it is called an over relaxation and if you are trying to restrict it do not change that much then it is called an under relaxation.

Now, this is useful in the iterated schemes that we are using, either for if you have strongly non-linear problems or something like that.

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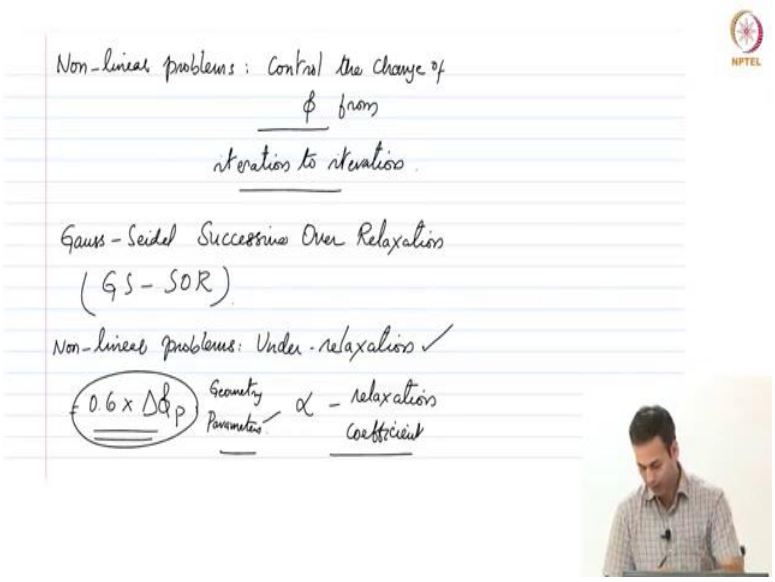

So, in the context of non-linear problems also, you have non-linear problems, now this could be either again in the nonlinearity could be either in source terms or it could be in the diffusion coefficients and so on. So, in these cases you want to control the change of the dependent variable ok. So, that is delta phi from iteration to iteration, ok.

So, that is where we use something known as relaxation, and if you use, if you accelerate the convergence then that is known as over relaxation ok. And you will find, if you use this in the context of Gauss-Seidel then it is known as Gauss-Seidel Successive Over Relaxation.

So, it is called GS-SOR this is essential if you want to accelerate the convergence this is used Gauss-Seidel successive over relaxation you have relaxation coefficient, that is what you would kind of tune, such that you can get convergence much more quickly.

Now, usually, over relaxation is not useful or not used in the context of non-linear problems. So, if you have non-linear problems, then you would go for more or often for under relaxation, than over relaxation ok.

So, you will go for under relaxation; that means, you do not want to have the change that is computed to be completely added. You want to kind of restrict the change ok, that is where the under relaxation part comes into play. So, for non-linear problems this is what is more useful. For other problems you can go for over relaxation to get the convergence quickly.

Now, what is the, how did we find about this over relaxation, any ideas? So, Gauss-Seidel was there, like Gauss-Seidel kind of was an iterative method. Now, how did we find out about this over relaxation in the first place. Completely by accident right essentially before the advent of computers people were pay to kind of do these Gauss-Seidel iterations manually, right.

And at that time somebody if they could instead of adding delta phi to 1 particular iteration, they started adding more than what delta phi that is being computed and they found that you are converging much more quickly. That is how the over relaxation kind of is useful and then there is of course, a lot of theory that was built in after this observations, ok.

So, in terms of over relaxation and under relaxation, but the starting point was where somebody found it by accident, if I just instead of adding this delta phi P, I would add 1.2 times delta phi P and then you convergent 60 iterations rather than 120 iterations, something like that ok.

Now, these are very common over relaxation under relaxation things in other software as well. You might see relaxation parameter for each of the equations right. So, you specify something like 0.6, 0.7 or 0.3 for these relaxation parameters.

What they had actually doing is, they are taking, let us say if you have computed whatever is your delta phi P in one particular iteration, they are only multiplying it with 0.6 and then adding it to the previous value ok. So, that is what they are only adding this much. So, essentially 60 percent of what is being given from the equations is added ok. That is what they are doing. So, essentially you are restricting the change, from iteration to iteration.

Now of course, this relaxation coefficient, we call it as alpha ok. This is not known ok, as such from any theory. So, relaxation coefficient is something that is, that you have to give from your experience of a particular problem. And this relaxation coefficient would depend on again on the problem statement; on the geometry and the other parameters of the problem. So, as a result it is not a constant value that works for all kinds of problems rather it kind of comes from your experience of running some kind of simulations.

However, there is some research that is been recently done on this alpha, we will come to that little later. So, let me go ahead with the theory of, what is relaxation as such.

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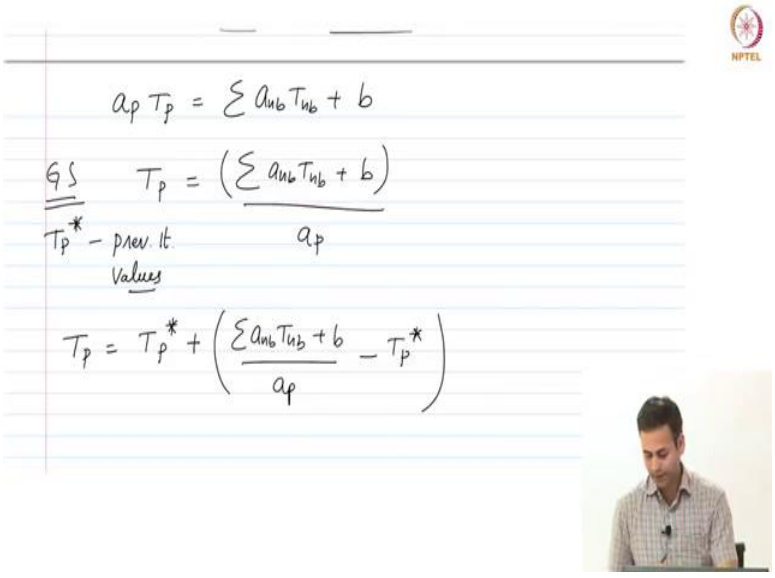

So, if we have a Gauss-Seidel, we have our original linear algebraic equation is $a_p T_p = \Sigma a_{nb} T_{nb} + b$. So, this is our original linear algebraic equation after discretization.

Now, we can rewrite this equation as $T_p = \left(\frac{\Sigma a_{nb} T_{nb} + b}{a_p}\right)$ I can write like this where we are of course, using some star values for the T n b, right; in our Gauss-Seidel method ok.

Now, what we can do is I can, if I have, let us say T p star is my previous iteration values and I can add and subtract $T_p = T_P^* + \left(\frac{\Sigma a_{nb} T_{nb} + b}{a_p} - T_P^* \right)$ So, I would write this equation . I just added and subtracted T p star on the right hand side. Fine.

Now, what is this quantity that is in the parenthesis here?

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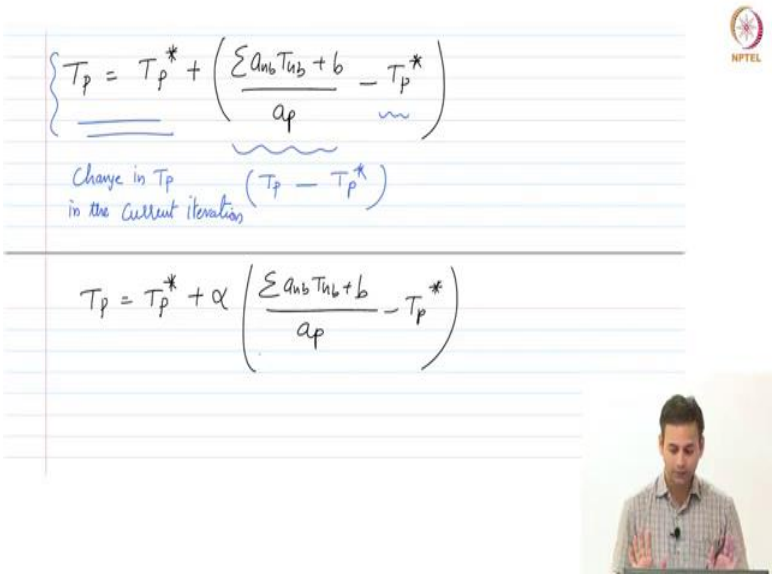

What is this? This would have actually given you what is T p, it would have given you T p if you had evaluated. This is T p. And what is minus T p star?

Student: Change.

So, this is a change in the current iteration that would have been produced, right. So, essentially this is the change in T p in the present iteration, right or the current iteration, fine. So, this is the change in T p in the current iteration for a particular cell p. Now, this is what we want to restrict or we want to amplified, ok. Now up till now up till this step until this equation, this is all correct, right.

Now, what I am doing here is, is actually changing the original equation. Because I am now restricting the change, right. So, that is actually changing the original equation we are solving, ok. We will come back to the point whether we will get the correct solution or not, by arbitrarily changing this delta T p ok. I will come to that.

So, essentially what I wanted to do is, in relaxation what we say is, we want to restrict this change; that is we want to restrict this term that is there in the bigger parenthesis. So, that will be T p equals T p star plus I want to multiply this with some alpha, this is sigma a n b T n b plus b upon a p minus T p star.

Now, this equation is not the original equation we have started off with, right. Until here, it is a same equation right, but then now I have multiplied with alpha which is some arbitrary constant, so as a result I have no modified the equation, ok. But we will come back and comment on it whether it is to do this way or not, fine? No? Fine, it is clear.

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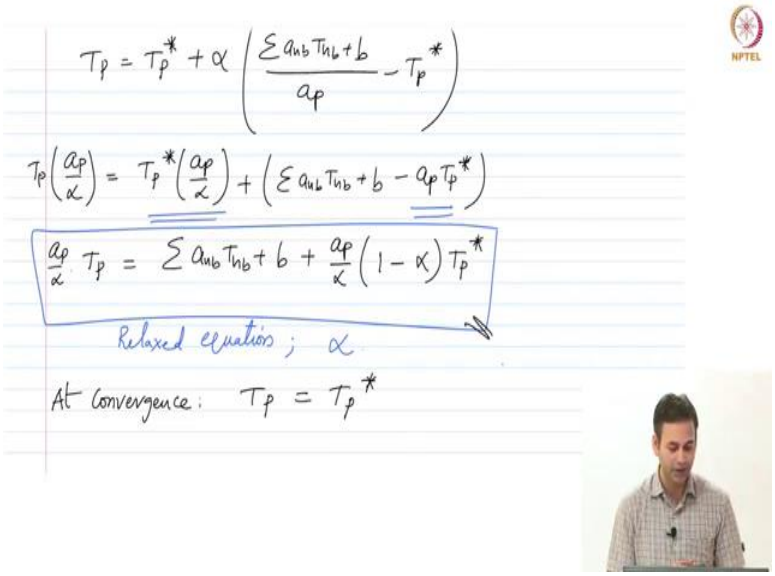

Now, let me rewrite this little bit. So, I will kind of take a p and alpha to the left hand side, essentially; multiply this, so this will be $T_P\left(\frac{a_p}{\alpha}\right) = T_P^*\left(\frac{a_p}{\alpha}\right) + \left(\Sigma a_{nb}T_{nb} + b - a_p T_p^*\right)$ is that, correct.

Student: (Refer Time: 24:29).

T p T p times a p by alpha, is that correct? Ok. So, we have a p by alpha times T p equals T p star times a p by alpha plus this entire quantity, right; ok.

So, I can rewrite this as a p by alpha times T p equals sigma a n b T n b plus b ok, plus if I want to take a p by alpha common, then this would be 1 and this would be minus alpha times T p star, right. Essentially, I am collecting this term and this term together. Is that correct? a p

by alpha times 1 and here it would be minus alpha right minus a p T p star a p by alpha T p star, ok; right.

So, we have done something, now we call this as the final equation. So, this is my under relaxed equation, ok or a relaxed equation for the original equation that we have. Which is a p T p equals sigma a n b T n b plus b. For that this is the relaxed equation, where the coefficient alpha is the relaxation parameter ok. Fine, now we have done something ok. Now we have to see whether this will give the correct solution or not, or we solving the correct system or not. That is what we have to see.

Now, let us say at convergence, what is the value, what is the difference between T p and T p star? At convergence your T p would be the same as T p star which is probably deferring by some epsilon. So, if that is the case, how will; what will happen to this equation? No not 0, this equation will be there right, there will be something here.

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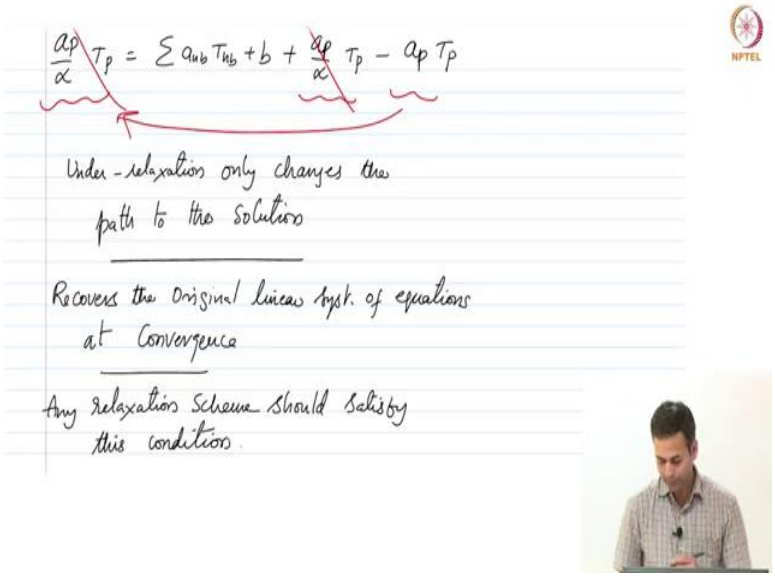

What do we recover? So, essentially if they are the same, then I will write it down here, you have $\frac{a_p}{\alpha} T_P = \Sigma a_{nb} T_{nb} + b + \frac{a_P}{\alpha} T_P - a_P T_P$. So, this is T p minus a p T p right. So, what gets cancelled? These 2 get cancelled right. And your a p T p, if you take it to the left hand side right, then what is the equation that is recovered?

Student: (Refer Time: 26:59).

That is the original equation. So, at convergence you are getting back your equation ok. You are still solving the same equation as you reach convergence ok. So, essentially, the T p he would obtained for every cell will still satisfies the original equation, ok. Now, so now what have we done, by doing this process of alpha. Have we change the final solution? No. Have we change the path to the solution? Yes, right.

Instead of climbing the hill directly, we have kind of taken a d route and then climbed it, right. Essentially, we have change the path to solution, right. So, we changed. So, under-relaxation only changes the path to the solution, ok. And it recovers the original linear system of equations at convergence ok, fine.

So, we get back the same system as we kind of converge. Now any relaxation scheme that you would kind of develop should satisfy this properties, right. It should, any relaxation scheme that you have, should recover the original system as you reach convergence. Otherwise that scheme is not useful, right. And also for any arbitrary relaxation parameter that you would use ok. So, that means, any relaxation scheme should satisfy this condition, fine. That is what we have.

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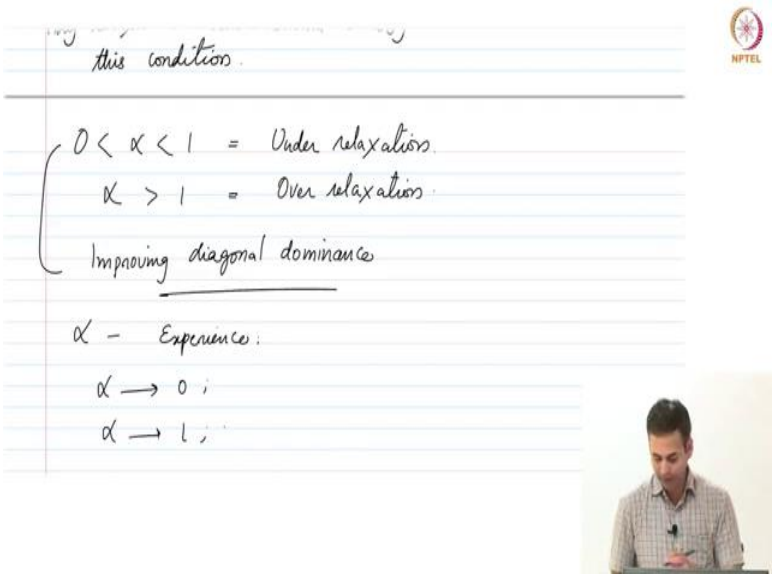

Now, if you choose, if you choose alpha in the range of 0 to 1 then this is called under relaxation. And if you choose alpha greater than 1 that is called over relaxation, ok. So, if you go back to the equation, essentially you are restrict in the change, right. So, if you choose alpha less than 1, this change will be only a certain percentage of what is actually given to

you by the equations. And if you kind of increase this, then you are adding 20 percent; 30 percent more of what is given by the equations ok, to get a faster convergence.

Now, what is actually alpha doing when you have under relaxation? From the equations, what is alpha doing?. Let us say if I have under relaxation, so alpha is between 0 to 1 right. What is it doing to the central coefficients? Diagonal dominance, right. Your a p is now a p by alpha right, as a result; it is now larger, right.

So, essentially, under relaxation is increasing your diagonal dominance, as a result it is actually helping you converge your system in a better way right. So, essentially improving diagonal dominance. So, under relaxation is your improving diagonal dominance or it is helping you satisfies Scarborough criteria, much more strongly right.

Now, what if you have alpha greater than 1? Is it helping you with diagonal dominance? No, right. So, it is actually making it worse, right. It will not be diagonal dominant, but you can still relay on other things which are still making a diagonal dominant for example, boundary conditions or source terms any things like that, right. For example, boundary conditions could still be probably be, ok; fine.

So, essentially that is the, that is the reason why I said if you have strongly non-linear problems then you have to kind of look at under relaxation, because the changes in the solution could be much more quickly, which the you know, it might be difficult for the system to kind of converge, ok. So, that is the thing.

Student: (Refer Time: 31:24).

Very good. So, essentially the question is where will we consider the T p star term? Yeah, so where will this, where will this go? This will go in to.

Student: b.

b term. In the constant term, ok. And your a p is. Now, a p by alpha that is there and this will any way be there will be some coefficients, ok. Now, this is this pattern is or this equation is what you have to kind of use, if you want to under relax or over relax any equation, ok. You have this extra coefficient, that is a p by alpha here ok. And then you have you are adding a p by alpha times 1 minus alpha times the unknown here, on the right hand side.

So, this is your relaxation relaxed equation for the original equation. For the a p T p equal sigma a n b T n b plus b fine.

So, you would use this equation whenever we refer to a relaxed system, for a given equation ok. You have these two extra terms. Which is you have 1 by alpha on the left hand side and you have this term a p by alpha times 1 minus alpha times the dependent quantity for that particular cell.

Now, as we discussed before there is no thumb rule for how to choose alpha ah, but it kind of comes from experience, ok. So, usually you can take, if you want to under relaxed you probably take it as 0.6, 0.8 or whatever. Or it could be much smaller as 0.3, 0.2 as well. So, essentially you are restrict in the change, right.

So, if you make alpha tend to 0 what will happen to the convergence? It will be very slow; right. Because T p would not be very different from T p star, it will be very slow right. So, if you make alpha close to 1 then essentially you are making it same as the original system, right. If it is greater, then you have a larger changes coming into play ok. So, these are the limits.

Now, recently there has been some renewed interest in coming up with, how should I calculate alpha, ok; or should alpha be the same for all iterations or can I change it with iterations?.

You can change it with iterations, but we just do not know how much we change it ok. So, , but we can certainly do that, we can start off with the alpha which is small value 0.3 or something after; let us say you feel, solution has evolved somewhat into a better you know physically correct solution, then you can increase the alpha. So, that you can reach convergence quickly. All these things can be done.

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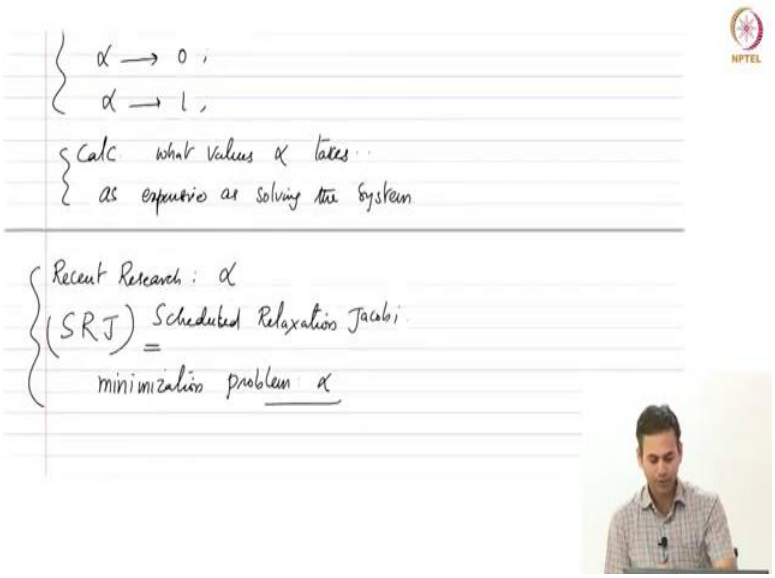

But many a times calculating you know, what values alpha takes could be as complicated as actually calculated in the solution itself, ok. That is the actually solving for two problems.

So, that is why you do not kind of dwell too much into, what alpha should be and how should I use it; what should I take it after. So, many iteration. So, that will basically make it computationally as expensive, as solving the system ok. So, rather you are more happy just go and solve the system with some alpha then actually trying to see what is this alpha value should be, ok. As (Refer Time: 34:35) you are solving the system.

But there has been some recent renewed interest and then some recent research, have shown that we can actually have certain values of alpha ok. So, essentially what alpha value should I take and how do I kind of change them for a given problem, this was this is recently known as the Gauss-Seidel scheduled or the or not the Gauss-Seidel, it just called scheduled relaxation Jacobi it is called SRJ, Scheduled Relaxation Jacobi, which will tell you what alpha to use for what kind of problems, ok.

And then how should you kind of, pattern it you know. What are the values you should use as you go with iterations? Now, this basically will, will come out as a if you kind of go through this research papers that are there, if you look upfor scheduled relaxation Jacobi, you would get them. Essentially, it will basically become a minimization problem to calculate alpha.

So, this was done in the past and it was shown that, if you pattern alpha in a certain sequence then the scheduled relaxation Jacobi will converge much faster than Gauss-Seidel, SOR and things like that ok. So, we are not going to this as of now, but it is kind of information for you, if you are interested you can go and look up all these things, fine. Otherwise the kind of depends on experience and the problem you are solving, ok.

So, in your assignments or projects also you will use under relaxation, when you have strong nonlinearities, ok. In which case you will modify the system like this and you would use some alpha value. Which will be either told to you by you know in the assignment or something, fine. Questions? Questions on this part, on the under relaxation over relaxation? Yes,

Student: (Refer Time: 36:40).

In case of?

Student: (Refer Time: 36:44).

So, what is the upper limit for alpha in case of over relaxation, essentially there is no upper limit. But the issue is, if you use very large values, it will just diverge you know, that is a thing. The solution will diverge. Usually, you do not see more than like 1.2 or 1.1, because it is kind of adding 20 percent more than what it is supposed to be, right.

Student: (Refer Time: 37:06).

Right.

Student: (Refer Time: 37:08).

Yes, you can. Yes, that is possible. So, essentially can I increase the diagonal dominance by using under relaxation? Yes, we can; right. Essentially what is the advantage of under relaxation? It will help you satisfy diagonal dominance and strongly non-linear problems can be solved, right; because you are changing the path to solution as a result your algorithm will not diverge right.

Your diagonal dominance will be preserved at every stage, ok. You will see that actually (Refer Time: 37:40) mean assignment if you have one problem we can see if you set alpha equals 1, you will see that it start diverging after certain values, because the diagonal

dominance will not be preserved. Otherwise you will have the diagonal dominance maintained throughout the solution, right. Other questions?

Student: (Refer Time: 37:59).

Right ok. So, can I try over relaxation, knowing that this Scarborough is satisfied with by other means? Yes, that is quite possible. That is where actually over relaxation will be used. You know that you have a kind of a not very strong nonlinearities and you have boundary conditions are helping you with the Scarborough, then you can go for some kind of over relaxation that is possible right, ok.

So, that is about the under relaxation part as well. If we write a an algorithm yesterday for the source terms, source term linearization or we calculate, I guess we did not, right. We just calculated the source terms and how to kind of do it. We did not write a an overall solution algorithm we did not do, right.

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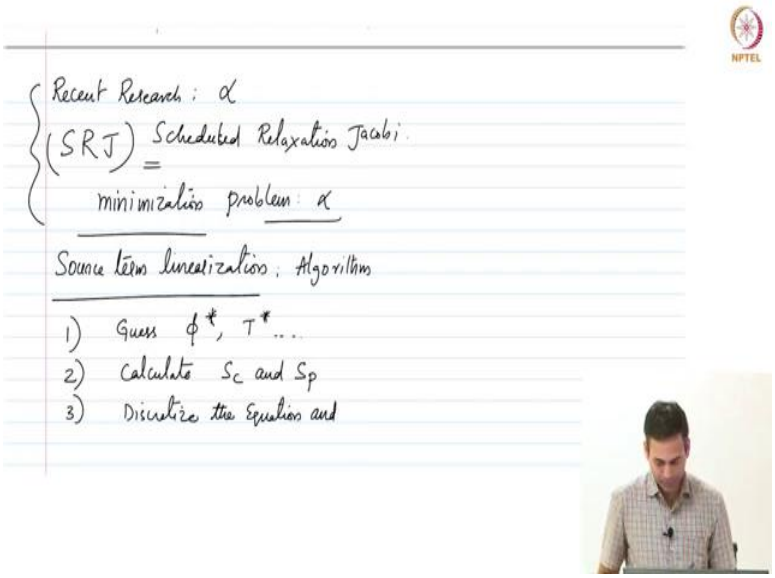

So, let me kind of finish that part. So, essentially if we have source term linearization, this is important, because you have to update the coefficient S c and S p as part of the solution loop ok. So, it is a small algorithm.

So, if you have, then how do we do it? Essentially, you first guess either the phi star values right or the T star values depending on the problem. Then you would calculate S c and S p

right. And of course, you would also calculate what is your, essentially also discretize the equations and come up with the linear system, right.

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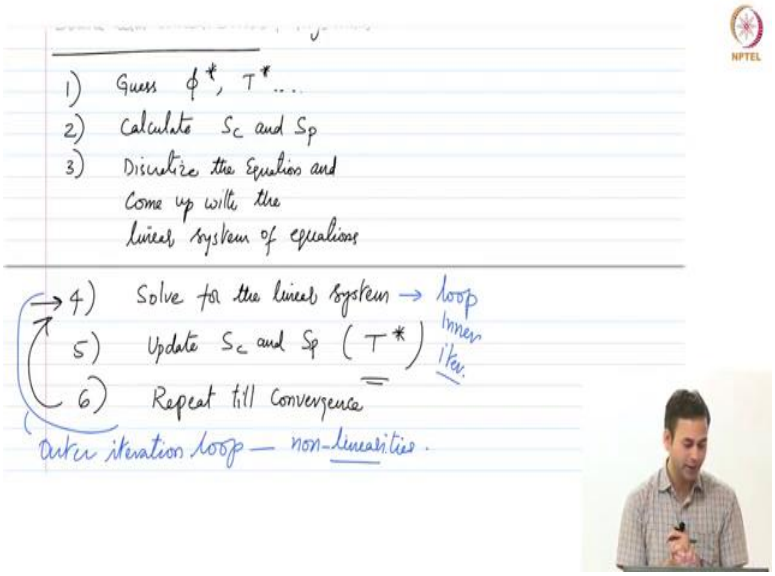

Then, what you do? You solve for the system of linear equations, ok. Solve for the linear system ok. Then once you have solved it, what do we have to do? We have to update the values of S c and S p right.

So, now this step 4 here is not convergence. Why is it not convergence? Because S c S p were are no modified, right. We were given some non-linear equation, we have comfortably made them linear and then we solved something.

So, as a result the solution we get out of step 4 is not the solution to the original problem we wanted right. As a result, you see we are now introducing a an iteration, right. On top of the iterations we might have had in step 4 right, if you had used a Gauss-Seidel you are doing iterations in step 4, right.

Now, I have another iteration that I am doing, which I would like to call it as an out iteration, right. Which is coming out because of nonlinearities in the problem, because we have linearize the nonlinearities. You see that. So, essentially here what I do is I update, S c and S p, because now these are now functions of temperature, right; these are now functions of temperature and the temperature just got updated in step 4 right. So, I will go here.

Now, what do you do, I kind of repeat from repeat till convergence right, so; that means, I go back and then once with the updated values I will go to step 4 and then update the coefficients. Then solve for the system and come update the S c S p and so on.

And then once you reach convergence you can stop. So, do you see two loops here? There is one loop here, which we call it as an inner iteration loop and there is an outer loop that is this guy. This is the outer iteration loop. This is coming because of the nonlinearities, ok. Or essentially this is coming, because of the nonlinearities, right; which we cannot avoid as long as you have nonlinearities.

Now, we had 1 question, I think in the previous lectures, let us say if I use a direct system to solve x equal to b, I would use you know, then where will that go? That only goes into the inner loop, right. So, this will be a direct solver, but still I have to have a iterations, right; for the outer loop right, because of the nonlinearities, ok.

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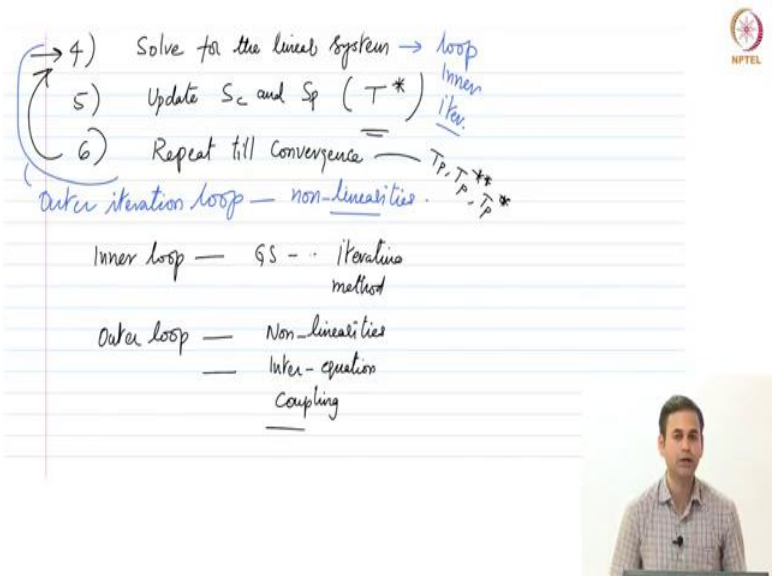

So, keep these things in your, I will just come to that. So, actually we have loops one is inner loop, this is basically you because of your Gauss-Seidel or because of an iterative method we are using.

And we have this outer loop which is, because of required because of the nonlinearities. Now, later on we will see that this outer loop is required not only because we have nonlinearities, but also if you have inter equation coupling.

For example, let us say you are solving for momentum equations in x and y directions, let us say you have u and v equations for Navier Stokes, then while solving for v equation you are using u, but u are actually guessed values. Similarly, while solving for u equation you have a flux coming from v. So, v are guess value. So, if you have inter equation coupling then you still have to use iterations.

We can solve for, let us say a system of linear equations you know; linearized system, but still if they are if there is inter equation coupling, then the coefficients are provisional, right. So, whenever you have coefficients are provisional then you need an outer loop to converge it to the correct solution, ok.

So, outer loop comes even if you have inter equation coupling. Usually, if you have more than 1 equation there will be some kind of coupling between the equations, right; otherwise there is no point in solving for 2, right.

So, there will be some coupling. So, inter equation coupling also gives raise to outer loop, nonlinearities in the source term, diffusion coefficients or in the terms itself. For example, convection term; right, it is a non-linear term, right; you have del dot rho u u bar right. So, the convection term all this things will give raise to an outer loop. Yes, questions?

You have to check for, no convergence is basically you will always check for between T p and now there is a, now there is not T p and T p star there is also another T p outer, right. You see that? So, there is one T p T p star to check for convergence of the inner loop, but there is one more T p old that is what you have started off with, right.

That is also there, because if you check with T p and T p star everything is fine. You have converged, you will just come out right. That is not correct, see that. So, you have T p old which you have started off here, right. Essentially, you compare with this T star right, and then we can call this inner loop as a T double star; ok, you see that.

So, that value should be converge, right. Should not change more than some (Refer Time: 45:20). Is that clear? The difference between outer loops and inner loops and things like that, ok; fine.

Then, we have kind of finish these part. So, that next the concept is basically going back to the diffusion equation, up till now we have applied it for Cartesian meshes, right.

Now, the same methodology that we have developed can be applied for other orthogonal coordinate systems. For example, if you have polar coordinates or if you have axis symmetry problems in 2 d, the same methodology can be extended to all other orthogonal coordinate systems.

So, we are going to see how to solve it on a, in r theta coordinates for a polar you know, in polar geometry and also for axis symmetric geometries and things liked in the coming lectures. After that we will move on to unsteady and then to unstructured meshes and so on, ok. We are still dealing with a diffusion equation, ok.