Indian Institute of Technology Madras
Presents
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National Programme on Technology Enhanced Learning

Video lecture on
Convective heat transfer
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Lecture40
Natural convective in other configuration

So good evening and we will look at the last part of natural convection today before we move on to turbulent convection so far we have focused on the methods to obtain analytical solutions through similarity methods approximate methods like such as the square solution and all these have been focused on a very simple vertical flat plate configuration okay so now what happens if we deviate from this plane configuration we will see that it is not possible.

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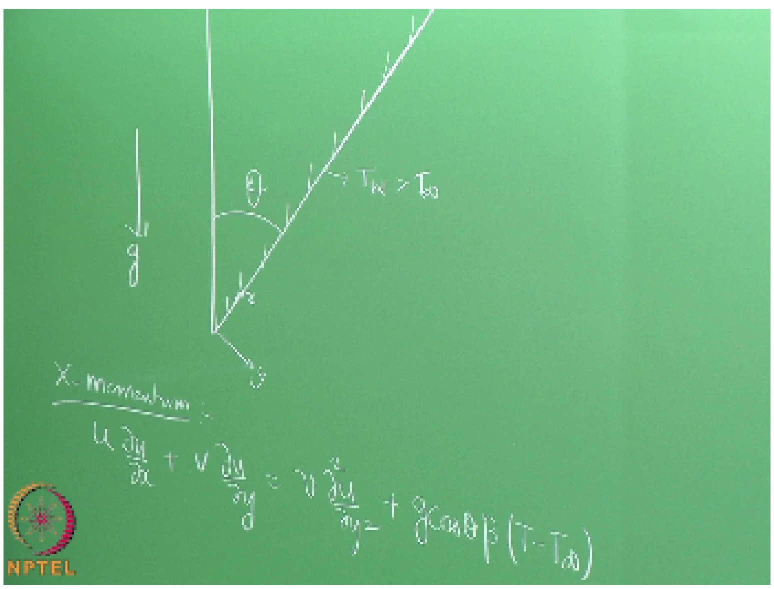

To get analytical solutions for more complex geometries so we will quickly go over some of the other configurations and I will give you the empirical correlations which have been obtained through experiments and may be you can just use them when you are whenever you solve natural convection problems involving complex shapes so one deviation from the simple vertical flat

plate is that if you tilt the plate so let us say that this is your plate here okay and your gravity is of course acting downward vertically and let us say you have an angle.
Theta with respect to the vertical direction so this is the tilt of the plate facing downward so if you heat this plate such that tw > T infinity still the same natural convection boundary layer will form but the only difference is the strength of the gravity will be slightly reduced in this case okay so the same governing equations will be applied for example if you denote this coordinate as X along the plate and perpendicular.

To the plate as y you can still write down the momentum equation this is the momentum equation along the plate length this is what we call as the X moment and we have new d square u /dy square plus the only difference here is that we have cos theta multiplied to G okay and the energy equation is still the same okay so in that for in such a case we can have the same solution whatever we did for the vertical flat plate and we will only redefine.

The definition of gash of number okay so we will just include G cos theta into beta D w- T infinity into X cube/ M square so we will just include the inclination angle cos theta directly into the definition of gash of number so that whatever solution that you already encountered whether it is the similarity solution or the approximate solution is still valid for this case with the slight modification in the gash of number okay so but now we have to be careful that if you have a limiting case where you tilt it facing upward that is your theta in the minus direction and you make it - PI / 2 okay so what happens so when you have a configuration where you tilt it this way so that is the plate is facing upward instead of downward what happens when you tilt it such that theta becomes minus PI / two in this direction becomes a horizontal plate okay so you will have a heated plate position horizontal gravity.

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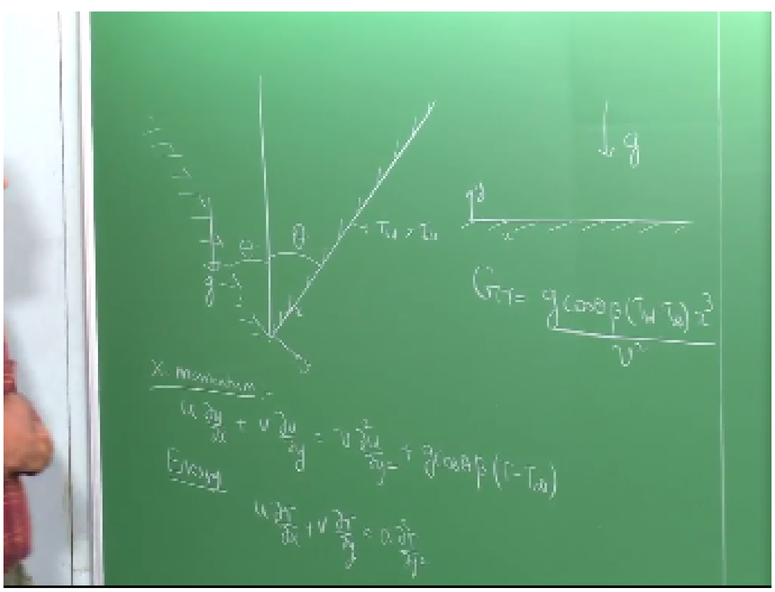

Acting downward okay so can we use then the same equations here so you have X and this is your y the limiting case where your theta becomes - PI / two so you have a horizontal surface now can we still apply the same equation to this configuration we cannot because as you see the buoyancy is now acting in the Y direction there is no buoyancy at all in the X direction so Therefore we have to be careful okay.

For reasonably small inclination x' the earlier approach is okay but if you have a completely horizontal plate then we have to redefine the governing equations so in that case what will be the X momentum okay so in this case you cannot include the bossiness i approximation in the X momentum because there is no gravity in the X direction source you just write your boundary layer equations so this will be your X momentum now the effect of buoyancy will be felt in the Y momentum equation okay.

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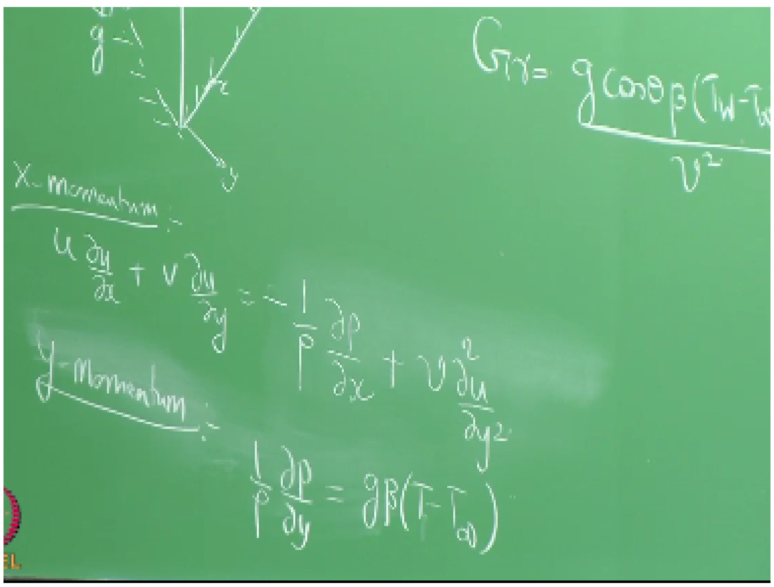

So therefore although you neglect the Y momentum in the advection part there will be a pressure gradient in the vertical direction which will be balanced by the buoyancy force so this will be equal to therefore G beta into P- T infinity so this is the modified set of momentum equations for the horizontal heated plate configuration okay so for this also we can find some similarity solutions which I am not going to go through in detail but I will only give you the final expression.

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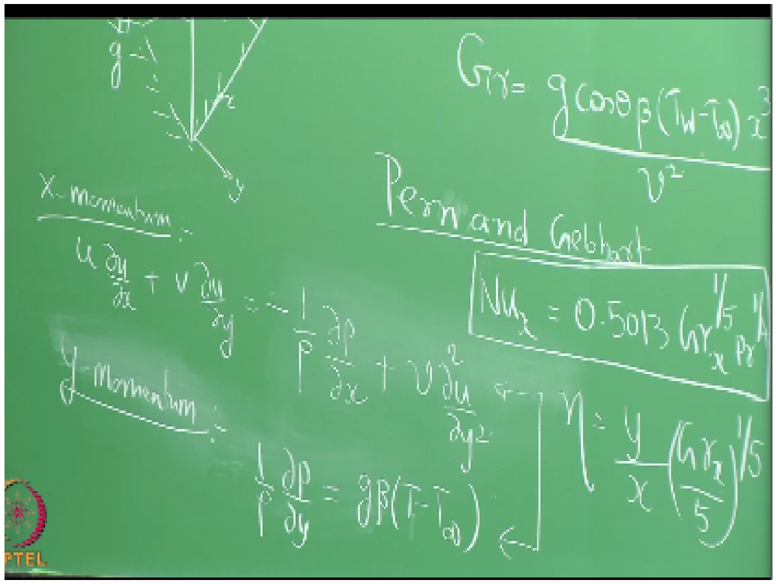

For the nusselt number this was done by Fernando Gephardt so this is burn and Gerhardt and for the isothermal case they have given the four following so this is coming from the similarity solution 0.5013grash of number to the power 1 / 5prantle number or 1/ 4okay so to derive this they used a similarity variable ETA which was y / x rash of number / 5 the whole raised to the power 1 / 5 so this is the kind of similarity variable that they used okay.

So you see that the advection component the momentum is neglected but in this case you do not have DP / dy is 0 so you have the pressure gradient in the vertical direction balanced by the buoyancy force okay whereas in the axial direction you do not have any buoyancy force but the most dominant momentum will be in the axial direction so this is the kind of you know configuration which is different from what we have seen so far so now.

All these are assuming that the plate lengths are infinity right so what happens if you talk about finite plate lengths so if you for example make this finite so then you have edge effects coming into picture okay so let us discuss this configuration which is also kind of common so this is the case of horizontal finite plate the horizontal directly so we have a case in which we keep the plate in the horizontal configuration and this is finite in length okay.

So one case a we heat the plate okay and we can have case B where we cool the plate that means T w is < T infinity yes but in the most important component which is governing the variation of nusselt number along the x direction is your momentum in this direction now as you can see the nusselt number here is calculated as a variation along the plate length so therefore it is actually the axial momentum which is the reason why the nusselt number is itself varying in the direction so it has been made an approximation that we can neglect the Y momentum in the advection part and we can still solve this to get the profile variation the variation of the u momentum in the X direction which actually influences this but this is an approximation of course you can you have To do a finite difference or finite volume method based solution in order to solve this exactly you cannot neglect also the vertical momentum there will be some patterns stream line patterns which will rise due to the buoyancy in the vertical direction and that will definitely cause some vertical momentum but while deriving the similarity solution it has been assumed that that is not going to impact the local heat transfer okay so of course we have to make some assumption in that case if you also include the vertical momentum you cannot find a similarity solution right so let us come back to the finite plate configuration.

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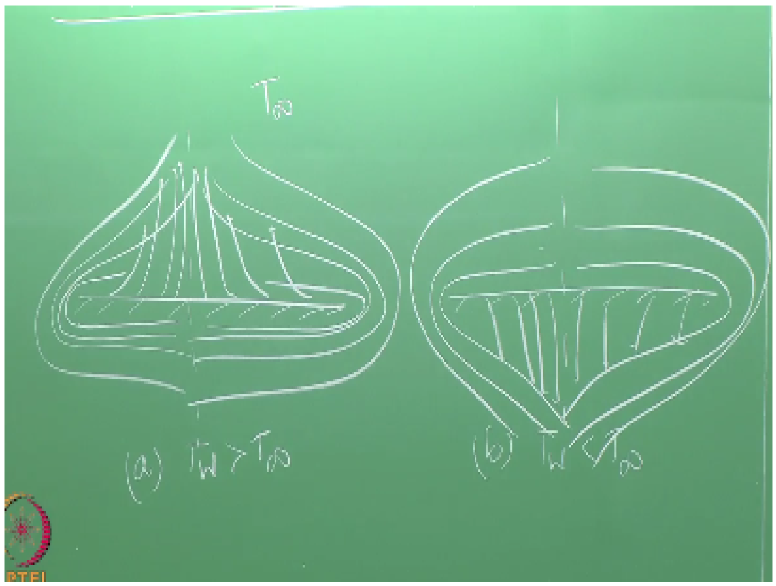

Where tw > T infinity now what do you think will happen when I have a heated plate and surrounding is T infinity and this is finite length so naturally the fluid in contact with the plate which is he getting heated up as a lower density and its tendency will be to rise up so therefore since now you have a finite length so this will try to sneak in through the edges of the plate and this will be symmetric from both the edges so if you draw a vertical line.

So you will have a symmetric patterns in the flow about this line so you will generally see that so this is the kind of patterns that you will end up okay so that is that is the hard fluid which is in contact with the hot the plate at the bottom surface will now have a tendency to escape through the edges if this was infinite so this hot fluid will be just staying there it can have a rise up but due to the finite length so the edge effects now cause the flow of the hot fluid.

To rise up and also the hot fluid on the top surface what will happen so this will also have a tendency to rise up correct so this is a very complex flow pattern that you will have okay so in this case you cannot neglect the y moment so everything will become important so therefore you cannot find analytical solution now what is the case where you cool the plate so in this case again The density of the air surrounding the plate will be heavier compared to the ambient density so therefore what is the tendency it should be the reverse it should try to come down sneak downwards therefore if you draw a vertical line so about that you have a pattern which is something like this and at the same time the heavier fluid at the bottom surface here will also tend to move downwards so in one way the flow pattern at the top surface.

Of the heated plate and the bottom surface of the cold plate are identical and vice versa the flow patterns in the bottom surface of the heated plate and the top surface of the cold plate are also identical okay so therefore the empirical correlations have been developed for one pattern which is this one the other pattern which is this one and the same pattern for this can be used for this also and similarly the nusselt number for this pattern can be used.

For this also an obviously should understand when the nusselt number for the top and bottom surfaces will be different because the flow patterns are locally different okay so I will just give the correlation these are obtained from some experimental observations okay so let us identify this region to be so I will call this as region 1 region 2 here so that is region 1 is the upper surface of the heated plate region 2 is the lower surface of the heated plate and 3 is the upper surface.

Of the cold plate 4 is the bottom surface so for regions1 & 3 so these 1 & 4 so these are identical flow patterns so the average nusselt number based on the length of the plate so you can call this plate length to be some capital L this is calculated as 0.5 4 times Rayleigh number to the power 1 / 4 okay so how do you now so far we have not defined aline number right we have used everything in terms of gash of number so we will define Rayleigh number now Rayleigh number.

Is nothing but another non-dimensional combination of gash of number into prandtle number so many a times you see that gash of number cantle number come together so many times you can kind of say see that in some complex configurations the power of gash of number and prandtl number might be identical such as this case okay both graph number and prandtl number raised to the power 1 by 4 so in that case they can be grouped and you can have one single.

Non-dimensional number called the Rayleigh number so this will be nothing but G beta delta T so now if you define this based on the plate length this will be L cube by nu into alpha right there is the product of garage oven plant alarm so similarly for regions to 1/3 the region 1 & 4 so region 1 is the upper surface of the heated plate 4 is the cold set for the bottom surface of the cold plate now for regions 2 & 3 that is the bottom surface of the heated plate and upper surface of the cold plate we have again identical flow patterns so therefore we have another correlation for that this is given by .27 times L a number raised to the power 1 / 4 okay so this region 2 1 3 so they seem to have a smaller value of heat transfer ate compared to region 1 & 4 ok so these are some correlations.

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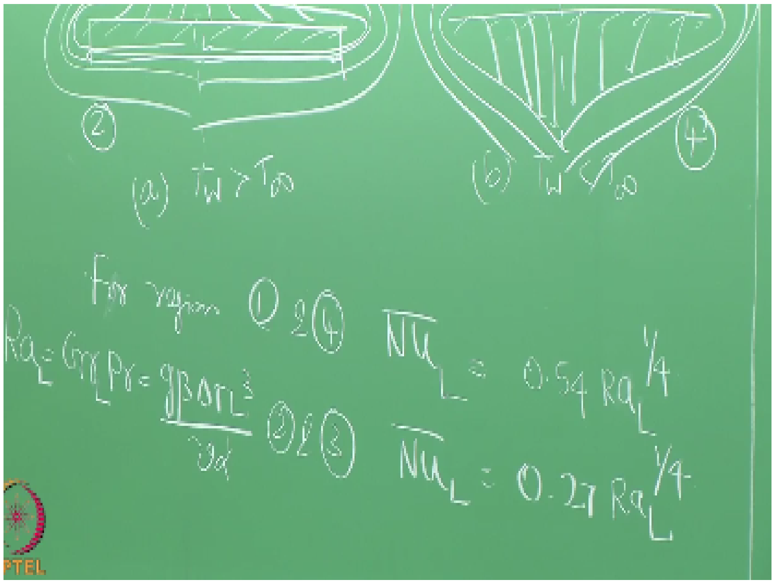

For a more complex configuration like this we can also look at other configurations for example the case of a heated cylinder okay heated cylinder which is the suspended the ambient where T w> T infinity okay so in this case how does the flow pattern look so again you will have something like this okay so you have rounded edges now instead of the sharp corners here so therefore the heated fluid will tend to rise up due to buoyancy and you will get nice flow patterns looking like this okay.

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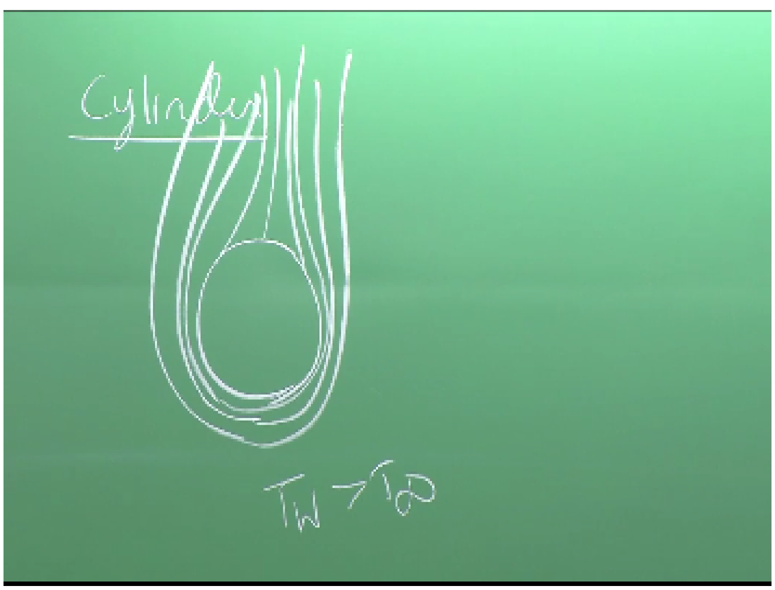

So from the upper surface you also have some flow patterns which are rising similarly for cylinder also for similarly for sphere this is this is cylinder so that means you do not have any variation in the depth it will be uniform within the perpendicular to the board but if you look at sphere there will be variation in the third direction as well okay so but nevertheless if you take a cross-section of that you will find a similar onion bulb kind of a flow pattern so for these there are some correlations available.

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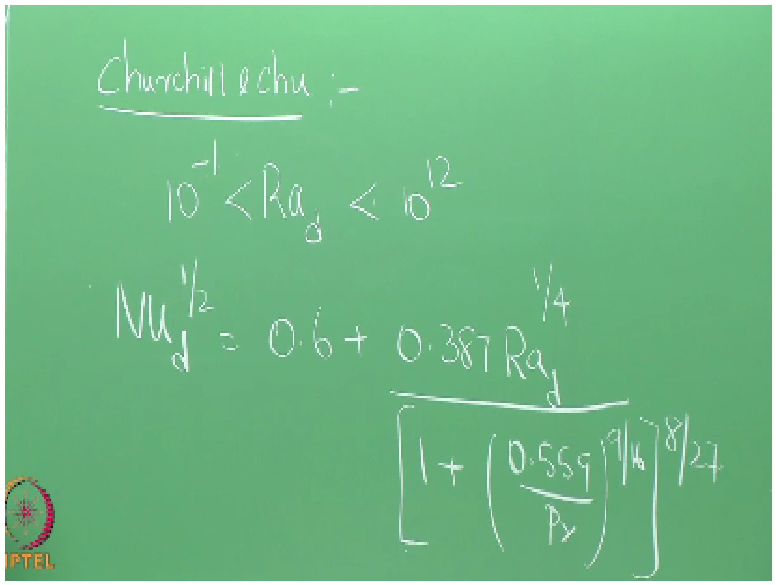

The most comprehensive correlation was developed by Churchill and chu so this is the most commonly used correlation and this is valid for valley number now in the case of cylinder and sphere the Rayleigh number is based on the diameter okay so this is valid for Rayleigh number > 0.1 and<10 power12 that means a very wide range so13 orders of magnitude variation so this has been correlated over a wide range of experimental data.

So this is one of the most popular correlations it is a very complex looking dependence on rally number and prattle number but this is the same kind of formulation which is also done for the vertical flat plate because as you can see the vertical flat plate whatever we have done so far is for laminar flows now if you want to extend it to turbulent flows again we cannot use a similarity solution so the same set same people Churchill and chu they have developed an empirical correlation for wide range of rally numbers so I will also give you their correlation for vertical flat plate which is of the same form.

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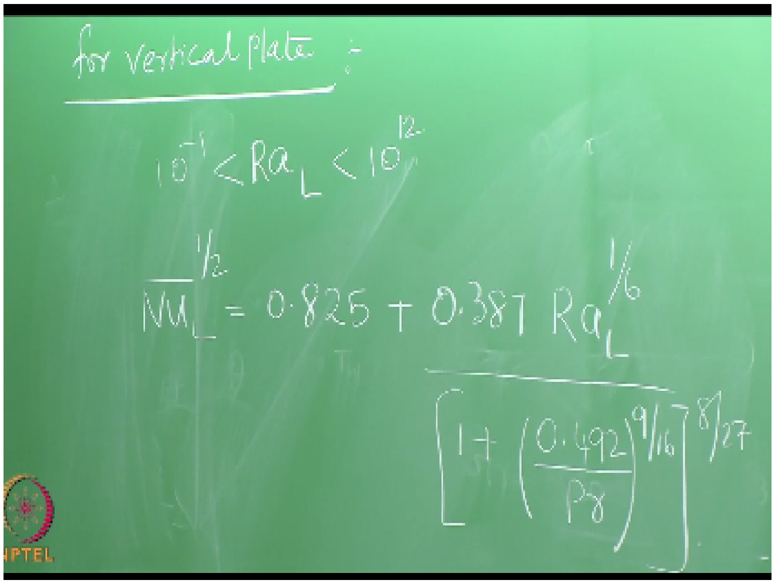

So for vertical plate this is valid for a line number from10 $^{-1}$ to 10 ^{12} and this is the correlation for the average nusselt number based on the length of the plate not the local Nestle sum this is something similar form for instead of the 0.6 you have 0.825 but this also is the same this becomes 1 / 6 here divided by 1 plus 0.49 - number 9/ 16 8/ 27 a very similar form to the cylinder case some smoke coefficients are slightly different and RAL is raised to the power 1 by 6 whereas that it is raised to the power 1 / 2 so again for general heat transfer calculations this is a very accurate for a wide range of dialing numbers all the way from laminar to turbulent right and in the laminar regime this will be recovering the correlation base or stretch okay.

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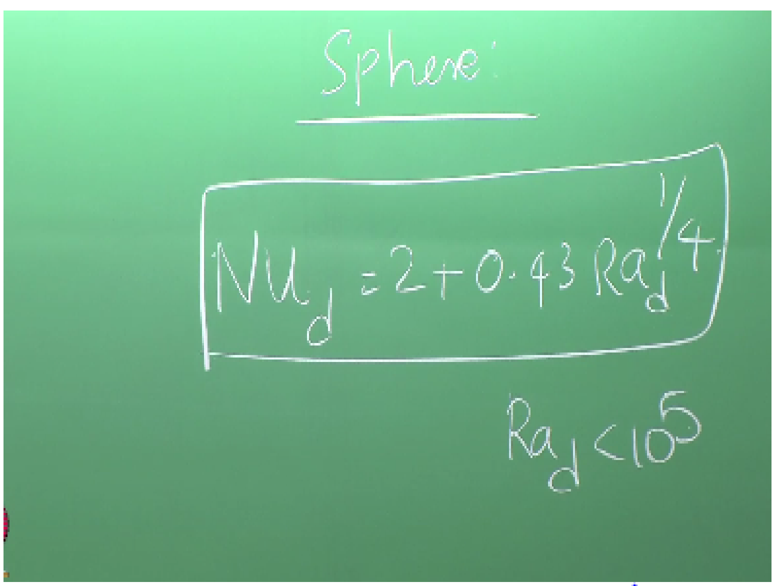

So this is agree very well with our start solution now for sphere for the case of sphere we have a simpler correlation which is fairly accurate to point 2+ 0.4 3 again valley number based on the diameter of the sphere but this correlation is valid only in the laminar regime ok so let me just quickly check its validity because this seems to be quite small valley number range so looks like this package does not have oh yeah that is yeah so it looks like this is valid for a small valley.

Number regime that is correct < 10 power 5 okay we do not have a very accurate correlation for sphere for a wide rally number range okay so these are some common complex configurations so you might encounter anything close to a cylinder you can use the cylinder and three dimensional object you can use the sphere and so on so kind of with these kind of correlations you can approximate slightly more complex surfaces and you can apply these correlations.

So next what we will do is quickly look at the presence of natural convection in internal flows okay so all these have been on external flows there where you place a plate in a free stream and you observe the buoyancy forces which aid the development of the boundary layer now what happens when you confine this completely okay so that means you can look at simple objects like cavities and where you do not have per se anything like a external free stream temperature and density so what happens to these natural convection boundary layers within enclosed spaces.

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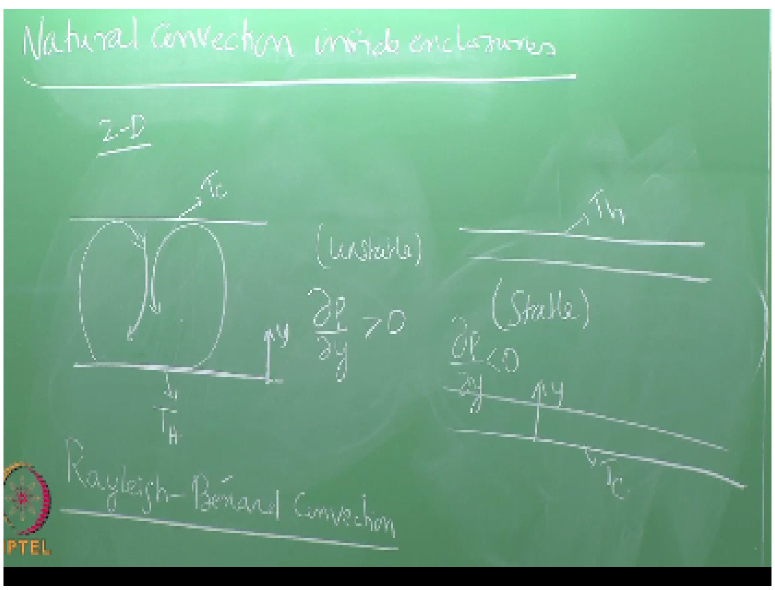

Therefore there is a separate discussion on natural convection inside enclosures and in fact these are far more interesting than the external natural convection with external natural convection we are doing in detail because of the availability of solutions but the more interest in phenomena occur when you have natural convection in cavities so one of them is that you can have two parallel plates in 2d configuration okay you can for example heat the bottom plate and keep the Top plate cold okay and what happens now again due to the buoyancy forces so this will be an unstable configuration the bottom plate will heat the air and the top plate will cool the air and therefore the tendency will be to rise up and this will be to descend downwards in the end you will getting some kind of structures vertical structures like this where the hot fluid tends to rise up cold fluid will come down and this can happen symmetrically okay.

So depending on therefore the length to the height ratio so the aspect ratio of this configuration you can have several number of these vertical structures appearing so this kind of vertical structures are attributed to what is called as Rayleigh Bernhard convection okay so essentially this is happening due to an instability that means if you say this is your Y so what should be your d / dy should be should it be positive or negative it should be positive.

So then only this can be a unstable configuration now what happens if you feed the top plate and cool the bottom plate so in this case what will be D Rho/ D Y negative so what do you think will happen in this case this will be a stable configuration okay the heated fluid will be stratified here and the cold fluid will be here okay the stratification will happen the hot fluid will stick to the top surface cold fluid to the bottom surface but you would not have any of this convection.

Patterns so this is a stable configuration and no convection cells will be formed right so in order to produce the rally Bernard instabilities you need to have unstable configuration where D Rho / DY is >0 so this you can very nicely see the vertical structures happening and you can also see this dimensions in three dimensions you will see a honeycomb kind of a structure but the lighter fluid will be only can rise up it cannot go down so therefore you cannot have a convection.

From the hot to the cold in that case okay so it will always this is always buoyancy aided convection okay so you cannot have a convection which is opposing the buoyancy and going only the buoyancy has to wait it so the natural direction for buoyancy 2/8 is d rho / dy >0 right so for this case these are very complex instabilities and you can to some extent do some linear stability analysis and predict the onset where the instability is can become rally but not convection cells but beyond that you cannot use any analytical methods okay so these are usually predicted by numerical methods okay you have to solve the entire set of nervier-stokes equation with the business approximation and you will be able to capture this kind of rolls convection rolls so I will not have a very detailed discussion.

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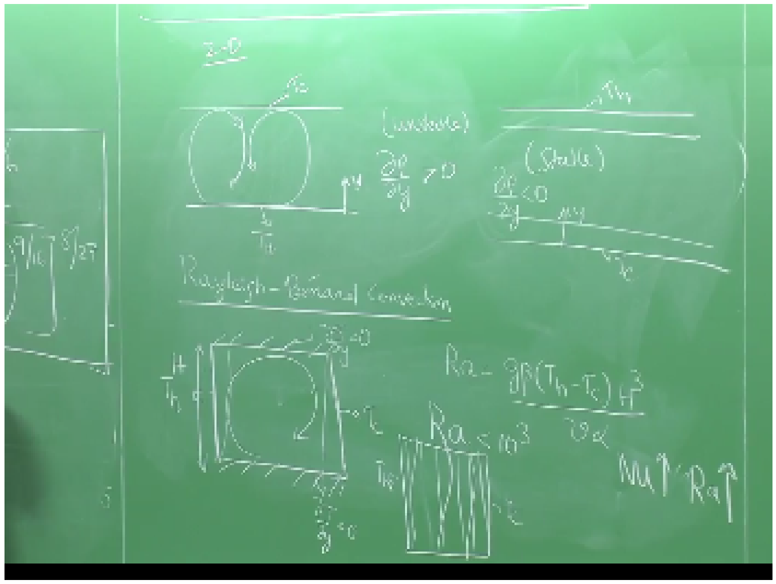

On this research topics beyond the regular classroom thing but the other interesting configuration in 2d is to have an enclosure that is a cavity which is closed on all sides okay and you can heat the left wall of this cavity and you can have a cooler right wall the top and bottom being insulated okay so this is a another interesting configuration okay so now what happens when you have a differential heating between the left and the right walls again.

So you will have a boundary layer which can actually grow okay and again a boundary layer which will descend in this direction so what will happen together in an enclosure will be the net effect will be a vortices which will be driving a convection cell in this direction the clockwise vortex okay from the hot side it can only rise up and from the cold side you can discern down so you will be ending up with a clockwise vertex pattern okay now very small temperature.

Differences if you define for example Rayleigh number based on this temperature difference between the left and the right walls and let us say the cavity is a square cavity with sites dimension H so for small Riley numbers less than say10 power 3 in this case the temperature difference will be too small to produce a convection pattern so it will be only conduction which is happening okay so therefore when you look at the is other.

Isotherms you will find that there will be as imply a linear variation from the hot temperature to the cold temperature so once you increase the rally number the about 10 power 3 that is when you find the nonlinearities in the isotherm so the isotherms will now start deviating like this correspondingly you will find this vortex pattern appearing so as you keep increasing your rally number you find the vortex pattern becomes more complex you might get more than one vortex there okay and what will happen to your nusselt number there is nestled number on these two walls they will increase that your rally number that rally number increases you nusselt number Also consequently increases it increases the convective motion and therefore higher heat transfer rate okay now this is again we can find the solution only by numerically solving the equations nervier-stokes equations okay so what I suggest is that this is a very interesting problem for you to solve equations as such so I would suggest that you follow the solution procedure as given in this book by gusto eosin and Naylor convective heat transfer okay.

So I will give you also the section number where he has explained the complete numerical solution to this cavity problem so he has given a wonderful explanation section 8.9 that is chapter eight 8.9 in Chapter eight natural convection so the topic is natural convection heat transfer across a rectangular enclosure so there he has listed down the equations the boundary conditions and also how he is solving them with finite difference methods huh.

So this is introduction to convective heat transfer by oosthuizon Patrick who is too is a nun David Naylor actually I think this is an e-book now which is available for free I think most of you can all of you can access this is an open access e book okay so this is available on the if I remember right this is part of the open access thermal fluids the thermal fluids they have an open access of journals and books so you should they are more thermal fluid central discard okay so there of fluid central if you go to their website they have some free open access eBook scan although originally it was a published as a hardcopy now there they have a free eBook as a soft copy

Available which you can download okay so especially you look at this section he has very nicely shown the way he is solving these equations is by what is called the stream function vortices method so rather than directly solving the continuity and the momentum equation because if you use stream function it makes the continuity equation redundant so therefore rather than having so many equations you have only two equations one for the stream function the other for vorticity.

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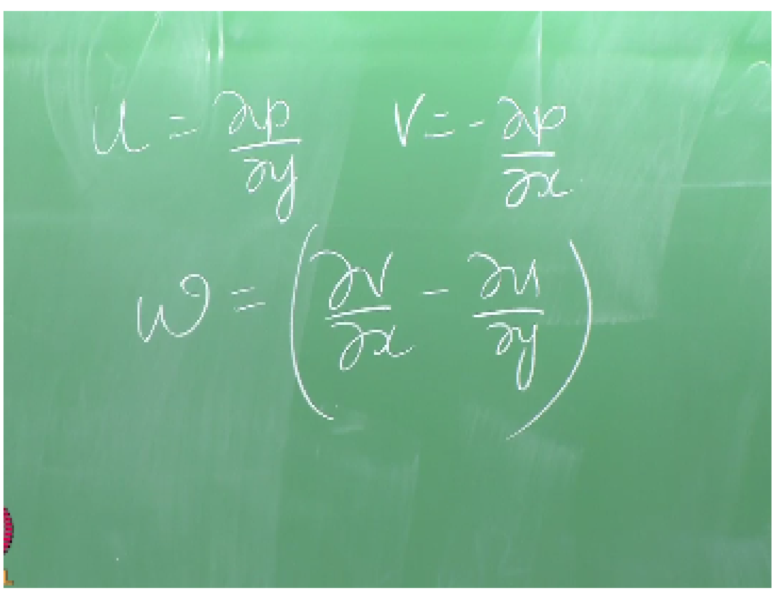

Right so you introduce the definition of your stream function d / dy v is = -DC / DX and similarly vorticity this is DV / DX -D U by so this is then cast the equations the nervier stokes equations both the x and y momentum are then rewritten okay we do some manipulation there like for example we take the derivative of the X momentum with respect to Y and Y momentum with respect to X and then we subtract so we can actually recast these equations.

In terms of a vorticity equation and since the stream function satisfies continuity okay so we will have a simple stream function vorticity equations which can be solved and the appropriate boundary condition and the procedures are all very nicely explained so you can just go through and he is using only simple finite differences I am sure most of you would have already gone through some numerical methods course.

So we should be able to quickly understand how to solve the finite difference equations okay so you can use an iterative scheme for example like Gauss sidle okay and try to find the solution so it is a very interesting problem because once you code it so you can change the relay numbers and you can look at the different streamline patterns so I suggest that you please take this up as part of your final project okay look at relay numbers in the range from 10^2 okay till 10^5 that is three orders of magnitude variation.

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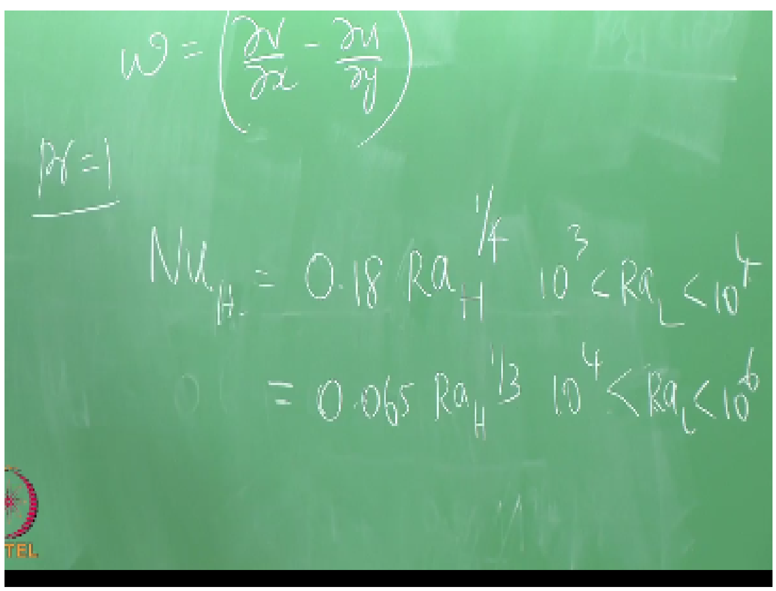

In the relay number and you can actually use the following correlations to compare your numerical solution for the nusselt number you can assume that your parental number is =1 okay and so for the rally number between 10 power 3 & 4 this is the correlation and for rally number between 10 power 4 & 6 the correlation becomes point .0 6 5 la number or 1/3 so you can actually try up to 10 power 6 if you can run the code if you cannot run up to 10 power 6 you can stop it till 10power 5 okay so you run the code get the results for nusselt number.

And compare it plot them as a function of Rayleigh number so your average nusselt number as a function of Rayleigh number so you plot your numerical results and also the correlation should be able to match very close and also you can look at the stream line patterns you can plot the stream lines because already we are solving for the stream function so in mat lab you should be able to plot the stream function isotherms all this for the different relay numbers okay.

So yeah so this is a very interesting exercise of all the things that we have done in force convection and so on because you see flow patterns which vary drastically with the changing value number okay so you please try to submit this project when you come for the final exam okay so that that will be mostly I think third me third may is that in the d slot exam so we will have the submission of the project also on the same day along with your final assignment.

The assignment final assignment I posted it has both natural convection and turbulence we will be covering in another four classes so meanwhile so you will have time to work on the natural convection part so all this together the assignment which I have posted assignment 5 and a project together on the day of your ensign exam ok these are this is a very standard problem if you go through some literature survey you should be able to find many papers to validate and programming is given very nicely explained in this book ok so any questions so in that case we will conclude one last topic which I would talk very briefly a few minutes is the case of mixed Convection ok I have mentioned about this in the beginning of the natural convection part but I want to just emphasize because most of the practical heat transfer problems are you know having the combination of both natural and forced convection you cannot ignore either of them so in that case what happens to the flow patterns.

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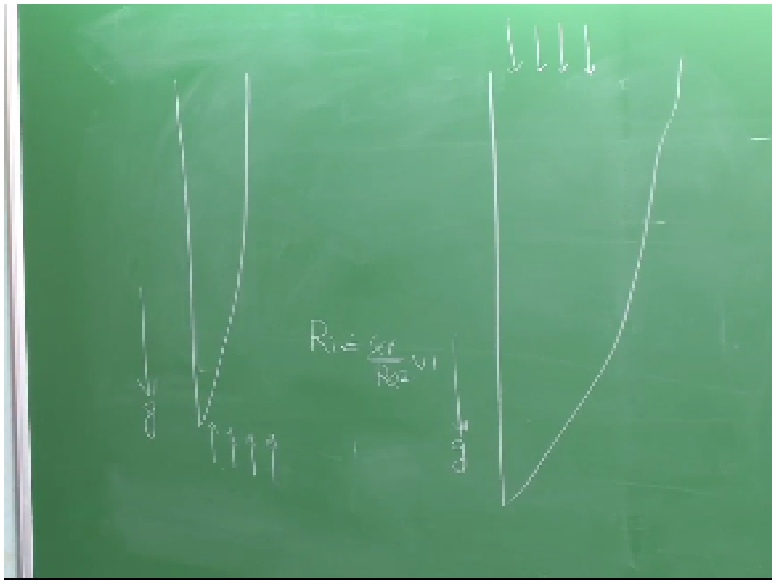

So for example if you talk about gravity acting downward and you have a forced motion bulk motion of fluid and along with that a natural convective boundary layer okso this is one case the other case where the buoyancy is acting upward but the forced motion pushes the fluid downward ok so in this case you will have a boundary layer growth ok you should understand that since there is a forced convection motion in the up direction.

So this will kind of tend to compress the boundary layer compared to the case where you push the bulk motion downwards so in this case the boundary layer will look more stretched out okay so when we do these kind of flows now what is the way to calculate the similarity solution is there a similarity solution for this problem well the thing is what is the correct similarity variable to use because when you talk about forced convection boundary layer.

We use the Belasis variable but natural convection involves rush of number so which similarity variable can be used to describe this problem so therefore if you look at the cases where the ratio of Richardson number that is your gash of by re square is of the order of 1 we cannot find any similarity solutions because both of them are very dominant but for the limiting cases for very small Richardson numbers okay so there the force convection similarity variable might be the correct one and similarly if you go to the other limiting case where Richardson number is basically very large so there the gash of number using rush of number and the similarity variable Might give you the similarity solution but these are only in the limiting cases okay but most of the regimes where both are dominant we cannot find any similarity solution to this again we have to go for numerical solutions but as I told you so the simpler way of doing this is to just blend the nusselt number from four Stern natural convection by using a simple power law expression like this so wherever you have a forced convection.

Which is assisted by buoyancy so you use a plus sign and wherever you have a forced convection opposing the buoyancy you use a minus sign so naturally the case where you assist the buoyancy will be the one with a higher nusselt number because you compress the boundary layer and therefore the ball temperature gradient will be higher correct so this is a very simple hands-on approach if you want a very detailed flow patterns then you have to go for again numerical solutions so I will try to quickly cover the turbulent convection also ok so I think it should be sufficient ok so thank you very much and please take note that you have to also complete the project along with the assignment when you come for the end semester right you.

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