Indian Institute of Technology Madras

Presents

NPTEL

National Programme on Technology Enhanced Learning

Video Lecture On

Convective Heat Transfer

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Lecture 41

Turbulent Convective Heat Transfer:

RANS Equation – Part 1

Very good morning to all of you so the last four classes we will be looking at a new topic a very important topic for practical engineering applications which is the turbulent convective heat transfer okay so although we do not have too much of time since we were covering other topics most of this semester so we will spend at least four lecture hours to emphasize the nature of turbulent convective heat transfer.

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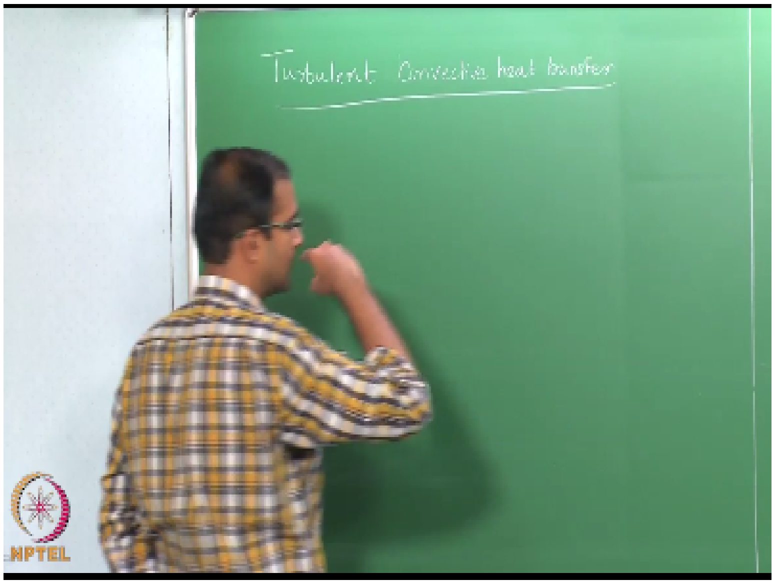

Because most of the practical applications that you are all dealing with I am sure are turbulent in nature and I am also sure that many of you are already taking a course on you know some introduction to turbulence or turbulence modeling or part of this is already being covered in a fluid mechanics course okay but the approach in heat transfer is very similar except that for simple applications simple arguments we will use certain analogies and one analogy's which we have already shown for the laminar external forced convection is there in all Sinology.

So we saw that since the structure of the momentum and energy equations are very similar so once you get the expression for skin friction coefficient we can directly derive the expression for nusselt number using the Reynolds analogy so to start with the simplistic kind of arguments in heat transfer turbulent heat transfer state that we can actually use certain analogies like these like the Reynolds analogy we can extend these analogies to turbulent flows.

Also okay under reasonable accuracy they you know in a very less time you can get a good prediction of the local Nr and things like that without having to solve them rigorously using computational methods okay so let us quickly look at the corresponding range of the flow parameters which will classify the flows as either laminar or turbulent and as you all know in the force convection case this is the Re so if you look at external flows.

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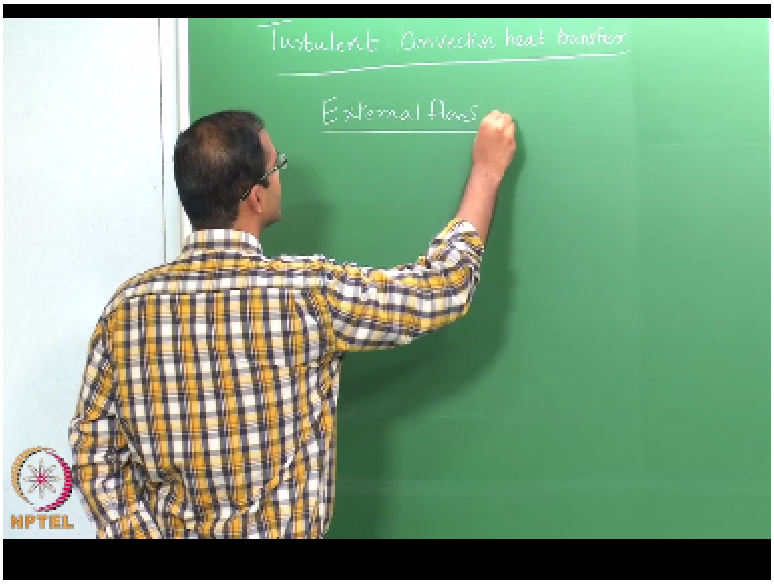

Okay so we use the definition of Re say let us say local Re and generally if this local Re is > 5 x10 power 5 this is for the flat plate so the flow tends to change from the regular streamline laminar pattern to a more chaotic turbulent pattern okay so similarly if you go to internal flows so let us say flow through a duct so we use the Re based on the diameter of the duct and typically if this is > say 2300 so the flow is said to be turbulent.

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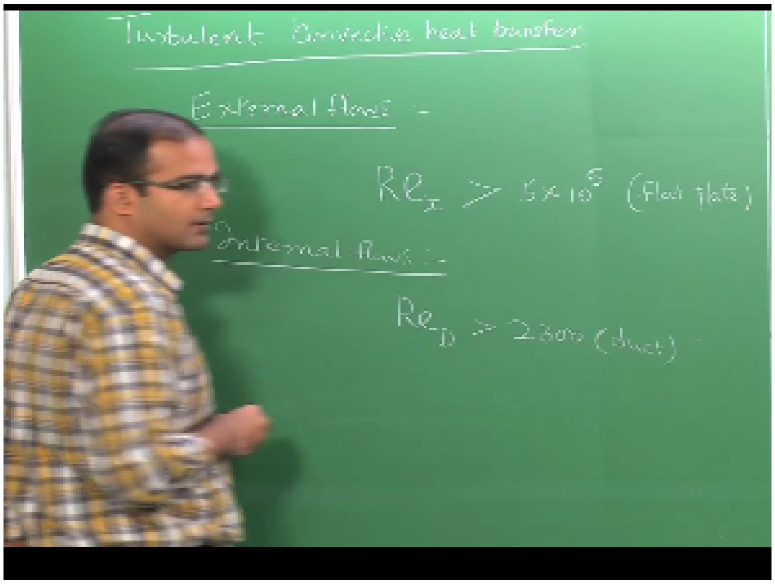

And what about the natural convection so natural convection how do we classify what would be the parameter huh we can either use Gr or Rr okay so generally people use Rr which is the product of Gr on Pr and if you take the case of a flat plate or a cavity an enclosure so typical values of Rr > 10 power 7 these are classified so this could be a vertical plate or enclosure cavity this is no natural convection in a cavity so that is why in the project I asked you to work out rally numbers < 10 power 7 okay up to 10 power 6 it is still laminar.

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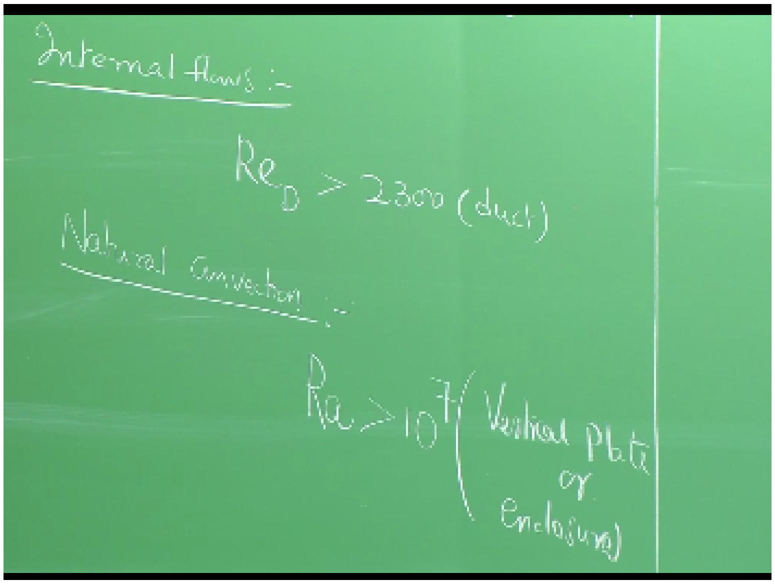

Now what happens if you cross these thresholds either Reynolds number or rally number so when you put a probe okay so typically it could be a hot wire probe which is used to measure the local velocity instantaneous velocity at a particular point suppose you have the flat plate okay so let us say that till certain position where your Re is < 10 power 5 so this is where our Blazes solution and everything is applied okay so now after 5 x10 power 5 you have a small transition zone in which instabilities will start will change the pattern of flow okay so once these instabilities start you will find that now the turbulent region.

Where Rr is > 5 x 10 power 5 you will see motions which are definitely not streamline and not only that you will see several structures several large Eddie's it is of all length and time scales.

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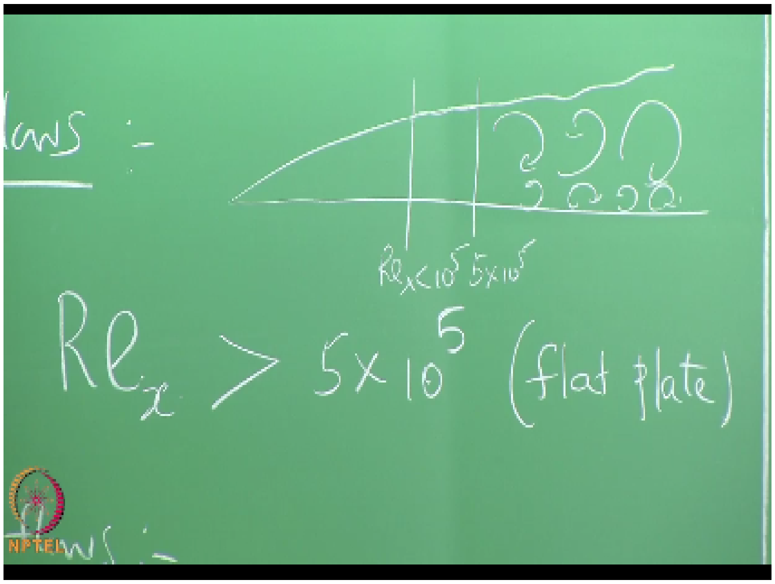

Okay so large it is typically and of a definite structure these are called coherent structures okay now you can have linked scales which are quite disparate right from you know order of magnitude to 5 or 6 orders of magnitude difference in the length scales depending on the Re okay so the smallest Eddy will be naturally lying closer to the wall okay so as you go from the turbulent boundary layer edge towards the wall the size of the eddy's will naturally become smaller and smaller and finally close to the wall the flow will be only laminar okay so essentially near the wall you have the effect of viscosity which is molecular viscosity which is predominant as compared to the turbulent motion and therefore this will be referred to as the viscous sub layer or laminar sub layer.

Okay so now the action of the wall therefore is to damp the effect of turbulence okay so without the confinement without the wall you do not have a laminar sub layer therefore and you only have a range of scales in turbulent flow so in the case of a typical boundary layer profile the effect of wall is to damp the large scales and finally the large scales cascade to smaller and smaller Eddie's and finally it becomes laminar flows to the wall okay now only in this laminar region you have the significant effect of molecular viscosity.

So outside this it is all turbulent and the turbulent motion will dictate the velocity profile temperature profile and the characteristics of flow field and temperature field okay so the effect of molecular viscosity and thermal conductivity will not be significant outside the viscous sub layer okay so this is a very interesting you know physical phenomena mainly because it is very challenging to observe so many length scales and also F we have therefore different regimes different regions where the effect of the molecular viscosity can be significant and in other places where it is insignificant okay so now if you therefore look at these structures so what do we how do we capture these different length scales so if you put a probe for example let us say a hot wire probe and try to measure the velocity at a point here so how do you see the variation with respect to time okay so let us plot velocity versus time so you might start with some value at time T = 0 and then you might see some fluctuations.

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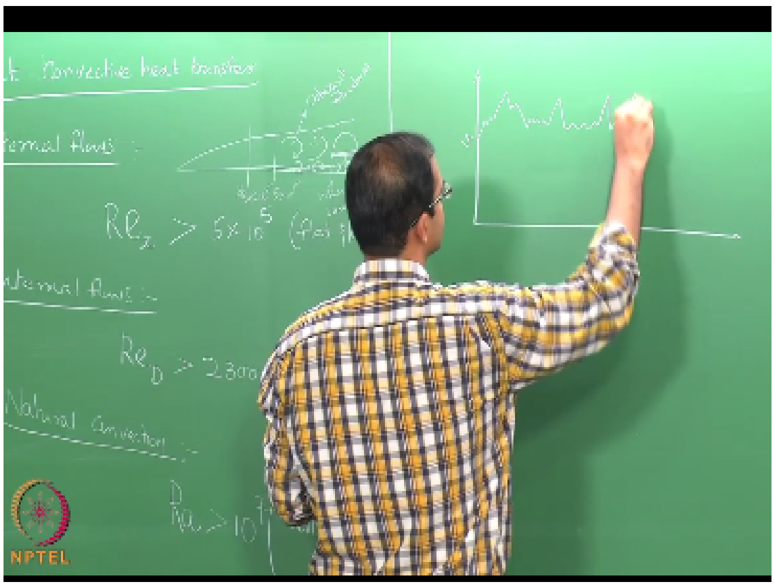

Will be anything but you will definitely see lot of fluctuations like this it could be periodic need not be periodic okay now this measurement is called the instantaneous velocity that means this is at a particular point and over a time instant that you keep monitoring continuously.

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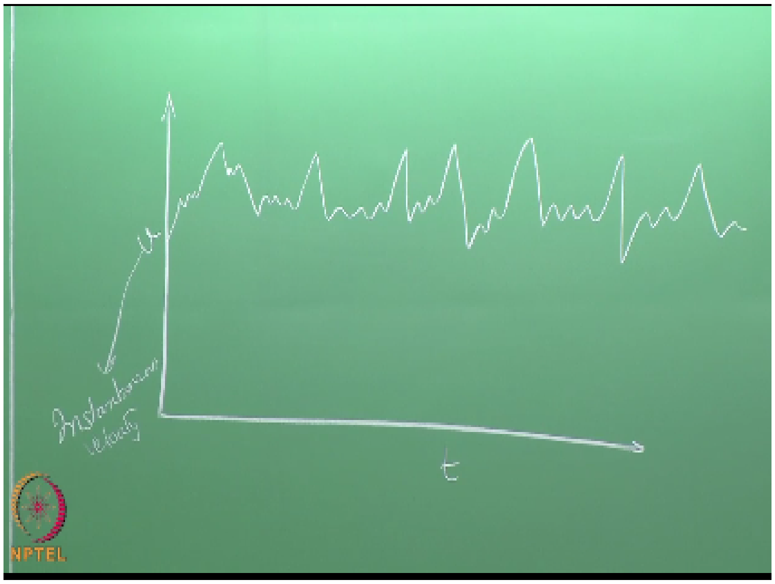

Okay and you do not apply any filter to this so whatever comes out to the probe and if the probe is quite sensitive you have to be important that pay attention that the probe is sensitive to all the timescales of turbulence and therefore it should be able to resolve these fluctuations okay now these fluctuations represent that there is inherent turbulence in the flow field okay in a laminar case you wouldn't see these fluctuations it will be a constant value.

At that point in time okay so initially there could be some unsteadiness but finally once it reaches a steady state condition it will not change but in the turbulent case there is nothing like a true steady state there will be always an inherent fluctuation that you will observe and therefore so this represents the nature of turbulent flow regime now that you can capture this we how do we then post process so what we do usually is take an average of this readings okay so we can classify this entire instantaneous data into an average and a fluctuation about the mean okay so that we can assume that is a fluctuation which will represent as u Prime.

It could be positive or negative about the mean and therefore we can decompose the instantaneous velocity as a fluctuation which we will represent is an average which will represent by an over bar okay this average can also be a function of time we will see that plus you have the fluctuating components.

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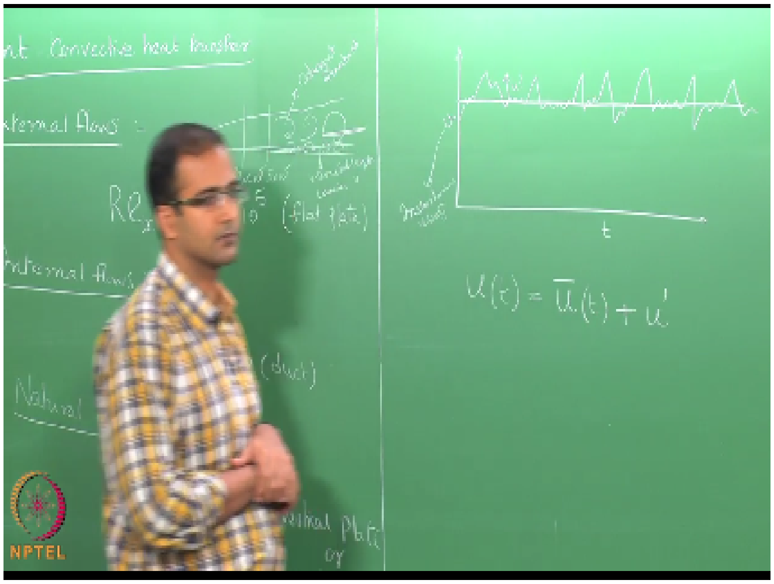

So this is a simpler way of basically representing the instantaneous velocity because the instantaneous the last has too much of data which we may not need okay so in order to simplify the on post process and also solve the equations we look at the mean velocity and a fluctuation about this mean and therefore we assume saying that the instantaneous velocity is = to sum of the mean and the fluctuation so now this mean velocity also can be a function of time depending on that kind of filter that you apply.

So when we say mean velocity here how do we calculate this mean velocity this is nothing but integral some time T to T + Δ T U of T DT 1 over Δ T okay so we can do some kind of an averaging.

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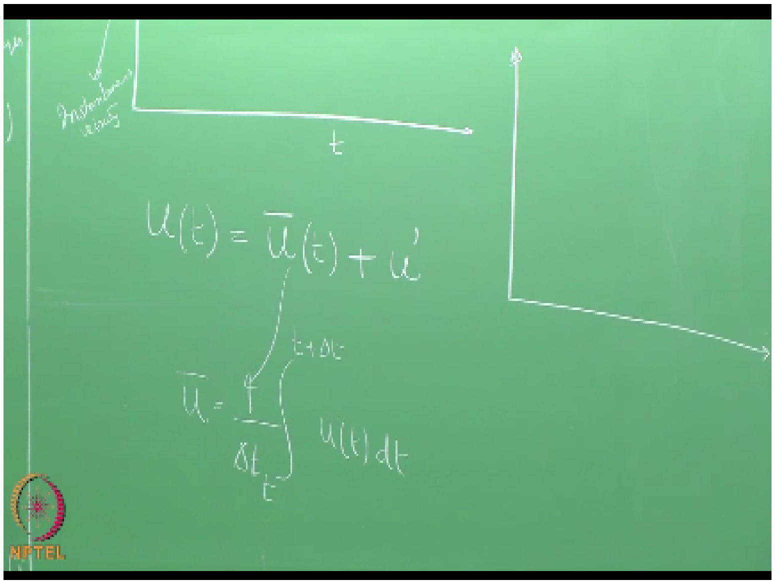

In this case I have done what is called as a time average okay so we can average the instantaneous component over a period of time with a certain filter with this which is Δ T here and we can if you do this average you get what is the what the mean velocity we have represented here okay now if you apply the filter carefully enough for example if you have an instantaneous velocity something like this so some kind of waviness which comes in time but it is periodic and repeatable so if you apply the averaging filter carefully you should be able to construct a pattern which is repeating and periodic understand so these are not small fluctuations.

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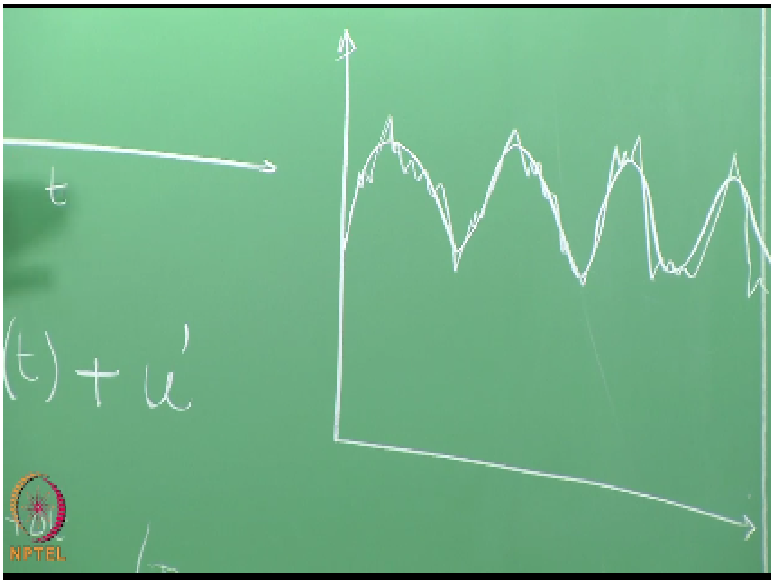

So you should distinguish the waviness which is actually the time variation of the mean velocity from the fluctuation which is your RMS velocity or root mean square velocity so if you plot the instantaneous velocity here and if you carefully apply the time filter okay such that this time filter is smaller than the waviness okay so suppose this is your time scale of the unsteadiness so your Δ T should be definitely smaller than this if you apply a Δ T larger than this time scale you will be only getting something like this right so it has to be smaller than the waviness time scale but it should be larger than the fluctuating time scale.

Okay so these are the time scales of fluctuation which you want to smooth out so in order to smoothen the fluctuations you need to apply a filter which is larger than the fluctuating time scale but smaller than the time scale of the unsteadiness okay so if you select that Δ T appropriately then you should be able to reconstruct you mean which is actually a function of time so this is showing that there is a periodic unsteadiness in the flow pattern the mean flow itself and this is coming after you apply the appropriate smoothing and therefore the correct representation will be to say that an instantaneous velocity can be decomposed into a beam which is a function of time okay plus the fluctuating component now there are different ways of doing this averaging.

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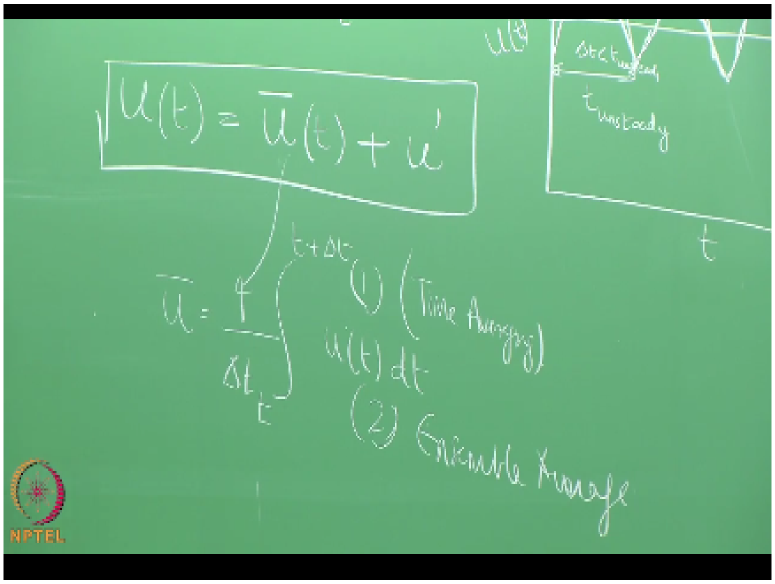

So what I have described here is time averaging you can also do what is called as and sample average that means you take certain data at a certain location you take it repeat it again you repeat it again you repeat it again so you have several sets of data okay and then do a statistical averaging of all these different repetitions so that is called as ensemble averaging okay anyway so for the simple cases of and also considering incompressible flow regime people say that does not matter whether you take a time averaging or ensemble averaging they both lead to the same set of equations.

Okay so we do not have to worry too much about this for incompressible flows for compressible flows we do what is called as Fabre average that means we also have to account for the variation of density.

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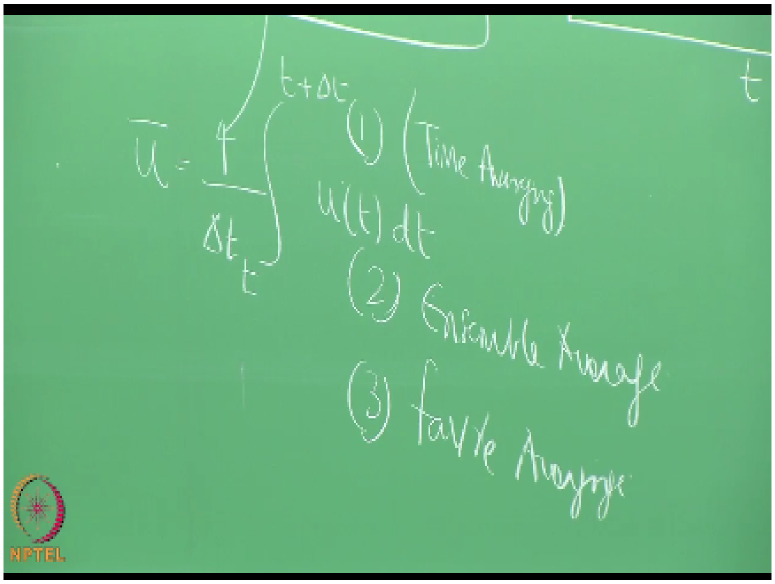

Okay with time and also the fluctuation of density whereas it incompressible flow we assume that density is constant although there is a turbulence the turbulence is affecting only the velocity and temperature is not the density the density as a property is constant okay whereas in a compressible flow the density also has fluctuations. So therefore we have to average Rho U for example in compressible flow not just you okay so this is called Fabre averaging which will not be worrying about here you can also do what is called a spatial average.

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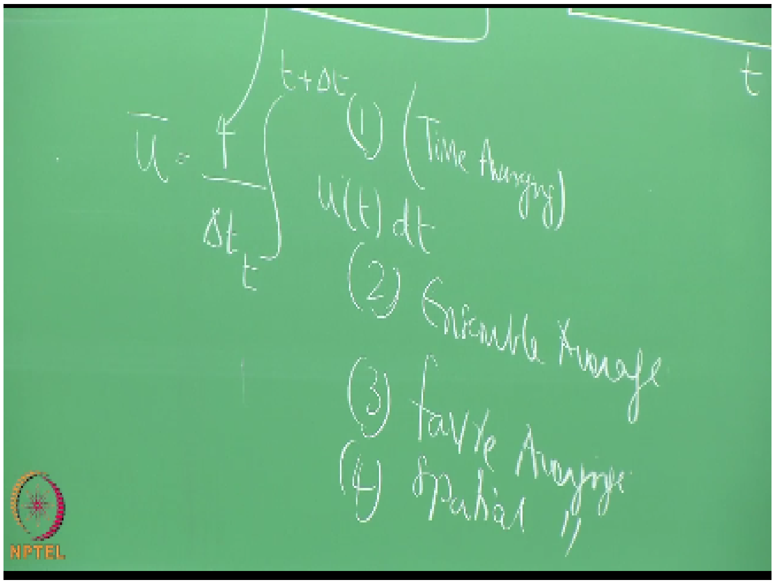

So you can use a filter which is not time but the grid size for example so this is typically done in what is called as larger D simulations. Okay so you use a spatial filter which is actually the grid size which can resolve thee which can actually resolve the smallest ad possible beyond that all these smaller ID have to be modeled so you apply typically in Elias the spatial filter is the grid size Δ okay so like this you can talk about different ways of doing this average but nevertheless after you average you say that the instantaneous component is therefore a superposition of the mean component and the fluctuating component.

So like this if you do the averaging so how should you model or how should you capture all these scales so as you can see that these fluctuations are a result of the small Eddies okay so the time scale of these Eddies are resulting in this fluctuation so therefore how do you really resolve this so for this what we do is general the most generic case we solve the navies-stokes equation as it is okay and we should be able to when you solve navies-stokes as it is meaning there is no political solution to that you only do it numerically.

But we use a grid size which can actually resolve all the links kills length and time skills okay so this kind of treatment is called direct numerical simulation so popularly referred to as the DNS so this is the precise most precise way that means we just take whatever navies-stokes equation because they are valid irrespective of whether the flow is laminar or turbulent and solve them exactly but the only constraint is how much you can actually physically resolve numerically you can resolve okay so that is limited by the grid size but you should understand that this is not very trivial because most of the times the ratio of the largest ad to the smallest ad can be > 10 power

3 okay so the smallest length scale is called the Kolmar grouse links the largest length scale could be of almost the size of for example here it could be almost comparable to the boundary layer thickness itself okay the smallest length scale is the length scale before which it finally gets dissipated into heat okay so therefore this is called the Kolmar length scale and if you have if you if you plot what is called the energy spectrum so usually when you are doing a DNS the first thing that you are asked to show is the energy spectrum.

That means it tells you how much the turbulent length scales turbulent energy scales have you resolved okay with your grid so you will be able to show here energy cascade okay so that shows that the energy is kinetic energy turbulent kinetic energy is transferred from the largest scales finally to the coal Kolmar scales okay and finally that energy is dissipated as heat at the wall due to the viscous dissipation finally they all the stuff turbulent momentum coming from the larger Eddies this actually handed over to the smaller and smaller Eddies and finally the smallest ad will dissipate this there is nothing else to dissipate take away this energy from the ad so but it dissipates itself into heat due to viscous dissipation effects.

And the wall okay so generally when you do a DNS you plot the energy spectrum and show that you have resolved all the energy scales of turbulence right from the largest scale to the smallest scale and this length scales can be at least 10 power 3 the ratio of the largest to the smallest links gate so that means you should have a grid size which is 3 orders of magnitude difference so if you are therefore doing a domain which is 1 meter the smallest grid size should be of the order of 1 millimeter okay this is an example.

Right so that means you should have enough grids to resolve anything from 1 millimeter all the way to 1 meter and sometimes it even falls sub millimeter scale okay 10 power 4 if it goes 10 power 5 because you are actually resolving micron scale eddies okay so this becomes a very challenging task especially in three dimensions this I am talking about in one dimension okay if you need resolution in turbulence is inherently three dimensions so if you are doing a three dimensional simulation.

Then it will be really a humongous task you were talking about measures of 1 billion typical ok so to do even basic turbulent simulations so which will be very computationally challenging therefore the alternate option is doing some other ways like larger D simulation or the more practical method which is called the Reynolds average. So I think what we will do is spend the next lectures only discussing about the practical method which is the Reynolds.

Deriving the Reynolds average navies-stokes equation and then how do we find simple solutions to the Reynolds average navies-stokes equations so now I mean the Reynolds averaging assumes first you decompose your instantaneous velocity into the mean and fluctuating velocity so this is called as a naught stigma position so this is the starting point of deriving the Reynolds average navies-stokes equation which is also popularly referred to as the Reynolds equations now before we do this we also have to decompose all the flow parameters not only the velocity.

But also what are all the other things in the navies-stokes equations pressure for example and paddling the energy equation we have temperature you can do the same thing for V velocity okay we do the Reynolds composition.

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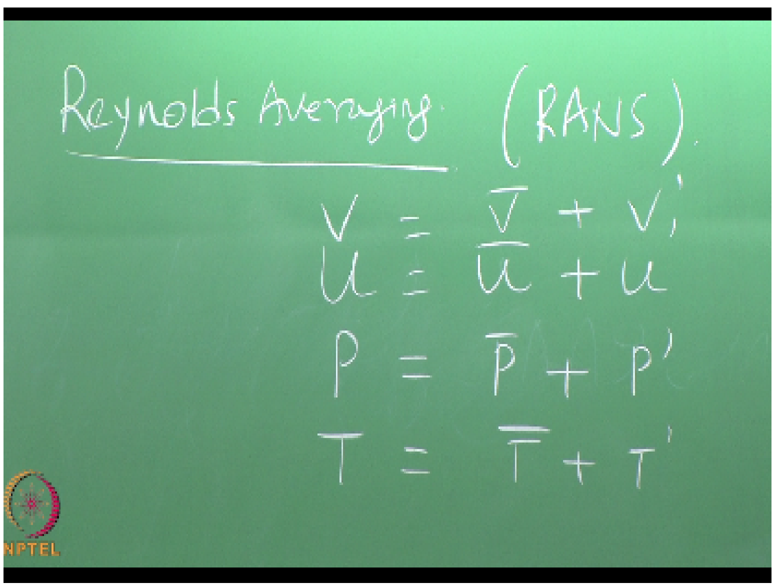

For all the flow parameters here by constructing a filter which could be either a time wise filter on and in sample wise filter and then we decompose that into a mean and a fluctuating component now before we do the derivation there are certain rules that we have to list down so I will just state what are called the rules of averaging okay so these have to be satisfied.

When we apply these averaging to the entire navies-stokes equation the first thing is that when you take the average of the fluctuating component that is you would have a mean here now if you want to find the average of this what it will be it will be 0 okay so that that is because you take the mean of the instantaneous that should be the same as the mean velocity itself so therefore the mean of the fluctuating component should be 0 so you have a positive fluctuation negative fluctuation so when you apply this over average it over the Δ T it should cancel out so that is the first route.

And therefore what does it tell you now when we apply a mean to a mean when we average the mean component over the same Δ T it should give back the same mean so it does not matter how many times you average the mean component.

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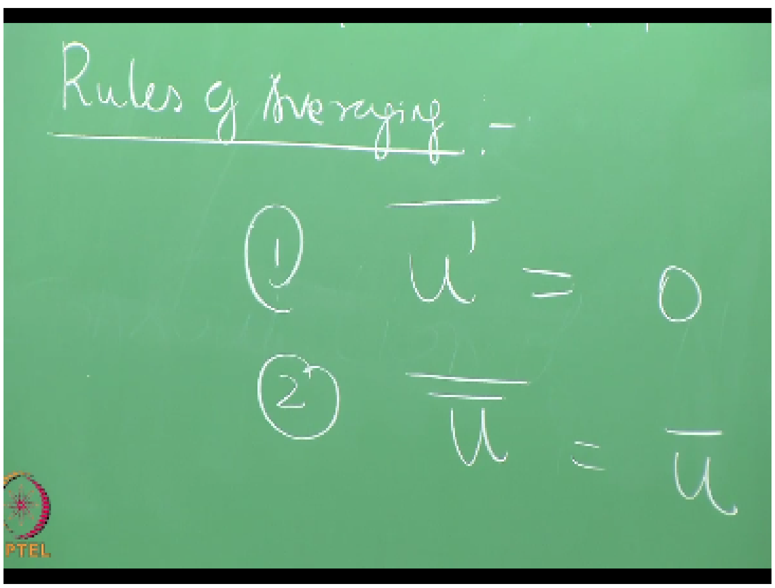

It still get back the same mean that means you still apply the same Δ T over that okay so therefore it does not make any difference to the average okay now there is a product rule which states that if you take the product of mean and an instantaneous quantity okay and then you average this component you can break this into a mean of the mean which will be nothing but the mean times the mean of the instantaneous what is the mean of the instantaneous is nothing but the mean again the mean component okay so since because the instantaneous average is 0 so if you can substitute as you mean plus you crime okay so this will be simply u Prime this u bar u bar + you have u bar u prime since u prime average is 0 that will okay so the fourth rule is the summation rules which says that if you have for example two mean values you add them and then you take a mean of that it is the same as taking the summation of 2 means okay so that means if you take the sum of 2 means and again take a mean of that it will be the same as just the sum of the 2 means independent okay so number 5 apply for example mean to the integral so that means I am averaging the entire integral of an instantaneous component so I have say integral u DX and I want to average this so this averaging operation of place only to this you are not to the integral.

Okay so this can be written as therefore what so if I apply the averaging to the instantaneous component it returns the mean so u / DX the same thing can be applied to the differentiation operator holes so if I say D / DX of U and then I apply the mean to this entire thing it will return d /DX of U / okay so now the other component the other rule is an important rule it says that if I take for example a product of the mean and the fluctuating component and if I apply the averaging of this what do you think will happen 0 because this can be split as according to the product rule u bar into u prime bar and since u prime bar is 0 so this should be 0.

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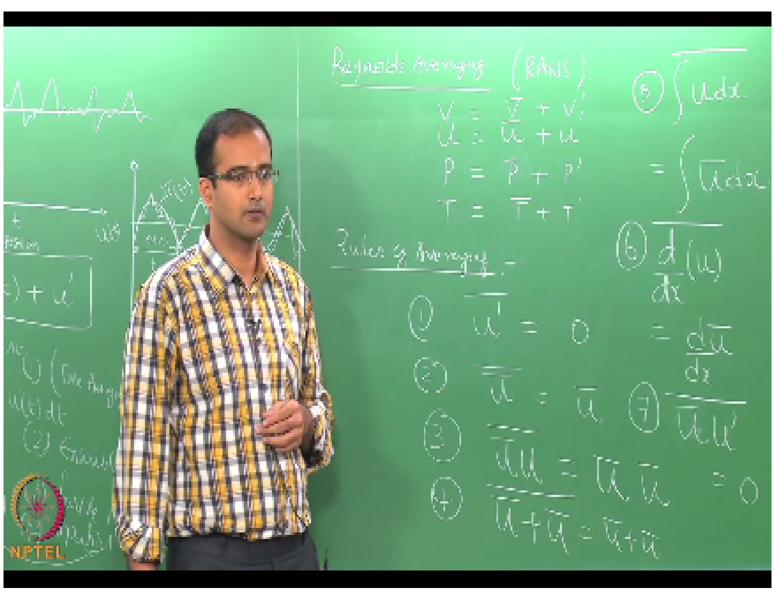

Okay so these are the rules that you have to keep in hand when we start deriving the Rans equation so let us quickly go over the derivation part so once you apply the rules it becomes much easier so you please write down the 2 dimensional steady state incompressible navies stokes including the energy equation okay so first is the continuity all in dimensional form then the X momentum so we are writing this only for forced convection write now.

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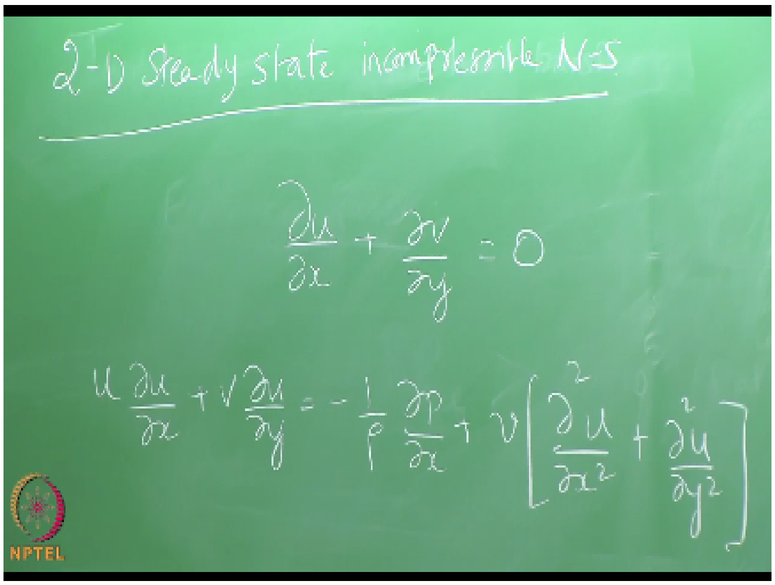

Okay without the body force term but later on you can also derive this for the natural convection case right so now what we will do first is apply the NOS decomposition okay so we can decompose all these are instantaneous quantities in turbulent flows okay so we have to therefore do a Reynolds decomposition of the instantaneous quantities and then we have to average these equations so we have to apply the averaging operation over the entire equations so then only we get what are called the Reynolds average navies-stokes equation.

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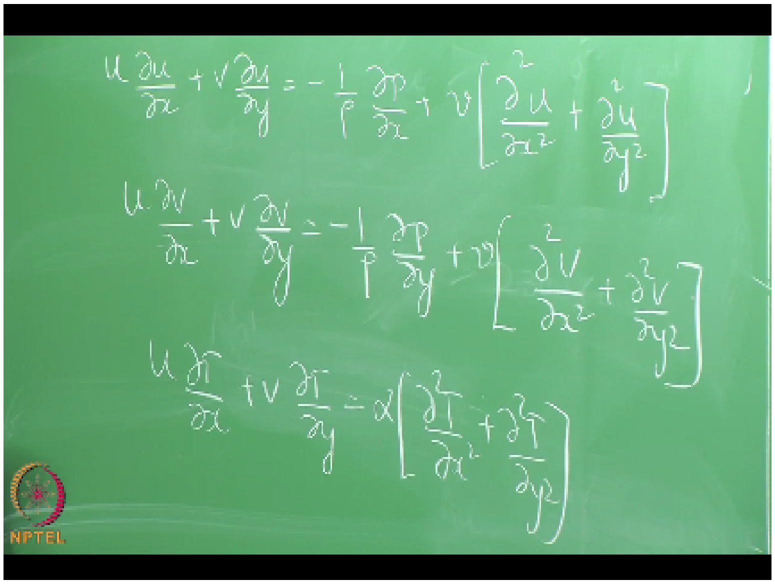

Okay so can you quickly do that for the continuity equation and tell me what you get so I have written down the NAS decomposition here so you should be able to so you can write the continuity equation as T by DX of u /+ u prime D by d y of V /+ V Prime and now average of this entire equation so usually some textbooks follow some conventions for example time average is given by an over bar ensemble average is given by this kind of parenthesis okay so you write this means it is an ensemble average okay so if you write like this is time average so some textbooks follow this convention.

So now how do you write this use the rules of averaging and tell me so this will be so this operator D by DX of U u bar Plus u prime bar so that will be what D by DX of u bar so we can of course write this as like this plus D U prime bar by DX + PV bar by d y + D V prime bar / d y so according to our rule this is what this is also 0 right so therefore you have a continuity equation for the mean flow field so there is a continuity equation which satisfies the mean velocities.

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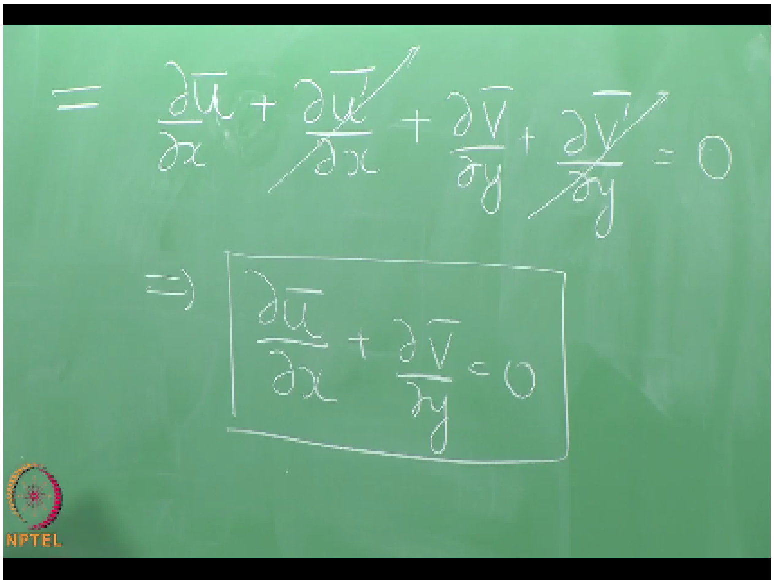

And therefore there should also be a continuity equation satisfying the fluctuating component because since you have a continuity equation for the instantaneous flow field and there is a continuity equation satisfying the mean. So if you substitute back you can actually write a continuity equation satisfying the fluctuating component also so the velocity fluctuations u prime v prime also should satisfy their own continuity equations.

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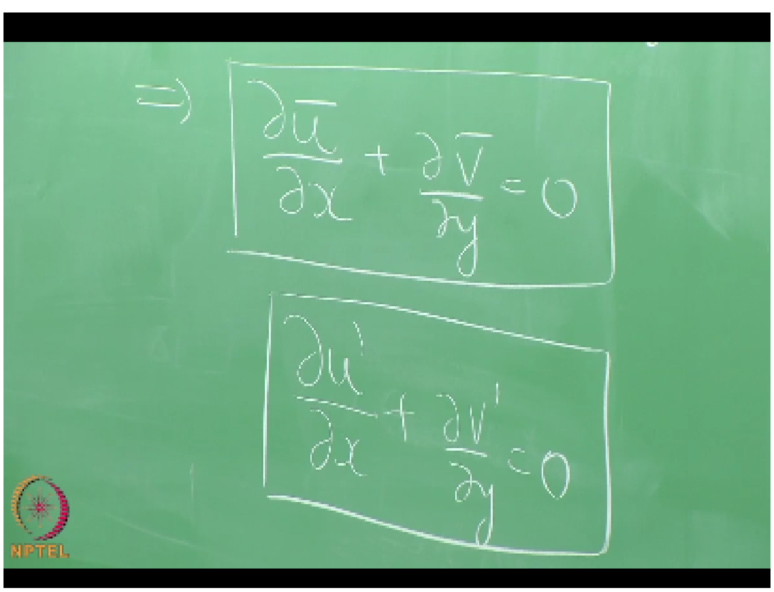

Now please do the same thing for the X momentum equation do their Gnostic composition and averaging and tell me what are all the terms so I will parallel e do it but you also do it on your own and then verify so therefore if you look at the right-hand side this is okay everybody must have written the step on the right-hand side this will be this is incompressible so there is nothing there is no averaging operator on the in density so 1 /Rho D /DX of P /+ 3 P prime bar P prime bar will be 0 similarly if you do the averaging here this is u /+ u prime bar so u prime bar is also 0.

Similarly here right so therefore on the right hand side it is straightforward what do we have after averaging the equation you have V P bar / DX v square u bar / DX square + d square u bar / d y square so the instantaneous quantities are now replaced only by the mean quantities so let us look at the left hand side first okay so u bar +u prime x D by DX so how can we write this we can split into four terms so one is U bar D /DX of U bar averaging operator applied to that + you have u bar D / DX of U prime averaging operator applied to that okay please remember how.

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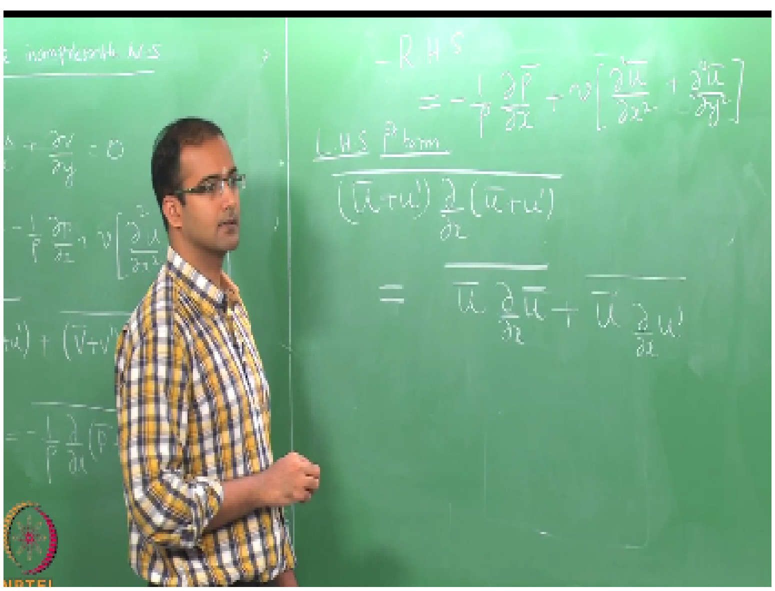

I am writing it okay so it is very careful I have not because we cannot now split the averaging operator separately to this term and this term okay so we have to first multiply these 2 terms and then apply the averaging operator you understand it okay it is not simply something like this so you have this term + x multiplied by D by DX of this okay there is no rule which says that this can be written as the averaging operator of over this multiplied by D by DX averaging operator applied over this okay we have separate rules for D by DX hand separate rule for the product not a combined rule okay so therefore we have to first multiply the terms and then apply the individual averaging operator.

So now we will go back to the averaging rule which says that we can apply the averaging operation to these independent things okay so now we have two more terms left one is what u prime x B u bar / DX whole bar + we have Q prime D u prime / DX the whole bar okay so therefore now we can apply the averaging rule so what it will be for this case you bar D by DX u bar which is fine + this will be 0 so you have u bar D by DX of U prime bar which will be 0 so when it comes to this now we have the last term okay we have u prime bar multiplied by D u prime bar / DX but we do not have an averaging rule.

For that okay which says that this should be 0 okay in fact when you say this can be actually returned you look at it as d / DX we will see that u prime u Prime and then apply the average so when you take the product of the fluctuating component multiply it with itself with either u prime u prime R u prime V Prime and then take the average this is not 0 you have to be very careful because this is the one which comes as turbulent stresses so in this case this is non0 quantity we will just write it as u Prime into D u prime by DX okay we can put the averaging there but that that does not matter so we can actually leave it as it is okay so therefore we can just keep the averaging operator like this.

So if you take the second term we have V D / d y of so what do you think will come out of this second term on their larches so we have V Bar d u bar by the way that is correct the mean components will come as they are now fluctuating component when they are multiplied with the derivative of the mean components they will be gone and you have another component which is V Bar D u bar by d y and the mean of this will not be 0 okay so therefore if you write the combined equation.

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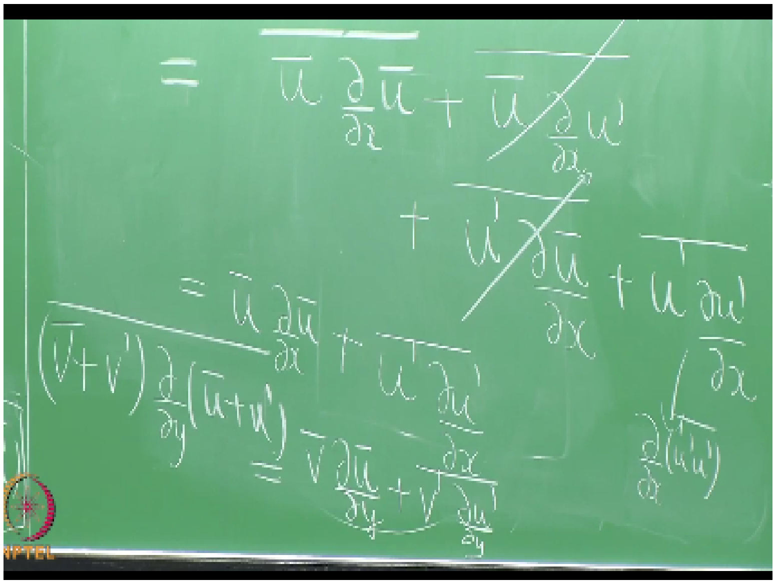

On the left hand side so you have you bar D u bar / D X + V bar D u bar / D Y so this looks similar to the instantaneous terms convective terms + additionally you have the term which is u bar u prime D u prime / DX averaged +V prime D u prime / D Y averaged the right hand side you have all the mean quantities so now we are going to introduce the continuity equation for the fluctuating component which says that D u prime by DX +DV prime by d y is = to 0 so therefore we can write this as D / DX of U prime u prime bar +D by d y of u prime V prime bar - what do you have d / DX.

What do you have one case you have u prime d u prime / DX + what is the other term u prime DV prime / d y okay if you take u prime common then we can we have actually D u prime by DX + DV prime by d y which will be satisfying the continuity for the fluctuating component so that so this will be 0 you understand so similarly I asked you to repeat the same thing for the V momentum also okay so therefore some the V momentum what do you get you have if you repeat the same averaging exercise you will have the mean components in the advection term Plus you have additional terms you have u prime V prime bar + D / d y of V prime V prime bar the other terms will be 0 on the right hand side.

You have V P bar / d y + x square V by X square so all these are mean components okay so now therefore we will stop here so in the turbulent trans questions you have similar terms in the advection component as the instantaneous but they are replaced by the mean quantities apart from that you have two additional terms on the left hand side okay so these are the u prime u prime derivative and u prime v prime derivative so if you write the Y momentum equation also you get similar derivatives of the product of the fluctuating components so these can be actually taken towards the right hand side okay and they can be clubbed to the existing shear stresses the shear stresses are nothing but tau xx tau XY and so on apart from that you also have u prime u prime bar u prime v prime bar and so on so these are called as now turbulent stresses okay so although the nature of these stresses are originating from the advection term okay they are named as turbulent stresses just for the ease of grouping these together.

So they are grouped along with the viscous stresses and then they are called together as the total stress so it seems that there is an additional stress which is coming in turbulence but that is are generating from the inertial terms and that is why when they are taken to the other side they are given a negative sign okay because they enhance the momentum so they are not in a conventional sense that they are not stresses.

Which impede the flow so they are the ones which are actually promoting the exchange of momentum and energy therefore so they have a negative sign because they are originating from the inertial terms so these are called as turbulent stresses so what I suggest you to do is do the same thing for energy equation also and see what does he then also averaged energy equation the tomorrow will list down all these Reynolds energy equations together and then see how we treat these turbulent stresses okay thank you.

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