Indian Institute of Technology Madras

Presents

NPTEL

National Programme on Technology Enhanced Learning

Video Lecture on

Convective Heat Transfer

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Lecture 43

Analogies in Turbulent Convective Heat Transfer

Part 1

So very good afternoon and today we will look at the different velocity profiles the last class we had quickly derived the velocity profile in the laminar sub layer in the log layer we will summarize that and then quickly move on to the heat transfer so far we have not really focused on heat transfer per se this is a very generic discussion on turbulence okay which you will find in any basic book we will now slowly transition to what happens when you want to consider a turbulent heat transfer particular focus.

On that and how different it is going to be from modeling the turbulent flow field okay so in fact we will use some kind of analogies the same way that we did in the laminar flow and as I told you the most rigorous way of looking at turbulence is to therefore solve the rants equations okay we have the Reynolds equations for the momentum where you have the turbulent viscosity and this has to be solved either / a very simple mixing length approach provided you have a boundary layer.

Which is always attached okay this is the parental idea hypothesis of how the turbulence cascade happens and then you can solve the energy equation with introduction of a turbulent parental number so once you have the turbulent viscosity you can extract the turbulent thermal diffusivity from this and therefore use that the solution of the energy equation so this has to be done numerically right but if you want to pass the solution to these Ran's equations the only other option in fact which works reasonably well for basic flows such as we flow through ducts and then boundary layer flow you know so we will use some kind of analogies which we will look at in the next couple of classes okay.

So today just let me once again summarize the nature of profiles that you find indeed within the boundary layer of a turbulent flow so usually these are derived either for a simple case like a flow past a flat plate where you have the turbulent boundary layer and you have a sub layer okay or a simple fully developed turbulent fully developed flow through a duct okay so where you talk about the boundary layers merging and then you also have a growth of a small viscous sub layer the turbulent boundary layer is much but that is a small viscous sub layer which is actually still growing in the case of a turbulent fully developed internal flows.

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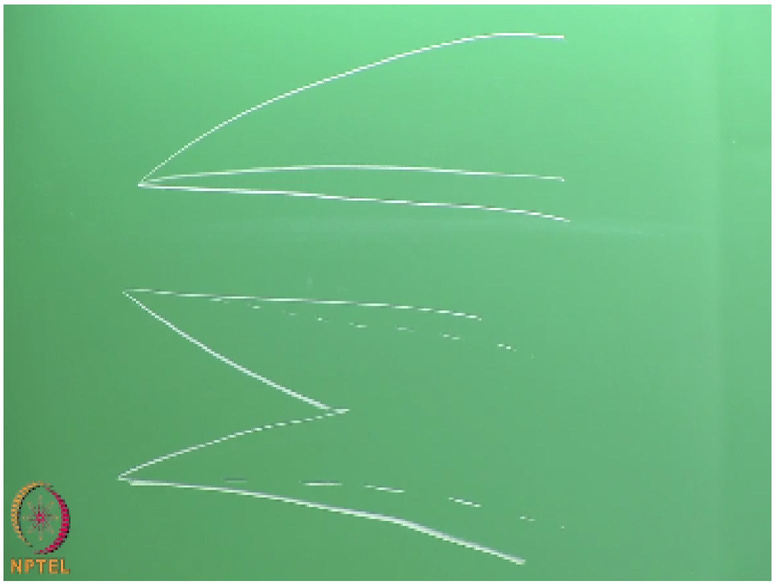

So in these cases you classify this is a laminar sub layer and this is your fully turbulent boundary layer okay.

So we have therefore from the expression for the total shear stress that is τ is = to τ wall which is = we had μ + τ w / P we can use and then say this is your kinematic laminar kinematic + turbulent kinematic viscosity times D u / d y so from this we arrived at the expression in terms of the non-dimensional velocity profile and the non-dimensional y ρ coordinate which is y + we had 1 + μ T / μ this is = 1.

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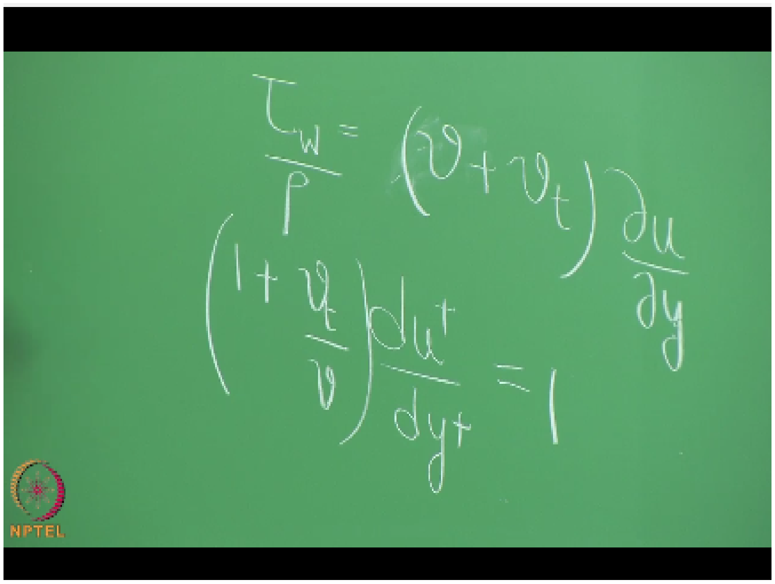

So we had just manipulated this equation so that we can cast it in non-dimensional u + + y + where the non-dimensional y + is = to the dimensional coordinate x u τ / μ this is your frictional velocity which is √ of wall shear stress / P so these are your turbulent parameters you know there is nothing very logical why we should use a friction velocity here but it is supposed to scale well for different kinds of Reynolds numbers if you use y + define in terms of friction velocity okay and similarly with respect to u + we have used u / u T so from this so we have differentiated x a laminar sub layer.

So for the laminar sub layer case you are new to buy new will be very small compared to 1 and therefore we get a linear variation in the velocity profile so therefore here we got the profile which was u + is = to y + whereas in the turbulent boundary layer if you neglect the laminar sub layer and assume only a turbulent boundary layer which is present throughout we got the fact that new T / μ is much ≥ and therefore we substituted for μ T from the parental mixing length model ok in the parental mixing length model the turbulent viscosity was assumed to be a function of what the length scale which is the mixing length and the velocity scale essentially you have that for LM square x d u / d y and a kind of a very empirical crude empirical guess for the mixing length will be the fun Carmen constant times the actual distance from the vertical distance from the wall Y ok so if you consider the effect of laminar sub layer there will be a damping function if you consider only a purely turbulent boundary layer.

Then you don't have that so therefore we just substitute for this x this and when we perform the integration we come out with 1 / K non of y + + a constant so this is the nature of the so this should be yeah absorb correct you're right okay so the prantle so this is this is what was originally conceived / Prantle himself so this linear layer and the large layer were derived / prantle using his mixing length model and the constant here this is called the Fon Carmen constant one Carmen is actually student of Brant on so this is given as 0.41 and this constant C do you remember what it is so this is actually 5.5 so now if you plot this profiles let us say on the x-axis you have you are plotting y + in fact you can use a log scale and plot this on the x axis and on the y axis you can plot the logarithm of U +.

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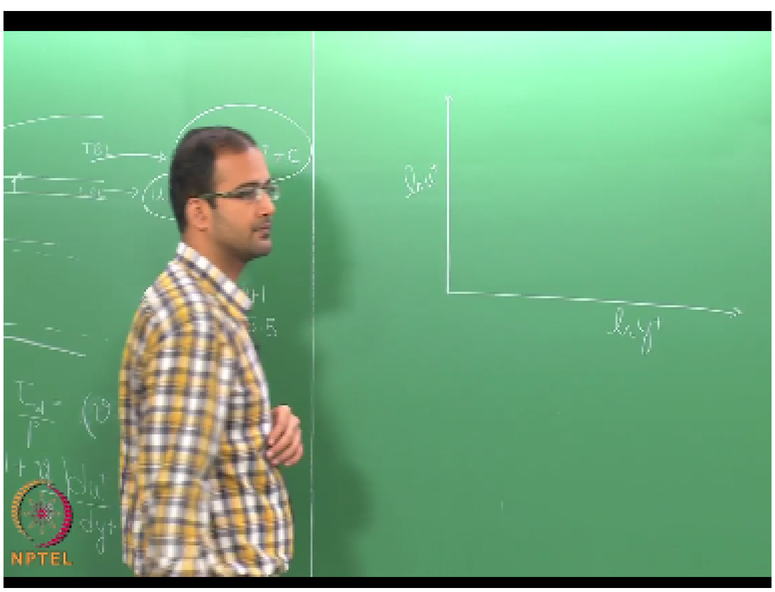

Okay you should understand this y + very several orders of magnitude within the laminar sub layer you are talking about the order of y + is around one whereas in the turbulent boundary layer it can extend up to a y + of close at the edge of the turbulent boundary layer so therefore three orders of magnitude we cannot plot on linear scale so we have to use a log scale to plot this and similarly the variation of U + also will be quite significant so if you therefore plot this on a log scale what do you have for what will happen to this profile u + is = to y + so this is a linear on a linear scale.

This will look like a straight line but now on a log scale it will look something like this okay so this could be the edge of this will be somewhere around you + need not be on a logarithm scale we can actually plot this directly on a linear scale itself but the y + will be on a logarithmic scale because y + variation is quite significant for example if you start from 0 this will be five okay and I am just plotting the actual y + how it looks okay suppose you plot it on a log scale directly okay and then you have somewhere here variation of about let us say 50 and then we have about 100 and maybe 500 some kind of variation like this.

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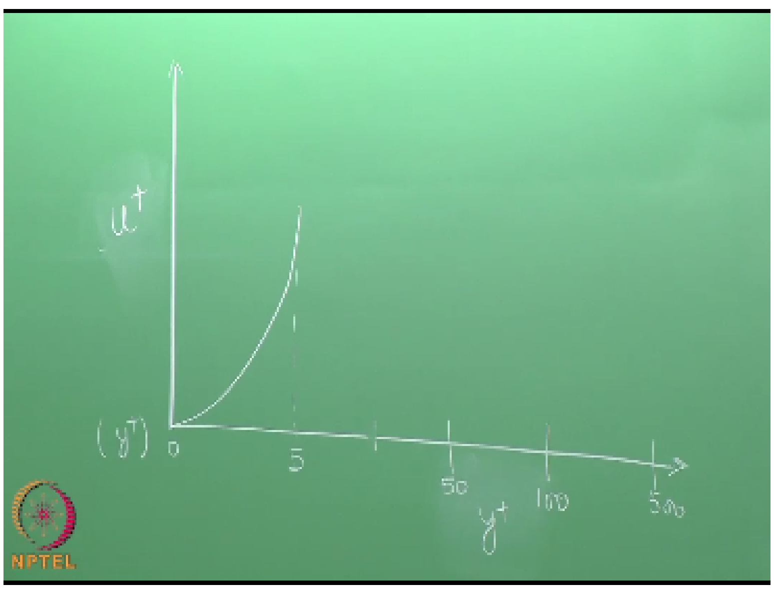

Okay so if you plot the linear profile so this will be looking like a curve on the logarithm scale of the Y ax x axis and then if you plot the log profile here this is y + is = to lawn of u + is = log of y + so that will look like a straight line okay so but this limit will be a straight line from somewhere like this okay so the reason being if you look at the extent of the laminar sub layer it will be usually the order of 1 to 5 and this is not a sudden jumps from the laminar sub layer x the fully turbulent boundary layer that is a transition in between when you do the experiments okay you find that it does not directly this . directly does not transition to this okay so the time fully turbulent boundary layer will actually start from y+ which is greater than 30 okay so you can actually look at the corresponding x axis it will be around 30 so there is a discontinuity discontinuous region between the laminar sub layer and the fully turbulent boundary layer.

So in order to patch this later on Carmen's student of prantle himself he used a simple correlation again empirical not rigorously derived like the way we are doing this so he just made sure that there is a smooth transition from this region to this region in fact let me draw the slope the slope will come out to be something like this in the log line so this is your u + is = to y + this is your u + is = to 1 / 0.4 1 Lon of y + + 5.5 okay now in order to patch this to this one Carmen introduced another intermediate layer a buffer layer so this is y + but I have plotted with a log scale.

So if you take a graph with a log scale on the y axis and the x axis okay so then you have a distribution of .s like this ok you have 0 to 10 and then suddenly 10 to maybe 100 then 100 mm so the order of magnitude will quickly increase so on this scale since you have your plotting directly lawn of y+ on the x-axis so this will be a linear curve right so now the intermediate buffer layer is supposed to provide a transition from the edge of the laminar sub layer to the starting of the fully turbulent boundary layer.

So this was actually called the buffer layer red / von Karman and it uses a similar kind of logarithmic variation but with different slopes so this turns out to be _ 5 times Lon of Y + _ three . 0 five and this is valid for a y + ≥ 5 on ≤ 30 okay so this is valid the pure log lay law is valid for y + greater than thirty and the linear law is valid for y + less than five okay so this derivation is just to make sure that your u + recovers at y + = to five the value of the laminar sub layer and at y + = to thirty it should recover your Rosina log value okay so it is just like a buffer between your turbulent boundary layer and your laminar sub layer of course.

I know you cannot really visualize this or we cannot really see this in experiments but this is how it is conceptualized okay so definitely the discuss a blur exists okay but how the transition happens it's all concepts ok so I mean this is what has been widely accepted because it is reasonably a good mo I mean a good patch work rather than complicating it further okay so this is a classically you know accepted variation of velocity profiles.

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From the wall all the way to the fully developed fully turbulent boundary layer okay so and also this has to make sure you do not have any adverse pressure readings so if you have adverse pressure this is therefore derived for the case of a flat plate boundary layer if what happens if you have a favorable or adverse pressure gradient again those pressure gradients will change the nature of these profiles okay so then you have to do an experiment and then find out how these profiles way and then plot them okay so this is the basic you know idea about the variation of velocity profiles now one thing I just want to emphasize from the rants equations.

We can again write down the boundary layer equations for turbulent flow because those ran's equations again are a generic navies-stokes equation can apply that to you know any kind of flow the pressure gradient without pressure gradient now with all kinds of diffusion in different directions but if you want that to be solved specifically for say the flat plate boundary layer okay so we have to write this in a classical boundary layers form so can you try writing down the turbulent boundary layer equations from the ran's equation we use the same order of magnitude analysis that we did earlier for the turbulent boundary for the laminar boundary layer same conclusions come out the diffusion in certain direction is predominant over the other the y momentum can be neglected and these same facts can be used and only we are applying this to a Ran's equation.

Okay so therefore the continuity equation remains the same except that you replace the instantaneous components with your main components and out of the 2 momentum equations the dominant momentum equation will be in the X direction okay so this is your X and this is your Y so therefore we can write this as u bar the u bar / DX so the advection terms will be as they are and we can also include the pressure gradient term and what about the diffusion term now so you remember that we have 2 diffusion terms and we have also combined the turbulent stresses as a diffusion along with the molecular diffusion so we have diffusion in the X direction and in the Y direction. So according to the order of magnitude analysis which is the dominant diffusion in the Y direction so we have to just retain D / d y off so we have μ+ μ T x D u / d y.

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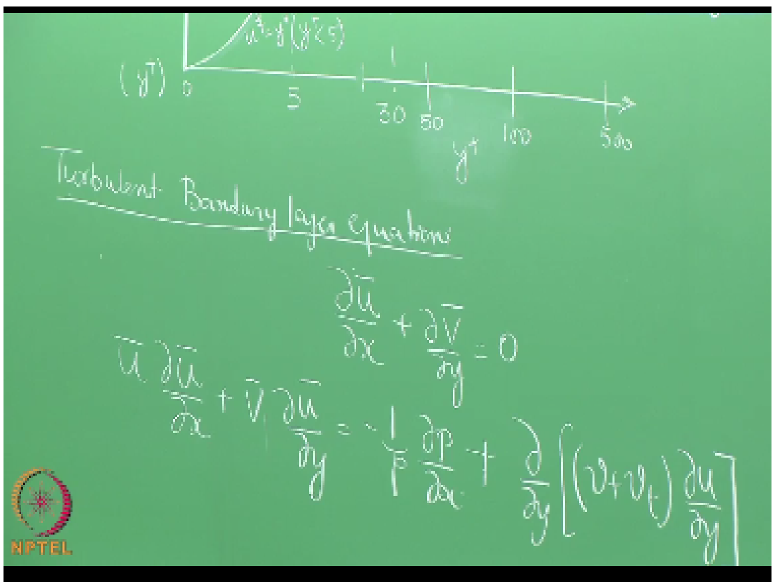

Okay now in the Y momentum equation y momentum itself is negligible compared to X momentum and therefore we have DP / DX is 0 there is no DP / d y 0 there is no variation perpendicular to the plate length so you can use bars everywhere sometimes you know it is understood implicitly that when you write this equation they are all for mean flow so you can actually omit them becomes painful to use bar every time right and finally the energy equation also for the turbulent boundary layer becomes α.

Now when we are using here we have to be again careful D / d y of α + α T x DT bar / d y right so we have the dominant diffusion both momentum and thermal diffusion happening in the vertical direction that is where the boundary layer is very you know so well again do you think that we can find analytical solutions to this like the Blazes solution it looks that we have simplified the Ran's equation for the boundary layer but the major problem is μ T it is not a thermo physical property we cannot assume that to be a constant and wherever flashes has used μ we cannot replace that with $\mu + \mu$ T okay so now this is where the problem comes so we cannot simply find similarity solutions for the turbulent boundary layer the same way that we have found it for the Blazes equation for laminar boundary layer flows.

Okay there have been some attempts to find you know kind of similarity solutions but the kind of effort that we put x it is not worth it so it is better to go for approximate methods okay so usually the easiest route to finding the solution to these equations is to use integral equations momentum integral equations approximate methods where you solving momentum and energy integral equations so we will have the same kind of equations that you had earlier except that wherever we had new we will now have $\mu + \mu$ T.

Okay now how is it going to simplify it's not going to simplify that but the kind of profile that we are going to take for velocity we cannot take a linear profile or for the matter quadratic or cubic so you are left with only one option which is to assume what is called as a 1 / seventh power law variation that is in the case of flat plate we assume $u / u \infty$ as Y / D to the power $1 / 7$ so this is the call the famous power law equation it is a reasonably good assumption for a turbulent velocity profile.

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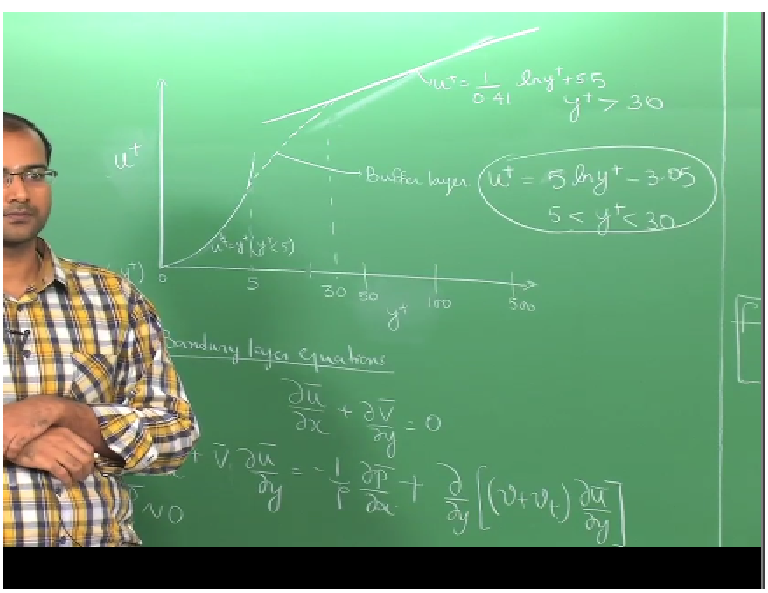

Okay so this one seventh power law however has a difficulty unlike your quadratic or linear profile what is that? so, what happens to the derivative of velocity at the wall you will have a singularity correct so this is a problem so that is why you cannot use this profile to calculate the wall shear stress or the gradient of velocity at the wall because you need that when you have integrate this on the right hand side you will have new + new TD u / d y at y = to 0 and D u / d y at y = to 0 you cannot determine from the 1/7 power law.

So then what do we do so we do not have any other option but to use some correlation for the wall shear stress okay although this is the best variation of velocity for turbulent boundary layer we cannot use it to integrate right up to the wall at the wall it becomes a singular solution therefore we will have to use some kind of an empirical formulation for the wall shear stress and usually the variation is used from the C F = 0. 046 times so this is re λ to the power _. 2.

So this is the kind of correlation that is the most commonly used for approximating the w shear stress in turbulent boundary layer okay so this is nothing but τ w / 1/2 ρ ∞ ^{2} so therefore if you take the half on the right hand side this becomes zero point zero two three and this Reynolds number is based on the boundary layer thickness okay so therefore this will be u ∞ Δ / μ to the power _ 0.2 okay so in the idea and the integral method then is to substitute this for the right hand side okay at the right hand side we have this entire term is going to be a w correct.

Since we cannot use that profile we will directly substitute for τ wall from this and now we will have an equation which we can solve and find out the expression for boundary layer thickness Δ okay so that is the same thing you do in your classical momentum integral equation except that the right hand side you integrate the profile up to the w but in this case you cannot do that so we have to therefore patch it with a correlation on the right hand side find out the expression for Δ so Δ has a function of Reynolds number okay the same expression can also be used for internal flows so an internal flows.

You will be replacing your CF with your friction factor and this friction factor has to be which one Fanning or Darcy fanny because the w /1/2 P u ∞ ^{2} is the Fanning friction factor Darcy is defined based on pressure drop okay ,so the equivalent will be using the Fanning friction factor once again which will be τ w / ½ P ∞^{2} which is the same thing .046 now only difference is the Reynolds number will be now defined based on the diameter of the duct are not based on the boundary layer thickness now this is for the fully developed turbulent boundary layer so there since your boundary layer thickness does not vary.

So we will use the hydraulic diameter rather than the value of Δ okay so these are the most famous relations that are used you know if you look at in fact the Moody's chart for internal flows the turbulent for the turbulent region it is also plotted as a function of the roughness of the surface so that is a more detailed expression because in turbulent boundary layer surface roughness also plays a very important role and these correlations are basic they do not account for surface roughness right so therefore the Moody's chart also expands this basic correlation to account for the effect of surface roughness right so next therefore now that we know the w shear stress in turbulent flow is turbulent boundary layer flows is obtained from this kind of an empirical correlation next is to go for the solution to heat transfer okay so how do we therefore get the solution to the heat transfer now what do you think I mean if you talk about the approximate methods.

You will use the momentum integral method find out the expression for Δ and obviously then you want to go to the energy integral equation find out an expression for the thermal boundary layer thickness but will that give you the nusselt number well in a laminar boundary layer okay so your Prandtl number is governing the ratio of your molecular diffusion ratio of the momentum and the energy diffusion but what happens in a turbulent boundary layer okay if you talk about this boundary layer this is actually your turbulent boundary layer thickness so this is governed by the turbulent parental number okay.

So therefore now the question becomes I mean whether I can use the energy integral and integrate it up to Δ T and what is Δ T here is it only within the laminar sub layer or within up to the edge of the turbulent boundary layer so that becomes a problem so therefore what we will do is we do not actually solve the energy integral in this case but we will try to find an analogy between the momentum and the energy transfer we already saw in laminar boundary layer there is a clear analogy between the two if you replace u by u ∞ with θ the structure of the profiles as well as the equations become identical and especially if you have tantalum be equal to 1 they are exactly the same right.

You are d u / dy at the w and D θ / dy both the both the Skan friction coefficient and the slope of the temperature gradient they are identical so what happens in the turbulent boundary layer we will quickly use these kind of relations and derive the analogy for the turbulent boundary layer so the first analogy that is quite popular popularly used it is cause they are in also knowledge so now we are talking about analogies between heat and momentum transfers.

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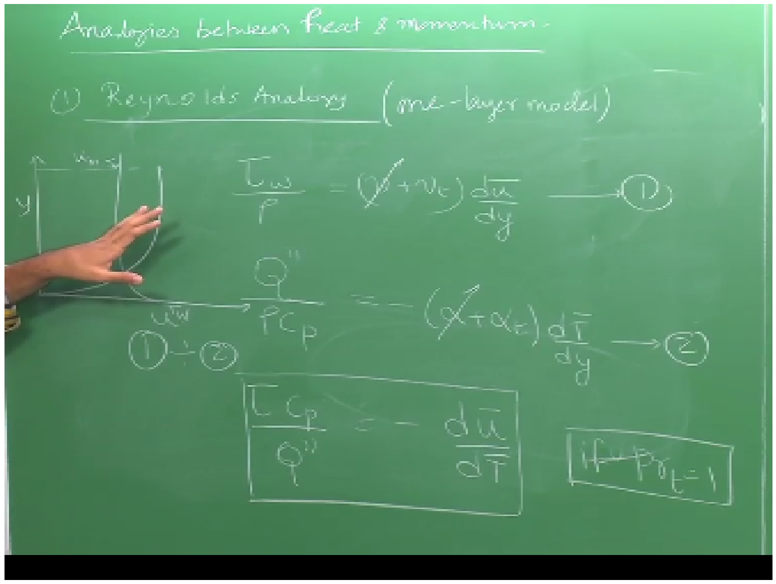

And the most popular is there our most basic also it is called the Reynolds analogy now we have the picture of the turbulent boundary layer here which has three layers in fact okay according to the hypothesis that we have a linear variation in the laminar sub layer we have a buffer layer and then we have a turbulent boundary layer okay.

So now we can actually look at all the three layers together and that brings a very complicated picture okay but to start with we will assume the entire boundary layer is turbulent there is no laminar sub layer there is no buffer layer so this is called a one layer model okay and the nonce analogy is derived based on this one layer model so only turbulent boundary layer extends all the way from the w to the edge of the boundary layer okay so in that case let us again write down the expression for the shear stress from which we derived this law of the w so this is your new + μ T x D u / dy okay but and how about for the w flux heat flux so this is for the w shear stress similarly for the heat flux.

We can actually use Q ’ ‘ by P CP and that should be equal to - α + α T into DT bar / dy so this is your extension to a basic furies law of conduction we also now include the effect of the turbulent thermal diffusivity into this right so therefore if you just assume only one layer which is completely turbulent so there will be no effect of the laminar or molecular diffusivity so only the turbulent diffusivities will be dominating throughout okay so in that case we can therefore divide let us call this as equation one and this is your equation two we can divide one over two and we will be therefore so one divided by two you have τ C P / Q ’= _ D u bar by DT but if frontal number turbulent Pranle number =1 okay so I am just making an assumption that right now my turbulent prantle number =1.

So that the ratio of turbulent the momentum to turbulent thermal diffusivity =1 okay and therefore I get a very simple relationship between Q one shear stress so this is how we are building the analogy so if you therefore integrate it so for example if you draw the velocity profiles and temperature profile so this is some kind of you let us say mean velocity outside this is T w and this is TM and the edge of the boundary layer okay this is the variation of temperature profile vertically and velocity profile therefore if you integrate it at y equal to zero your U is zero and y = and the edge of the turbulent boundary layer your U becomes um okay I am using μ because the μ can be u ∞.

In the case of external boundary layers internal boundary layer this is your mean velocity similarly your temperature can vary from w temperature to mean temperature so I am will just integrate it and upon integration I can get therefore so if I integrate the shear stress what happens so I have at the edge of the boundary layer the shear stress is 0 whereas at y = 0 it is τ w okay.

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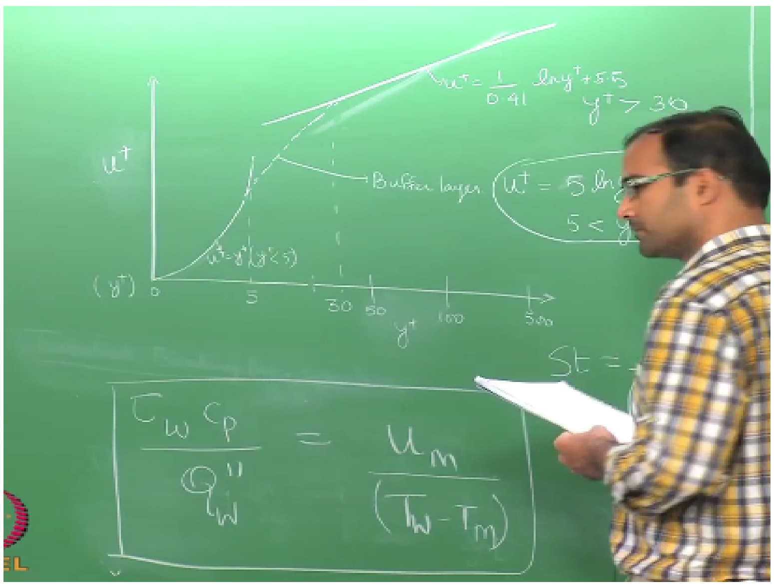

Therefore I have the w CP by similarly the heat flux right this is become w heat flux here on this side I have / T w - here okay please check that you integrate it from the w to the edge of the boundary layer right okay so now I am going to introduce I already introduced the definition of Stanton number to you which is nothing but H / P CP um this is nothing but nusselt number by re into PR correct so can you cast the equation let us name this as equation number three here the integrated one in terms of Stanton number okay and therefore tell me the relation between Stanton number and non dimensionalize the shear stress w shear stress in terms of the friction factor the finding friction factor.

Yes so give out a relation between Stanton number and F use this equation should be able to multiply it with some terms so that you can write this in terms of standard number and hitch is nothing but Q all ’’ / T w _ TM okay by for example ρ numerator and denominator so Q / T w _ TM / ρ CP I can multiply numerator and denominator by μ right so this will be H by I take this ρ CP μ so that will be standard number okay and then I have towel by ρ^2 that is nothing but F by 2 right so therefore finally.

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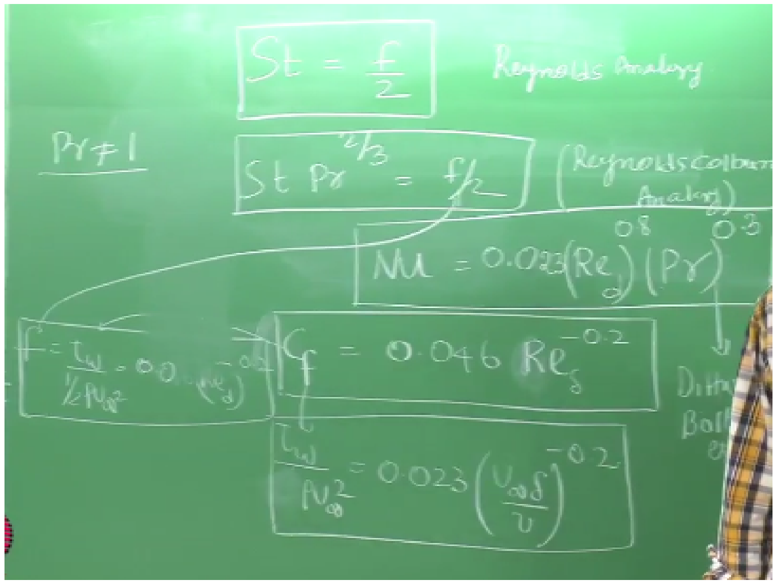

I get the relation in terms of standard number F /2 and what is this is your analogy remember you derived the same thing in laminar boundary layer. We derived it from the expressions for nusselt number and skin friction coefficient that we got from exact solutions and we got the same expression in laminar flows now in turbulent flows also we are getting the same analogy assuming that the entire boundary layer is turbulent and also your turbulent antal number is 1 so it is not hard to visualize it if your entire turbulent boundary layer entire boundary layer is turbulent and both the diffusivities are same so once again if you look at the nature of this equation therefore your velocity gradient and temperature gradient have to be having similar values okay so therefore the Reynolds analogy is also applicable for turbulent boundary layers now if you used for internal flows when internal flows already we have the relation between F and the Reynolds number.

So please substitute this okay now one more thing so this is your Renault analogy later on it has also been extended to various prantle numbers so it is for the case where parental numbers not equal to one it has been extended and this become the Reynolds Colburn analogy where you have prantle number factor also so this is your Reynolds Colburn analogy okay so the effect of the molecular prandtl lumber is also brought in into this okay this is not this is not turbulent prantle number this is molecular prantle number ok the effect of molecular prantle number which we have neglected here has also been brought in later by coal burnt and extended to different values of molecular prantle number.

Because at the w finally molecular prantle number is important we cannot claim that we can predict the nusselt number okay nusselt number is a quantity at the w without accounting for the laminar diffusion so therefore later on coal burnt adjusted this four values of molecular prandtl so now you can therefore substitute the expression of F into this and get the expression for nusselt number as a function of Reynolds number and prandtl number can you do that you already have the announced number dependents here okay so what will be this constant zero point zero I am just substituting directly for F 0 .0 23 I have re D $^{0.2}$ but Stanton number is nusselt number by re PR so I have point two this is _ 0.2 into 0.8.

So you will have into Reynolds number so it will be re $^{0.8}$ right and what about prantle number power so this is again our EPR okay so 2 / 3 _ 1 / 3 so 1 1 / 3 and then this is the export point 3 okay, this is therefore a correlation for nusselt number directly derived from the analogy so we are not solving your energy integral or anything here okay so this correlation directly gives you a very simple approach directly from the analogy you know the expression for the w shear stress therefore we use the analogy and directly get the expression for nursultan this correlation is popularly referred to as latest bolter equation very fundamental equation that many people know in heat transfer turbulent heat transfer in ducts flow through ducts no fully developed turbulent flow through ducts but you may not know how it is derived you have used this as a correlation without knowing how it is derived it is nothing but derived from a simple application of Reynolds Colburn analogy okay.

It is not very rigorous but it has been found to be a reasonably good approximation within + or _ 20% to the experiments so men people do experiments on fully developed turbulent pipe flow they compare it with the data sorter and find that it is very close so now how do we deal with the external flows so this is for the internal flow you have the data sorter now in external flow your Reynolds number here is defined based on boundary layer thickness then what do we do so we use the momentum integral get the expression for boundary layer thickness as a function of your local Reynolds number and that is putting into this substituted into the Reynolds Coburn and you will get a similar expression for the external flows okay.

So only thing there you have local Reynolds number in fact the variation is similar same you have instead of re D you will have re X $^{.8}$ prantle number 2 $^{.3}$ this constant will be slightly different it comes out to be something like point zero to nine an external flows because you are substituting Δ from the momentum integral so that constant will be little different but the dependence on Reynolds and prantle number will be the same okay so for the internal flow case it is a very straightforward thing whereas for the external flow you have to use the momentum integral solve it then use the Reynolds Colburn analogy okay.

So therefore although the turbulent flows are considered you know theoretically very complex so very reasonable simplifications like these have healed and useful results okay so sometimes it looks that they are oversimplification but when you do experiments measurements and compare so these correlations match agree very well and they are there therefore they have been well accepted even in industries okay so this is one simple analogy but most often used there are analogies which are taking into account multiple layers so from one layer we can therefore transition to two layer accounting for the viscous sub layer and the turbulent boundary layer so that is called the prantle analogy prater tailor analogy and finally.

We can also include all the three layers including the buffer layer so that is called as the one Carmen analogy so we will derive these two analogies at least I want to derive the Pant anal analogy parental tailor analogy or the two layer analogy then I will just give you the expression for the three layer analogy so most of the turbulent heat transfer is dealt with using these analogies okay, so if you go for complex flows where if you have flow separation or very strong pressure gradients then these analogies will not work okay, so in that case you have to solve the Rance equations and again model the turbulent viscosity whether you use a one equation model two equation model so those depending on the level of complexity.

In the computational time you can therefore solve them more rigorously okay but for simple boundary layer kind of problems these analogies are the most commonly used okay so we will stop here and following Tuesday Monday they do not have a time in the studio we will meet on Tuesday at our regular time 10 º'clock there is the last class so we will complete the other analogy also and with that will complete our turbulent boundary layer convective heat transfer any other questions or discussion if you have time. I think for 10-15 minutes we can do that Monday is following Friday timetable is it you.

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