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Video Lectures on
Convective Heat Transfer

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Lecture 44
Analogies in Turbulent Convective Heat Transfer
Part 2

Good morning all of you and welcome to the last lecture of this course most of you must have been quite happy and relieved also you know you have to go through a lot of assignments I know and also computer assignments as well as detailed derivations probably putting lot of effort compared to the other three credit course but this is course where you learn / applying you know so I mean the classroom lectures are just kind of helping you to understand in a broader sense but as long as you solve a particular problem.

Yourself we will not be understanding because we have so many different types of problems in convective heat transfer it is not a one particular type of solution and then you just change the boundary conditions and then you find you know a different equation solutions and so on so we have external flows where we have a particular type of solution and then internal flows all together again different regimes you see how the ends up from starting from a simple no integral equation all the way to solving a PD anything we can do within the internal flows okay.

And again when we are talking about natural convection again the solution methodology changes because you have coupling of heat and mass transfer right so then we say method the approximate solution are for example the similarity solution when we applied to natural convection has to be slightly modified the kind of unknowns that we are dealing again when you talk about turbulence it is altogether different it looks--wait different from the kind of analysis we have applied to the laminar boundary layer equations we have not looked at any similarity solutions in turbulent flows.

So whatever was possible in terms of simple analogies you know we have been trying to derive so you know in that sense this course convective heat transfer has lot of things to be covered you know so it is quite a bit of challenge for me also and to make justice to this course still in spite of that we have not covered topics like you know porous media convection in porous media and we have phase change condensation and boiling so these are also parts of you know really speaking convective heat transfer.

But I would suggest that you know VA since in our department we are also offering specific courses related to those topics no such as boiling and condensation porous media so anybody who has taken this course now should be able to easily you know get on with get along with those courses as well the foundation has been a very well established so the rigor will be more or less similar it is only you are exchanging the approach a little bit when you do it for porous media for example.

If you have additional resistances which have to be countered for boiling and condensation is also highly empirical subject so the kind of analytical solutions are very limited to extremely simple cases especially in condensation in case of boiling you cannot find any analytical solution more or less everything has to be empirically you know it is not even a rigorous derivation but the thing is the physics of boiling is very complex so there is a combined heat and mass transfer so far we have not looked at mass transfer in this course again when we say convective heat it is also convective heat and mass transfer there is a there are cases.

Where you know mass transfer is an analogy to heat transfer just like we are doing an analogy between heat and momentum through Reynolds granted all these analogies so we are solving the flow field and finding the heat trans solution same way we can also find an analogy between heat transfer and mass transfer so we introduce another non-dimensional number called the Schmidt number.

Which is the ratio of momentum to mass diffusivity and once we know this then once you know momentum futility you can extract the mass diffusivity and a similar kind of analysis instead of Nusselt number we define a Sherwood number things like the mass transfer coefficient instead of heat transfer coefficient so it becomes very similar instead of Prandtl number you have a Schmidt number which becomes the governing parameter there but there are cases where the heat and mass transfer gets coupled that is a good example is the phase change evaporation for example.

So in evaporation you also remove certain amount of mass from one phase and you know add it to the other phase so that becomes more complex now when you couple heat and mass transfer you cannot find analytical solutions okay so that requires a very rigorous treatment even numerical solutions are kind of you know hand-waving approaches you know whatever we have you know in a commercial computational fluid dynamics packages and so on they are not very rigorously established models and we are still Lu illusive okay.

So I mean so boiling and condensation in that sense it is a highly empirical subject you have to deal with lot of correlations and experiments which have been conducted and the understandings of that okay so yeah overall this is a very vast course you know so I think it is really hard to cover all these topics in one semester if at all can be done but it can be done but you will not be able to appreciate the depth of this subject .

No so which I mean I think already most of you are aware of the breadth of this course so when you take a heat transfer course I am sure you are exposed to the breadth of heat transfer but unless you probe deep into a particular topic we will not be able to appreciate the fine aspects of finding rigorous solutions especially analytical solutions okay so when it comes to turbulence again people have different approaches to it my personal approaches also I do not look for analytical solutions everywhere but you should understand that for certain problems there are analytical solutions available.

Which you should not overlook okay for example I have seen cases where people solve the flat plate boundary layer numerically and they do not know where to look for the Blasius solution to validate okay so you should know now / now for which cases you can actually compare with exact solutions even when you talk about turbulent flow flat plate boundary layer so we have correlations derived from approximate methods okay.

In the last class I was talking about the use of analogy the Reynolds analogy so in that case we have established an empirical relation for the skin friction coefficient right so if you remember what was the relation for CF.

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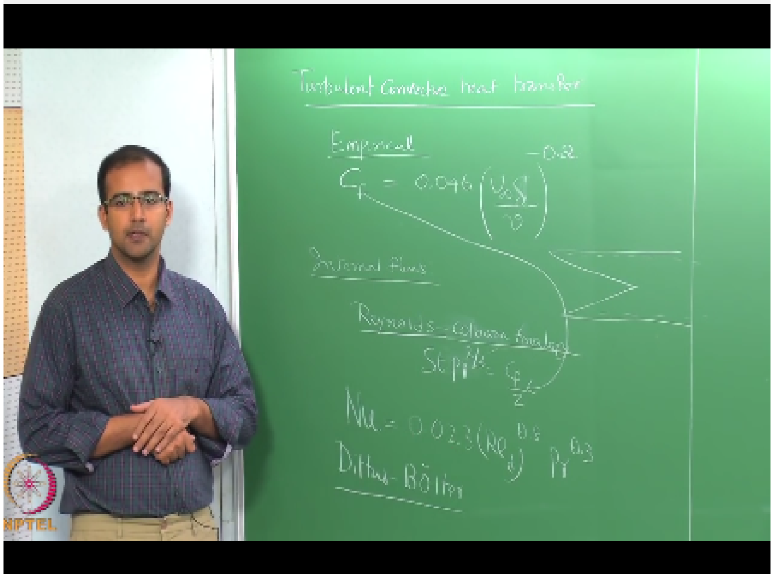

So this is B this is an empirical one okay this we cannot rigorously derive this okay this is coming from set of experiments okay for the hydrodynamics part do you remember what this correlation is 0.046 into now when we define re now rather than defining it as RE because RE you might actually misinterpret the link scale used so here it is whatever U MR U ∞ depending on whether this is an external or internal flow if you are dealing with external flow you have u ∞ and then the length scale is now the boundary layer thickness divided / what is the – 0.2.

So this is a kind of most commonly used relationship for internal flows it becomes easier you replace your Ð with D right and then the next step is suppose you take the case of internal flows the simplest analogy that we have derived so far is there a nice analogy right which considers the entire boundary layer to be purely turbulent ok so for this case there also analogy what does it say your standard number is = CF / 2 for this.

In this case you are actually neglecting the role of molecular diffusivities compared to the turbulent diffusivity which you know that for that is an influence of molecular prandtl number even in the turbulent Nusselt number case because this is a w phenomena and near the w the molecular diffusivity s cannot be neglected right so therefore Colburn extended this Reynolds analogy to also cases where he considers the effect of molecular prandtl number okay .

Then it becomes the Reynolds Colburn analogy and what does it say your standard number X PR power 2 / 3 is = CF / 2 now when we substitute the expression for CF into this therefore we find we can derive an expression for the Nusselt number they remember what it is 0.023 times RE Lee powers 0.8 PR power 0.3 okay so this is called the Dittus Bolter okay so Dittus Bolter correlation many people think it is a completely you know based on empirical formulation it is semi empirical.

You can say that this skin friction coefficient is empirical but after that we use the Reynolds analogy to get this and again in the textbooks there is a variation of exponent of prandtl number for heating we use value cooling we use point force so but it does not matter overall the structure of this comes from the Reynolds analogy okay so then it has been slightly modified depending on the heating or cooling case because the prandtl number dependence is not a very exact dependence that is been introduced later on / cold 1.

But it depends whether you have a heating or cooling that is been shown to be a slight difference but not too much okay some people use an average value of 0.33 and they are still okay with that right so now for the external flows however you have in terms of the boundary layer thickness Ð right which has to be first determined because we know that in the external flows boundary layer as Ð itself is a function of Reynolds number so therefore we cannot simply define Reynolds number based on Ð and stop here.

So we have to therefore how do we do this we have to use the approximate methods okay so the only way to find Ð as a function of the local Reynolds number is to use the approximate methods and then substitute this for the w shear stress and then we will have an expression for Ð and then we apply the Reynolds analogy then we find the Nusselt number correlation so there also you will have something similar except that this constant gets slightly modified and the

Reynolds number dependence still remains the same only this becomes a local Reynolds number okay.

And the Nusselt number also becomes a local Nusselt number may be causing the external flows your boundary layer thickness keeps varying so therefore the Nusselt number and Reynolds number have to be defined based on the local coordinate okay so from this you can understand even for internal flows in the laminar case your Nusselt number was constant in the fully developed region but what this says is that even if it is fully developed in the turbulent flow this is still a function of Reynolds number and prandtl number correct.

Because what is fully developed it is only the turbulent boundary layer when you say these two boundary layers these are the turbulent boundary layers so only these are purging but the viscous sub layer will not merge this is very close to the w it will it will not much okay so therefore there is an influence of Reynolds number now because Reynolds number controls the amount of this viscous sub layer thickness okay which in turn affects your Nusselt number so although you are turbulent boundary layers are fully merged your viscous sub layer is still remaining which is now governed / the value of Reynolds number and therefore that comes into the expression.

And so is the case now since this is a heat transfer case Prandtl number will also be important okay so please remember that in the turbulent fully developed flow you do not have a constant value of Nusselt number like in the laminar case laminar case does not depend on Reynolds and Prandtl number because you have only laminar boundary layer once it is merged there is no effect of what you are putting up stream whatever announced number may be once the two boundary layers are merged you will have the same profiles and only thing that will decide the Nusselt number will be boundary conditions.

Whether it is a constant w temperature or constant heat flux only that alters the nature of the temperature profile whereas in the turbulent case still this is dictated / the Reynolds and parental numbers okay so this is a very popular correlation you know so you find in most of the engineering applications people use the data spotter correlation without having to worry what is the accuracy of this how it has been derived and still it gives a reasonably good prediction within something like + and + or - 15 to 20 percent of the experimental data right.

So within our simple flows like pipe flows you know flat plate boundary layers these people have been successfully using this right for more complex flows they have to look at numerical solutions there is also we should however you know we should look at modification to the basic analogy okay we know that although this is a very useful analogy widely used but nevertheless since this considers the entire boundary layer is turbulent it is not the most accurate representation of the turbulent boundary layer profile so we have seen that.

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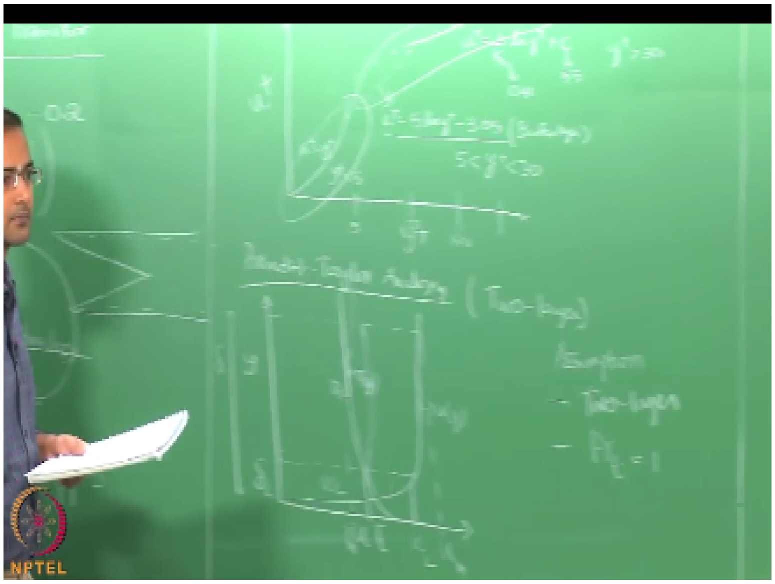

That the entire turbulent boundary layer profile can be divided into three regions so if you plot u + and y + y + on a log scale for example right so for example if you are talking about so you have 5 here you have something like 50 let us say you have 100 but I say this is like 500 and so on so based on this we have identified a profile in the viscous sub layer which is linear plotted on a logarithmic x-axis becomes a curve like this okay.

So and then we have another expression for the log layer which is extending above y + of 30 so that can actually if you extend it goes something like this okay and here it is given as a logarithmic profile you remember how the profile looks + a constant and this is your foreign carbon constant which is 0.41 and this is 5.5 okay this two we have derived from the basics okay now also there is a patching layer which has to be connected between the linear and the logarithmic variation another logarithmic profile so this is called the buffer layer okay.

So also represented with a logarithmic variation you remember what the profile is 5 Y +, - 3.05 so this is your buffer Layer so this is valid for Y + < 5 > 30 right this is valid for Y + > 5 and this is valid for Y + < 30 these are all kind of you can really measure to discuss a player and the log layer very clearly okay when you when you have a very fine probe which can resolve the viscous sub layer you can actually measure and show that the data also Falls with these so this has been generally well accepted.

Again variations will happen when you have pressure gradients that when you have flow separation okay then when the boundary layer itself is detached so then the this kind of profiles will not be there so according to the Reynolds analogy we altogether neglect this and only consider the entire turbulent log low profile okay and then we make use of the calculations of no turbulent w shear stress and also the heat flux everything to be having only one component which is nothing but the turbulent diffusivities.

We integrate it all the way from the w to the edge of the boundary layer okay we ignore completely the molecular diffusion and then we derive there now Sinology so naturally then extension to this is called Randall Taylor so I am not really sure about who really did it but looks like it has been also named after GI Taylor so it should have been I think maybe a joint kind of intuitive discovery of both prandtl and Taylor so it has been credited to both of them so in this case the point is that we assume a two layer models.

Where the inner layer which is the viscous sub layer is also considered which is the important thing you know the molecular diffusion has to be dominant near the w it cannot be node so therefore so we consider the viscous sub layer and we consider the rest of the boundary layer to be turbulent so when we draw the velocity for example as a function of Y and also the temperature so we will have a variation like this for example so till a certain location so let me let me give it a much so till a certain location this will be dominated / viscous effects from the w and from this point let us call this UL.

Where it transitions from a laminar sub layer to a fully turbulent layer okay the corresponding thickness of the viscous sub layer let us call this as Ð okay so above this the profile becomes governed / fully turbulent diffusivity right and finally at the edge of the turbulent boundary layer okay so you reach the velocity which is either the mean velocity inside the duct or it is a free stream velocity in the external flows okay so this is at Ð right.

So therefore now you have two regions one is your viscous sub layer and the other is your turbulent boundary layer similarly if you draw the temperature profile something like this so your temperature profile is governed / again let us say till up to the laminar sub layer you have a value which we will call TL okay and from that it becomes fully turbulent and then outside it reaches the value TM so this is your temperature variation with Y and this is your velocity variation with work.

So this is the picture of a two layer model which was which was used / Prandtl to derive the two layer analogy so naturally the Assumption here is that so we are picturing the entire within the turbulent boundary layer is divided into two layers okay one is the viscous sub layer near to the w and we have the turbulent boundary layer outside okay and again when we derive it we also use condition that turbulent Prandtl number is = 1 so when we say turbulent Prandtl number is = one we say that your ð is = ð T okay.

Because even though if you assume turbulent Prandtl number is = one all the processes at the w are governed / the molecular diffusion okay so therefore the assumption of ð T is = Ð outside will not affect what is happening near the w okay so the same thing was is also used in the Reynolds analogy right so in the Reynolds colburn analogy also we use turbulent prandtl number = 1 and then only we derive the analogy okay.

So the assumption is that inherently this is not bad and especially when we are doing even turbulence modeling most of the time we assume this turbulent prandtl number to be close to 1 okay so the starting point of this is.

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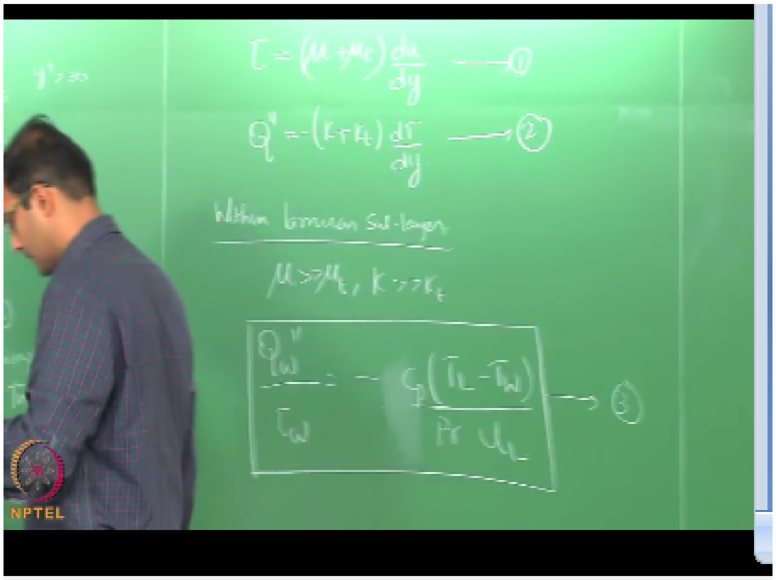

Now we have the shear stress which has now both the molecular and the turbulent diffusivities so we can write this as μ + μ T into D U / DY and similarly the heat flux also consists of - K + AT DT / D Y okay so if you are looking at Renault analogy it assumes fully turbulent so therefore we neglected μ compared to M T K compared to K T and then we divided we integrated from the w to the boundary layer and then divided one over the other and then we ended up with this .

So now we have divided this into two regions so we will first apply this for region 1 which is the laminar viscous sub layer ok so within the laminar sub layer can you integrate let us say equations 1 & 2 within the laminar sub layer where we can neglect the turbulent diffusivities integrate your velocity and temperature profiles from your w till the edge of the laminar sub layer Ð and then take the ratio of 2 / 1though that is the heat flux divided by shear stress first integrate it and then you divide it.

So what do you get if you do that what will happen to Tao so at w it becomes Tao all right and this is a mutates neglected you have μ and then the U gets integrated from zero all the way to UL okay and why all the way from zero to Ð the same way here also you have Q w and this gets integrated from T w to TL and then this is also zero to Ð L and then we divide 2 / 1 so Ð L gets cancelled so essentially we will have therefore Q w divided / Tao w is = - K / μ into TL - T w / UL okay.

So therefore we can write this in terms of molecular cantle number multiplied and divided / CP from you CP / K so we can therefore write this as CP divided / lanthanum okay so let me call this as equation number three so now next we will apply equations one and two in the turbulent boundary layer okay.

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So for turbulent boundary layer we can of course do the other approximation where the turbulent diffusivities are much larger than the molecular diffusivities okay and therefore we can integrate from Đ L all the way till Đ right so you do the integration of this profiles from Đ L all the way till Đ and then divide again 2 / 1 okay let us see what is the expression that you get so let us say again you have Q double prime / Tao will be = - CP into TM - TL divided / prandtl number into UM so where do you have prandtl number here because this is turbulent so this is yeah PRT is = 1.

So therefore we can so therefore now we have this is your equation number four so for continuity the same w shear stress or w heat flux has to be taken all the way from the w to the edge of the turbulent boundary layer correct so we are right now assuming a one-dimensional transport of heat so whatever you apply at the w has to be transported all the way till the edge of the boundary layer so therefore this has to be the same as the w quantities yeah correct there is no discontinuity in the heat flux or shear stress will also be felt by the turbulent boundary layer same with the w shear stress okay.

So therefore we can just replace Q and tau with Q w and now the unknown quantity in this we know for example for a given isothermal condition we know what is the w temperature T w okay but what we do not know here is TL the temperature at the edge of the laminar sub layer okay now UL is also obtained from measurements okay we will see how do we get it but then we are solving the heat transfer problem TL is difficult to obtain so therefore we will eliminate TL by using these two equations so therefore eliminating TL between 3 & 4 so how do we do that we just add 3 + 4 so if you add 3 + 4 you have - CP you have + CP TL there you have - the PTL so the TL gets eliminated right away okay.

So therefore you please do 3 + 4 if you do that so you will be ending up with the following expression which is Q w double prime by okay so I just take this to the left hand side okay and then this also to the left hand side and then I add three and four so on the right hand side I have CP into T w - T N okay so this is let us say equation number five in this case there are TL is eliminated so what I ask you to do now that we have something like Q w / Tao w okay you can take this Tao w to the right hand side and write this in terms of Stanton number okay

So if you write this in terms that means Q all / T w - TM there is nothing but H , H / Ro CP into UN is your Stanton number so write this in terms of Stanton number on the left hand side take the other extra terms to the right hand side and see what kind of expression you get okay so you should be able to get an expression for stanton number is = function of the other terms including prandtl number.

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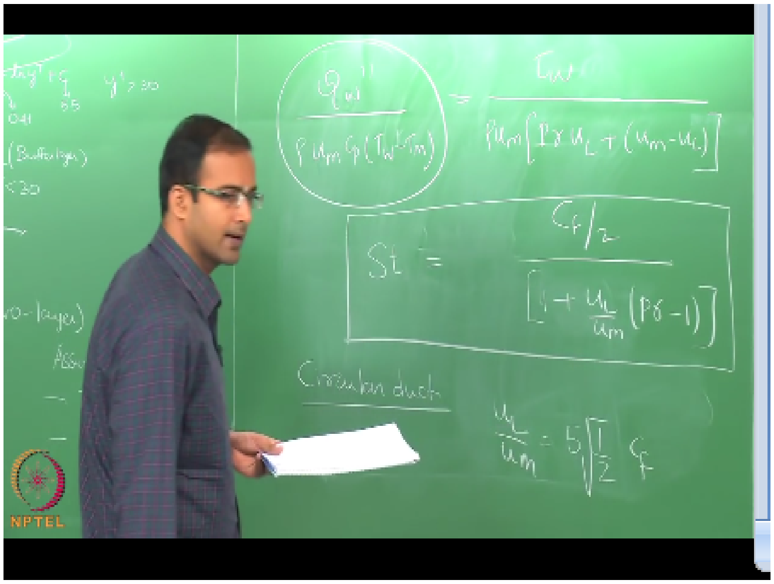

So now this entire term here is standard number the Q w / T w - TM is H divided / Ro CP UF okay so your standard number therefore is = now what I am going to do is multiply and divide / UN okay so therefore I will have towel / Ro UM^2 is nothing but your C F / 2 okay so this is your C F / 2 and when you take UM common so you have prandtl number into UL / UM correct + I have I will just rewrite it 1 + UL my um into prandtl number – 1.

So finally I end up with a relation between standard number and CF and you see in this case naturally the Prandtl number is coming so I am not forcing prandtl number to come in like the case of Reynolds Colburn analogy since we have not neglected the viscous sub layer we have also considered that so naturally the Prandtl number is built in inside this analogy right so now the only thing that needs to be close to so UL / UM so UL / UM actually is measured for turbulent flows turbulent boundary layers and we have some correlations for that.

Now let us see for the case of a circular duct for example so the UL / UM is expressed as a function of the CF written as 5 into √ of 1 / 2 CF so this is the kind of correlation let us call this as equation number 6 and this is number 7 so if you plug in equation 7 into 6.

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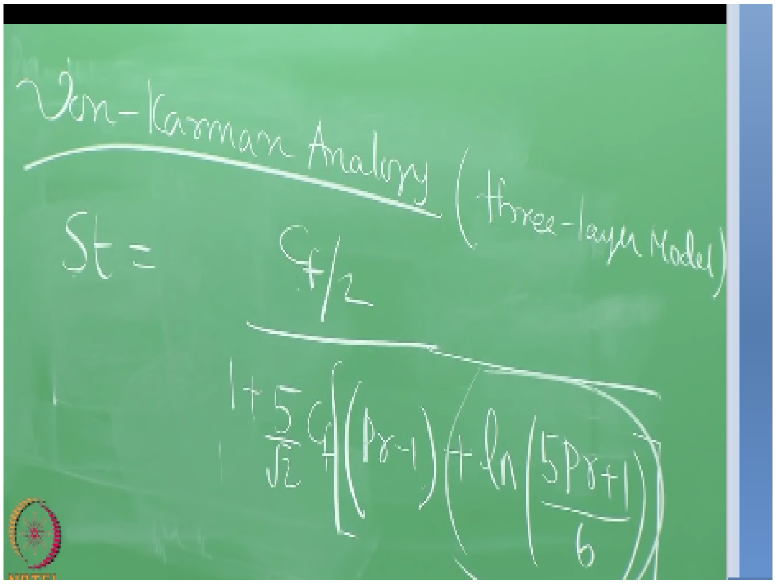

$$St = \frac{C_f/2}{1 + \frac{5}{\sqrt{2}} C_f \left[(Pr-1) + \ln\left(\frac{5Pr+1}{6}\right) \right]}$$

so you get the expression for a circular duct you get the analogies CF / 2 divided / 1 + 5 times CF again into frontal number - 1 for example so this is the analogy for this our product similarly if you are talking about maybe a rectangular duct you will have a different relation between UL / UM and CF which you will substitute and get the final analogy okay.

So therefore once you know the CF we can use the previous CF that we have written down okay for the Reynolds analogy that can be used substituted here and we can find an expression for Nusselt number right so this is how the Prandtl Taylor analogy is developed so for the case limiting case where prandtl number = 1 where we have a fluid which is having a prandtl number exactly = one the molecular and a molecular laminar and the thermal diffusivity is are identical so what happens to this reduces to the Reynolds analogy.

So in this case this gives you that Stanton number is = so you see that Reynolds analogy is not bad after all especially when your Prandtl number is = 1 that is your molecular Prandtl number = 1 then also analogy is the correct analogy because all the complex analogies are finally collapsing to that okay it only makes a difference where you are molecular prandtl number is either law very large or very small that is when you start deviating from the accurate results.

So most of the gases you do not have to worry so you can actually therefore most of the gases you'll find in all Sinology it is reasonably accurate whereas if you apply it to very large parental numbers or very small cantilever liquid metals and so on very small parental numbers so then you will find that Reynolds analogy is more accurate than Reynolds analogy is that clear so this is more complex expression naturally so the final case will be to consider all three layers including the buffer layer right.

So if you also consider the buffer layer then you are tracking the transition more accurately so I will not derive that but I will only give you the final analogy so this three layer analogy is called the von Karman analogy so finally the most complex analogy that you can derive so this is based on the three layer model which includes the laminar the fully turbulent and the buffer layer and this case the expression for the final expression for standard number for the case of circular duct okay so you have the same term till now till now this is the same as the frontal layer analogy okay now the additional term comes because of the buffer layer.

So this additional term is coming in due to the buffer layer okay so this is the difference between the two analogies and again for the limiting case of prandtl number = 1 what happens same this becomes 6x6 frontal of 1 0 this again becomes 0 so you have CF / 2 so finally it collapses to the Reynolds analogy again right so for more complex cases where none of these analogies will work okay so we cannot find a simple analytical solution again then we have to use numerical solutions for heat transfer okay.

So we will kind of stop here because all the turbulent heat transfer involves lot of modeling I do not want to talk about in detail because these are covered in other courses related to turbulence modeling and so on but you can learn how to use different models for different problems and they are also available in many of the numerical solvers okay so we can you can also go through those documentation try to understand which kind of turbulence model work for different kinds of problems now practically.

When you are working with turbulence you will not be using too many too much of analogies the most of the complex engineering problems cannot be solved with analogies so there you have to find numerical solution so you will be using some kind of turbulence model right okay so we will I hope that you know I could cover it with the limited number of hours at least some overview of about turbulent heat transfer okay thank you so much.

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