Indian Institute of Technology Madras
NPTEL

National Programme on Technology Enhanced Learning
Video Lectures on
Convective Heat Transfer

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Lecture 7
Entropy Generation and Stream function-Vorticity formulation

Okay so good morning again yeah so I hope you are all getting used to the number of derivations that we are doing so far I think this entire course we will be doing such derivations and finding similarity solutions okay so it's not so difficult except that you should get used to the notations and things like that okay so ten self notation is one of them which we introduced yesterday I think if you have time you please read up a little bit on the tonsorial notation it is not that difficult to understand also you should have some basic knowledge of solution to ordinary differential equations because in all the similarity solution problems.

We will convert the partial differential equations into an equivalent ordinary differential equation and find the similarity solution so that has to be done numerically and I expect I will be giving some assignments where you have to solve those do is numerically by some techniques okay so otherwise if we just stop there and give you the solution you may not appreciate exactly how the solution has come and it will all look too mathematical okay unless you do it yourself you will not appreciate the solution process so we will look into the second law of for a closed system so we were till now deriving the conservation equations we were looking at two different approaches we started from a Cartesian coordinate control volume and derived the other one.

We had a coordinate free representation applied anoles transport theorem to a coast system and converted the closed system rate of change of properties to rate of change with respect to an open system and that is another way of deriving in a coordinate free representation so after we have looked at the first law it is also very important to look at the second law because as we all know most of the times we stop with the first law without too much of violation to the second law but if you look at any practical heat exchanger device entropy generation very important you know there are a lot of irreversibility within the system.

And we have to quantify to some extent the extent of these irreversibilities and we should also identify what are the sources of the irreversibilities okay so that is where we are going to apply the second law and we are going to derive a conservation of what is called as conservation of entropy okay just like you have conservation of mass momentum and energy so that is also a possibility of deriving conservation of entropy and that is what we are looking at here so we start with applying a Claudius inequality for a closed system.

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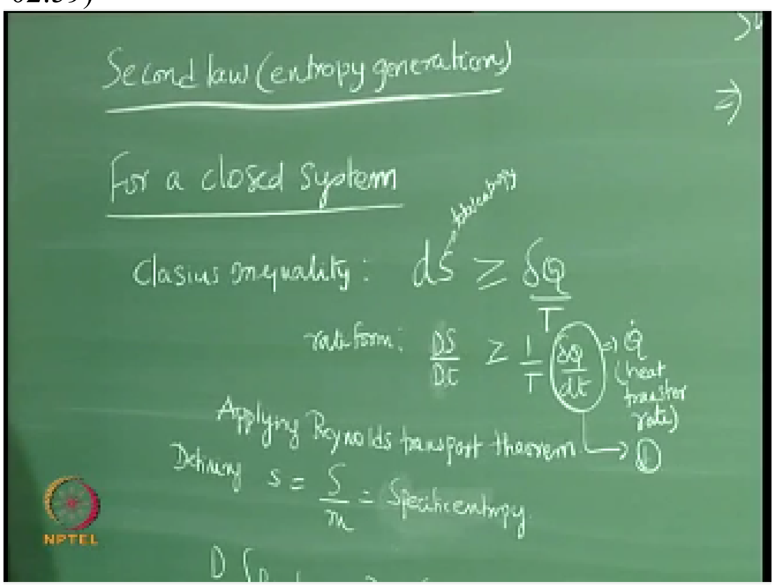

Okay and if you express this in the form of a rate equation so you know that the total derivative d d s by DT should be greater than or equal to 1 by T into the differential amount of heat transferred okay so I will just represent this as Δ Q . okay now let us apply the Reynolds transport theorem to express this total derivative for a control mass and with respect to the partial derivatives in a control volume.

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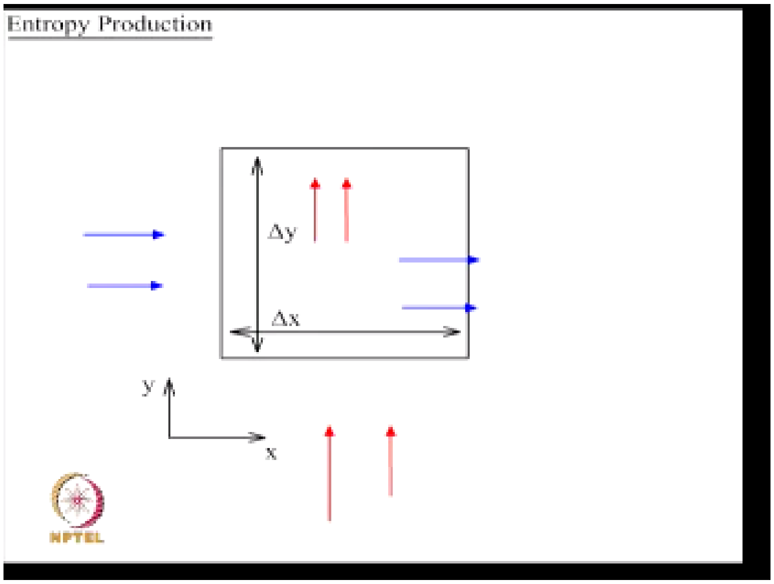

You so you can define a property called a specific entropy okay and in the Reynolds transport theorem the α is a property per unit volume right so then that that can be expressed as P times the specific entropy okay so we can expand this using the Reynolds transport theorem like this now the right-hand side applies for an open system okay so now for an open system therefore if you substitute this Reynolds transport theorem expression into one so this is your equation which defines the conservation of entropy for a open system okay so this is nothing but conservation of entropy.

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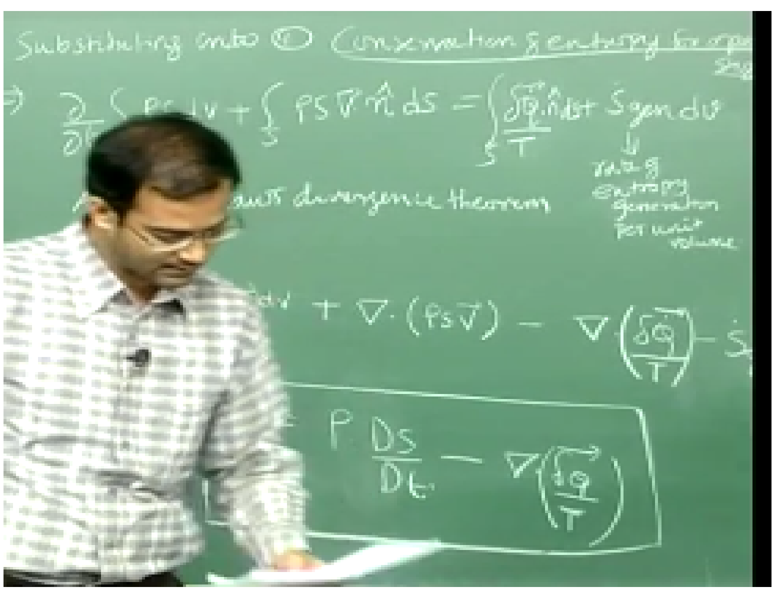

For open system okay or you can also if you do not want to look at this as a conservation of entropy you can also look think this has an entropy balance okay entropy balance equation okay so now what we are going to do all the surface integrals can be converted to volume integrals by Gauss divergence theorem and therefore we can further put this I can take the integral over V out so D by DT P is DV + this will be again a divergence operation right here this will be ∇ . P s V okay and I am going to bring this to the left-hand side here so this will be again _ ∇ ∇ Q okay so this is actually a vector right here so I am just going to give this vector notation okay so this is all integrated over the differential volume.

Because I have already applied the Gauss divergence theorem so on the right-hand side that that should be equal to the rate of generation of entropy okay so therefore I can I can also say that if I express this in okay or maybe I can also look at this way I want to look at the entropy generation for the entire volume so I can also bring this inside - a s . gen okay DV =0 right so that I can separate out the integrand okay and write a finally a partial differential equation okay so strictly speaking this entropy generation is per unit volume here it of a rate of entropy generation per unit volume.

So if I can multiply by Δ V okay so I can say this can be multiplied by DV okay so then I can put this on the left hand side group all the terms together and that can be equal to 0 therefore the integrand has to be 0 so then this will give me my a star gen should be equal to D by DT of P is DV + this which I am going to write in terms of the total derivative notation which is P D s by DT okay _ ∇ . Δ Q think this should be Δ Q by T okay I think I omitted my T because that that is there in the clashes in equality okay so this should be ∇ so this divergence operator is on this entire term okay so this is how I finally can calculate the entropy rate of entropy generation now provided I can also expand this particular divergence operator on ∇ Q by T so I can rewrite this a little bit.

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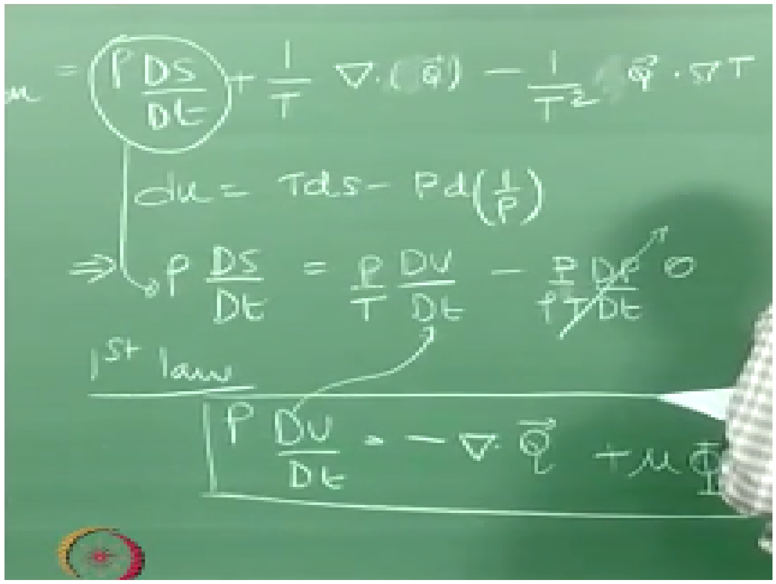

So I can express this as estrogen is equal to P the s by DT + 1 by T the so I can expand this I can split it up into I can take 1 by T out ∇. I can write this as ∇ Q that is 1 the other term is 1 by T Square ∇ Q. ∇ T okay right so because this is the gradient operator so gradient this is a divergence operator on a vector so this has to be a scalar so therefore you should make sure that you only get scalar operators outer so this is again a vector .ted with the vector u you basically get a scalar here okay so you make sure that the final resulting operation also is consistent with the original operator is that clear the splitting up is clear okay.

So I am just so if you if you have a gradient the same way that you write you have to have a write the same way for a divergence operator also only thing you should make sure the resulting operator is also square giving you a scalar okay so therefore now what I can do here so now I am going to evoke the Gibbs theorems one of the Gibbs theorems if you can remember so for a closed system again you know that D U = TDS _ PDV okay so I am going to write as P D into 1 by P okay so this is coming from straight from your first law right where you substitute TDS for Δ Q and you have PDV work okay so this I can write in terms of the rate equation and you can tell me how they should look if I want to write BS by DP okay.

So if I if X + D s by DT so I can write this as d u by DT so I can take 1 by T on the other side - so this should be if you can express this as D P by DT so what should be the remaining term P Square and T right so I am taking this T dividing it all the side so I am now going to multiply this by P okay throughout so that this u can be written as also this that should be a P here P by P T right so this is a little bit of manipulation to suit my convenience here.

Because now I have to find some relation for P D s by DT so I am just expressing this from the gifts equation okay I am connecting that to the change in the internal energy and the change in density with respect to time now if you look at incompressible flows okay for that this has to be zero right so there is no change in density with respect to time therefore the change in the entropy has to be directly the change in the internal energy of the system that is directly linked there so let us also express the first law now if you write the first law so you write D P D u by DT should be equal to _ ∇ . Q okay + u φ okay so I am writing in the final coordinate free representation okay.

So instead of saying ∇. Q you can also say ∇. ∇ Q in fact if you if you want to maybe rewrite to just avoid some confusion you can also write this in of yeah okay just to avoid some confusion you can express this as clue instead of ∇ Q here so that finally it is consistent with that maybe you can do the small change okay so both are consistent all right so therefore what I am going to do now is to substitute for P D u by DT from this expression into this and this I am going to substitute into this okay.

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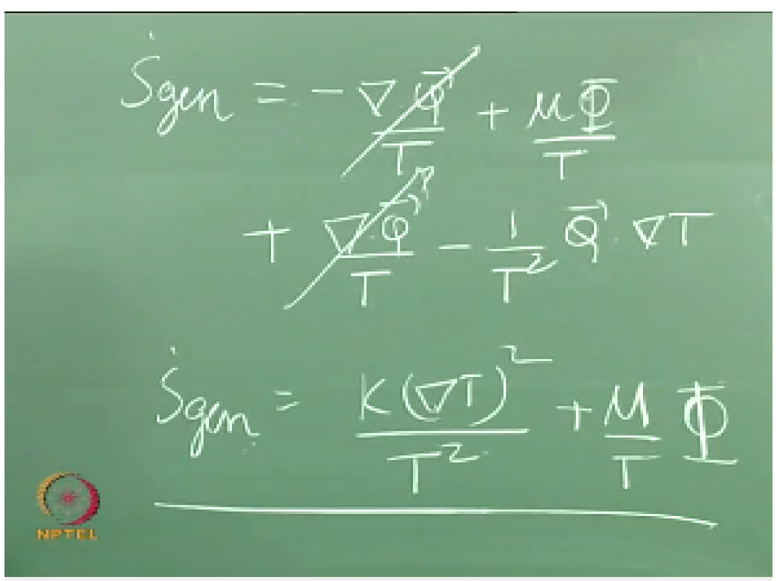

So the resulting expression will be S.gen n will be so this will be basically P D s by DT is nothing but P by TD u by DT which will be _ ∇ . Q by T + μ Φ by T okay so that is this term

+ you has these two terms right + ∇. Q by T _ 1 by T^2 Q . ∇ T okay so these two terms cancel off so therefore your final expression for s. Jen turns out to be if you can also now substitute the four years law for Q as K δ T and express this as - okay so this is _ of Q. which is again _ K δ T so that will become K δ T^2 divided by T^2 okay + μ by T C okay so therefore the components of entropy generation are two.

So one is coming from the entropy generation due to the conduction part okay so within the system the conduction of heat the other is due to the viscous dissipation okay so this part till here I think straight forward only after this we apply the Gibbs law and just manipulate a little bit so that we can eliminate some of these total derivative terms okay so finally we write everything in terms of the heat transfer by conduction and the viscous dissipation so now there were there was this person called Adrian Bejan.

I think that is one textbook also which you are referring convective heat transfer okay so he came up with a very ingenious method that he wanted to give a non-dimensional number which is actually referred to as the Bezier number okay so this is called Bejan number so notation is given as be h wanted to look at the contribution of the entropy generation by means of conduction okay as a fraction of the total entropy which is generated okay so that he has taken this term on the numerator divided by this entire thing on the denominator okay so this is called as Bayesian number in fact the idea of entropy generation due to heat transfer versus vs. idea so this derivation actually is done by him.

And that is also a paper in 1990s you know with the student where he has derived this expression for heat transfer reversibility due to heat transfer okay and I think the Bayesian number was credited to I mean it was basically given due to his contribution for this work particularly and this measures if your baser number is some value say about two it tells you that the majority of entropy generation is through by conduction okay and if it is much lesser than two it tells the majority of entropy generation is through viscous dissipation okay so basically this can be plotted just like you plot your isotherms and heat flux lines you can plot the entropy generation due to each of these and you can visualize and see how it looks okay it is a very useful to learn I mean and you can see where the in which location.

In a particular system your entropy generation is by the conduction part and where it is by viscous dissipation and if you can probably try to reduce the viscous dissipation by some method so that I will also reduce the entropy generation yeah both are dominant kind of both are equally important okay so any questions. I think I wanted to just finish the governing equations therefore I wanted to touch upon the second law also apart from the first law which you are already familiar okay so I think many of the textbooks do not talk about the second law conservation and things like entropy generation much okay.

So any other questions on this any ∂ any anything that requires some clarity or is that okay fine so then we will proceed to a new topic now still we are in the introductory portion of this course where we are deriving the conservation equations and so on so you have seen that the nervier-stokes equations are quite complicated so we cannot solve them analytically except when we are making some approximations now we can also bypass this and we can introduce another set of equations which can be solved in place of nervier-stokes equations especially for two dimensional flows and two dimensional incompressible flows okay.

So for 2d incompressible you know we have to solve if you solve the nervier-stokes we need equations for UV and P okay so UV p and if you are solving energy again temperature okay now we don't have a separate equation for pressure okay we have equation for u and V and continuity equation which is like a default equation so therefore numerically there are some techniques to overcome this hurdle where we construct an artificial pressure equation and things like that if you want to overcome that we can rewrite the nervier-stokes equation into what is called as a stream function vortices formulation okay.

And that is very useful when you are looking at 2d incompressible flows okay there you have only two variables two equations you can solve that straight away and the equation solution is also slightly simpler than solving the navier-stokes okay so we will just quickly derive those formulation today and in fact in the project that you are supposed to do you will be using those 2d stream function verticity equations with the energy you will be solving them numerically okay I will send you some reference papers and you will apply that to a problem of natural convection okay so now as in a natural convection will be covered you can because this is a nice rectangular cavity so the taking you know finite differences will be much easier alright okay so we will do that now.

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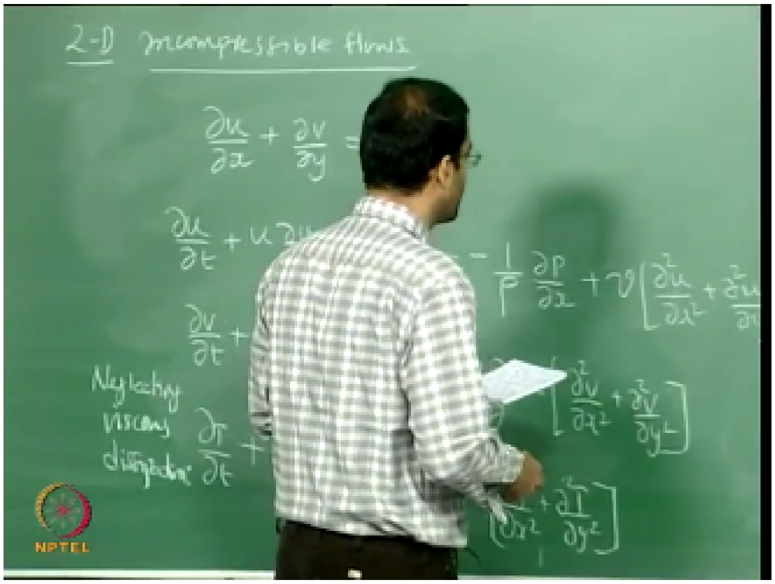

So the next topic will be for 2d so this 2d is the approximation that is required okay we cannot write this in 3d again okay 2d incompressible flows okay so let us write down the navier-stokes equations first and from there we will try to derive the stream function verticity equations so from the continuity equation so you have to tell me for 2d incompressible flows what is the continuity equation let us assume Cartesian coordinate system okay so D u DX + DV dy equal to 0 so now the complicated equations slowly will get simplified as and when we go through the solutions okay and the X momentum D u by DT + would be equal to _ what about density has to be divided right + dynamic or kinematic Dynamite ink.

This is a laplacian operator right so and similarly your Y momentum which will be DV so we will also write your energy equation after we derive the stream function vortices formulation we will quickly go back to the energy equation and simplify that ok so what is your 2d incompressible energy equation we can directly write for temperature okay + = α because that is K by P CP which is nothing but the thermal diffusivity okay the thermal diffusivity α + okay.

So now I am going to neglect the viscous dissipation now okay I do not want to put too many terms into this so I can safely neglect the viscous dissipation okay so this is neglecting okay so now this is more familiar to you have been working with these equations in your earlier courses now what we have to do we have to think a little bit see that is a pressure term in then as the momentum equations and we want to somehow eliminate it because as you know that we do not have a separate equation for pressure and when we want numerically solved that we want to simplify this problem so how can we probably do that take derivative with respect to Y here respect to X subtract these two right so that will eliminate this pressure derivative term so exactly that is what we are going to do okay.

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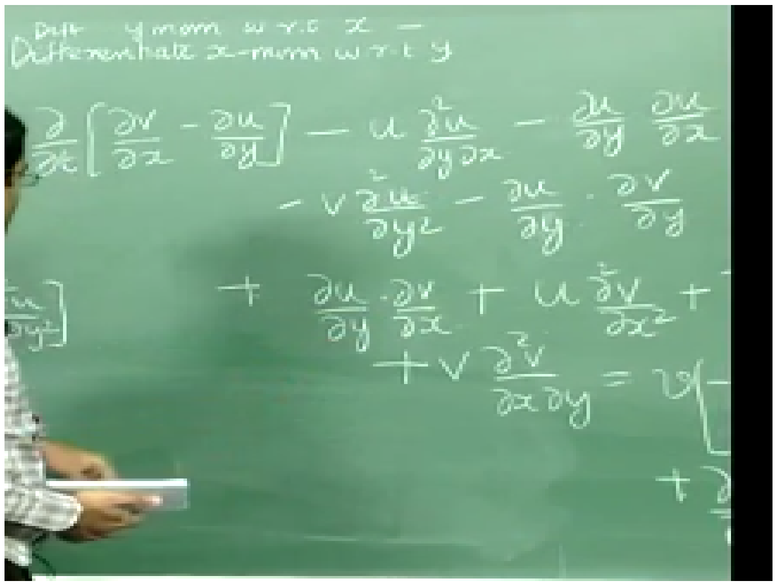

So we will differentiate X momentum with respect to Y okay and so I am going to say differentiate Y momentum with respect to X - differentiate X momentum with respect to Y okay so if you do that so you can group these two terms the temporal term you can say D by DT of so you can write this as DV by DX _ D u by dy is that right okay so I am taking D by DT common so I am differentiating with respect to X - differentiating this with respect to Y all right so the other terms you get lot of terms here because now if I say I am going to differentiate this with respect to Y so I have to split it up again right.

So this will be - you have to tell me now so this is d / dy of u D u / DX okay so I can write this as u into d square u by dy DX - yeah D u D u by dy into D u by DX okay similarly this term can also be expanded that will give you _ V into D square u by dy square right - d you by DX into DV by dy they should be eyx D u by dy I am sorry let me check all the terms again yeah okay so the same thing now I am going to do differentiate the Y momentum with respect to X okay so this _ this right so this terms will be positive here so + you have D u by dy into DV by DX + you have u into d square V by DX^2+ I am going to differentiate with respect to X into

DV / dy + V x v ^{2} V by DX dy okay so when I differentiate this with respect to X this and subtract they are going to cancel okay on the right hand side.

I will have nu into _ d ^{3} u by dy DX ^{2} _ d ^{3} u by dy ^{3} + so I will have these two terms right here d cube V by DX^3 + D ^{3} V by DX dy ^{2} yes one two three four five six this one do u by do X yes you are right because we are differentiating with respect to X so all the other terms are correct please do this and check once so I am now going to group these terms together in some particular fashion and we will see that that grouping will help in reducing we can actually define a new function which can be substituted for those grouped terms.

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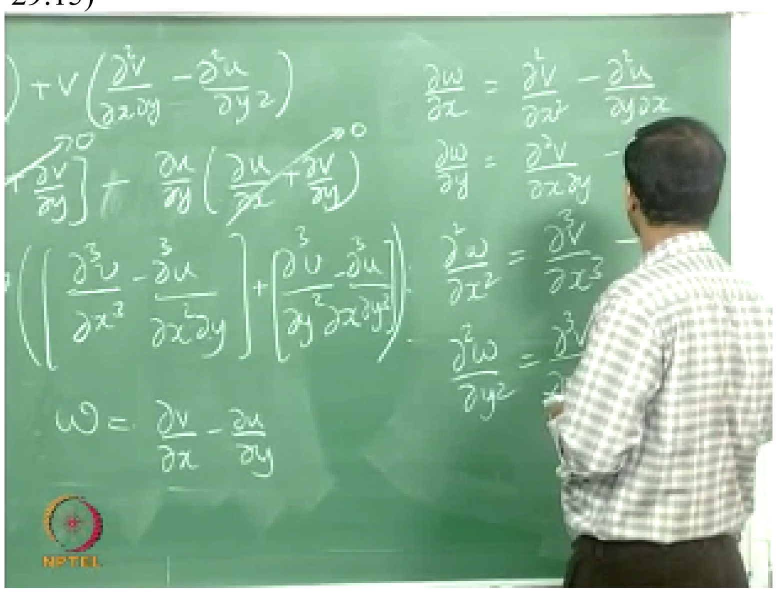

So I am just going to group like this so U take common and what are all the terms common to you so you have this term you have this term so I can write this D ^{2} V /DX^2 _ d ^{2} u by dy DX so this has the term is common to you okay so again + V now you have to tell me which terms I can group those square V by do X doY - all right so we have somehow one two three four okay so four terms we have taken care and before that okay I am going to add that M temporal derivative term so that is d / DT of DV by DX _ V u by dy + these terms right so this term is also taken care now we have one two three four terms which are still on the left hand side that have to be grouped together.

So what I am going to do is I am going to take DV by DX common okay so if I take DV by DX so that means this comments DU by DX + DV / dy which is nothing but two-dimensional continuity which is automatically satisfied okay this is a nice trick to eliminate all the unnecessary terms okay + next I am going to take D U so this is actually _ D u by dy out so d u by dy so you have d u by DX and DV by dy so that will also be satisfying continuity so this goes this also goes on the right-hand side.

I can just group them as d ^{3} V by D X ^{3}_ D ^{3} u by DX ^{2} dy okay so I am grouping these two terms these two terms okay + B cube V by dy^2 DX okay _ D ^{3} u by dy q all right so I am just grouping them now what I am going to do is to introduce a function called the vorticity which you are all familiar right so how do we define what is it e many of you can recollect from your incompressible flows in fact you can get a clue from the terms that we have grouped DV by DX - yeah exactly okay now you can see why we have grouped those terms because they are going to be directly in terms of the vorticity.

That we have defined and now we are going to take derivatives and check it will come exactly to those terms okay so for example if you now take the derivative of T with respect to X now you please tell me what the term should be that should be exactly the first term on the special derivative term right so that is ∇ ^{2}V by ∇ X ^{2} _ ∇ ^{2} u by ∇ y ∇ ∇ X okay so d Ω dy should be what d^2 V by DX dy _ d^2 u by dy ^{2} okay that is this term right here okay so now if you take second derivative d square Ω by DX ^{2} can you tell me what the second derivative will be okay so this is this term right here okay so regarding this no QV by okay and this is ∂ ^{3} u by ∂ y cube okay that is this term right here okay,.

so therefore you find that our grouping all these terms make sense you can write them in terms of the vorticity and its derivatives therefore I am going to substitute in terms of vorticity.

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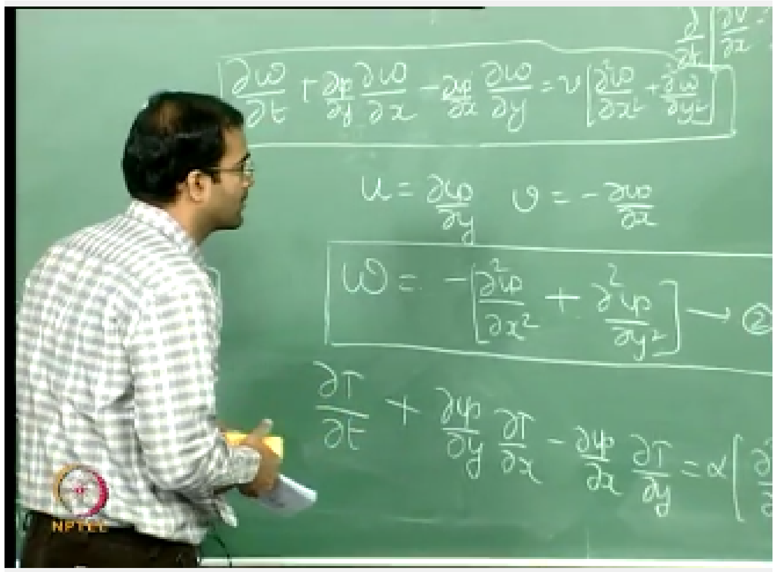

get so your Ω will be so if I take DV by DX this is _ d^2 y by DX^2 and d u by dy _ so _ of three squares I by right.

So you have you can now substitute this in terms of the stream function therefore you have eliminated the velocities okay so this is your equation number one this is the conservation of vorticity equation number two is now since you have introduced stream function you have to solve for stream function okay and that is done by means of second equation which relates your stream function - vorticity okay so this is your second equation so now you have two equations two unknowns right one for sigh and the other for Ω so this is much better to solve rather than the navier-stokes correct okay.

So where you have to solve for three equations and you do not have an equation for pressure so that so that is why it is very popular technique for numerical solution to two dimensional incompressible flows okay most of the journal papers that you take for to 2d incompressible flow still until recently okay where you know do not did not have powerful computing facility to solve the full navier-stokes equations they were employing the stream function vorticity technique okay so and this is what you are going to dry out also now the same thing in energy equation if you substitute okay.

So apart from the flow field you can also calculate for temperature so the energy equation becomes you can substitute for u again in terms of stream function so this is your energy conservation okay so you solve for three equations for three unknowns Ω sy and temperature okay so these are the equations that you exactly how to solve in your particular project and I will be giving the papers you have to apply the corresponding boundary conditions to solve this problem so for a particular problem you have to do it and then you can you can probably look at steady-state solution where you do not have to consider the time derivative okay only the spatial derivative and you can do this iteratively so there are a couple of techniques like gauss or Gauss Jacobi iterative techniques which you can do it and you can use finite difference methods simple finite differences to basically discretized.

These derivative terms all right okay so with that I think most of the conservation equations we have seen and also different variants of the conservation equations what I am going to do now is to slowly get into the theme of this course which we are going to apply for both external and the internal flows okay now we have to make some approximations to the navier-stokes equations when we apply that to external flows we cannot solve the navier-stokes as it is okay so these approximations are called the boundary layer approximations as far as the external flows are concerned and before doing that first we will try to non-dimensionalize.

The navier-stokes equation we will try to define some non- dimensional numbers we will see what non dimensional numbers are governing the flow and heat transfer parameters and w will also identify the regimes based on these non dimensional numbers and finally when we go to boundary layers we have to use these non dimensional numbers to make certain approximations okay so once the boundary layer equations are derived from there we will start our process of solving for different configurations okay so we will I will just introduce you little bit we have some more time about seven eight minutes so I am going to talk about the different parameters different variables and how we are going to non-dimensionalize them okay so I think this is probably familiar to you because you have done this layer in your incompressible flows and also advanced written math so very quickly we will go over it and you can yourself try to non-dimensionalize.

I am going to write down only the final non-dimensional equations okay so here after we will be dealing only with incompressible flows and that also in two dimensions okay so all the complicated terms will be dropped off one only we will retain terms in these two dimensions.

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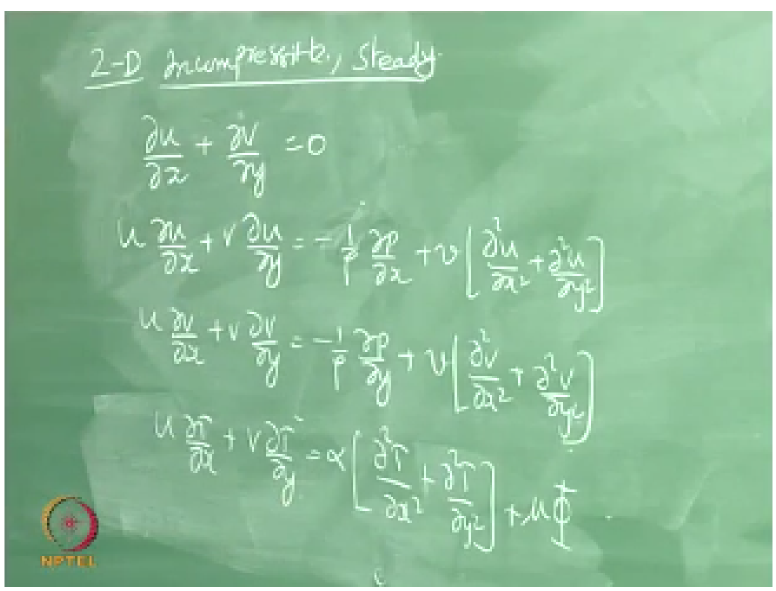

So 2d incompressible in fact I should not have it is the navier-stokes so the same thing applies here and anyway. I will just do it write it quickly again. I am going to also include the pressure term so here I am what I am doing this 2d also incompressible and steady so this is the approximation that I am going to bring here because after we non dimensionalized and apply this to boundary layer flows we are pretty much going to do this for steady-state solutions okay so we will stick on to this particular form throughout so all you are most of your solutions will be for 2d incompressible and steady state and also we are including the viscous dissipation here to non dimensionalize and see what kind of non dimensional numbers come out okay.

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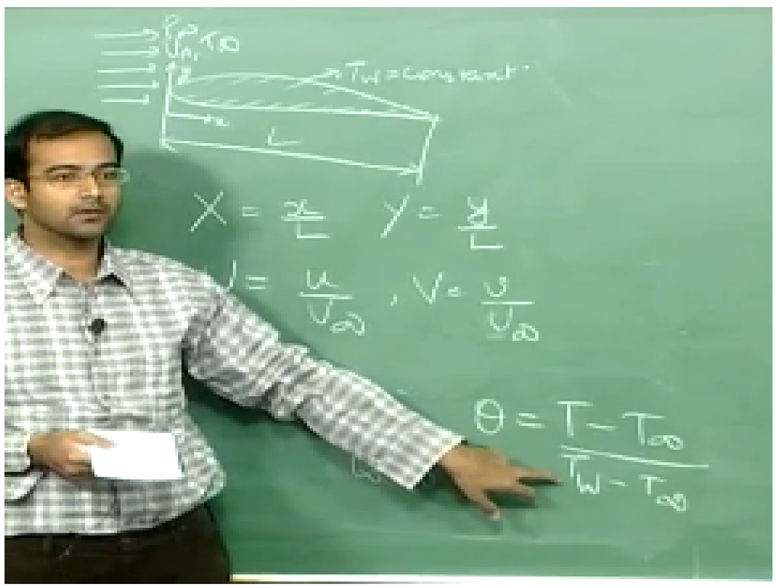

Suppose you take any flow say past an airfoil or past your flat plate okay so you have your free stream which is described by your free stream velocity and temperature and you can also maintain the surface of the airfoil okay at an isothermal condition first that your tea wall is constant and let us say the characteristic dimension of this particular soil is given by the chord length which is L all right so this is these are some of the variables that you have fixed you know you have your u ∞ u ∞ define okay the geometry is well-defined and also they are fixing the boundary temperature to an isothermal condition so with these parameters how are we going to non-dimensionalize.

So the as well as far as the coordinate system is concerned okay let us take a coordinate system x and y okay like this which are along the cart length and normal to the cart so I am going to define a non dimensional coordinate system capital X which is X by L similarly Capital y will be Y by L and as far as the velocities are concerned I am going to introduce U okay which is what u by u ∞ similarly capital v = V by ∞ okay now coming to pressure okay so velocities coordinates are done.

So I am just going to introduce the P which is equal to the small p dimensional P divided by so how do we non dimensionalize the pressure okay so I am going to cut the half term because it does not make much difference $P \infty u \infty^{2}$ okay so the free stream density is P ∞ all right and finally coming to the temperature. I will define a non-dimensional temperature θ such that at the wall the value of the non-dimensional temperature equal to one and in the free stream away so the value should be zero.

So how do I non-dimensionalize that t _ T ∞ by T wall _ T ∞ okay so that at the location where at boundary T equal to T wall this becomes 1 and at the free stream somewhere that θ goes to 0 all right so with this I just write the final equation you can in fact derive this and check for yourself if you substitute these non-dimensional variables into the dimensional navier-stokes equations so you are now going to tell me so we will write down only the final version so what will happen to the continuity.

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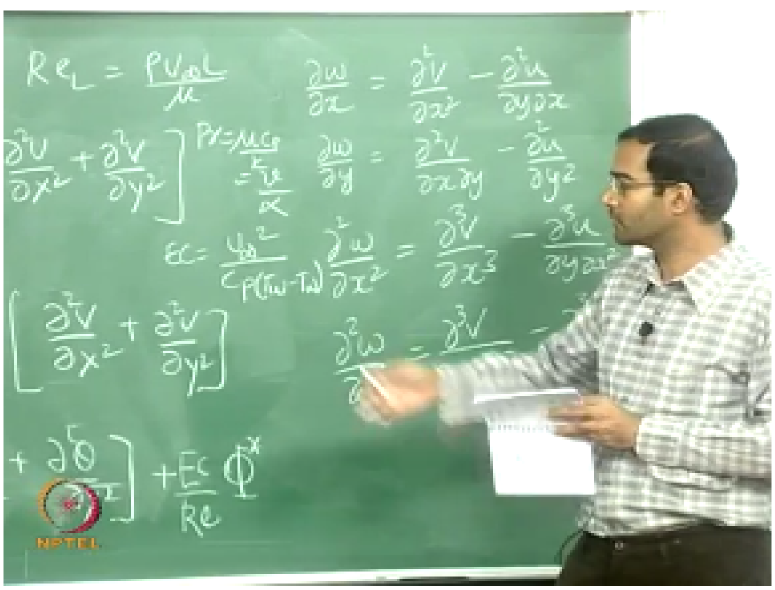

This is right and we are just substituting for the dimensional in terms of the non-dimensional variables there all right they cancel out everywhere okay as far as the momentum equation is concerned you be u by D X + V D u by dy I am writing all in non-dimensional terms is equal to _ D P by DX + something which you have to tell me what it is d ^{2} u by DX square + d square V by dy^{2} okay the same thing for the this is U dv/dx + V DV dy - so these are all in non-dimensional variables so you have to check and tell me what terms should be there and similarly finally you have D θ DX + I am going to introduce a non-dimensional risk viscous dissipation free star.

So you have to fill in the blank how what are the terms that are going to appear here okay so what should be the term right here announce number so if you are an number is very large so what happens if this term is more important than the convective term is that correct it should be 1 by Reynolds number right 1 by re based on the length so where you are re L is nothing but P u ∞ L by mu correct similarly the same thing comes here 1 by re L now what should be the term here just 1/2 minute we are done okay.

So I am just going to give it you please check it 1 by re frontal number okay where your number is equal to μ CP by K which is also μ by α ok and finally here I am just going to introduce a non-dimensional number which probably you have not encountered so this is going to be what is called as a curtain umber by Reynolds number here where your record number is defined as u ∞ /CP xT /1 _ T ∞ okay, so what I give you as a homework is to please substitute all that carefully and check whether we arrive at this now non-dimensional formulation okay.

I think you can do that directly you can see that that this comes directly straight away only this combination and this combination you have to check again all right so we will stop and we will meet on Tuesday we will start with the boundary layer approximation.

Entropy Generation and Streamfunction-
Vorticity formulation

End of Lecture 7

Next: Couttee Flow –Part 1

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