Indian institute of technology madras
NPTEL

National programme on technology enhanced learning
Video lectures on convictive heat transfer

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Lecture 9
Couette flow- part2

3^2 t /dy^2 +μ / K (du/ dy)^2=0 how did you get does that term from the fighter huh you have the fight term with you in that if you look at the fight you have the fighter with you or no so if you look at that all ∇/ ∇ x terms vanish ∇ / ∇ Z terms vanish V term vanish W terms vanish all you have is from that second (∇ u/ ∇ y + ∇ v/ ∇ X)^2 from that particular term(∇ u/ ∇ y)^2 remains so now look at the expression of the energy equation the entire set now all of them

Actually are ordinary differential equations this point number one you should notice the observe appreciate a complete set of nonlinear partial differential equations its entirety by in this particular case of this particular physical mo ∇ we have taken with certain fairly reasonable acceptable justifiable assumptions is brought down to the simplest we cannot getting get anything more simpler than this in convection and dv/ dy = 0 says V = 0 the solution is

Already there with the boundary conditions of course d^2 u / dy^2= 0 remains the y component is a hydrostatic pressure description W is 0 everywhere and then you are only two terms in the energy equation so now you can change all the ∂ u/ ∂ Y ∂ T/ dy it is simply du/ dy, dt / dy number one this actually tells you how before you formulate a during the formulation of the problem you you think what assumptions are justifiable the first thing you should you should

There is no necessity for directly hitting the complex partial differential equations also please understand these were all done when there was no computers they said how do we simplify the problem for our hand calculations computers came in only in the 40s in early 40s during the Second World War it was basically developed by the Manhattan Project basically the nuclear project before that what were they doing and also that was the time when heat transfer I was

Really picking up heat transferred as a subject to be taught in colleges came only in the 1960 actually if you look at the first book by Jacob Hawkins 1955 or so that is a very first textbook for heat transfer nobody people did not know what to put into that later it has now come to the incorporate divert beige on level action so assume making assumption simplifications was a necessity at that time and this apparently simple problem you can make in you do a three-

Dimensional thing if you do not put in the ∇ by ∇X tends to 0 /∇Z tends to 0 it becomes a right solution starting from the leading edge to the trailing edge and the two sides what you have done is I will not worry about the flow near the edges always you see even when you do the experiments you will see near they exist you will have two dimensionality three dimensionality you know what do you think is a one-dimensional two dimensionality what do

You think is a two-dimensional flow is a three dimensionality and boundary layer assumptions are not valid very close to the reading edge as all of you know there are certain complications both physical and mathematical so first take away from as far as I am concerned is first of all the procedure of convection problem starting from the physical model the second takeaway is how do I simplify before I start solving now you have computers and codes so you write down

The how many times have you really written down the physical model to really hit the mathematical model you simply so flat plate you start $\nabla u / \nabla X + \nabla V / \nabla = 0$ and start putting your boundary conditions boundary conditions if you look at this velocity conditions are fixed not much can be done either it is 0 it takes the velocity conditions of the pet but look at the thermal we have now taken and we did not write it down please write it

Down isothermal place that it is kept my mind both are isothermal but if you look at the thermal part of it you can put various kinds of boundary conditions on both the plates with various combinations both isothermal both non isothermal one isothermal one are also thermal one with constant heat flux both with constant heat flux one adiabatic how many kinds of boundary conditions you can have combinations each one of those boundary conditions will

Give you a different solution for the same set of equations any number of times they repeating this the importance of formulating the problem rightly formulation of the problem correctly is 50 percent of the solution especially this true important rather when you hit the numerical techniques when you use certain certain equations appeared to give you the right solution but they are really not the right solution because they were numerically equivalent that you have

Written do not really simulate the mathematical equation that you have written so whenever you write numerical you use numerical methods you should check for most of the time you check for only for convergence stability criteria to develop but third most important thing is consistency whether you were finite difference of finite element representation of the partial differential equation although d is consistent with that equation because of this round off errors

And truncation errors sometimes what you think is the right finite difference formulation will simply go off in a different direction but give you a convergent stable solution this has been experience of many people it would not happen in every case but there are situations where this happens you should bear in mind so formulation of the problem is extremely important simplification is extremely important I like this problem because in its entirety you can get this

Idea of what simplification all solutions were never Stokes equations is based on simplification what is particles of bounder layer here we did not invoke the bounder layer earlier not necessary we have solved the entire neighbor Stokes equations in order to then you can of course add the heat transfer roads that called heat generation term you connect we have kept a very important term in the energy equation the viscous dissipation we will seal it now write

Down your return the boundary conditions so these four equations with the boundary condition u = 0 V = 0 W = 0 or the wall bottom wall T = T not at the bottom wall u = ul at the top one v= 0 W = 0 at the top one these are the boundary conditions there are no initial conditions because you have taken as a steady state now let us proceed hope some of you have solid but let is go step by step solve the velocity equation in any heat transfer problem first you have to solve the

Velocity equation because you require that for the energy equation in the case of forced convection these are different in the case of free convection they have to be solved to do that so what is the solution $d^2 u / dy^2 = 0$, $y = 0$, $u = 0$, $y = L$, $u = u1$ so now this this also tells you how when you are actually applying a numerical technique to a partial differential equation what are you doing actually after putting in a finite difference for finite difference or finite element then

You go ahead with the solution point by point actually you are I want you to flow you are - the partial differential equation you are - doing this – no no when you get the solution every question what should you do for you to get the solution huh no that is a numeric class that is where you are directly jumping to new discretization because you want to use a numerical method even up to that when you really solve what is a PDE do partial differential equation let

Us take T for example dependent wait it is a function of XYZ and da 1 what is the difference between a PDE and a finite difference equation find a difference equation of that part PD what is the difference what do you get when you solve these things in terms of the solution tell me where where in the other one so you are now I will to save time you are integrating you are integrating the PD or to integrate now you do that here that is what I am saying $d^2 u/ dy^2$ twice

You have to integrate and two constants the two boundary conditions you must have at y= 0 please by looking at the equation should be you should be able to actually write it is nothing but.

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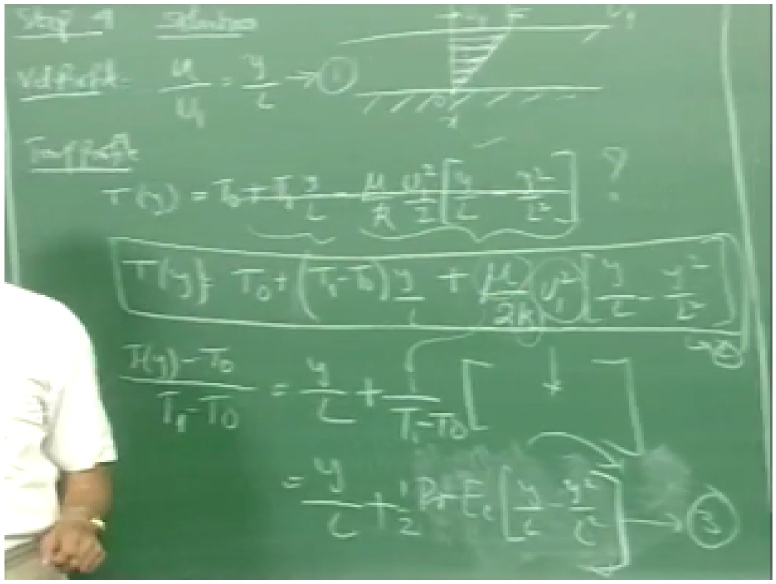

u / u 1 is = well now we will go to what is this step term solution now for a now we are actually going to this solution so the momentum equation that is well I will say velocity profile is nothing but u/ u 1 =y/l please see and what does this say what kind of equation is this now we use the word whatever you used earlier the solution now is linear so plot this please in your physical model what it says is at any given this is let us say X here so here this value here is U

1 and this is 0 again I want you to appreciate this such as originally what we thought was extremely non linear it is a linear profile basic so one more point V= 0 everywhere we have said W = 0 everywhere you is this profile now you come up with a name for this what kind of a flow is this it is not a potential flow it is not about a layer flow okay all the layers are can you look at this all the layer stream layers are to each other in what fashion are they oriented to each

Other huh it is parallel flow V= 0 W= 0 it is simply parallel for number 1 it is also a fully developed flow it is not going to change ∇ u/ ∇ X = 0 so it is a parallel flow it is a parallel plate geometry it is ok but you can have non linear velocity profile in parallel plates as you will see later and this happens only for DP/ DX = 0 please see that so therefore it is called a simple or plane quiet flow and therefore you get a linear velocity profile number one it is also called a

Parallel flow as different from boundary layer flow the streamlines are all parallel to each other this is something again you get from this profile very easy they the really exciting thing is go to the temperature profile write down the energy equation(d^2 t /dy^2+ mu/ K du/ $dy)^2$= 0 boundary conditions at y= 0, t = t not y= L T = t1 we have not said what are the relative values of T naught and T 1 leave it let us see what happens later now we have to integrate this twice I had

Yesterday asking you to do this but some of you have not done why I did not do it please this actual procedure that you follow this is a classical solution of classical complete exact solution no approximations you get two constants take two minutes give me the expression for T(y) =

what these function of only Y now finished can you give me the terms I will write down tell me $T0+T1y/l$ –meu /R $U^2/2[$ y/l- $y/l^{2]}$ something else is missing here mu u square by 2 K yeah this

Is okay this is minus a plus check it out or you should redo all of you get this this equation is important because you have to differentiate it later to three times tell me you know some manipulation of this can be done I am not sure I will right how many of you got this do not copy that do not I am checking whether it is right or not wrong but I will give you my the expression you check it out maybe slight rearrangement will be alright please check this out t

Not that is okay actually this is also okay here please see whether we would like to correct it please check whether something is missing there and then this sign also huh check it out please will reorganize it again later for something else something is missing there was it okay Ty = T0 plus mu by 2 K $u1^2$ Y/L Y- y^2/L^2 +T 1- T 0 Y/ L did you miss one term that check it out how is it somebody tell me this second one is right then I will proceed because you give me the first

Answer I am asking you everything is right except that that T0 was missing T 1/ L- T0 better just check it out ok now you see a little bit this is this is the equation which please keep it give it a number I do not know what is the number of this and instead of what was it can you give me a number okay today I will give you this number now you see you have got a Y/ L without you are trying for it you have got a non-dimensional distance can you get a non-dimensional

Temperature difference by remain by manipulating this let is look at that expression there is a t here which is what we want there is a t0 there is a t1- t0 so I can rewrite this expression as T of Y- T0bring T 1- T0 to the left hand side now you have a non-dimensional temperature difference is not non-dimensional temperature in non-dimensional temperature difference always non dimensionalization makes things easier for various reasons now so this is y by n is y

By l plus this whole thing divided by C now you have got a non-dimensional temperature different temperature in the non-dimensional form as a function of a non-dimensional distance which is very nice on its own it is coming out slight manipulation and as a function of that we sleep the fluid the viscosity the velocity the thermal conductivity we will do a small further manipulation and see something else very important will come out and then we will go for it

Can you temperature difference is non dimensionalized this is non dimensionalized therefore one more thing this term is non-dimensional please write down what this term would be mu $u1^2$ /2 K T 1- T0 I am a communicating here I am a communicating piece that term I am saying t minus T0 by T 1- T naught you have you are y/ L why do not we put the other see whether non dimensionalization possible what is that term µ u square by 2 K is there and you have divided by T 1 minus T naught how can you in all dimensions what is it that you can do here I will give You a hint you will get a very well-known non-dimensional number out of this by slight maniple you are jumping fine fine will get the cut but a more familiar non-dimensional number you have a mu here ok how do you get a parental number there multiplied divided by CP and now you see what you are getting only now you get their cut number so now you will see the non-dimensional temperature difference can you give me the second term here t – T0 / T1- T0=

Y / L + μ CP / K is now prattle number then what else you have there so you have mu CP by k half is there of course you have u 1^2 T 1- T0 is there you are multiplied and divided it by CP now look at that prattle number you know now you are coming out with a new non dimensional number you did not try for it it is coming on its own it is slight manipulation and what is that number this number is the occult number we did not plan to get this we did not say

We will get a cut number even when they did it first term they did not plan for a cut numb but it was that name was given there is a different matter later but you see how from again and again I emphasize this from very simple problem you are coming out with prattle number you knew that of course a cut number is very new and it comes out because of you will look at the expression u 1^2 / CP ∇T what is the physical significance of a planter number mu see P / K

What is the physical signal of the cut number u 1^2/ CP ∇ T CP ∇ T is a impose a temperature difference so CP is imposed thermal energy kind of a thing and what is U 1^2 is the kinetic energy so kinetic energy divided by imposed thermal energy form there CP into actually the CP T not- CP T 1 so there is a certain in a problem in this problem there is a certain energy in terms of CP T that is now being related to the velocity of the pitch if the plate were stationary

Could not have come up in fact that will be 0 obviously so a new number has come up now called occurred some books give you the symbol as easy some give you as a symbol E you use as you like but now you just change the final equation as Y / L + 1/2 prattle occurred in to Y/ L – $y^2$by you do not have to do that but this gives you a new you can visually see the new non-dimensional number and you also realize that it is important only when you one is significant

You one can be 0 then it is not significant you can you one can be very small value it has its own role but you also know it changes with $u1^2$ so from 5 meters per second you want you go to 10 the effect is square actually so that means a small change in the velocity finite chain will have a very that power law dependent effect on the cut number which is nothing but the manifestation of the u 1^2 the kinetic energy if you go into the previous one here this this

Actually is much more according to me representative of the situation μ u 1^2we are talking about viscous dissipation if you look at 1 un^2 its kinetic energy that is also right but the real effect is viscous dissipation is mu u 1^2 and that can be in am you is already here in the Prattle number so you can try to give it a physical significance this is extremely important why do we Do all this we would like to know the temperature difference by the way suppose you have u 1 = 0 here what is the temperature profile linear same thing in fact the velocity profile is linear temperature profile is linear when u = zero that means it is simply conduction this conduction that is happening the moment you start adding u1 convection is coming into picture the profile is becoming nonlinear this is a nonlinear term the second term is u 1^2 is being introduced now

This is the temperature profile after getting the temperature profile in a convection problem what is it that we want to do find out what is the heat transfer at the top plate what is the heat transfer at the bottom plate get the expression how do we do this therefore you should first get now I will say this is step whatever now I have lost track of this heat transfer.

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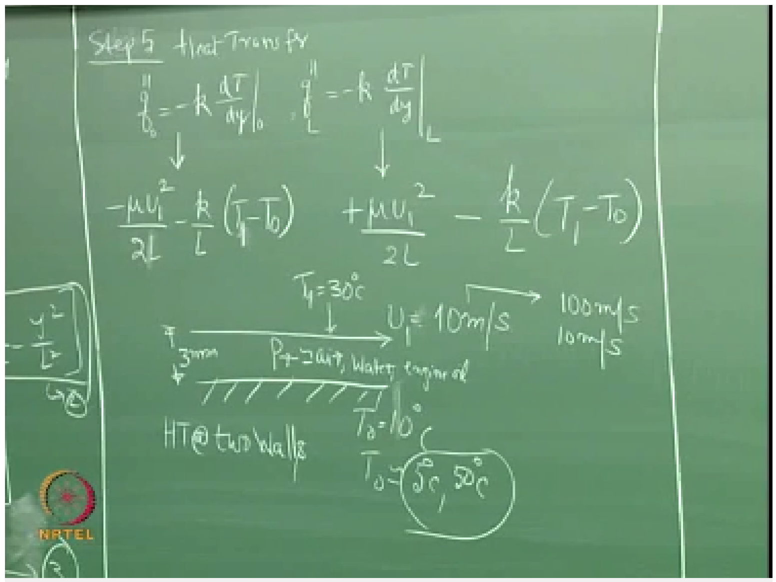

heat transfer Q I will say flux =- K DT I can easily read DT by D when there is no problem not necessarily   ∇ T / ∇ Y this is a general expression if I put Q 0 here this is the wall so I have to put DT / dy at y = 0 and flux at yell at the top plate is - K DT/dy at y = L can you get two simple expressions first first get D DT / dy and get two expressions please that will complete this part of the code problem there are two more important things which I want to refer to get a

General expression for DT/ dy from the T equation you know how to do this is not it what I will do you do it it takes some time I like you the expressions check it out but I want you to do it separately q0 ∂bled as a heat transfer at the wall this is equal to minus mu u 1^2 / 2 K 2 L sorry - K by n TL – T0 T 1- T - mu u $1^2$by 2 n- K by L T 1- T 0 and the heat transfer at the top wall is +mu 1^2/ 2 L- please check it out so you get the temperature profile then take DT / dy DT by

dy at 0 multiplied by K - K you get the heat transfer at the bottom wall and then heat transfer at the top 1- K DT / dy at y equal to here this is all fine these are all equations but you have to have numbers final so what I will do I will give you a very simple problem I want all of you to write a simple program and now study in this solution for the effect of various variables here tell me looking at this expressions what are the important variables that you that effect the heat

transfer number one huh okay so all other conditions being the same you one controls the heat transfer influences d transfer then what   else suppose I keep you one constant   fluid prattle number okay then right of course you have to say yell but we will pick cell now I want you to do the following please take a problem I want you to write a small program for this we have these two plates distance small we have never talked about what is L actually let us take this L

e= three millimeters and this is u1 please switch off your mobiles please ten meters per second yeah we will see the parental number here this is at ten degrees yeah t0= 10 t1= 30 is everything there Parental huh now I want you to do this take air here parental lumber take water take what is referred to as engine oil different kinds of oil are their pure loyal engine oil that gives a pendulum with these conditions please get the temperature profile but basically the heat

transfers are the two ones find out what is the effect of prattle number then in the same program please change this u1 up to 100 meters per second or so in 10 meters per second intervals find out what is the effect of this velocity okay now keep it at 10 m/s change the bottom temperature t0 take 5 degrees centigrade and 50 degrees centigrade you get the point what I am trying to do here prattle number 3 prattle numbers air water and oil higher velocities

10 minutes per second is very small temperature you see what I have done the first problem I have given is 10 degree centigrade keeping that at 30 why have given you 5 and 50 okay if I take 50 here what is the difference between this problem and that problem this is 30 and 10 so T 1 is > T0 actually what you could do you could also take 30 degrees T 1 = T0 and then T 0> T 1 see the difference in heat transfers but actually there is a very important application of this

and therefore another factor is extremely important for us to calculate the moment there is an non-linearity in the temperature profile can you can you visualize another factor which comes into may come into picture and there kind of a temperature within the fluid by the way please understand all these temperature profile is for the plate plate temperatures are fixed if there is a non-linearity what would you like to look at as a means of as a way of understanding the

problem linear you know how it is now there is a nonlinear thing whenever there is a non-linearity will to find out one particular I do not know whether I am able to communicate this question properly you want to look at something else in terms of the temperature profile how does it will it continuously nonlinearly vary in one direction I mean I am trying to give a hint or could there be a in the curve what do you expect you have ten degrees at the bottom and thirty

at the top linear profile you can easily see how it is how would it be the nonlinear profile I will come to that you have to plot it then only maybe I understand okay maybe I am not able to come in let me assume I'm not able to communicate the problem I will state it what is the maximum temperature in the fluid find it out find out if there were to a maximum temperature because it is a nonlinear problem exactly the answer is actually my reason for asking you to do

this how do you find out whether there is a maximum or a minimum what do you do to the equation huh what differentiate what so dt/ dy should be what find out if there is a dt / dy =0 at what condition how do you find out whether it is a maximum or a minimum do that see if there is a maximum and if it is there find out if there is a maximum what would you like to find out first of all find out whether it is a maximum if there is a maximum what is the next question that should arise in your mind.

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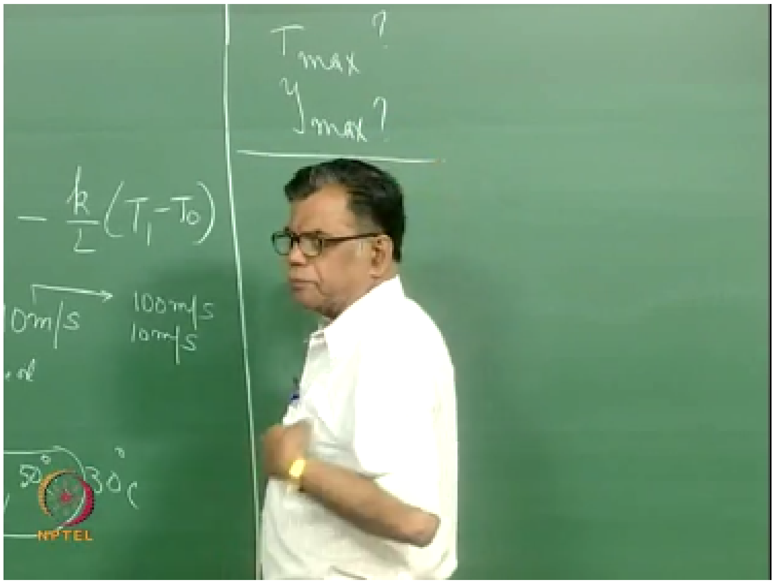

Okay but something else related to this please when you serve this very important and therefore I want you to see what is the T maximum for air for water for oil oils can be anything planter number from thousand to fifty thousand you can check it out but take any one or two planter numbers it is okay and please write a small program and prod all these things we go back to the temperature profile how do we plot plot the temperature profile what should be the ordinate and

The abscissa what can be look at the physical problem and how do you want the profile to be plotted what should be the abscissa or should be the ordinate that is fine so where do you want tita we do not want to looking at the problem which would be okay for you so you please plot what will be the maximum value of y buyer of course this is 0 what is the maximum value of y / L what is this what is the linear profile if there is no u 1 u 1 = 0 static plate how will this be

Here to where will it go there what I want you to do is really plot when I say u 1 = 0 look at this expression please theta if this is Theta if we want to refer to it theta equal to Y / n that profile is for second term being absent the moment I bring in u 1 that means a cut is coming into picture tell me you mean the effect of prattler card number that will give the effect of the you are more importantly viscous dissipation what numbers can it take you do not know you can assume no at this point in time I am going to suggest to you use this expression now it is a very nice non-dimensional expression it came up just like that plot Y / L and this for various plant like odd

Numbers Brantley occurred numbers and I want to convey to you unless you plot it you will not understand what I want to talk about in the next class please do this not only plotted think about it what is happening I have to still bring out two important aspects of this whole problem we will do it on Monday no problem I am not I am not in a hurry I want you to understand this especially one two three concepts will come out of this which you can then relate it to normal

Convection so please plot this Y / L plotting is no problem it is the same thing way by Y / L and theta are the same u by u 1 = Y / L theta = Y by L so there is no problem but for varying planting a cut number make sure that is when you do that you will know there is a maximum now I am doing you actually the answer but I want you to do that it is a maximum find out where that maximum is why think about it there is one aspect of the solution we have if a

Maximum comes into picture what is the effect of this on the heat transfer not the rate heat transfer direction at the two plates we have when I went be said how to plot you said something when I asked what is the maximum temperature you said t1 you will find it is not given it is something different you it is t1 if there is no u1 because of u1 something is happening that something is there is a maximum checkers I am giving you the answer find a rod at some

Location but then what is the effect of that maximum coming into first of all why and what is the effect that will effect gentlemen the direction of heat transfer except this is extremely important point I want to make little bit with little bit more force in the next class but for that I want you to plot the theta with respect to parental a card also it is not simply occurred it is partly occurred when you then fix up a parentally occurred numbers will change that is

Different but give a number to product like a ties one two three four five six seven eight in particular tell me what happens at promptly occurred equal to two study that also therefore find out what happens after equal to 2 what happens means what happens to the heat transfer that was what I am looking at you will find very new information coming out of this is very extremely extremely important I will ask you something else now before I leave you look at the

Word problem what does that look and look at that please look at that there is the bottom plate fixed there is a gap there is a top plate which is moving there is a temperature here that is U e=0 you are called even what does this this physical model remind you of which you are more familiar with you will be you that is because you are not thought which is ok no problem we have said there is a plate suppose there is no plate but there is a velocity what would that problem boil down to look at the physical picture look at the boundary conditions Baba flat

Plate flat plate okay flat plate or what is the flow there for the type of flaw do not simply say flat plate add something to that what type of a flow on the flat plate sometimes you know you have to give the teacher the answer that he wants but in different ways you can discuss about it this point you have not thought of so far I am making you think requesting you to think rather you are right but flat plate boundary layer flow do not simply say lambda that is also correct

That is why I want the bright words to come look at this problem quiet problem you thought it is all restricted but one of the takeaways from the curve flow is this can be related to a boundary layer flow very simple but a bit fixed y =0, u = 0, T = TW, y = Y, L is y = ∇ u = u infinity whatever that is T = T infinity means one small qualification what could that you are right now you can write a sketch of the boundary layer flow and compare these two where will this quit

Flow situation will be really applicable in a boundary layer flow in what zone I will tell me where it is not applicable ∇ is always very you that is a function ∇ is why by its route or not x near the entry it is not valid I say I wanted that word that is actually now the quiet flow is nothing but a representation of a fully developed boundary layer flow fully developed I would say and a flat plate still but you can extend it to the pipe flows later where after that nothing is

Changing it is a constant okay it varies as ∇ / X is y / X root Reynolds X. X power half it varies but as you go along half it the rate reduces so away from the leading edge the flow is very similar to quite well once you appreciate this the effect of viscous dissipation you can actually super impose upon the boarder airflow post-conviction water and therefore any information that comes out of it it is just a mod if it has to be modified for a regular flat paper

This is one of the major take away from looking at a quilt flow in some detail we could have done it by same I could have given you the equations and then you say solve it you solve it but I want you to please think about these things so that is another take away I will stop here now Monday we will meet I will draw the profiles solve this problem I know you have other subjects but also please do this you have what is it this is a quiz week but do this as much as

Possible at least for one part number especially for oil you do mu this the hint you know why I am saying this this problem okay tell me in machines in mechanical equipment where would you have given you a hint at where would you apply this especially therefore look at the size of 3 millimeters that we have taken this is a journal bearing problem actually but very general meaning problem how can you will go to infinity I mean a constant you one a plate moving at

You one actually that is replaced by the journal so that you can take it as a flat plate moving continuously and then there is a bearing in between is now tell me why T max is important not simply for you to do some calculations here and say sir I got T max know related to journal bearing problem why is T max important that is why I asked you to do for oil general bearing if I put oil what is that why do I do I put the oil huh what is the word used for that lubrication and

But you are putting oil why it has a certain kind of what viscosity now how does temperature affect viscosity find out you have to maintain the temperatures properly for the oil film for you to have that lubrication effect lubrication will fail under these conditions so find out what is T max now you see the squared problem has a practical application although not a very great it is important actually but none earth-shattering application but it is a very important application

You will know how to formulate the problem how to make simplifications how to make it the simplest of equations in convection but how much information you can get out of it especially in the next class after you draw the temperature profiles we will discuss a little bit further and see something very interesting that is happening thank you.

Coquette flow- part2
End of lecture 9

Next: Coquette flow-part 3

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