Indian Institute of Technology Madras
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Video Lectures on
Convective Heat Transfer

Dr. Arvind Pattamatta

Department of Mechanical Engineering
Indian Institute of Technology Madras

Lecture 5
Energy Equation

Okay so good morning the whole of you yeah so in the last class we were looking at the derivation of continuity and the momentum equations and we started off with the derivation of the energy equation in a Cartesian coordinate framework and today we will contiμe and complete this derivation so as we had seen we have taken a cubic or a you know a Cartesian coordinate control volume and the contributions to energy if you look at so you have primarily the efflux of internal energy as well as the kinetic energy.

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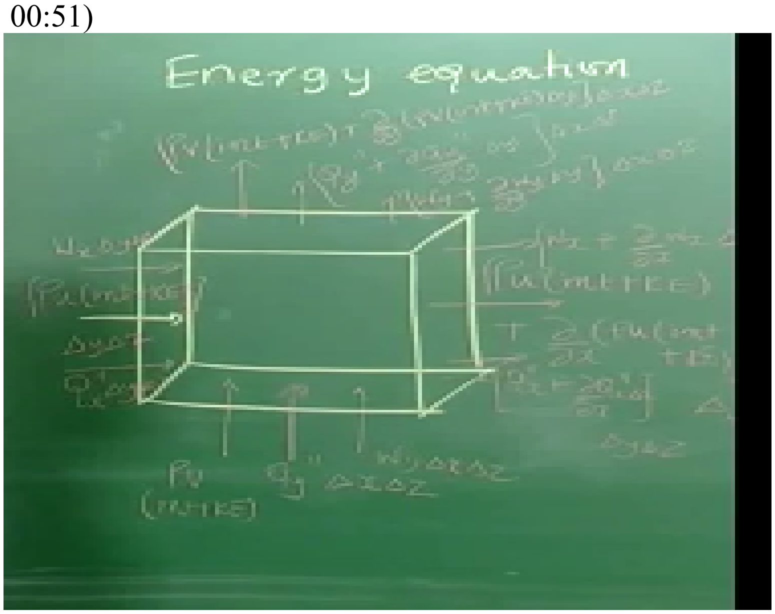

Which is crossing the control volume boundaries in all the three directions? I am just representing here on only two directions you can also extrapolate that to the third direction apart from that you have the reflex of heat by conduction okay so that is this Q double ″ so you have heat conduction which is essentially transferring the heat by diffusion process and you have energy transfer by means of reflects of the internal and the kinetic energies and also you have work transfer so all the three are simultaneously these are these are reflexes or effluxes of contributions of internal kinetic energies the work and the heat which is all acting on the control volume boundaries.

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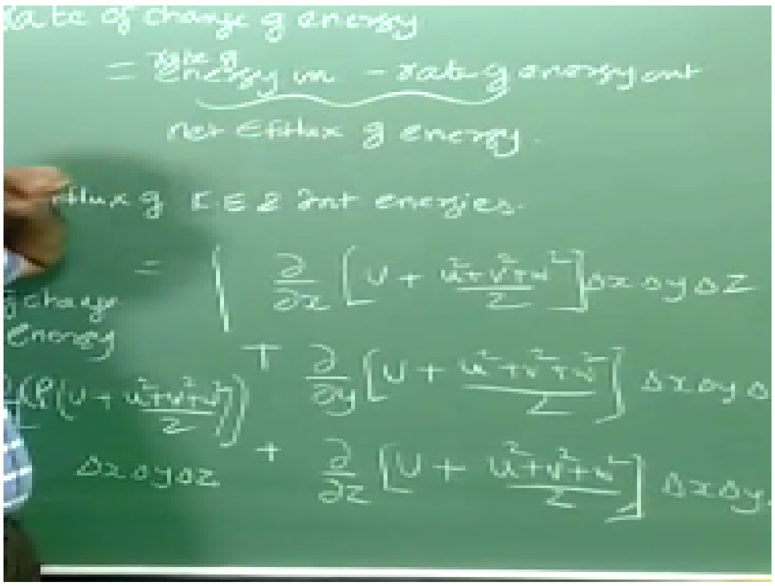

You and as far as the change within the control volume is concerned so we know that we can express the change rate of change of energy in the control volume to the native efflux of energy across the control volume boundaries okay and now we are also looking at when we are looking at the work we include the body force terms which are acting on the entire volume into the work terms okay so they are the potential energy terms which we are adding as work so if you look at the rate of change of energy essentially so this is the energy per unit mass okay internal energy per unit mass is this u capital u or small U which you want to use.

If I use small u that conflicts with my velocity therefore I am using capital u and this is your kinetic energy per unit mass so multiplied by the mass which is your density times the volume so this gives me the energy of the system and therefore the rate of change of energy will be d of this divided by DT okay, so therefore if you look at on the right hand side which is the net efflux of energy we have to balance the energy which is coming in from the left boundary - the one which is leaving and similarly in the Y direction and the Z directions and if you can do that and expand by Taylor series.

If you calculate the energy rate of change of rate of energy in - rate of energy out so you end up with these three terms which we have written down last time so similarly we can also express the efflux of heat as well as the work you know the Flex of work is a little bit more complicated which will lead to additional terms which are coming in but we can also try to expand and see how those terms look so let us write down the net efflux of heat.

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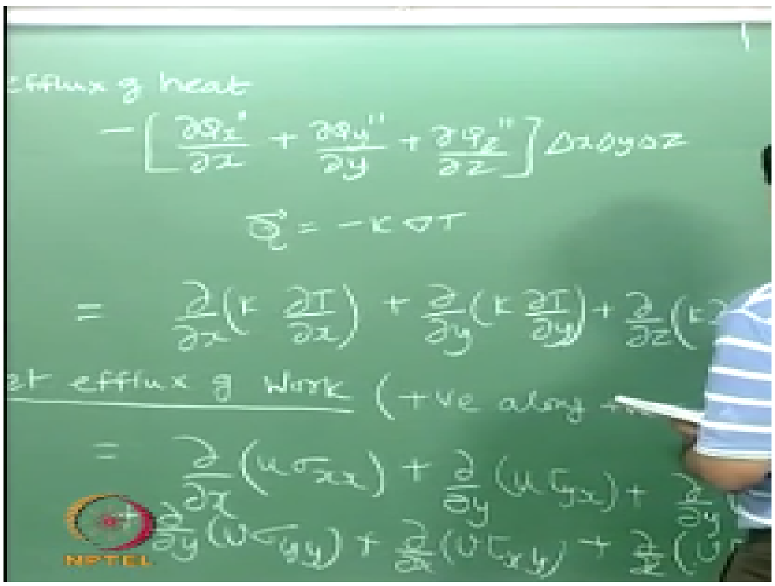

Okay so do you think that this is correct reflects of kinetic and something is missing here sign - there should be a - here right so that is there are efflux of kinetic and potential energies coming in - the one that is leaving so there should be a - sign here right and when we write the net efflux of heat okay so let us write it down can you say probably how it should look - of DQ X DX + V Q double dy + V Q Z ″ by DZ so this are your effluxes into the volume finally so anyway you have for each of these derivatives you have δX and multiplied by the area so everything comes out as a volume you have δ X δ Yδ Z okay.

So we can now introduce the Fourier law of heat conduction okay so you know that your Q vector is equal to – K δ T right so that can be introduced and when you say how this can be written so we can the - - sign can be cancelled off you can write this as d by DX strictly speaking the thermal conductivity could be anisotropic that means you can have different thermal conductivities in the different direction so you can keep it within the partial derivative if you assume that is isotropic and constant property.

So this can be taken outside the partial derivative okay so I am assuming here the thermal conductivity is isotropic but could be a function of position okay because of its dependence on temperature so I can express this okay so much so for the energy efflux of heat so it is looking very straightforward now let us move on to the net efflux of work now this is a very lengthy expression because you have to consider all the terms which contribute to work okay, so if you do the same analogy and apply that to work so how does the work terms appear can you think about so you are writing the net efflux of internal and kinetic energy is heat and in analog.

This way so what are all the terms which contribute to work the forces okay all the forces the surface forces the volumetric forces they are all included under the work term okay so if you go back and revisit your momentum equation so you look at what are all the surface forces and body forces acting and then you multiply them with the respective velocities okay so that will give you the amount of work okay so that is basically the rate of work right the force into displacement divided by time so that is the rate of work so all these are in rate terms alright so if you do that so the net reflects of work in all the three directions.

So I am just going to write down and you just please verify if they are correct and also the sign convention is that so I am assuming the forces which are all acting the net forces acting in the positive direction or all positive work okay they contribute to the positive along a positive x direction so this should be looking like this D by DX from the X momentum equation you have D Σ X X X by DX so I am just multiplying by the corresponding velocities in that particular direction so you Σ xx right you can verify that from your momentum equation expression okay u τ YX so these are the stresses the normal stresses normal viscous stress stresses.

In the tangential stresses which are acting along the X Direction multiplied by the corresponding velocity in the direction alright so this is the contribution this is the net contribution to the in efflux of work so we have W into τ Z X okay so this is in the X direction okay so this should be you here right so you have to sum them in all the directions okay so you can say + you can just write that in the Y direction as well you have V and you tell me what this term should be Σ YY + D by DX into V into τ is it YX X Y okay so you look at this the last subscript here denotes to the direction.

That we are talking about in which you are calculating the work or momentum okay + your D by DZ of V x τ Z Z Y okay so this is a good exercise for you + you have D by DZ.

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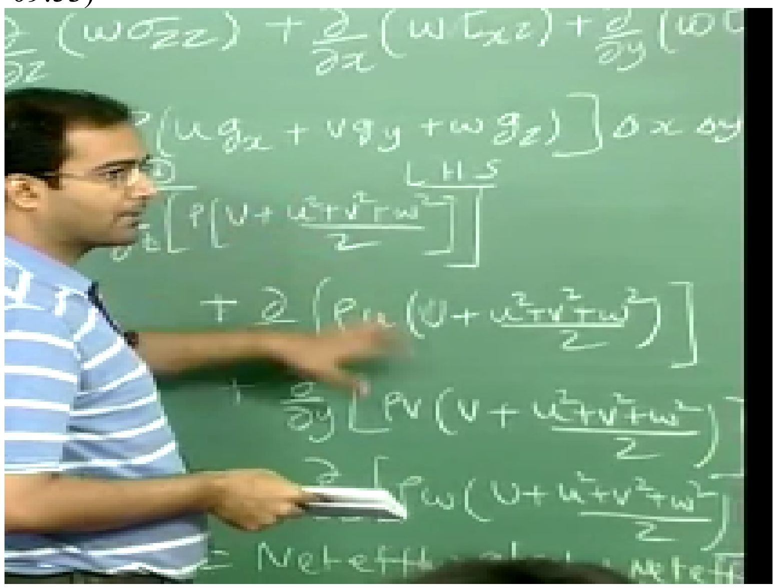

Of W into Σ Z Z + D by DX of W into Σ what should what should be this term τ XZ + D by dy of W τ why is it okay so these are the contributions to the surface forces so what should be added to this what are the other contributions toward body forces okay so that also we can yeah so that is coming under the body forces okay so that is the potential energy contribution that is added under the work so this will be P U so I am taking P common because P is common in all the three directions you have u GX b g y + w v g z okay now this entire expression right here.

I have to multiply by the volume and same with respect to this because this is a volumetric first you can see directly this is multiplied by the volume the other ones you know from the Taylor series expansion we have this δ X δ Y so any questions on this , I hope you have identified the different contributions to the efflux terms as well as you know rate of change of energy within the control volume is straightforward so now we will just balance that using the principle of conservation of energy and we will write down so this will lead to a big expression here which can be written as d by dt okay.

Let me number this let me call this is number one here okay so from one so I am going to substitute for all these terms into one so this is d by DT of P U + the kinetic energy so this is the rate of change of energy within the control volume + you have the terms which are so actually strictly speaking what we could have done we can combine the rate of change of energy and the efflux terms together into a total derivative so I am just going to first write it down and then combine the convective terms and the rate of change terms into one single total derivative for a compact notation.

So right now I will just write it in the full form so this is D by DX P you probably have missed the row here okay so this should be a row here right because this is the efflux okay so that should be mass efflux multiplied by the area which will give you the flow rate of this particular quantity alright so you can you can write this in terms of P u so u + u^2 is v^2 + W^2/ 2 okay + you have d by dy P V so I have taken all the efflux of kinetic and internal energies towards the left hand side and combine that with the rate of change of energy.

So this is my left hand side terms now this should be equal to I am not going to write down it is taking too much of space here I will just mention this is equal to native efflux of heat right so you can just write those terms here native flexor feet + what net the efflux of what okay so now on the left hand side term I can I can just say from the LHS.

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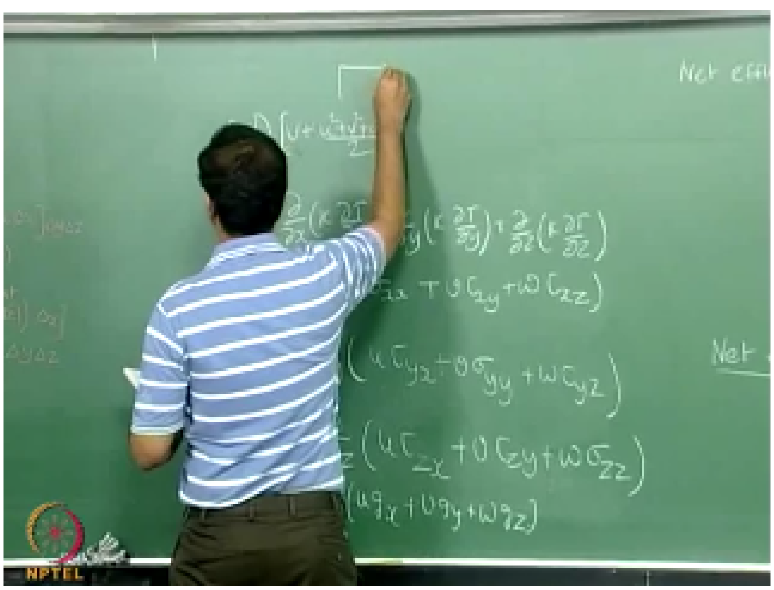

So the term right here the time derivative and the spatial derivatives can be combined into one total derivative if you also observe we can also simplify this a little bit we can say take U + u^2 + v^2+ W^2/2 common so this will be D by DT of P + D by DX of P u + this + this so what is that that is a continuity equation okay.

Which will be automatically satisfied so that will go to zero and therefore you can say that you can take P out and we can write this is d by D by DT of this + D by DX of U into this + D by dy fu into this okay so that can be written as P into the total derivative of D by DT with respect to U + right because the continuity is satisfied so I can take the P out okay so I am not making any assumption here. I am not making any incompressible assumption but you can see that if you expand this the compressible form of continuity itself is satisfied so I can write this as P into the total derivative of the internal and kinetic energies so the LHS is simplified.

So now I can equate that to the other terms okay so that will be equal to my d by DX of K DT by DX + D by dy + I have the work terms so what I am going to do here I am just clubbing these terms D by DX terms here in the from the X momentum the Y momentum and the Z momentum together okay so I am taking D by DX common and writing all the terms inside that so that should give me u Σ xx + V τ XY + W τ XZ please check these terms are right + D by dy of u τ YX + V Σ YY y z ZX + V τ that Y DZ + I have the body force term P xu GX + v gy okay so this is my complicated form of the energy equation now what I am more interested finally is the I should write down.

An equation for the change in internal energy of the system more than the total energy which includes internal energy + the kinetic energy so I want to somehow eliminate the kinetic energy part and write down an equation governing only the internal energy which is the direct indicator of the temperature change of temperature of that particular system so how can I do that how can I eliminate the mechanical energy component from this can we use the momentum equation somehow and we can construct an equation for mechanical energy from the momentum equation.

You know the momentum equation is written for U V and W ok we can multiply those with UV and W in each of these directions we can sum them up together and you will get this particular form on the left hand side that can be subtracted from here and this component can be eliminated straighter right so if you go back and see how your momentum equations are written okay now this is the let me call this as equation number one I already have one so I will call this is number two so how are we going to do it if you look at the momentum equations on the left hand side term how do we have the total derivative can you go back and check.

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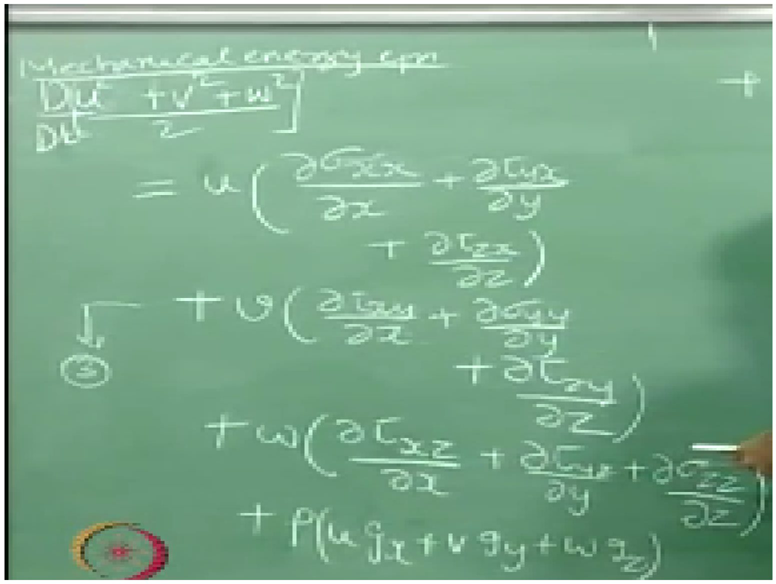

d by dt or d u by DT for example in the X momentum right ok so this is the left hand side term correct so if you multiply this by you okay so this will be UD u by DT which can be written as D u square by 2 correct this is nothing but UD u by DT so the same thing we are going to do in all the three directions multiplied by the respective velocities and sum them up so what you will get + v^2 + W^2 / 2 so now you see the left hand side term of the energy equation so this and this are common right so if you subtract this directly you will eliminate kinetic energy so we will retain only the internal energy component.

So now we also have something on the right hand side of course when we subtract we have to subtract both sides on the right hand side terms what you will get can you and you just go back to the momentum equation and multiply and tell me so you have u into D Σ xx by DX + D τ YX by dy + d τ ZX by D Z so this is the X momentum which are multiplying by u + what the Y momentum V into D τ XY by DX + D Σ YY so what should be this term third term D τ in which direction it is y direction and the derivative Z + you have D τ X Z + D τ Y Z + D Σ Z Z by DZ okay + if you multiply your body force terms also you have P x u GX v gy + okay.

So this equation is called the mechanical energy equation okay so this is nothing but something like conservation of mechanical energy from the momentum equation okay so if you express write your momentum equation automatically that satisfies this okay but you are explicitly now writing another equation for conservation of mechanical energy so let us call this as number three so now if you subtract three from two that means you are trying to eliminate the kinetic energy term and only retaining the internal energy so that is going to get us where we want to go in terms of calculating the temperature of a system.

So far although you are more familiar with an energy equation involving the temperature we have not introduced strictly speaking an equation for temperature although that appears on the right hand side term here on the left hand side term still you do not have something for the temperature. So we are slowly going there okay.

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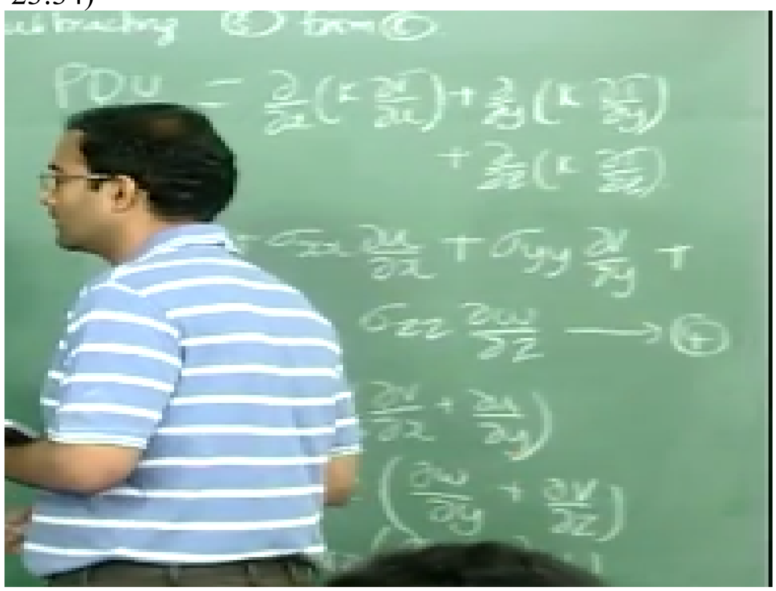

So subtracting three from two on the left hand side term you have row the you by BT okay ,so this is your change of internal rate of change of your internal energy of the system on the right hand side terms you retain your reflux of conduction reflux of heat by conduction so this term comes as it is d by DX and now coming to the other terms if you look at typically this term here and there so you can eliminate so you can expand this has your D by DX Σ xx D u by DX + u into D Σ xx by DX so that cancels so what should what should be the remaining terms after you cancel off you have Σ xx D u by DX right + τ YX into D u by dy okay.

So what I am going to do I am going to combine the like terms together so I am going to combine the normal stress terms together first so you will be getting DV by dy okay so I am looking at this particular term here and writing them together + I have Σ ZZ DW by DZ now the other tangential stresses .I am combining them I am also using the fact that τ YX = τ XY okay and τ Y Z = τ z zy and τ XZ = τ ZX so if I use that I can combine τ YX and τ XY terms together and that will be DV by DX + D u by dy because if you if you look at τ XY here so this will be DV by DX okay and this will be D u by dy.

Okay so these two term can be combined together and τ XY is equal to τ YX similarly the τ Y Z DW by dy + DV by DZ + τ XZ what should be the derivative inside D u by DZ + DW by DX okay so therefore you have another nice equation which is still not simplified completely but this is an equation for the conservation of the internal energy of that system alright so now what we are going to do so how do we simplify this further however written okay so this is coming from the momentum equations have you know the XYZ momentum equations right.

So multiplying each of those momentum equations by the respective velocities and summing them together okay so for example on the X momentum you have UD u by DT so I can write that as D D / DT of $U^2/2$ correct and similarly in the other direction I sum them and also the right hand side I am doing that summation so this is, s okay so now how do we further go ahead and let me write this as equation number 4 still you have some unknowns on the right-hand side right you have terms related to your stresses which have to be closed now how do we close those terms Stokes hypothesis.

Yes so we have to first derive a we have to use some relationship between the stress and the strain rate so Newton's Newtonian fluid approximation first and of course Stokes hypothesis now also we I am if you do that you will derive that for a compressible fluid but we are more interested in incompressible fluids in this course so I am directly going to substitute the Newtonian fluid approximation for incompressible fluid now directly okay because otherwise we will accurate more and more terms.

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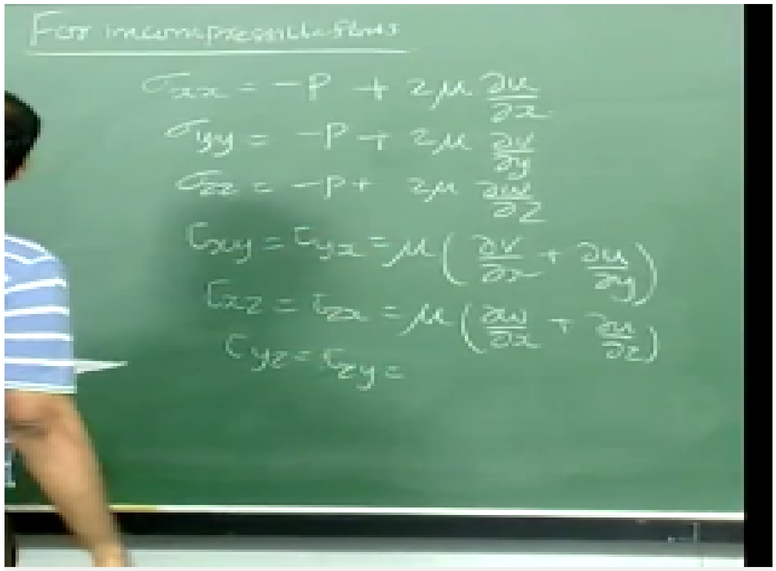

So for incompressible fluid or for incompressible flows so incompressible flows so what I am going to do is can you tell me for example Σ xx should become - P this is a pressure force + okay after the Stokes approximation so - 2 by 3 μ into divergence of U now for incompressible flows that is going to be 0 right so that term can be neglected so what is the other term + 2 μ bu DX right so that is why I am going to bring in that approximation that that divergence of views can be eliminated straightaway so one term this actually simplified so I can write this in the other direction.

So now my XY and τ YX is going to be the same for compressible or incompressible fluid okay so that is μ into DV by DX + bu by D Y so and also you can do that for τ Y Z is equal to τ zy please you can fill in okay now you can substitute this into for and we can write down in terms of velocities okay so what I am going to do just only this term right here which I can probably highlight so this highlighted term alone I am just going to express in terms of velocities.

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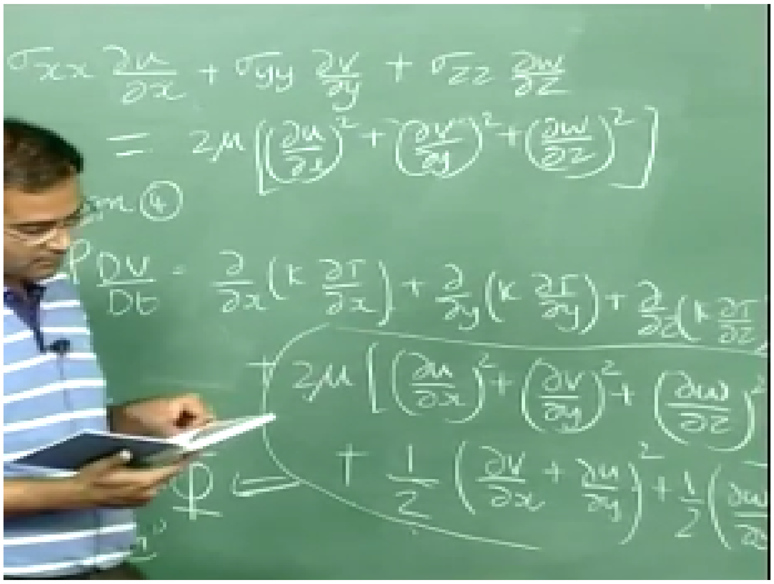

So my σ XX d you by DX + σ YY DV by dy + σ z z DW / DZ so if I substitute this so I am going to you can see directly. I can take P common d u by DX + DV / dy + DW / DZ so satisfies incompressible continuity okay so if I substitute for this okay so that time I am going to eliminate and you will have 2 μ x D u / DX x D u by DX + 2 μ into the same thing so finally you will have 2 μ D u by DX ^{2} DV by okay so the incompressible approximation is going to simplify this particular the normal stress work okay so to a great manner so therefore if you look at this terms here τ XY you can see that is already new DV by DX + d u by dy so this and this multiplies and this will be again whole square okay so this these are relatively straightforward terms.

So if I combine that I can once again express equation four as P D u by DT should be + 2μ and I have this term right here D u / DX the whole square okay + I can take two new common for these terms also I can take one by 1/2 factor out so that this is two and two cancels and it is new and what should be the other terms if we substitute for τ XY into whole square of each of these right so DV by DX + D u by dy ^{2} + half of my DW by dy + DV by DZ ^{2} + half of BU by DZ + DW DX ^{2} okay so this is my equation number five and now it looks more familiar to you I am going to group this set of terms contributed due.

To the work has one single term which is denoted by the symbol right here and this is called as the viscous dissipation see all the surface forces you can see the body force terms get cancelled off finally the body force terms do not affect the energy in any way all right so only the viscous forces are playing the role the surface forces and they are all grouped together as what is called as a viscous dissipation contribution to the energy internal energy of this system okay now some people make an approximation that under certain criteria and conditions this is going to be negligible and therefore you can see the familiar form of the energy equation.

Okay so we will make a few approximations now and simplify this equation for a couple of conditions so the first thing if you introduce the fact.

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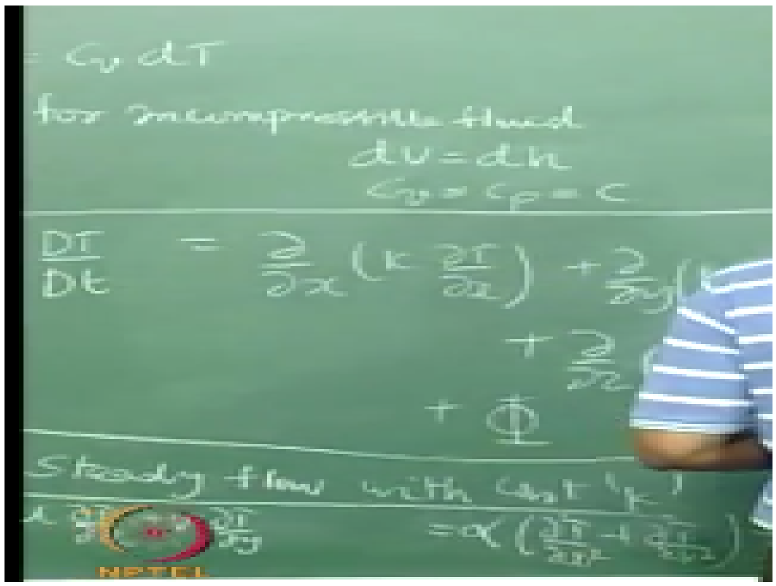

For that your d u that is your change of internal energies related to your temperature okay so if you say that it is CV DT and for an incompressible fluid it does not matter which specifically to use whether it is specific heat capacity at constant pressure or constant volume they are the same okay so for incompressible fluid okay so your D U is the same as the change in enthalpy therefore your CV is equal to approximately CP okay so this is just one constant specific heat I can use for incompressible fluids and therefore I can express that in terms of temperature so I can write this as P C DT by which is equal to C okay so on the right hand side.

I have this term conduction terms + I have the viscous dissipation term so this is the familiar form of energy equation for incompressible flows okay now we can also make some more further approximations to this okay so because this two one two should cancel up the term is actually new into this movie was common to all of these in fact I should put another big bracket here and yeah you are right yeah so two new is common to all these huh no I am calling this entire thing as five okay this is a single term which is the viscous dissipation term all right so for steady flows first approximation.

I am going to reduce this incompressible energy equation for steady flow under constant property assumption that my thermal conductivity is invariant of position with constant or uniform property everywhere so this is going to be through the time derivative is going to disappear you can write this as u DT by DX + V DT by dy + W DT by DZ that is equal to K by P CP which is nothing but the thermal diffusivity okay α so I can write my K by P C P C as α D t by DX ^{2} + B ^{2}t by dy ^{2} d ^{2} t by DZ ^{2} + P by P C okay so for two-dimensional flows I can make further approximation that the third dimension is not important + three by P C where V becomes even more simplified okay for 2d flows.

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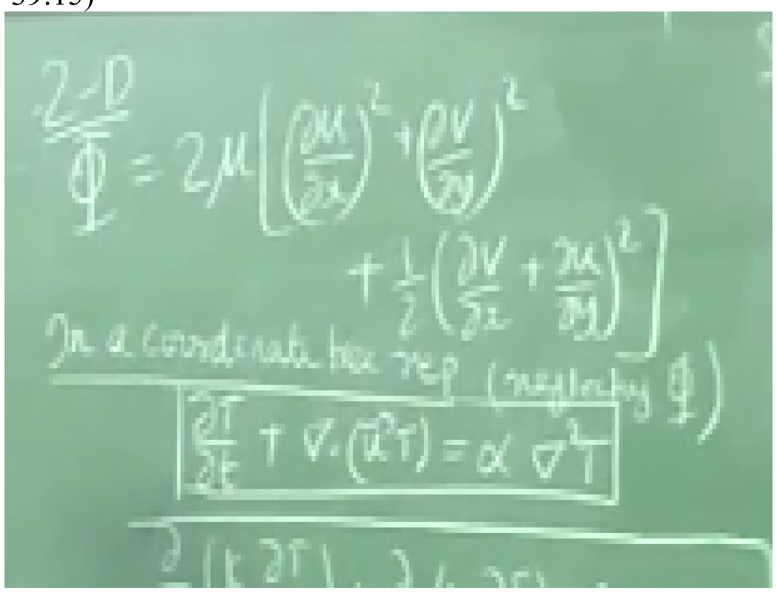

I can expand my so that will be only two new into D u / D X ^{2} DV / dy + ½ of DV / DX + D u / dy ^{2} alright so finally when we are working with the energy equation in this course for doing the nautical solutions we will be looking at these approximations okay, incompressible flow and mostly in two-dimensional we are concerned only with 2d flows for analytical solution so this is the form that we are going to work and you can see the viscous dissipation also gets substantially simplified okay in a coordinate free representation you can also do this and you cannot suppose you neglect your viscous dissipation terms.

In a coordinate free representation neglecting fee how can you write this okay so suppose you don't make a steady-state approximation but still it is an unsteady you can write this as DT by DT + ∇ dot u vector times the temperature which is equal to α ∇ square t so this is my equation for temperature right so this is your advection term this is your diffusion term right here so they are represented in terms of your divergence and the laplacian which you can substitute for the corresponding coordinate system okay so this is a common representation you can if you want to write this in cylindrical or spherical you please substitute.

In that particular coordinate system the corresponding divergence operator and the laplacian operator we will we'll come back to that okay so right now I am just concerned only with the derivation we will non dimensionalize this equations then we can then only we can find the criteria where we can neglect those terms otherwise when everything is dimensional we cannot find out exactly you know when this is important and when this is not when you non dimensionalize you find in terms of non dimensional numbers okay there is a particular number associated with that is called the ax-cut number the ratio of Eckert number.

To the Reynolds number is very high that terms become important viscous dissipation otherwise you can neglect okay so usually for given in the incompressible flows if the flow velocity is high or if you are going to the other limit where you are you are looking at extremely slow flows that is creeping flows under that is called Stokes approximation so under these two categories that term becomes very important for most of the intermediate velocities and intermediate Reynolds numbers so that term can be safely neglected okay so very quickly in the remaining time.

I am going to introduce to an alternate way of deriving these equations so far what we have done is taken a control volume which is probably in a particular align to a particular coordinate system and we have derived it nicely so what if you want to apply this to an arbitrary control volume of any shape and if you want to reach to this particular form this is a coordinate free representation okay so the way to do that is by what is called the Reynolds transport theorem so I will give you some brief introduction into the Reynolds transport theorem.

Now and probably in the next class we can quickly do only the energy equation derivation from their nostrils with a momentum equation I think you can try it out yourself so any questions so far on this okay I think some of this is already familiar to you okay.

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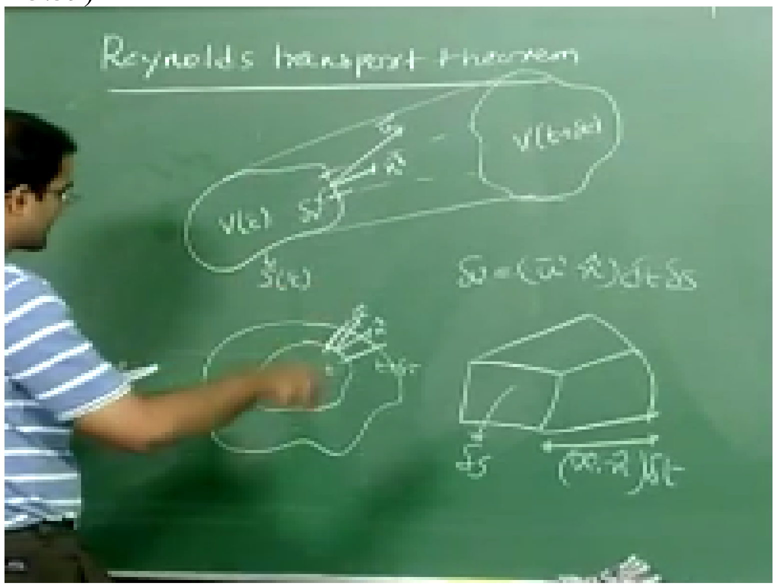

So this is through noise transport how many of you are familiar with the Reynolds transport theorem one only one I think whoever has taken the incompressible flows probably this was introduced to you right okay so I mean if you look at the undergraduate level usually we do the derivation in the Cartesian coordinate system that is the easier one to visualize whereas most of the graduate fluid mechanics the governing equations are directly started on a coordinate free control volume so let us take some kind of an arbitrary shape control volume so let us say the volume of these total volume of this at a particular instant of time with V okay.

And let us say there is a differential volume δ V okay there is a surface normal which is pointing outward and there is a velocity which is also oriented in some particular direction okay so this control volume is actually moving okay and if you take a differential amount of volume you draw the surface normal and you see the velocity is pointing in some direction okay now the surface of this is S given some value S at this time T now after some time you will find that this volume is changing after a period of time.

So if you go from T to T +δ T this volume deforms into some shape like this okay so not only the shape but also the volume is different so I am going to write this as t +δ t the corresponding volume and also your corresponding differential volume also changes right so if you overlap these two volumes so this is the volume at t +δ T and this is your previous volume say so this differential volume right here has changed from T to T + δ T and if I look at only this particular differential volume and I draw the full view representation of that so it is going to look something like this okay so this is my differential surfaces all right corresponding.

To this volume so I am just drawing a 3d representation of this particular segment alone okay now you know that this has a velocity vector like this and this has a normal like this okay so if you align the velocity vector in the direction of the normal so this distance between these two volumes that is the displacement from this position to this position how can you calculate that gives you the amount that it has actually displaced from this time to T +δ T velocity into time so my velocity in the direction of normal will be u dot n into δ T okay so my total deformation of this δ V is actually going to be u . n dt x d s okay this is going to be the total deformation of this differential volume correct okay.

So this is the basics of this so from here maybe I will take a couple of minutes two or three minutes and then go few more steps so I can say if I want to define any property α okay.

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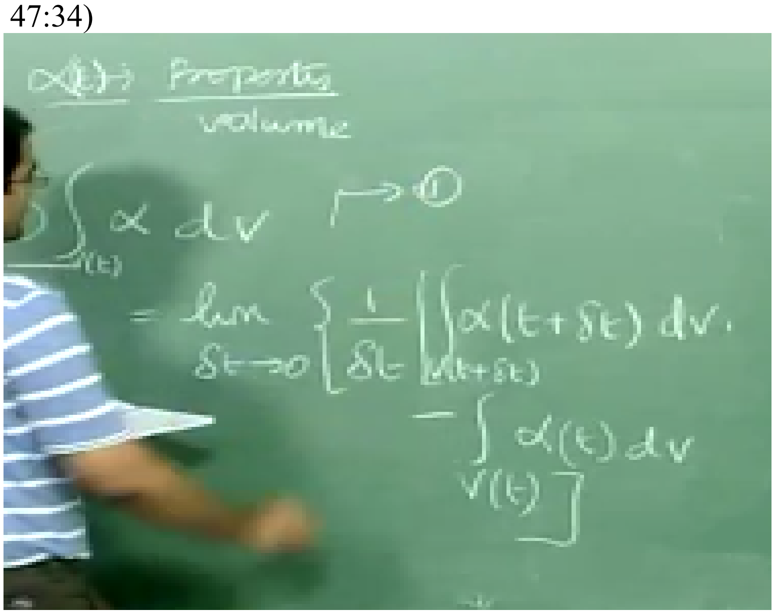

So this is the property per unit volume and now the volume is changing but this is a control mass so that means the mass is constant but the volume is deforming so therefore I have to define a property per unit volume so that this α is going to be the same here or here okay it is per unit volume property and if I want to see how this property is evolving over time okay so I can write that I can calculate the change of this particular property over the entire volume I can multiply this by the volume so that is α x DV and integrate this over the entire volume okay so this is property per unit volume.

So for a differential volume I multiply by that I integrate it over the entire volume this will give me the property for that particular control volume okay so I want to see how this property is changing with respect to time so I take a total derivative of this okay so this is the rate of change of property over this control volume V alright so this I can express as limit my δ T going to 0 I can write this as 1 over δ Tx the property at time T +δ T - so this is at volume T +δ t - the integral of this specific property at T right so this is a corresponding to volume T.

So if I take this difference from the property at T +δ T for this volume - the property at T for this volume so this difference divided by δ T is going to give me what is the change of this property ok due to the change in the volume and over time okay so this property therefore is only a function of time ok this is not a function of the volume.

Because it is already property per unit volume so it is only a function of the time so this is the starting point for the Reynolds transport theorem so we will stop here and we will let us call this as equation number 1 so this is basically to express the change in the property with respect to time in terms of the final and the initial States okay we will start from here and derive the transport theorem tomorrow.

Energy Equation
End of Lecture 5

Next: Reynolds Transport Theorem

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P.Baskar

Camera
G.Ramesh
K. Athaullah

K.R. Mahendrababu
K. Vidhya
S. Pradeepa
Soju Francis
S.Subash

Selvam
Sridharan

Studio Assistants
Linuselvan
Krishnakumar
A.Saravanan

Additional Post –Production

Kannan Krishnamurty & Team

Animations
Dvijavanthi

NPTEL Web & Faculty Assistance Team

Allen Jacob Dinesh
Ashok Kumar
Banu. P
Deepa Venkatraman
Dinesh Babu. K .M
Karthikeyan .A

Lavanya . K
Manikandan. A
Manikandasivam. G
Nandakumar. L
Prasanna Kumar.G
Pradeep Valan. G
Rekha. C
Salomi. J
Santosh Kumar Singh.P
Saravanakumar .P
Saravanakumar. R
Satishkumar.S
Senthilmurugan. K
Shobana. S
Sivakumar. S
Soundhar Raja Pandain.R
Suman Dominic.J
Udayakumar. C
Vijaya. K.R
Vijayalakshmi
Vinolin Antony Joans
Adiministrative Assistant

K.S Janakiraman
Prinicipal Project Officer
Usha Nagarajan
Video Producers
K.R.Ravindranath
Kannan Krishnamurty

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