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Video Lecture on
Convective Heat Transfer

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Lecture 42

Turbulent Convective Heat Transfer
RANS Equation – Part 2

So good morning so today we will look at will complete the derivation of the arnica equations that we started today so I asked you to work the energy equation derivation of the rants energy equation I hope you have tried to do that anyway let us just once attempt it here and then summarize the equations together so when we look at the energy equation.

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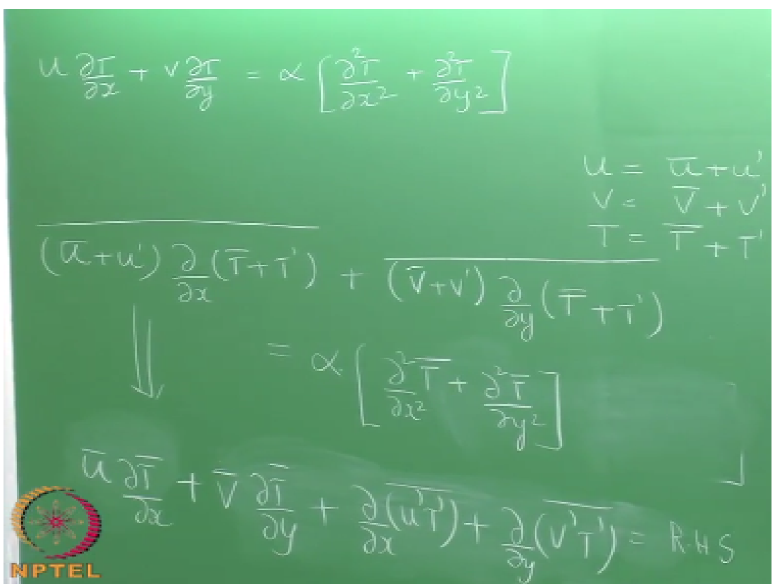

So which is nothing but u DT / DX + V DT / du dy so these are the two dimensional steady state incompressible energy equation α into T by DX square that is square dy square right so now when we do the Reynolds decomposition substitute for u and T so we have seen that the Gnostic composition for you instantaneous velocity will give Q Bar + u ' V is equal to V bar + V ' similarly the instantaneous temperature so therefore if you put a thermocouple the same flat plate you can consider that we discussed yesterday the now you heat the bottom wall and you insert a thermocouple somewhere within the thermal boundary layer similar to the velocity fluctuations have drawn you will get temperature fluctuations okay.

Therefore if you apply the proper filter you should be able to average the instantaneous temperature using this filter and decompose the instantaneous temperature into a mean and fluctuating parameter okay so now you can substitute for all of these variables and then average the entire equation then also average okay so what we are doing here is time averaging so for the 2d incompressible steady state doesn't matter whether we do a time or ensemble average so it will still be the same on the right hand side you have ' D^2 and DX^2 the right hand side is easy you can you apply the averaging rule and separate this into T bar + T ' T ' bar so T ' bar is zero okay.

So the mean of the fluctuating quantity is 0 therefore this can be simply written as now once again looking at the left hand side we have four terms here we have four terms here and again when you split this the way we did it for the momentum equation okay so for example this can be written as therefore u ' do u ' by sorry TT ' by DX okay so this we have averaged + you have u ' dt u bar dt ' by DX average of this + you have u ' dt bar by DX the average of this + you have u ' dt ' by DX average so out of this if you apply the averaging rule.

So this will be 0 and this will be 0 okay so essentially from this you are left with only u ' DT ' by DX average okay and this is nothing but by averaging rule U bar DT bar by leox similarly from the second term on the LHS we have V Bar BT bar by dy + V ' DT ' by dy average okay the right hand side is as it is so now we can write this as d by DX of U ' T ' _ T ' into D u 'by DX so therefore if you take T ' constant we will have d u ' by DX + DV ' by dy which will be 0 satisfying the continuity for the fluctuating component so essentially you will have therefore V V Bar DT bar by dy on the left hand side you will have D by DX of U ' T ' bar okay + you have d by dy of 3 v ' t ' bar is equal to RHS is it okay clear right.

So let us now therefore summarize the entire set of Rends equations momentum and the energy together so when I am going to write it I am going to take this term the fluctuating term I am going to use that towards put it towards the right hand side okay so I will keep the conventional advection term on the left and take all the new quantities arising out of the turbulent fluctuations towards the right hand side so we will see why we are doing it okay so for example.

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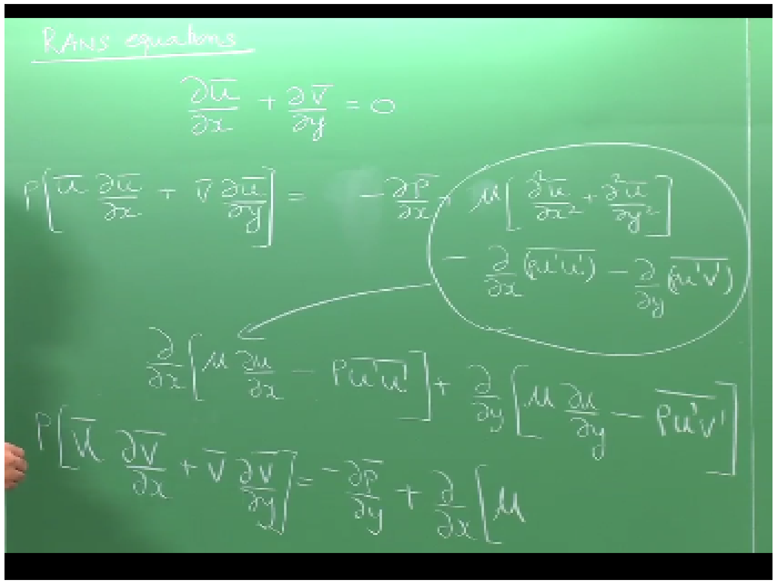

Now therefore the final set of Trance equations so the continuity for the mean velocity is satisfied and so is for the fluctuating component and then anyway we are not going to now solve for the fluctuating component okay.

We will only construct the equations for the mean component this is what we are going to solve we will see how we account for the fluctuating component could be and then the X momentum equation will be u bar V u bar by DX + V bar D u bar by dy so I am going to take the turbulent stresses towards the right I have _ 1by know DP bar by DX + now I have nu into v square let me first write this and then I will combine it now I have additional terms - what do I have - τ by τ X of U ’ this is in the X momentum equation u ’ u ’ bar right and - D by dy u ’ V ’ mod is if occur okay I'm just taking this to the RHS now if you observe the way that we have written this okay we can also write this let me multiply throughout by row first.

Let me keep the row here this is dynamic viscosity and therefore I can take P inside because this is incompressible doesn't matter this term the diffusion term this costs viscous diffusion and now the turbulent diffusion so the turbulent diffusion is although arising from the inertial term I have taken it to the right now I am going to combine it with the viscous diffusion yeah okay so therefore I can write this bunch of terms as d by DX common and this is nothing but μ d u by DX _ ρ x u ’ u ’ bar this is one set of terms + I have similarly D / dy of μ D u / dy _ Px u ’ V ’ so I am intentionally writing it in this way.

Because I want to combine the turbulent stresses with the viscous stresses and I want to group them together so that we will now draw an analogy okay similarly the Z momentum equation so P into we have u ’ u bar DV bar by DX + V Bar DV bar by dy on the right hand side we have DP bar by dy + again I'm going to have D by DX μ into what do I have DV by DX _ P into u ’ V ’ bar okay.

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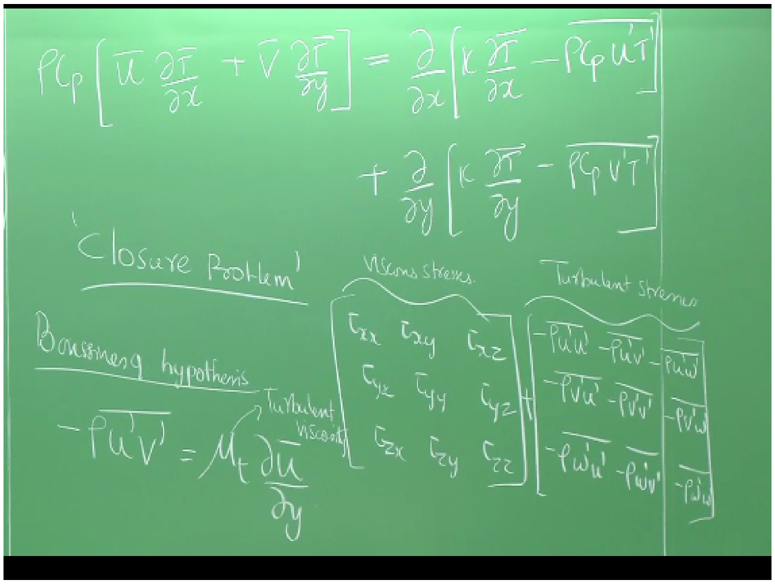

I am continuing this + so I am just continuing that so + D by dy of μ DV by dy and I have again _ P into V ’ V ’ bar okay so this will be the structure of my Y moment and finally the energy question also I am going to write it in the same so what I'm going to do this α is K by P CP.

So I am going to multiply throughout by P C so I have P CP into u bar DT bar by DX + V bar DT bar by dy okay and on the right hand side I have now d by DX of what K DT by DX and - - correct row CP x u ’ t ’ bar + I have D by D Y of K DT by dy _ row CP v ’ T ’ is it clear okay I'm just rearranging combining basically the turbulent diffusion with your molecular diffusion terms yeah all these are main quantities you're absolutely right all these are main quantities zeros okay so now I am doing this is I am going to close this problem okay, now we have a particular problem in this ranch models.

Which are called as closure problems so what it means is now when you did the Reynolds decomposition and averaged it you have created a turbulent stress which is like u ’ V ’ u ’ u ’ so on and so forth similarly in the energy equation okay so now you have your molecular diffusion which is giving all this τ xx τ XY τ XZ and so on and so forth you have your τ YX τ YY τ Y Z τ ZX thousand Y thousand so apart from that now you have therefore set of nine stresses coming from the turbulent inertial terms so we can therefore group them as what _ P u ’ nu ’ can you fill in all these elements _ P u ’ V ’ so if you write it in a three dimensional form so if you derive this for the XYZ momentum equations then you have _ P x u ’ W ’ so like that you have all the terms.

We have V ’ u ’ _ ρ v ’ v ’ _ P V ’ W ’ P W ’ u ’ w ’ so apart from your viscous stresses here arising due to the molecular diffusion you are now adding your turbulent stresses okay so when you do this you can therefore draw an analogy to molecular we name this as turbulent diffusion okay although this is not a classical diffusion term like molecular diffusion this is coming from the inertial term but we for the sake of convenience we would like to combine this with pure molecular diffusion and name this as turbulent diffusion okay so in doing so what we do is we want to avoid the closure problem.

Because now you have this term in order to solve for these equations you need to solve for therefore u ’ v ’ u ’ u ’ all the nine stresses have to be solved then only we can solve these equations but you don't have any equation for the stresses you don't have any governing equation so then what can we do we can construct a governing equations for the stresses by multiplying again the momentum equation with the fluctuating component for example okay so like this we can construct another governing equation for the turbulent stresses but they will have another term with the higher order moments.

You will end up with P u ’ V ’ u ’ P V ’ u ’ V ’ V ’ so all higher order moments will come so like this it will keep on going cascading there now if you construct again another governing equation for the higher order moments you will end up with one higher order moment okay so therefore this will be a never-ending problem this is called the closure problem so in order to close this we can actually do at this level itself we can close it at any level you can also construct a higher-order equation and then close the higher-order moment but that is not going to be practically useful so we will just try to close it here and how are we going to do that by doing what is called as a boussinesq hypothesis.

Now this boussinesq hypothesis is not the same as what you did in natural convection this is a different one so here boussinesq what he says intuitively is that the nature of these turbulent stresses or turbulent diffusion can be modeled analogous to the molecular diffusion only that will replace the molecular viscosity with what is called as turbulent viscosity okay so for example in this particular equation we can simply say _ ρ u ’ v ’ bar = μ T x V u / dy just analogous to the molecular diffusion but I am going to replace my dynamic viscosity molecular viscosity with what is called as the turbulent viscosity correct so in doing so now I have closed the problem okay.

So I can now solve this equation using this analogy but still what is unknown is the turbulent viscosity and turbulent viscosity μ T is not a property it's not a thermo physical property like laminar viscosity so again you have to it depends on the flow just like your heat transfer coefficient so for a given flow problem you have to actually understand how μ T has to be computed so for that we then solve additional equations and calculate the turbulent viscosity so like this we can you woke the business processes for all the remaining stresses and therefore we can combine the laminar diffusion with the turbulent diffusion so therefore for example in this case how do.

We replace this using the boussinesq hypothesis μ + μ T x D u /DX see how simple it is now okay accept.

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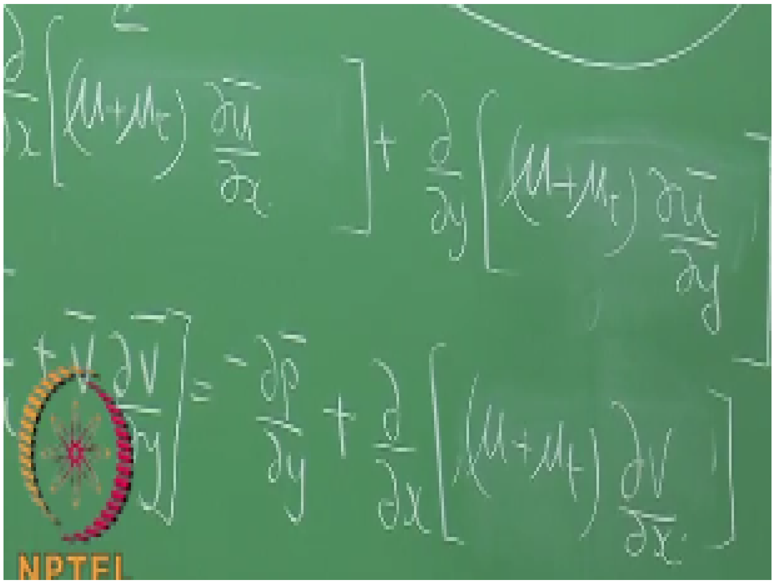

That μ T is unknown similarly here also this is μ + μ T times D u bar by dy no μtti will be a function of position it will not be a constant yeah so that we are assuming that this is constant right okay so we are not using different μting for the X momentum Y momentum its derivative yes because already this is an ad-hoc hypothesis which is reasonably good enough so there's no point in complicating it so let us make it simple and see if it works and most of the time with reasonable approximation it works okay so same way you can also use this as μ + μ T into this is DV by DX so similarly when you look at the stresses the thermal turbulent thermal stresses we have combined that with the molecular diffusion due.

To thermal conductivity okay so therefore similar hypothesis can also be evoked so what we can say is _ P CP x therefore u ' T ' can be written as what some turbulent thermal conductivity okay times DP bar / DX okay so we can actually therefore combine this as what K + K T x DT bar / DX and this will be a + KT into DT bar / dy so by this manner we have completely closed grants equations we have avoided the closure problem so next therefore when you now calculate the total stress we have to therefore include the turbulent stress along with the molecular stresses similarly with the heat flux.

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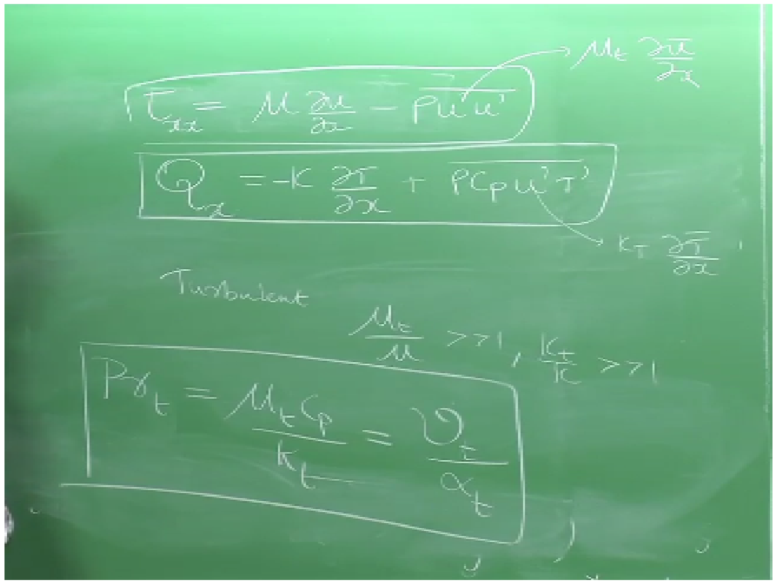

So therefore when we write τ for example when we say τ xx as I said we are now combining this + this is nothing but nu D u / dy you are combining with your turbulent stresses so for example τ X τ xx will be μ x du / DX _ P u ' u ' bar understand so this is the way we calculate the total stress the rotor stress has the molecular or laminar diffusion and the turbulent diffusion similarly for the heat flux so when we say heat flux in the X direction how do we calculate now this is K into _ K DT / DX right _ K DT /DX then + we have P CP Q ' V ' bar understand so when we say stress.

In the rants equations we include the combined action of the laminar and turbulent stresses and the combined action of the laminar and turbulent thermal diffusion together now in fact if you look at the turbulent highly turbulent flows and when we replace this with the boussinesq hypothesis so many a times the turbulent diffusion will be at least an order of magnitude higher than the laminar diffusion okay for in the turbulent region therefore most of the time your μ T by μ will be much greater than one and so is KT / K.

So most of the times they will be governed by only the turbulent diffusion the molecular diffusion will have little role to play in the turbulent region that is why it is very important to account then we cannot definitely neglect them under any circumstance so just like your laminar prantle number now we can define what is called as a turbulent Prandtl number since we have now a turbulent thermal conductivity turbulent viscosity we can now define a turbulent prantle um okay that is your μ t CP / kt or this is the ratio of your turbulent momentum diffusivity by turbulent thermal diffusivity understand okay so this is your definition of turbulent prantle number okay so I hope this part is clear.

So now what needs to be really computed is the value of the turbulent diffusion okay and in order to do that either the turbulent viscosity so once you calculate the turbulent viscosity we can actually fix the turbulent prantle number and therefore obtain your turbulent thermal diffusivity okay we do not solve for turbulent thermal diffusivity separated so usually we only solve for the turbulent viscosity then fix the turbulent prantle number usually around 0.9 or something like then get α t so now how do we saw for you P so there are different approaches to that so what we call as turbulence modeling.

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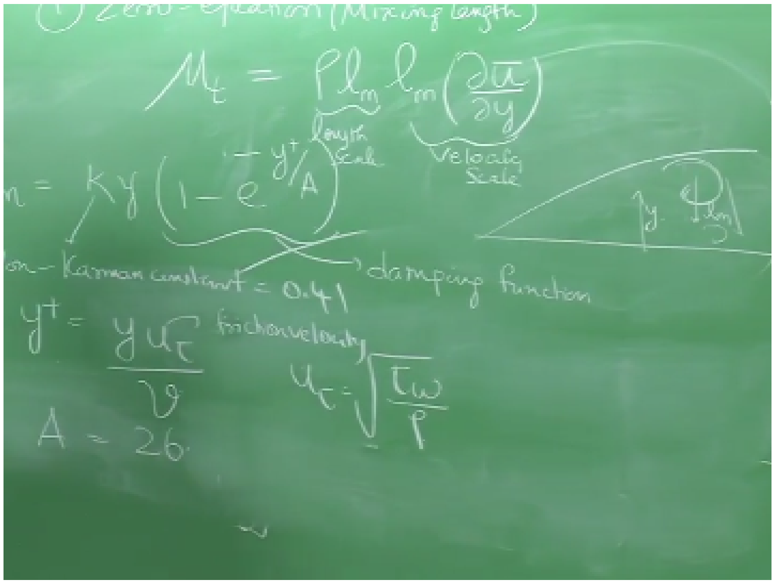

So I will not spend too much time because I am sure many of you must be already knowing how do we do this but just I will summarize it so we start from the simplest model which is called zero equation model or the mixing length model the mixing length model was originally conceived by Prandtl himself okay this is one of the simplest models so what he suggested was to assume that the turbulent viscosity is a function of some length scale and velocity scale so and when you look at the effect of turbulent scale so you have these largely DS which actually transfer energy to the smaller Eddie's through what we call as cascading okay.

So there is a certain length till which the large Eddie's maintain that size after which they break down into smaller Eddie's okay so this length is called the mixing length so Prandtl came out with some kind of intuitive hypothesis which says that this turbulent viscosity should be a function of the mixing length and some velocity scale okay so he represents the velocity scale to be a function of mixing length times the derivative of the mean velocity for example if this is your wife okay and you have a certain mixing length L then the corresponding velocity scale of turbulence is given by L M x D U ' /D u bar/ dy okay this is some scaling and some hypotheses.

So therefore he relates this turbulent viscosity by multiplying another length scale to do this so this is your length scale and this is your velocity speech okay so what is this mixing length so the mixing length if it is in a purely turbulent regime it is some constant so mixing length actually is some kind of virtual or theoretical length if you have an idea of a given size so it will actually break down into a small arity half after it travels down because of the action of the wall okay then finally it has to be dissipated as heat okay so there is a certain length till which it doesn't break though it retains its original size.

So that is called the mixing length okay you cannot of course measure all this okay unless it is the large it D for all the ADIZ you cannot find what is the mixing length the mixing length of course will be different but then he did not account for all of these so all he says is that we can relate the turbulent viscosity to a mixing length scale and a velocity scale and how do we calculate the mixing length so now that we have closed it so in the zero equation model we have crossed the turbulent viscosity to be a function of the mean velocity now only unknown is the mixing length so he is giving a empirical correlation for mixing length should be some constant times y.

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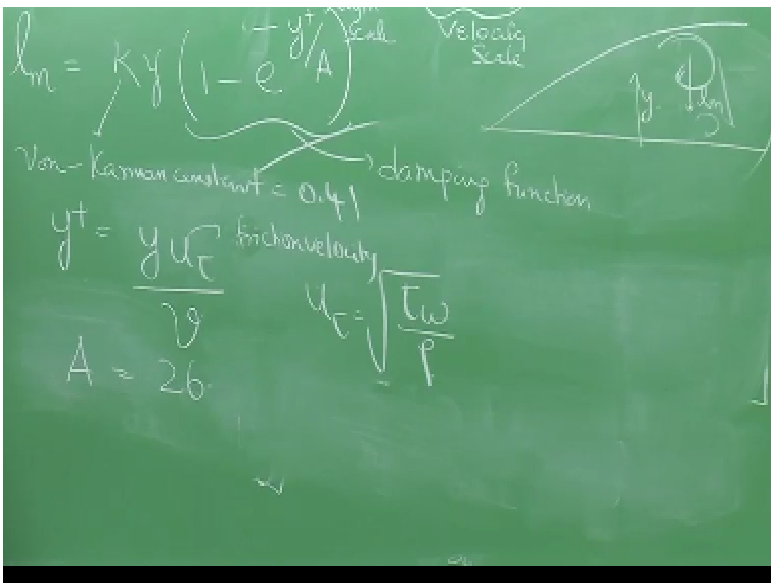

What is this constant this is called the von Karman constant? Which is equal to 0.4 ones now due to the presence of the wall in the if it is purely turbulent the mixing length will be just a constant times y but due to the effect of all this mixing length actually varies so as you go closer to the wall this mixing length will become smaller and smaller you can imagine that the effective wall gets very strong so the mixing length has to reduce it has to break down faster and faster so therefore to account for there is a damping function which is multiplied to this and he proposed a damping function like something like this so this is your damping function so what is y + so y + is a non-dimensional position which is Y times what we called as friction velocity divided by nu and this friction velocity is nothing but square root of wall shear stress by P and this constant a is nothing but 26.

So you understand that conceptually prantle had proposed this mixing length theory in order to simply close the turbulent viscosity without having to solve additional equations right so this works fairly well if you have a flow which is attached to the wall okay so like a classical boundary layer theory but what happens when there is a detachment flow separation so in that case then the mixing length model fails because there is no boundary layer okay you don't cannot locate the sub layer and so on okay so all this will fail so therefore then for separated flows are flows with pressure gradients this kind of simple approach will not work okay.

So then in that case you have to go for more advanced models which will now involve solving additional equations so these are your classical turbulence modeling equations we will just quickly list them so you have what is called as a one equation model.

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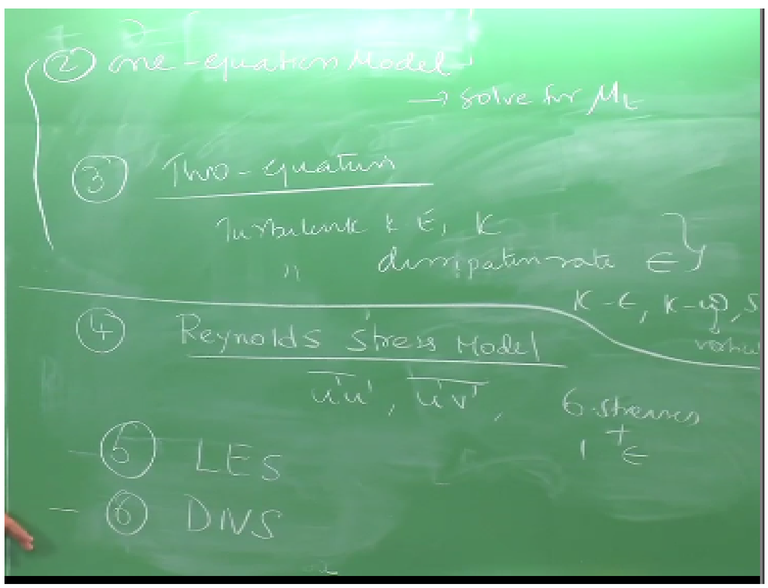

Which is slightly more complex than the mixing length model so in this case you solve for one additional equation for the turbulent viscosity so you construct one additional equation for turbulent viscosity and solve for multi so this will be additional partial differential equations along with your momentum and energy equations okay now for more complex flows recirculation flows with strong curvatures pressure gradient rotation and so on then we start going for two equation models.

So they involve solving additional equations one for the turbulent kinetic energy and turbulent dissipation rate Epsilon so this is denoted by K so we call this as K epsilon models for example so there are several bunch of these two equation models we have K ε we have K ῲ combination of these two which is called the shear stress transport model this is called actually some kind of vortices constructed out of the ε equation and modified so this now so you have several kinds of approaches into equation models okay and then you can also solve rather than applying a closure the boussinesq hypothesis.

We can construct equations for the turbulent stresses okay that is u ʼ u ʼ we can have a partial differential equation U ʼ V ʼ and then therefore we can solve for totally how many stresses so we have nine out of that three are symmetric okay so we have six stresses + one additional equation for dissipation okay so therefore we call this as in all stress mode so these turbulent stresses are also called Reynolds stresses so we can solve PDS for u ʼ u ʼ u ʼ V ʼ so on okay but anyway that has there has to be a closure for the higher moment which will be u ʼ u ʼ V ʼ so that will be closed in a different way right so therefore in three dimensions we solve for six stresses + one equation for dissipation rate and then without doing any of the Reynolds average we can actually do what it's called we you can directly resolve all the dominant structures.

It is by using a direct resolution by solving the navier-stokes as it is but we use a spatial filtering which is the grid size to filter out the eddies which are smaller than the grid size so those will be modeled okay the eddies which are larger than the grid size will be resolved so this is done in the large Eddy simulation and finally if you can resolve all these length and time scales without any filters okay and without any turbulence model we call this as direct numerical simulation so these are the increasing complexity of the turbulence models okay so most of the industrial problems are satisfied by stopping here okay.

So some of the research problems they want to go deeper they probe into alias and DNS but finally for practical purposes you do not need all these data although you are solving for the fluctuations and so on what are you going to do with that okay so we don't have µch ways of interpreting all the fluctuating components it may be looking good for understanding how they interact and so on but finally for practical engineering purposes working with fluctuations you know you ʼ T ʼ does not have any meaning therefore.

We are more or less happy with stopping with the two equation models so all these two equation models give you is the mean components that is a mean velocity mean temperature in turbulent flow field okay so once you understand the mean once you can predict the mean properties well they are enough to solve practical problems for example when you now define heat transfer coefficient it will be the gradient of the mean temperature okay if you if you know the gradient of fluctuation what are you going to do with it you know there is a fluctuation to the mean but that is not going to serve any purpose okay so as long as you can construct a engineering approach to solving turbulence.

I think that is a more sustained practical approach so although we have techniques using Elias in DNS I do not think these are still in the industry well-absorbed so they are mostly in the academic exercises right so most of the practical engineering problems have been designed with maximum with the two equation models right so I think now hopefully you have some kind of an understanding about the models that are used now what we will quickly do in the remaining five minutes we will try to derive certain relationship between the velocity scale and the position as we go across different layers in turbulence okay.

So very quickly we will do that and stop so as I already pointed out when you for example talk about turbulent flow past a flat plate.

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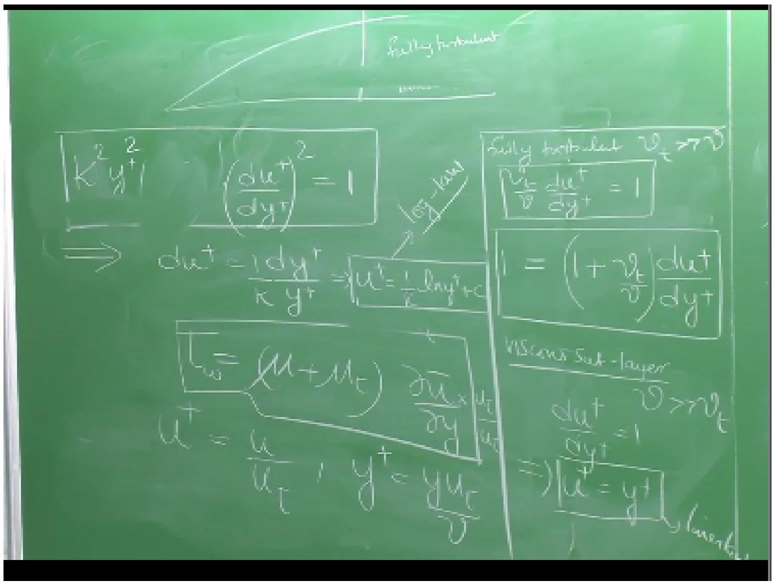

You have a turbulent boundary layer right and there is a viscous sub layer this is close to the wall so when you talk about the laminar prantle number for example okay so that is going to govern only the growth of this viscous sub layer whereas the turbulent Prandtl number is the one that will govern the growth of the actual bone delay and since your turbulent diffusion is much higher than the laminar diffusion okay so this will be the dominating pattern and the laminar sub layer will be confined only close to the wall right so now as you go from the wall upwards vertically.

So you will be passing through initially the laminar sub layer till a certain height and then into the turbulent region so this is a fully turbulent region here and this is the region where it is only laminar because of the effect of wall there is no turbulent effect seen there so now what we can do is derive a relationship between the velocity profile and the local coordinate so let us start with the business hypothesis which says that we can model the turbulent stresses with the analogy of the molecular viscosity so we can just write this as $\mu + \mu$ T x D U ' / V Y okay so if you what I would like you to do is write this in terms of what we call as a non-dimensional velocity u superscript + okay.

So this is nothing but u by you tout and you τ is nothing but the frictional velocity square root of τ w/ P okay and write this in terms of non-dimensional y coordinate y + this is y into u τ by kinematic viscosity so use this relation generic one and express this in terms of U + + y + see at the wall this will be equal to the wall shear stress the wall so one thing what we can do is multiply and divide by Newtown right so therefore D of u bar by u τ is nothing but u + okay so we have D u + and similarly we can combine the T wto this side okay so you should be able to write this also in terms of y + u tell me then what will be the multiplying factor.

I will also give you the left hand side should come out to be one so what do you get you will have 1 + kinematic turbulent kinematic viscosity by laminar kinematic viscosity okay so now so this is your equation coming out of the definition of the stress so depending on the regime now let us first consider the viscous sub layer inside the laminar sub layer your laminar viscosity is much more dominant than the turbulent viscosity so what will happen to this term 1 + nu T by nu so suppose let us take discuss a Blair discuss a Blair your μ is much greater than μ T right so therefore what will happen to this entire term 1 okay therefore this will reduce to 1 so if you integrate this okay.

So this is your profile of velocity within the viscous oblique now what happens in the fully turbulent layer so in the fully turbulent layer you are turbulent diffusivity is much greater than nu right so then what happens to 1 + nu T by nu will be nu T by μ into D u + by dy + will be equal to 1 okay so now you can substitute for μ T from the mixing length model okay so anyway since μ T is here you have to close it you τ square yeah if you are just erased this the same wall shear stress will be felt but only you are considering only the turbulent boundary layer not the laminar sub layer so you can call this in one sense you have a one layer model the other is a two layer model okay.

So in a one layer model you are considering only the fully turbulent region without accounting for the viscous sub layer okay it is the simplest approach that is what we are doing now if you consider of a fully turbulent region as the only layer okay so then we actually can write this like this correct so now you substitute nu T from the mixing length model so you have essentially LM ^{2} and what is LM ^{2} one carbon constant square times y square because in the fully turbulent region you are done do not have the damping function so that can be written as K square Y square into you have also do you so you have D U bar / dy / nu x D u + by dy + = 1 so can you now simplify this further we can also write this as d u + by dy + correct so what do you get if you write it so I want to write this as τ u + by dy + $^{(2)}$
So what will be the additional factor this μ should be absorbed okay so just check that so finally therefore when we take √of this and then we integrate it so what would be the profile so we will have d u + = dy + by Y by Y x K so we' wil have D u + = dy + by we can write this as y + 1by okay , so if you integrate this gets you + = 1 /K lawn of y + + another constant so this is called the log law okay.

So this is the linear law it means if you assume only within the viscous sub layer you have a linear variation of velocity with Y now if you assume only a fully turbulent region you have a logarithmic variation of U with respect to Y okay so the entire boundary layer can now be decomposed into profiles in the viscous sub layer you have a linear variation and in the fully turbulent you have a logarithmic variation okay so we will look at this we will draw these profiles and try to now next move on to heat transfer problem where we can use some analogies like the Reynolds analogy and calculate the expressions for nusselt number okay.

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