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Video Lectures on
Convective Heat Transfer

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Lecture 4
Momentum and Energy Equations

Okay so good morning to all of you yeah so today we will be continuing our discussion on deriving the conservation laws so we will again revisit some aspects of the momentum equation derivation and from there we will continue and complete it and then look at the derivation of energy equation okay, so let us look at the momentum equation.

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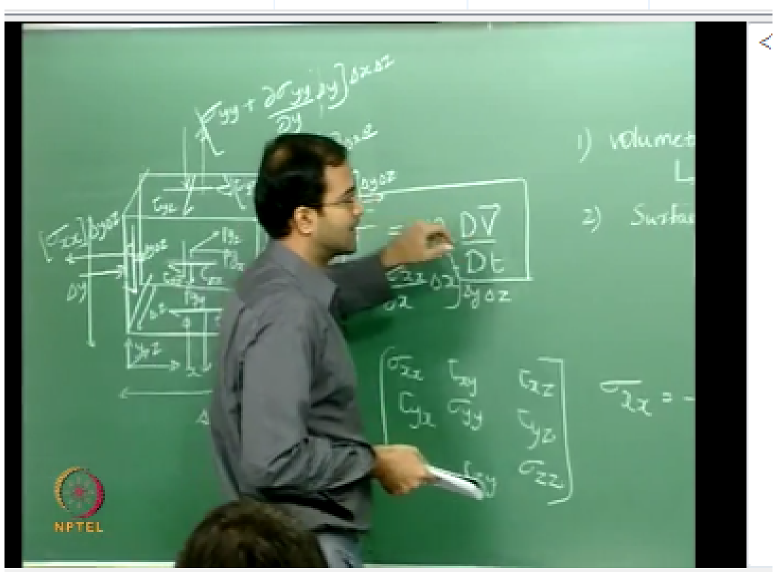

The derivation today first let us consider the control volume in rectangular coordinate system so now we have to apply the Newton's law which says that the force vector is equal to the rate of change of acceleration the acceleration times the mass so which is essentially required to balance all the forces which are acting on this control volume and we already have expressed the total derivative of velocity in terms of the partial derivatives so we will equate both sides okay.

So let us just quickly try to summarize the forces which are acting on this control volume so essentially you have the normal stresses let us put our coordinate system origin as X Y & Z okay so we have the respective dimensions of this control volume as δ X δ Y and δ Z the we will start with the body forces the body forces acting are G X F G X F G Y and F G Z okay so these are the volumetric forces so the acceleration so this is this is the force per unit volume.

That therefore you have the gravitational acceleration times the density and of course then you put these forces for the entire control volume you have to multiply by the control volume to calculate the net force on this control volume similarly when we look at the influence of forces as we discussed there are two kinds of forces.

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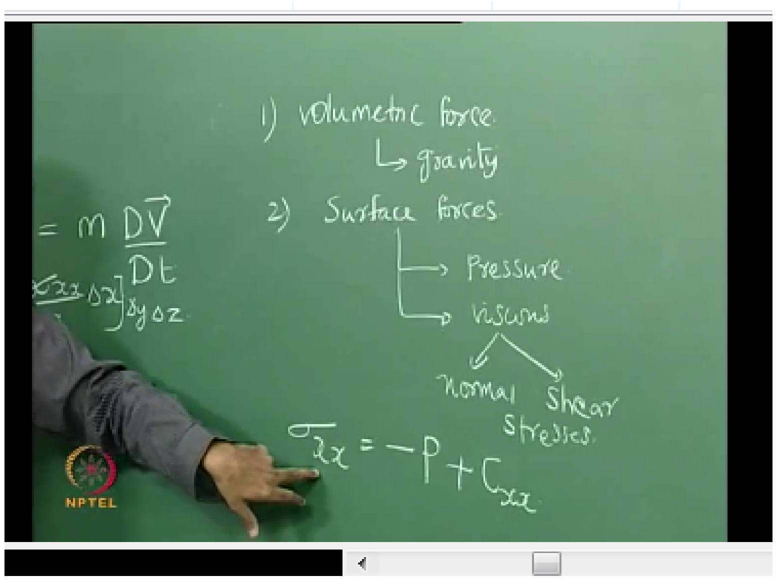

One is your volumetric Force which includes of course in our case the gravity number two are your surface forces okay so these surface forces can be split into two categories one is your pressure force and the other is your viscous forces which include your normal and your shear stresses so coming back let us include all these forces on this control volume so let us begin with the normal shear stress you have Σ XX acting on this face on the left hand side on an explained and on the right hand side on the explained you have Σ XX + D Σ XX / DX δ X of course this is the stress and to get the force you have to multiply / the corresponding area of cross section.

So I am going to show only the forces in two directions otherwise it becomes too cumbersome on this control volume so you can extrapolate and include the forces in the third direction also similarly we have Σ YY okay and Σ YY this is Σ over δ X δ Z this is Σ YY + D Σ YY / DY into okay so this is going to be δ Y into δ X okay similarly you have the normal stress in the Z direction now coming to the shear stresses now these normal stresses act in the direction of the area normal okay.

Which point away from the surface so if you include the pressure forces which also act normal but they are compressive in nature okay so the pressure forces act in this direction okay and now the representation that I am using for Σ okay XX this includes my pressure force and the normal shear stress okay so this is the complete stress tensor that I am using to denote both the pressure as well as the viscous shear which is acting normal to the face okay.

So similarly I will go ahead and include the viscous shear acting tangential to the faces now if you look at the notation as I said the second index here okay the last index corresponds to the direction in which the force is acting and the first index is AK denotes the plane on which the force is acting okay so if you want to for example say I want to calculate what is the shear stress acting tangential to this explained on the left side okay.

So let me say that this force is acting in the downward direction you can also say that force is acting in the upward direction and you will get a get to the same result but in our case we will say that this is acting in the downward direction so this will be τ so the last notation index should be Y and what should be the first it is okay so corresponding to that now this force should cause a turning moment in the anti clockwise direction so this counterpart on the right hand side should be pointing upward okay.

So this should be τ XY + D τ XY / DX δ X and once again this is stress you have to multiply by the corresponding area this is so on and so forth so similarly if you consider a stress which is acting tangential on the bottom surface so we will indicate this as τ can you say what it should be τ YX okay and similarly on the top so now this will cause a clockwise moment okay so this should be τ YX + d τ YX / DY δ Y and this should be multiplied / δ X δ Z okay.

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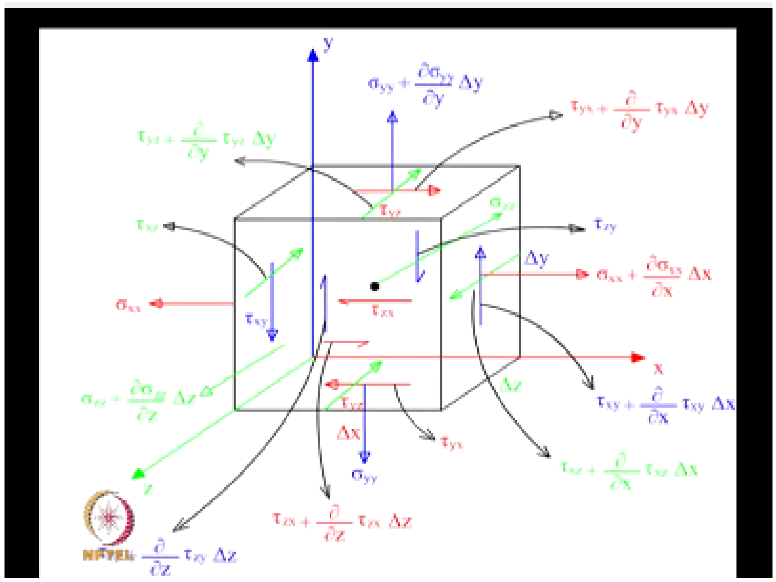

You so similarly you can fill up the other stresses so totally if you put all these stresses together in a tensor form you expand the tensor you actually get a matrix and these are the elements of the matrix you have the normal stresses which include the normal viscous stress as well as the pressure so this will form the diagonal elements of this matrix and you have the tangential viscous stresses which at the off diagonal elements τ XZ and this is , this is your τ YX τ Y Z so τ ZY and τ ZZ okay.

So there are totally nine components so we have three normal components and six tangential components out of the six so we I will quickly show which are the other components acting on these faces and you can fill up fill the rest so you have already to denoted here so the other will be you can take case where it is acting this way on the top surface and this should be what τ Y is it right and you can also take a plane which is in the Z Direction Z plane and you can talk about force which is acting in the along the X direction so this should be τ ZX okay.

And you can also have the same z plane you can have stress which is acting tangential pointing in the Y direction so this should be τ ZY and what else so you have 12345 so one more so you can look at this left and right plane so you can have again a stress which is acting in the Z direction right on and explained so this will be τ X Z okay so this completes all the six viscous stresses acting tangential and the three remaining normal stresses which are acting normal to the surface areas.

So with this we will go ahead and calculate what is the net force acting on this control volume okay so we know the force acting on each phase so we have to say this for the forces which are acting in along the positive x Y Z directions are the forces and the net forces in that particular direction okay so we will calculate the net force which gives you the left-hand side part of this Newton's law so let us calculate the net force in the X direction okay.

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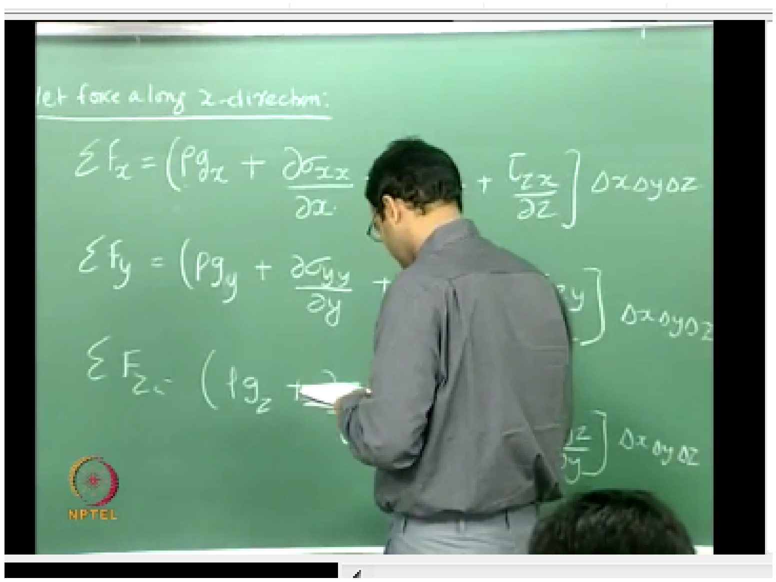

So if you balance these forces in the X direction what should be the net force of course you have your gravitational force okay per unit volume if you multiply it by the volume of the control volume that will be the net force acting on the control volume and what should be the other shear stress terms + D Σ XX / DX okay + D τ YX / DY + V τ ZX / DZ and of course this has to be multiplied / δ X δ Y δ Z right.

So similarly the forces in the other directions now you can fill them yourself so the thing is this you know you are if you look at your notation you can see that the force acting along the X direction will have the stress pointed in that direction and of course you have the gradient of these force in a particular direction in all the three directions okay you have in the X Direction y direction in Z direction so those have to be also accounted for okay.

So similarly if you look at the net forces acting along the Y direction you have Σ YY / why + ∇ τ so what should be this term right here XY / DX + V τ ZY / DZ so this should be the gravity in I am sorry this should be in the Y direction and this should be in the okay so this consists of the net forces acting on this control volume phases and we have also expanded the total derivative so let us equate them before we do that there is one more thing which I want to indicate that although there are six of these viscous forces which are acting tangential out of that we will use the identity that the forces at τ XY should be equal to τ YX .

And similarly τ XZ should be equal to τ ZX and τ Y Z should be equal to τ ZY okay so this will reduce the number of unknowns from 6 to 3 okay so why this is required I think I will leave this as an exercise for you can check you can take the case of two dimensional control volume okay you can balance the forces and you can see for yourself that for example in this case you have τ XY acting here you have τ XY + τ XY / DX into δ X and bottom you have τ YX and here you have τ YX + τ YX / DY into δ Y now this of course you know the dimensions of this δ X and δ Y.

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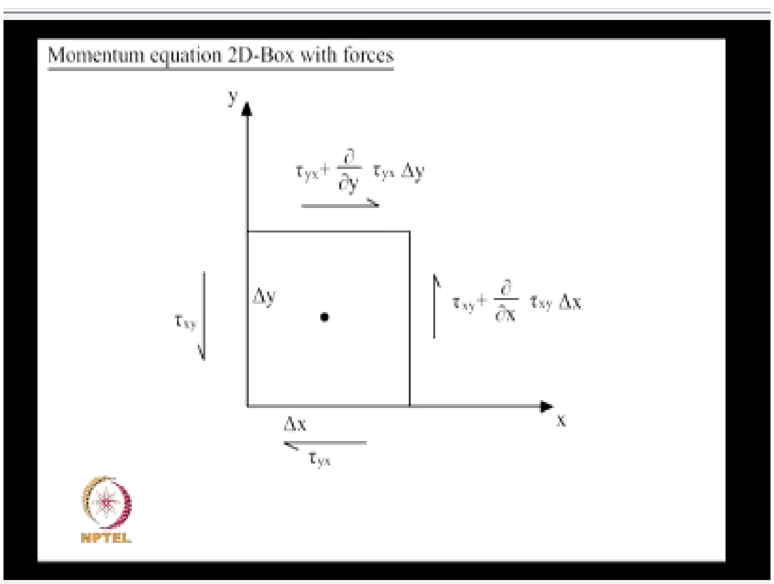

You now if you want to make sure that there is no rotational movement on this control volume okay that comes from the fact that the acceleration if you reduce your δ X and δ Y as δ X and δ Y = 0 so the acceleration of this the angular acceleration that we are looking at should not go to ∞ okay so if you denote the angular acceleration so this is your angular displacement and if you denote your angular acceleration as I am going to use the notation here as D^2 via α / DT square okay.

So this should not go to ∞ so this condition can be satisfied only if there is no imbalance in these forces so that that should be possible only under this condition only if your τ XY is equal to τ YX there will not be any imbalance which will rotate the fluid motion okay and therefore at the limiting case where your δ X and δ Y becomes a point the angular acceleration should not go to ∞ so under this condition this has to be satisfied and which means that there is no rotation to this control was caused by the imbalance of the forces okay

If you also implement that I think now you are number of unknowns in the stresses come down to totally six instead of nine and we will expand the forces and equate them to the total derivative the total derivative we all know that we can write your D U.

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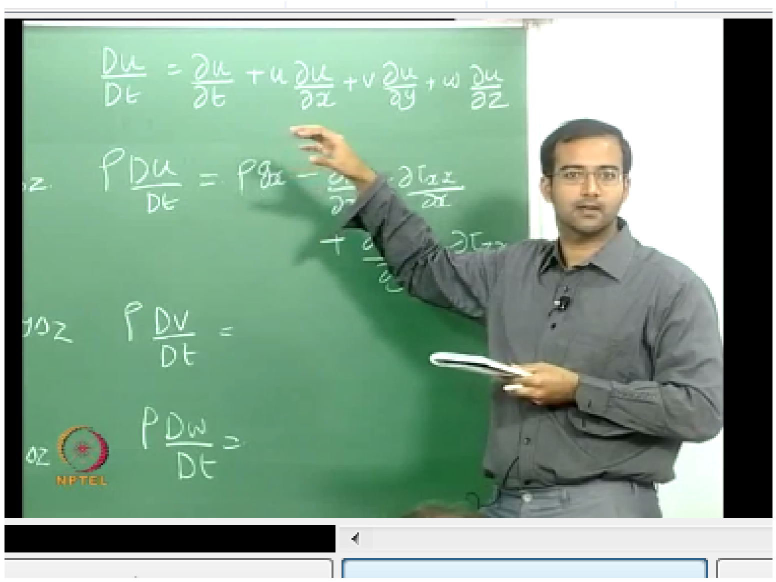

For example by DT as the partial derivative with respect to time plus the spatial derivatives like this okay so on so forth for the accelerations in the other directions as well now so this is converting the Lagrangian framework to and allarran framework so if you equate them to the corresponding forces in the direction we can write this as so we have already mass into acceleration mass can be written as density times the volume okay so we can cancel the volume in both the sides so we will have P be U / DT should be equal to this term for the balance of the forces which is P GX okay.

Now what I am going to do is I am going to take out pressure from the notation Σ XX and retain only the normal component of the viscous stress okay so I am going to say minus DP / DX correct + D τ XX / DX + D τ YX / DY + you have d τ ZX is DZ so on and so forth in the other directions I think you can fill in the blanks yourself so you can equate that to the summation of the forces and now we are going to have to introduce some kind of an approximation to equate the forces in terms of these shear stresses we have to rewrite them in terms of the known quantities which you are supposed to calculate and find out which are these velocities okay.

So the velocities are the quantities which you fundamentally calculate okay whereas the stresses are something which you do not know okay so we have to equate the stresses to these quantities which you probably know after the end of solution and so that is therefore there has to be a relationship invoked to equate the two to relate these stresses in terms of the velocity so that is what we are going to invoke this approximation as a Newtonian fluid okay.

So one of the important so far we have not invoked any great assumptions right we have considered all the forces we have made sure we have balanced them properly we have not neglected anything we have not made any fundamental assumption here except that we are working on a Cartesian framework but now the first assumption is going to be that we will take consideration for a Newtonian fluid and for a Newtonian fluid so we have to write down the relation between the stresses and the corresponding strain rates okay.

So these are given for a Newtonian fluid in the following manner.

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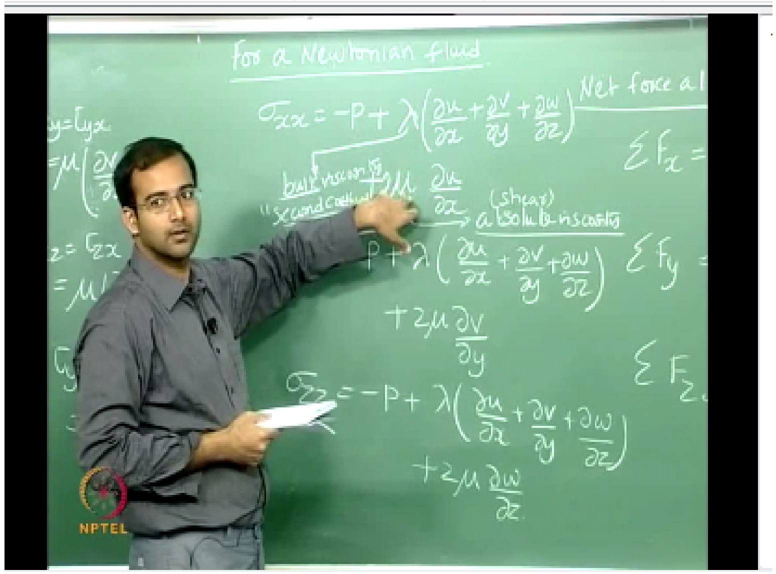

So your normal stresses Σ XX you can write your pressure plus your normal viscous stress that is your τ XX which is written in terms of two coefficient of viscosity okay so I am going to use the symbol λ for one coefficient of viscosity in fact it is usually called the second coefficient of viscosity okay so times the divergence of velocity so which in Cartesian coordinate system if you expand it will be like this plus you have your main coefficient of viscosity ν okay.

So there will be there will be a 2 ν D u / DX similarly if I go ahead and write the relationship in the other directions you can write the same thing the divergence term will be there as it is + you have 2 ν into so can you guess what should be ∇ V / ∇ Y okay similarly if you write in the Z direction okay so these are valid for a Newtonian fluid and how about the shear stresses okay so the shear stress can also be expanded in a similar form for a Newtonian fluid so let me raise these portions.

So we have τ XY we have already shown that for inducing the no rotational force on the control volume this should be equal to τ YX now that is given as a relation between the stress and the strain rate as this DV / DX + D U / DY similarly τ XZ should be can you guess the terms here so DW DX + D u / DZ okay good so you have the final term what is the final term you have τ Y Z τ ZY there should be new so this will be DV / for example DZ + DW / D Y so this is a closure now we got the closure problem and we closed it using the Newtonian fluid approximation where we are relating the stresses to the strain rates by this okay.

So this is also an approximation if you work with non Newtonian fluids okay then you cannot apply this kind of relationship but we know that is this is almost saying you have a linear relationship between the stress and the strain rate okay now what are these λ and μ here okay so λ denotes if you come to this right here so this new as you all know this is your absolute viscosity right so this is also called as your shear viscosity.

Sometimes which you are all familiar with when you say viscosity is the property of the fluid you talk about the absolute viscosity or the shear viscosity now you have also introduced another coefficient here which is the λ which is actually the bulk viscosity this is sometimes referred to as the bulk viscosity also it is more commonly referred to as the second coefficient of viscosity okay so there is still a lot of debate on the purpose of introducing these two viscosities and what is the role of the second coefficient of viscosity.

Anyway so in order to finally close this you also need to you know that μ is the property of the fluid so you also have to introduce a relationship between the λ and μ okay so that is introduced by what is called as Stokes hypothesis so we are introducing approximations one after the other.

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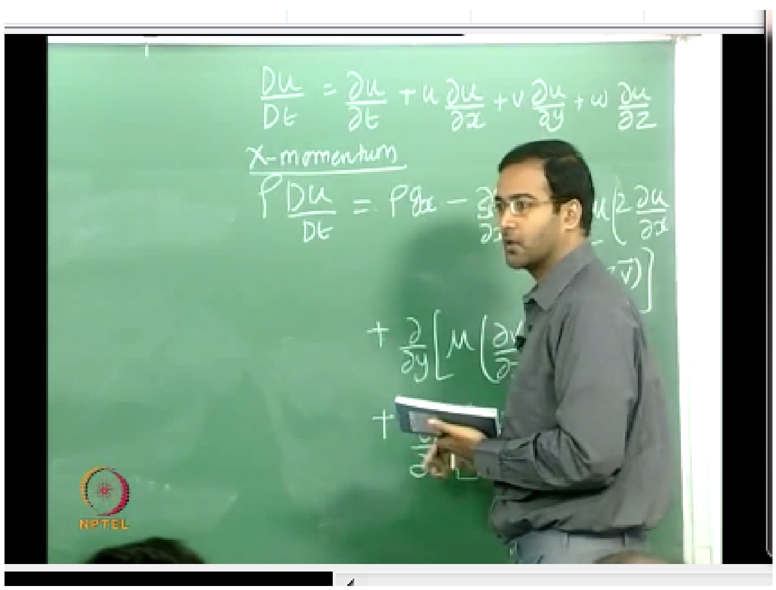

So the Stokes hypothesis States so this is an hypothesis you know it is you can see from the name you know Stokes hypothesis means in people have not exactly verified it its mathematical convenience to reduce the number of unknowns and what Stokes says that if you introduce the fact that three $\lambda + 2\nu = 0$ that means your λ is $= -2/3\nu$ okay so this will give you a nice form of the relationship between Σ and the strain rates convenient to suit your things to simplify a little bit and in fact when we do that we can we can also rewrite we can now plug in for these stresses in terms of the strain rates okay.

Now I am going to rewrite them in terms of the strain rates rather than the stresses so for the X momentum I will replace the derivatives of these stresses in terms of the strain rate so I can say D / DX okay so new into 2 D U / DX - 2 / 3 into ∇ dot P so which is the divergence the divergence is actually given by this term right here this is the ∇ dot V term okay and 2 / 3 comes because your λ is equal to - 2 / 3 ν okay apart from that you also have your D U / DX 2 ν D U / DX okay that is essentially coming from this term right here okay.

So I am just grouping the terms under the normal stress okay similarly if you expand that in terms of shear stress and write in terms of the strain rates you get D / DY into ν so what should be the term so you have your τ XY is equal to τ YX so I can write this as D / DY ν into what DV / DX + D U / DY so I can also go ahead and complete the third term D / DZ of τ ZX should be equal to τ XZ that will be DW that is ν times DW / DX + D U / DZ okay.

So it is a very lengthy expression so this is the X momentum equation so which is coming to a more familiar form right so if we started with putting the stresses and now we have invoked the approximation of Stokes and the Newtonian fluid and finally we have we are writing this in terms of strain rates so we will simplify 1 / 1 gradually.

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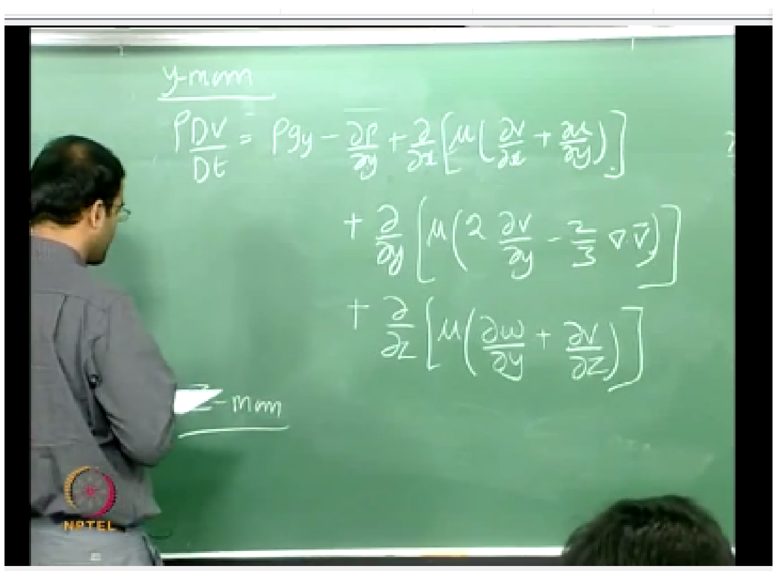

So we will write the Y momentum equation similarly you have P D V / DT so there should be P GY - DP / DY okay + D / DX of can you tell me the term that should come μ into there should be is there a 2 here or the Y momentum what is the term corresponding to D / DX in the Y momentum τ YX right.

So τ YX and τ XY have the same okay so this should be DV / DX + D U / DY all right + so D / DY so now you have your normal stresses okay so you have ν into 2 DV / DY minus 2 / 3 divergence of velocity + you have D / DZ μ into DW / is it DW / DX DY + DV / DZ okay so you can write these Z momentum similarly I am not going to do it I will give you a couple of minutes to write down these Z momentum also the same manner.

Yes it is been experimentally proven at least to be found close to the actual value laid is λ found to be close to minus watching you because you cannot simply accept such a yes that is true well Stokes hypothesis has not been proven directly but the final reduced form of the nerviest stokes equation which should get after invoking the Stokes hypothesis is definitely agreeing very well with the experimental data okay.

So that shows that Stokes hypothesis is pretty good now Stokes when he the hypothesis he did not do experiments to confirm his eye passes he just proposed bits based on some intuition and based on the structure of the nerviest stokes and later when you use this equation and solve them entirely so you are seeing that you get very good agreement with the experimental data so that itself confirms ok so now I hope that you have completed the Z momentum now what we will do is this is still not the simplified form of the nerviest stokes that we would like to use ok.

And as I said we will be focusing on primarily incompressible flows now this is valid for both compressible and incompressible flows will make introduce another approximation that we will be looking at only in compressible flows so for that.

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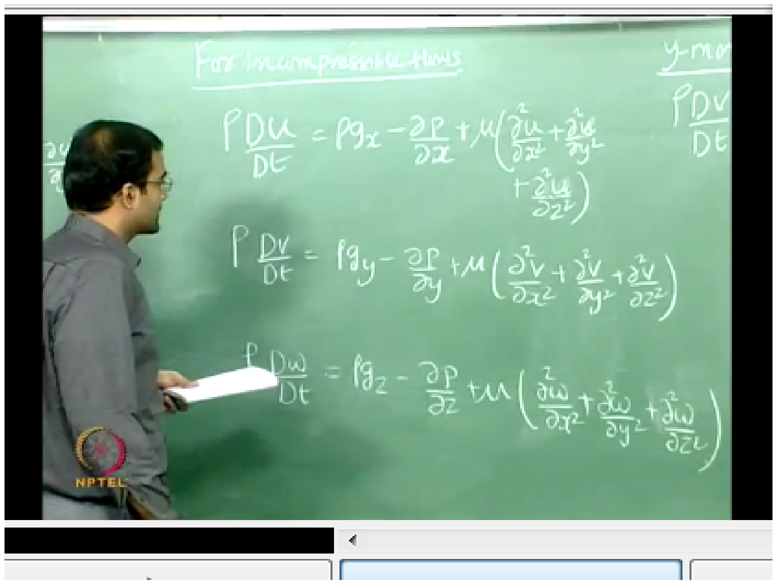

You will see that magically many of the terms drop out especially the divergence term should drop out because it satisfies the incompressible continuity equation right so you have your divergence term which drops out from all these equations and you can also group the terms in such a manner so here for example ok if you look at this terms right here ok you can write this as D / DX of DV / DY + you have D / DX of DU / D / DY of DU / DX + you will have similarly D / DW / DZ into D / DX so all these terms.

Once again you can take D / DX common and you have DU / DX + DV / DY + DW / DZ so that will again satisfy the incompressible continuity that will drop out so finally when you write only the rate and only the necessary terms you will have P D U / DT = P GX - DP / DX + ν times D^2 u / DX^2 + D^2 V / DY^2 + D^2 W / DZ^2 so this will be our X momentum term, your Y momentum will be so this will all be u in terms of U I am sorry okay.

So this will be in terms of V okay so I think you can relate and verify the terms once again okay so you should have this term knocked out and then you have D^2 u / DX^2 you have D^2 u / DY^2 you have D^2 u / so you can take D / DX of DU / DX DV / DY DW / DZ which will disappear so from each of these momentum equations you will have you know you will satisfy continuity here as well as if you group the terms together like that okay.

So hope all of you got it okay so the remaining terms will just be the lap ration operator okay so this is the more familiar form i guess correct when you start with analytical solution of equations you are more used to this form right for incompressible flows we can write also this form in terms of a coordinate free representation you know that is what we want to do if you want to apply this in all the coordinate systems.

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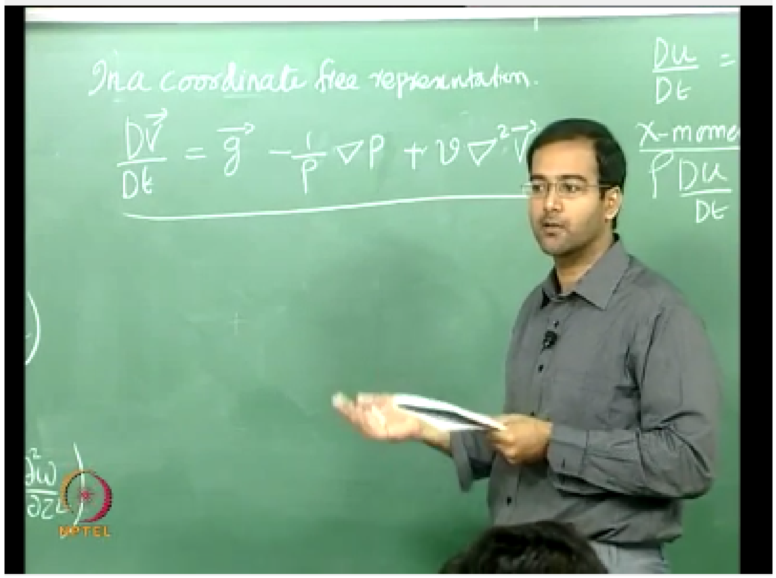

So in a coordinate free representation so we can write this as DV / DT and V is a vector here indicating velocity in any direction that should be equal to M dividing throughout by P so that will be G vector - 1 / P into how can I write the pressure gradient ∇ PY okay and the term here D^2 u / DX ^{2} and so on so forth this is nothing but the lap ration operator ∇^2 and the corresponding v vector so this is a very compact notation which indicates the momentum in any direction that you consider in any coordinate system.

You have to use the corresponding the derivative operator as well as the lap ration operator in that particular coordinate system and you will write those equations in either cylindrical or spherical coordinate system okay so what we will be doing mostly in our course is we will be focusing on the incompressible form of the nerviest stokes equation and will also apply this to two dimensional flows.

So therefore the third derivative will all be neglected okay so that will be the compact form of the nerviest stokes that we will be considering so with this we have completed the momentum equation derivation very quickly let me start the energy equation derivation okay which is more important because you have probably gone through the momentum and continuity in your earlier courses also any question on the derivation but I think it is very clear right you have already done it so it is just a reconfirmation of what you have learnt.

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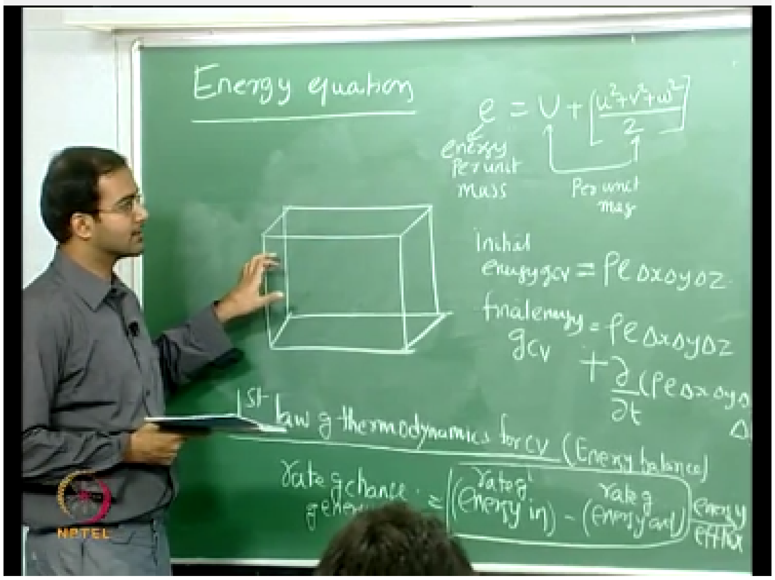

Let us start with the energy equation now once you understand the momentum equation clearly I think the energy equation just follows only you should know the contributions to the energy okay what are all the components of the energy so let me raise okay so when we come to energy we have to be very clear what is the balance of energy that we want to do so once again just like we have a mass balance we will write down the first law of thermodynamics for a control volume.

So which is nothing but the energy balance okay so what does the first of thermodynamics say for a open system the rate of change of energy within the control volume should be equal to the net flux of energy across the control volume boundaries okay so this is a very compact way of phrasing the first law let us expand upon the term a flux of energy so there are many contributions to that so what it says clearly is that your rate of change.

I am not using notations right now I will introduce the notations one by one rate of change of energy in a control volume should be equal to your energy in or rate of energy in minus rate of energy out okay so this is represented in rate terms so therefore I should say rate of energy right energy which is instantaneous so per time you know in a given time the energy which is basically crossing the control volume boundaries now when I talk about the this is nothing but the influx of energy right totally so this is your energy flux okay.

So now this reflects of energy has the following components okay so let us look at first the energy and its constituents you can use the notation E to denote energy per unit should it be per unit volume our unit mass per unit mass okay so that should have contributions from what so suppose if you look at the energy of a system this is the total energy has contributions from its internal energy let me call that as u this is the internal energy per unit mass + you have a kinetic and potential energy terms okay so I am going to put the kinetic energy term here $U^2 + V^2 + W^2 / 2$ this is also these are per unit mass okay.

So what I am going to do I have not included the potential energy term here because I am going to consider potential energy term under work ok because already you have the forces which are acting on the control volume and they do some work and gravitational force is one of them so I am going to consider that as a part of the work rather than as a part of the total energy contributing to the temperature of the system okay.

So anyway it will all come in the balance of energy so it is not lost anywhere so this is the definition of energy I am going to use so therefore so the initial energy of the control volume will be what so if I know this is the energy per unit mass for the entire control volume what will be the energy into mass so that will be density times the volume times the energy is right so ϱ e into δ X δ Y δ Z right so this is going to be the energy processed by the control volume at some starting time from which you are calculating the change in the energy after a time δ T okay.

So the final energy of the control volume after some time δ T you want to see what is the energy at that point okay so that will be P E δ X δ Y δ Z and so how should I so I should use the Taylor series expansion okay if I think that the time is infinitesimally small I can write the Taylor series expansion right so + D / DT of P E δ X δ Y δ Z into δ T okay so therefore your rate of change of energy of the system will be your final minus initial.

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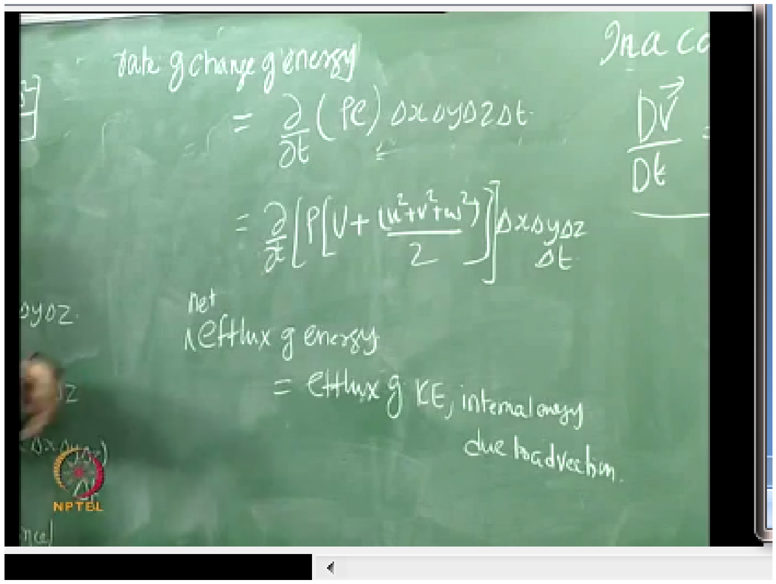

So you are so that will be what D / DT of P e okay and multiplied / δ X δ Y δ Z δ T now where you re is expanded into no constituents of internal energy and the kinetic energy in that manner so I can I can also write this as D / DT of P times U + U^2 + V^2 + W^2 / 2 okay into δ X δ Y δ Z so this rate of change from know you are calculating at later time minus the initial time okay .

So yeah if you have an energy which is depleted it will be negative if you have energy which is gain it will be rate of change will be positive right ok so this is the rate of energy change of energy which we are balancing on this on the left hand side with the energy flux terms now when you look at the flux of energy the net flux of energy so you have components of what so what are the things which can come in and leave the control volume.

So one is the components of internal energy and kinetic energy itself can be carried by advection process okay so you have a flux of your kinetic energy your internal energy right due to advection okay and you also have components of what apart from the components of the energy which constitute the total energy of the system what else can change the energy of the control volume conduction so that is basically a heat transfer process so you can have heat transfer which is happening at along across the control volume boundaries.

You can also have work transfer okay when you talk about energy you have both components of heat and work okay so you have a flux of these also you have + you have a flux of heat okay now if flux of heat is not happening due to advection okay it is purely a diffusion process for a conduction process okay so + you have a flux of work and now I how a flux of work is happening already you have these forces acting on the boundaries and so they do some work on the control volume boundaries okay.

Now the contributions of each of this now the flux of internal and kinetic energies is easy to write so I am just going to write it down.

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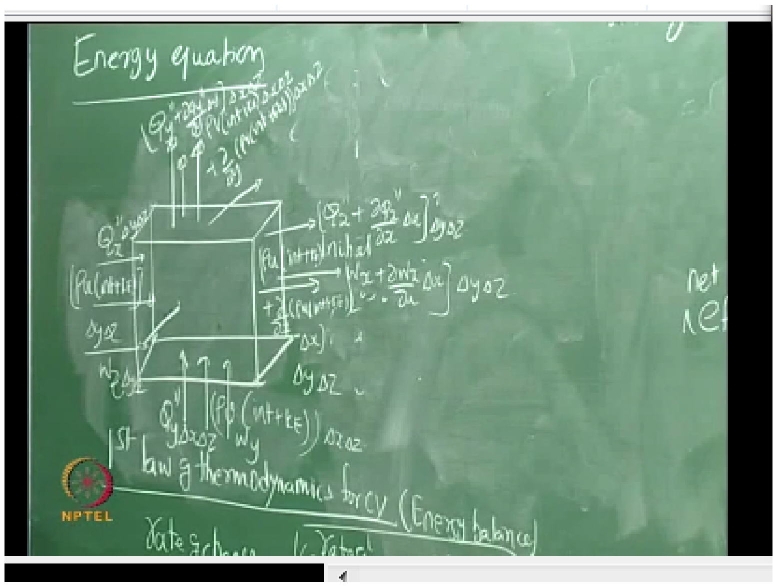

So can you guess how the structure of reflects of internal and kinetic energy should look so it is like I am saying if you take this control volume you have the flux of internal + kinetic energy which is entering for example on this left explained and on this side this will be P you internal + kinetic + D / DX of P u internal + kinetic into δ X right you have entire thing multiplied / δ Y δ Z just similarly in this direction you know you can also write this as P V into internal + kinetic energy into δ X δ Z this will be P V alright.

So similarly in the third direction you can write it apart from the Flex of the energy of the system in Flex of the energy which is coming due to advection you also have the heat transfer so you can say this is probably QX that is the heat flux which is crossing this particular left explained in the X Direction multiplied / δ Y δ Z ok so this is the heat transfer rate in this direction so on this particular plane this will be QX double prime + DQ X double prime / DX into δ X into δ Y δ Z okay so I will probably raise okay.

Similarly you have conduction of heat in the other directions as well so I am representing heat flux in this direction into δ X δ Z will be the heat flow rate or heat transfer rate and similarly you can express this as and also in third direction you can do the same apart from this you also have work transfer okay which you can write it down so work transfer in both the directions okay.

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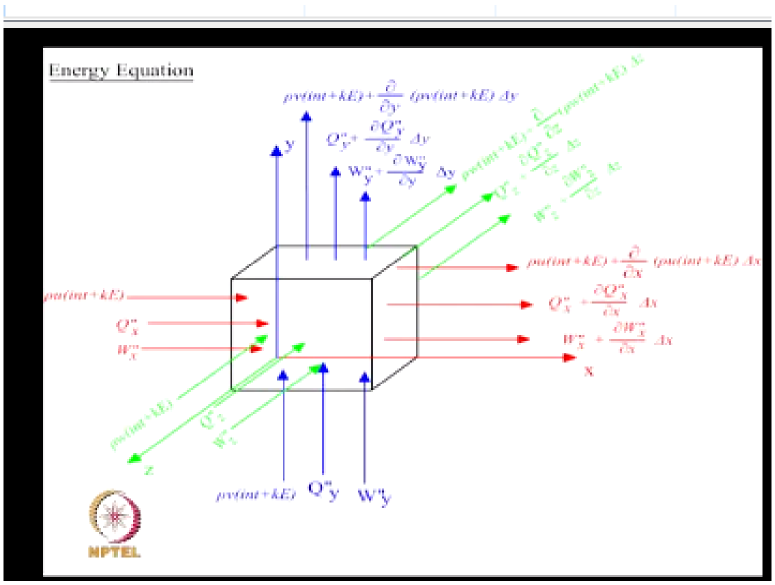

You so now so this is the complicated part of the derivation of energy equation you know you have heat transfer rates you have work transfer rates you have flux of energy everything coming you know into the control volume in a control volume and leaving at the same time so you have to balance all of them in one single equation so that is why it gets a little more tedious than know the continuity and X momentum and because the momentum as such is directional.

So you can very easily balance in each direction whereas here you have to include all the contributions into one single equation okay anyway so we will stop at this point and tomorrow we will continue and this is from the Arian point of view we will also do a derivation starting from the Reynolds transport theorem which is completely coordinate free and it connects the Lagrangian to the oil Arian so from there we can reach to the same conclusions okay so thank you so much.

Momentum and Energy Equations
End of Lecture 4

Next: Energy Equation
Online Video Editing / Post Production

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