Indian Institute of Technology Madras

Presents

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National Programme on Technology Enhanced Learning

Video Lecture on

Convective Heat Transfer

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Lecture 38

Similarity Solution in Natural Convection for Verical Is flux Plate

It is a very good morning so last week we were looking at the solution the constant heat flux case which was extended by Sparrow and Greg to the basic solution which was done by Paul Hassan and the solution which was obtained by awe-struck later on to the constant heat flux case and here typically we have to define what is called? As a modified because we do not know upfront what the temperature difference is going to be in order to calculate the Gr based on Δ T therefore we do a simple scaling and convert the Δ T in terms of heat flux.

Which is known okay so based on that we have defined a modified grand if we look at the same similarity variable which were housed in as you okay if you look at the H = to x times grashalf number by four whole power 1/4 so if you substitute the modified Gr into this you find that there is no dependence on X okay which cannot be possible because the similarity variable has to be a function of both x and y and therefore Sparrow and Grieg modified the definition of

H a little bit they just introduced to the power 1 / 5 instead of 1 / 4 and which is now going to be that function of both x and y so we will see with this modified definition of similarity variable whether we try to successfully reduce this into a similarity differential equation so before doing that let us also calculate what is the order of magnitude of the reference velocity okay so since we start from the basic assumptions in the similarity solution that u / u reference is actually a function of H so this is the similarity solution assumption that means if you plot a non-dimensional velocity profile so it should be only a function of H.

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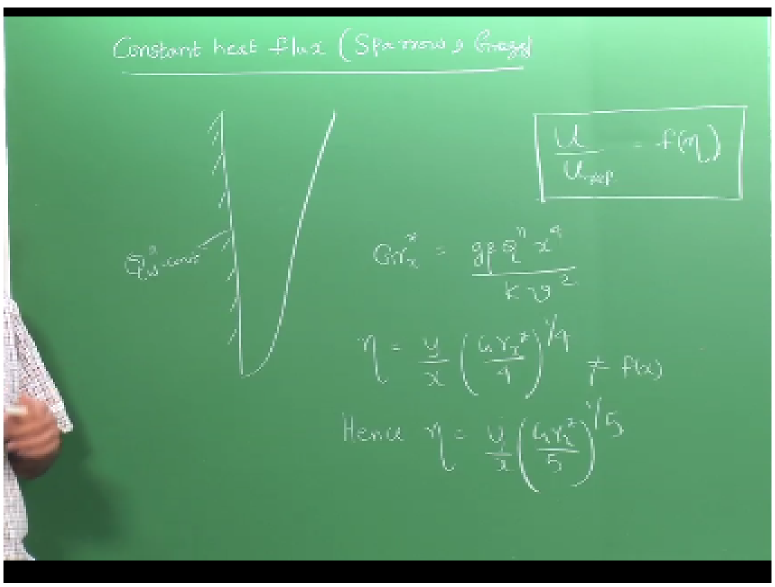

Correct so for this you need to know what the you references so how do we estimate you reference so this is the same way that we did for the constant w temperature case so there we equated the order of power of Gr so what did we get there the Gr power half is the same order as Reynolds number.

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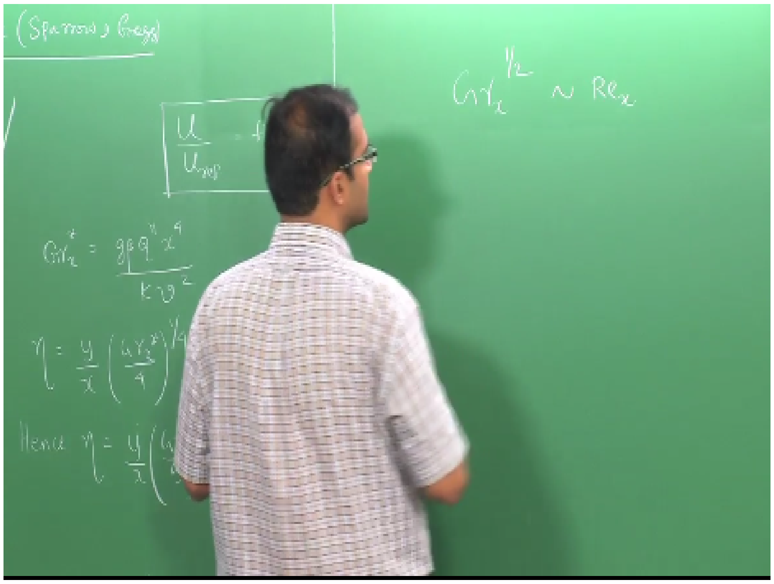

Correct so when we talk about therefore a Reynolds number to the power 1 / 2 this was 1 / 4.

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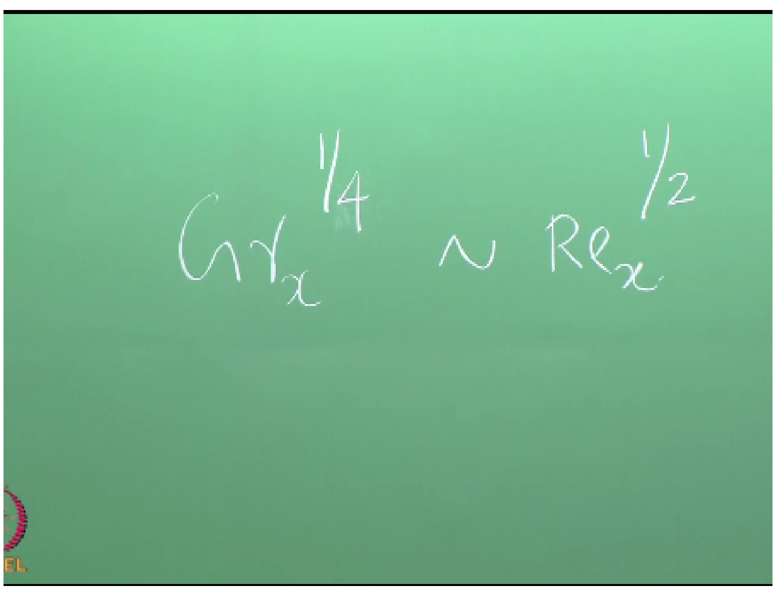

Okay so therefore wherever we had a similarity variable in the original Blazes equation if you remember this was y by x re x power 1/2 this is your blushes similarity variable so we replaced re X power half with Grote the power 1 by 4 here that is how Paul Luzon started so now in the present case out with the modified ration of number we have a modified similarity variable and now therefore how do we do the order of magnitude so in terms of Gr star how do we equate it this cannot be the same because now we have modified our similarity variable to be Gr star to the power 1 / 5 so only so 1 by Phi should be on the order of magnitude of re X to the power half okay according to the current modification of the definition of similarity variable the Sparrow and Greg so this should be the order of magnitude of your modified Gr okay so now can you please substitute our definition of modified   Gr calculate what is the order of magnitude of u reference.

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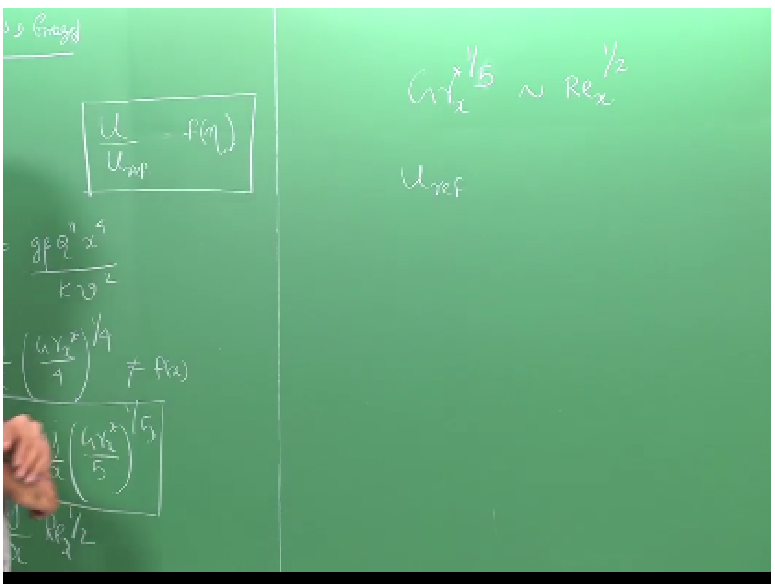

Okay huh that is therefore f of H here we started off with okay so you use to Jake vita okay fine use G of H and then we integrated G of returns called that as a forfeit okay it's a function of H that is all it is so now by equating these 2 you can find out what is the order of few reference tell me in terms of Gr because we have Gr here okay so you can directly tell me in terms of Gr in terms of modified - of now what do you get you get a factor of five here outside Gr / 5.

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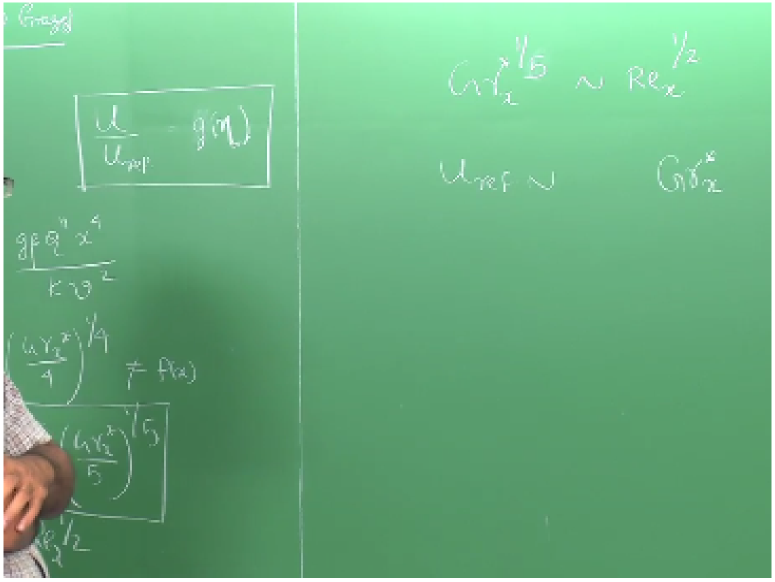

Raised to the power two by five right x **μ** / X is that okay right so this is you are you by this is your new reference now we can find out the corresponding transformation of the field from X/Y to heat up by using side how size is nothing but integral u d y so which we will substitute from this as u reference into we will write this as integral G H into d y as d y / D H x D H so d y / D H will take out x D the and integral G H D H is nothing but what you call as f of H is another function okay therefore size is = to UF x d y / D H x F of H.

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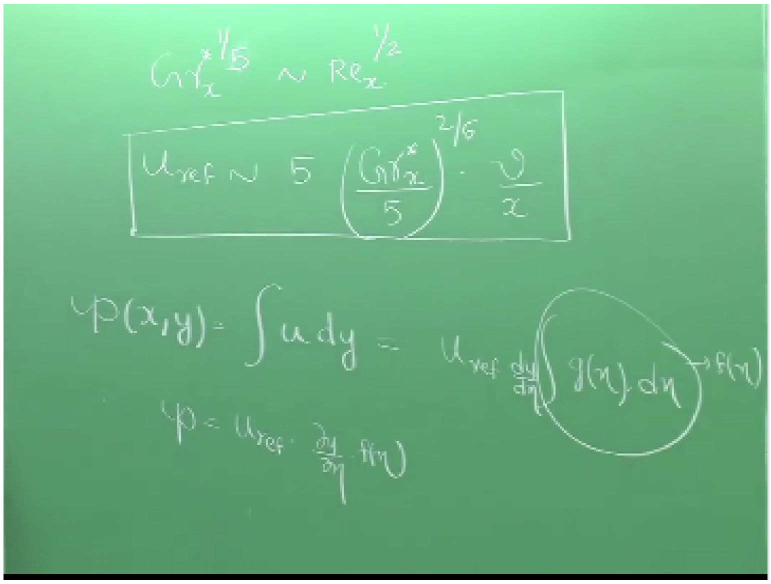

So we know d y / D H in terms of Grand new reference also we have calculated the substitute and tell me what will be a function side in terms of Gr the modified Russia what is d y / D H so X X cancels here so this gives 5 x rash of modified Gr / 5 the whole raised to the power 1 /5 into new I will put this new here okay so therefore once you find out sigh now we know the transformation between therefore sigh and so this should be multiplied by a for Peter so we know the transformation from XY the flow field in X y coordinate to H through the relation between s y and death okay also we know the basic transformation.

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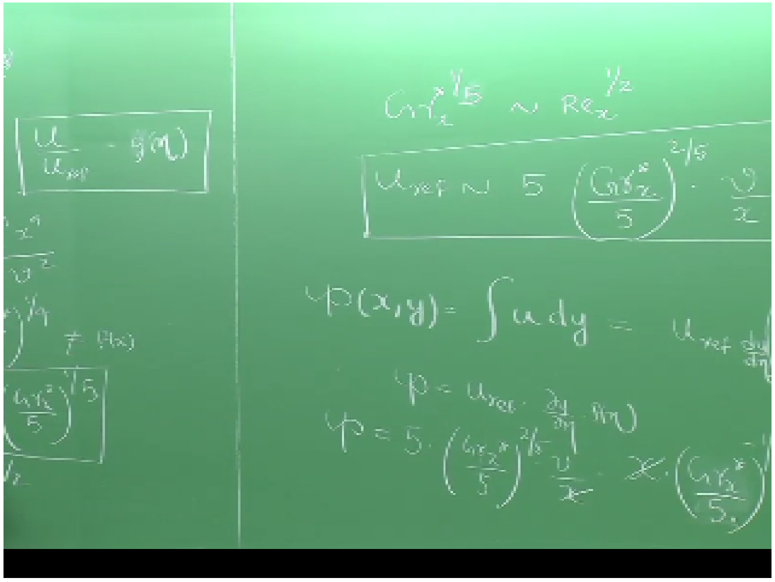

Through the similarity variable so therefore now we can substitute this into the governing equation the momentum as well as the energy equation the same way substitute for U, V, D. U / D X D u / d y d square u /d y square okay and you will be able to reduce this partial differential equations into similarity equations so before doing that also we need to find out the similarity variable for the okay so let us assume now the is a function of H but we have to find the right non dimensionalization here so in the constant w temperature case.

We use T_ T ∞ /_ T ∞ / the constant heat flux case we do not know the w temperature and therefore this has to be somehow converted in terms of heat flux again so we can again use the four years heat flux to do a scaling let us say that this is your Tw this is your T ∞ at the edge of the boundary layer so what is this distance Δ T this is your thermal boundary layer thickness therefore if you apply the four years law this will be K times T w _ T ∞ by Δ T so therefore we can find a scaling for T w _ T ∞ as Q double prime Δ T /K.

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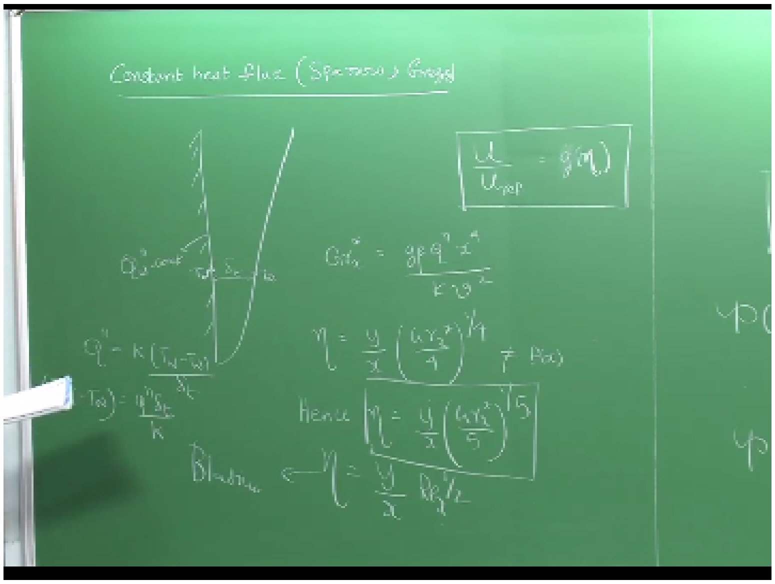

That okay so now the order of Δ T is the same as Δ right so we can just substitute this as T_ T ∞ by Q double prime Δ / K and what is Δ now how do you find out Δ hmm η is = to Y / Δ correct so therefore we can find this as d y by D H right so I am going to replace this as d y / D H.

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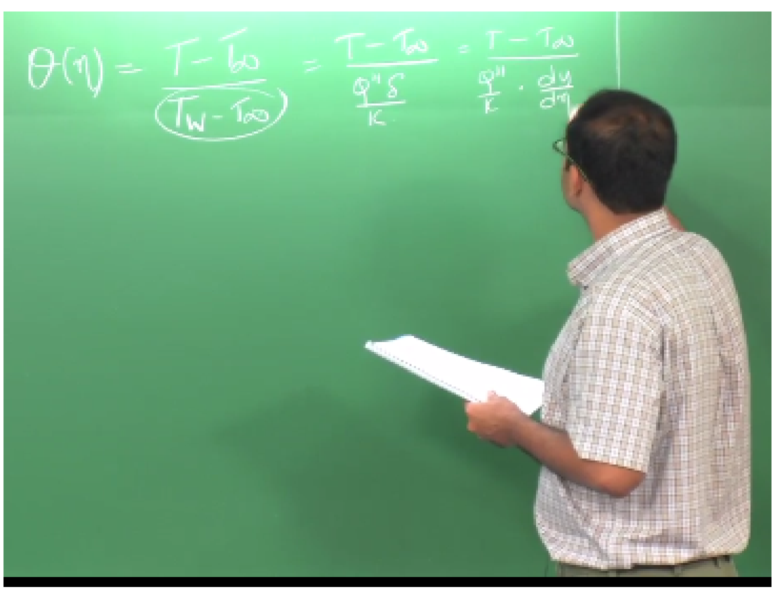

Okay so substitute this in terms of crash of number so therefore what will be the of so this will come out to be T _ T ∞ / Q wed by K x X to the power 1 / 5 okay so just check the transformation whether you can write it in terms of d y / Dt and tell me so rather than writing it out separately like this I will ask you to substitute directly in terms of Gr so what is d y / the H in terms of Gr X x rash of modified Gr the 5 power _ 1 / 5 so this is your transformation.

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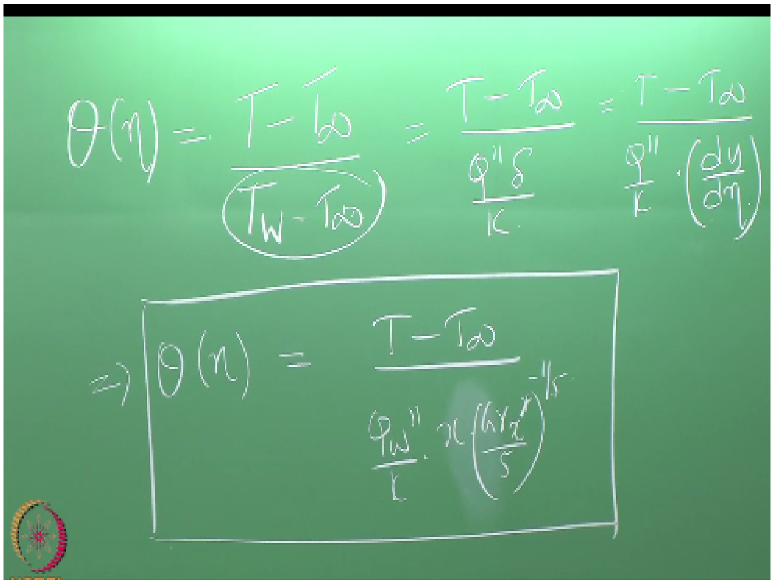

Is it clear so now you can go ahead substitute this into the momentum and the energy equations so you have u Du / DX remember the coordinate this is X is along the plate length and Y is perpendicular to that + the D u / D y is = to new e square u /d y square + Gr be x.

Okay so we are now t _ T ∞ is = to 0 therefore T is = to T ∞ + the of the x Q all prime by K so X into modified - of number verify the whole 4 _ 1 by 5 you can substitute for DT by DX as this you have D the b y so you have basically DT by DX has functional dependence on X directly and also through the ok similarly when you say DT by d y it is a function of Y only through H so like that you have to substitute for all the terms in the momentum and energy equation so G be T _ T ∞ will be again G be x Q w / K X correct so your T _ T ∞ is be substituted through this so similarly you complete the exercise and I will give you the final solution.

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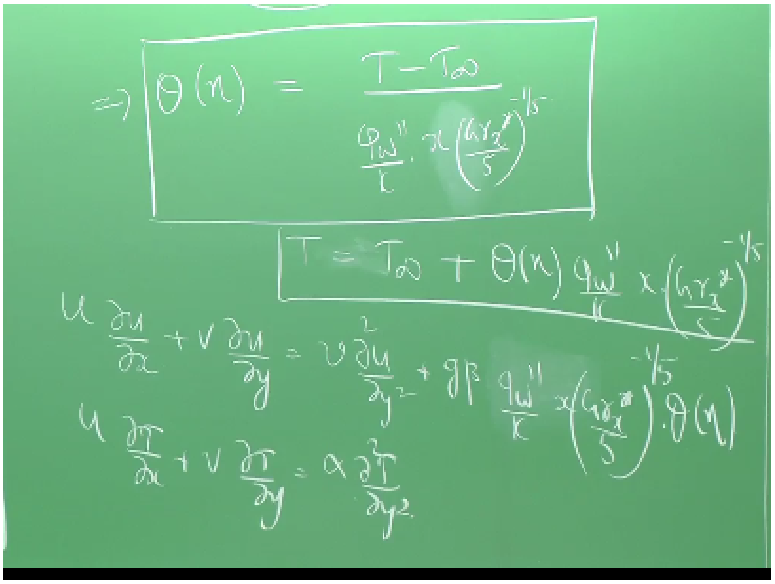

So you will find that this transforms the equations into a similarity ordinary differential equation as a function of only H therefore the momentum equation becomes B cube F by D H cube _ 3 times T up by dieter the whole square + 4 F into D square F / D H square _ the = 0.

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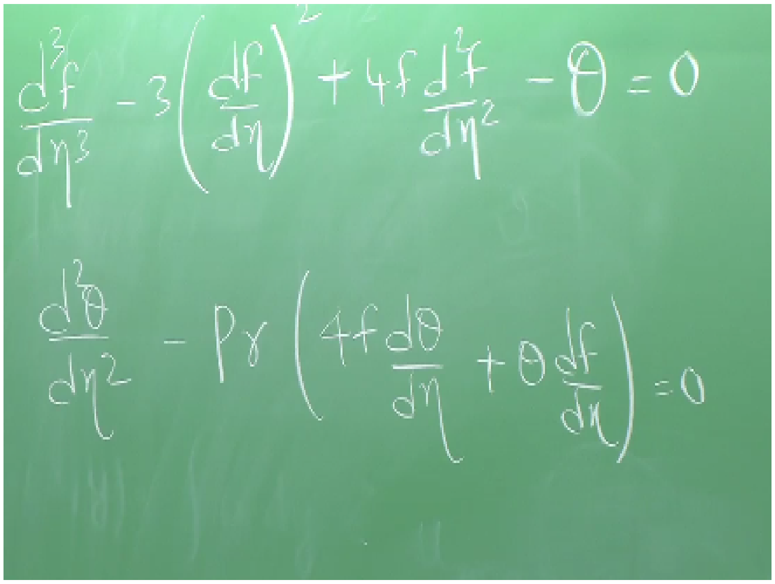

This is the similarity momentum equation and the energy equation is d square the / 2 square _ Pr x 4 D the / 2 okay so this is how you finally transform so you can try this at home so the same way that you did the constant w temperature case substitute and eliminate the common terms and this is the final set of equation that you get now we also have to make sure the boundary conditions have similarity that means they should not have dependence on X and Y so how do we do that so we know that the set of flow boundary conditions satisfy that at y = to 0 that is at H = to 0 both U and V are 0 which indicates that DF by D H is = to 0 and also F is = to 0 right and at H going to ∞ however what should happen you should approach u 0 okay so it is not like your brushes case okay so you should still approach 0 so therefore your DF by D H should also be 0 what either going to ∞.

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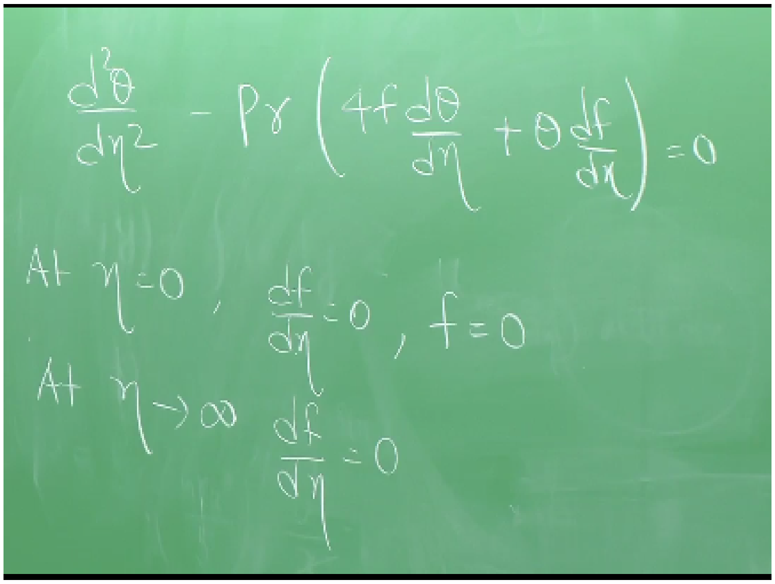

And apart from that what is the condition on energy equation that H = to 0 so let us look at H going to ∞ so there data will go to 0 okay now we need one more conditions at H = to 0 but we do not know the temperature at the w therefore four terms of Θ we cannot define but what we know is the heat flux so what can you define in terms of D Θ / D H so you have to once again perform the conversion so at the w you know _ Dt / so write this in terms of H Θ and H and therefore tell me what should be the condition for D Θ by D H at the w so what do you get for D Θ by D H _ 1 so therefore at H = to 0 D Θ by Θ is = to _ 1 right.

So you have all the boundary conditions required to solve the bodies here and once again you have to use the shooting method you have to start from the w okay and you need boundary condition at the w for basically both Θ and D Θ by D H but at w you have only boundary condition for D Θ by D H so you have to therefore guess the value of Θ at the w and then satisfy the value of Θ at Θ going to ∞ same way the iterative method okay so then you will be able to find the complete solution set and let me tabulate the final solution so naturally once again this is a couple set of hoodies you have to solve them simultaneously.

And therefore the flow solution is also a function of Pr for different values of Pr you will have therefore d square F by D H square at H = to 0 and what do you have for temperature Θ at H = to 0 so this will be the part of solution that you get correct so you know D Θ by DT at H = to 0 so u therefore guessed ΘD Θ= to 0 until you iteratively satisfy the condition that Θ going to ∞ = to 0 right so for different values of Pr 0.1 this is one point six four three four this will be two point seven five 0 seven rental number of one this is point seven two one point three five seven four and Prof ten this is 0.3 0-6 0.76 or for financial number of 100 so point four six five one two these are the different values.

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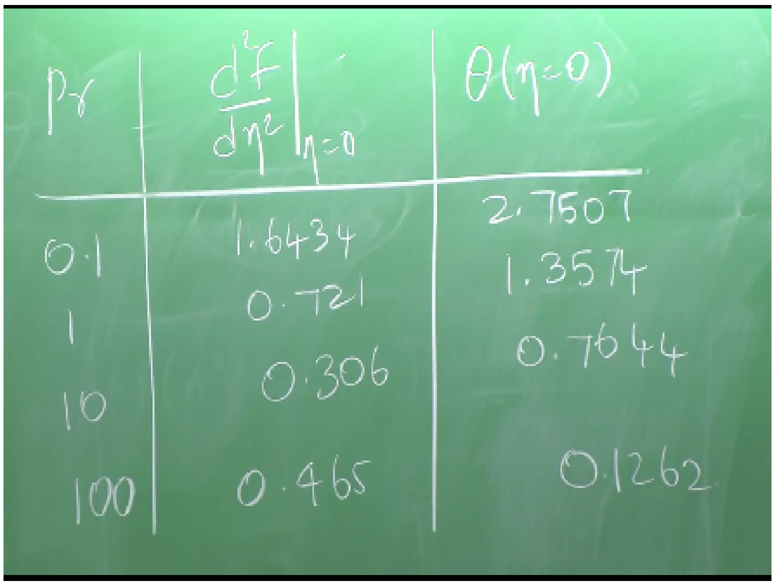

Okay so for a Pr of one for example what was the subsequent value of d square F by D H square for constant w temperature case if you go back approximately say we had for 0.72 it was 0.67 six approximately you can say about 0.68 or 0.69 so therefore this case the velocity gradient is slightly marginally higher than the constant w temperature case okay so since the flow and the temperature fields are strongly coupled you can see that the velocity gradient is also a function of the boundary condition should be point yeah okay I have reversed it point one two yeah you're right it cannot increase heaven 0.126 two and this should be 0.465 it is correct it should gradually decrease.

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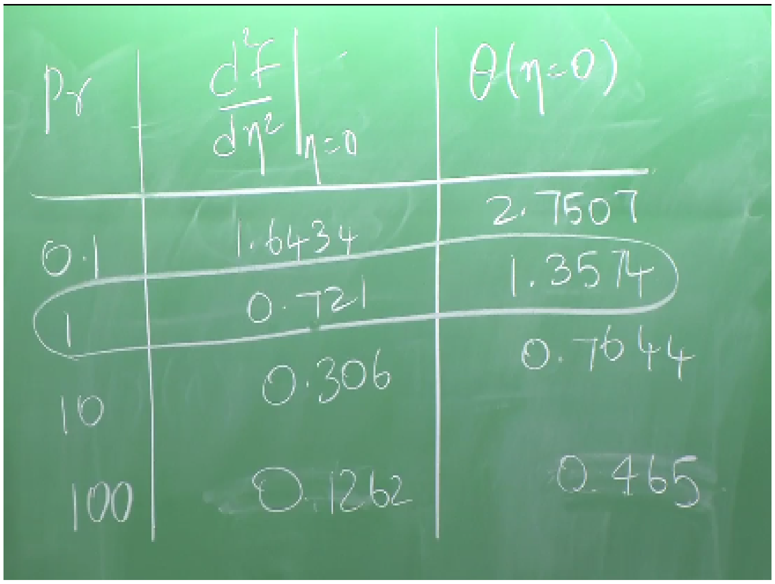

With increasing Pr so now having known Θ= to 0 how do you calculate the nusselt number you have to finally derive an expression for Nu as a function of the modified Grand P r so how do we get the expression for local nusselt number so you have once again HX by K which is Q w / T w _ T ∞ x X / K now in this case we know what is Q w but we do not know T w _ T ∞ okay so therefore put this in terms of Θ at H = to 0. Okay so our definition of Θ is I will write it again here so Θ= to t _ T ∞ by and this was crash half / 5 _ 1 / 5 and we also had X outside from this substitute for T w _ T ∞ that is nothing but Θ = to 0. So you simply get this as crash half number W i –F i H to the power 1 / 5 into 1 /Θ at H = to 0.

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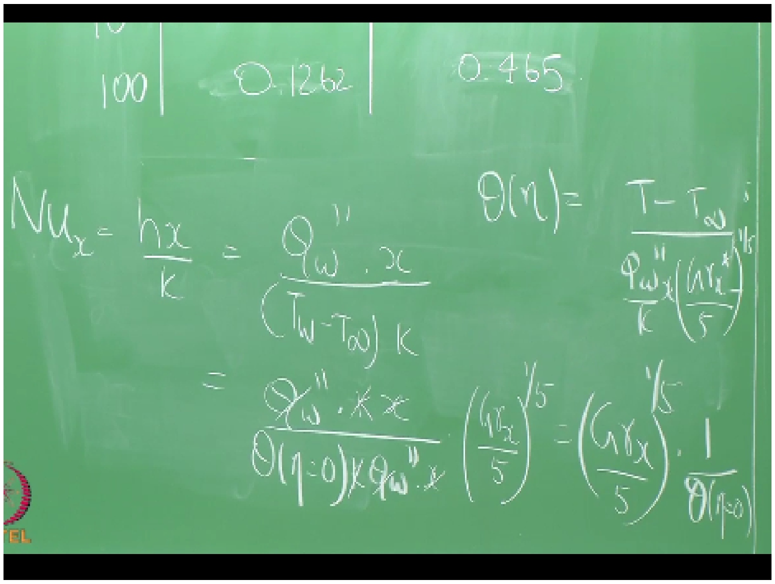

Correct so corresponding value of you can again fit a correlation as a function of Pr and you can substitute for Θ = to 0 as a function of Pr so you will get an expression for nusselt number as a function of crash half Pr. Okay so now what we will do next is to look at approximate solutions so these are the 2 exact solutions possible in natural convection when you talk about external natural convection when you talk about internal natural convection it is in a cavity and so on you do not have exact solutions then before going to mean numerical full set of numerical solutions we will also look at approximate methods.

Or the integral solution yeah this is all modified Gr correct so all this is star so this the same thing I am using this expression and putting this G r this is all modified so whenever you talk about constant w flux case you have to always use modified rush of them okay so next what we will do is look at the constant w temperature case first and try to derive the approximate or the integral solutions similar to the external force convection that we did so this is something less rigorous than the similarity solutions and nevertheless they give you useful solution which is kind of close to the exact solution.

So we will start off with the approximate methods so we will only focus right now on constant w temperature boundary conditions you can do a similar exercise for constant heat flux boundary conditionals okay so let us assume that now the w temperature is maintained constant you have gravity acting downward and you have a natural convection boundary layer okay the first point in deriving the approximate method is to derive the integral equations for both momentum and energy.

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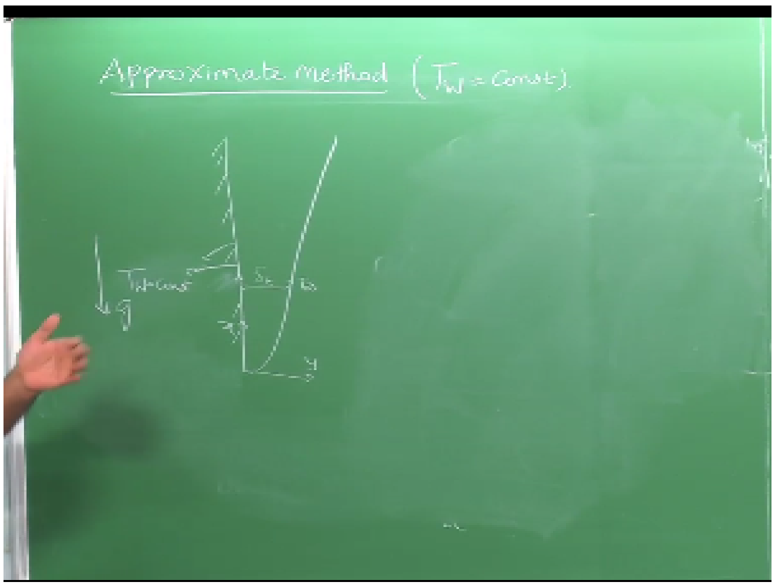

So let us try to find the integral equations by integrating the momentum equation and the energy equation across the boundary layer okay so now this solution was done by person called square it is called also square solution this approximate method.

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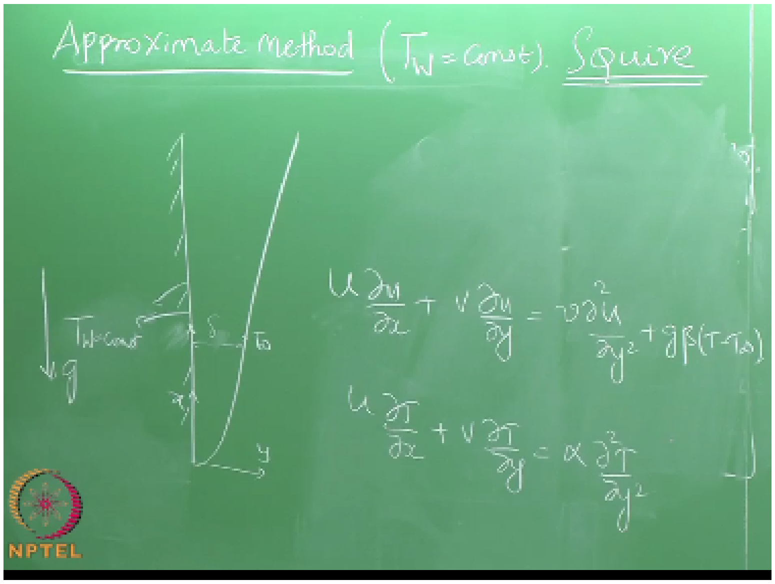

Now when we do the square solution you should understand that once we get the integral equation what is the next step we guess the profiles for velocity and temperature substitute them into the integral equation convert this into a simple Lodi which we can straightaway integrate okay now in doing that we need to know the reference velocity in the blushes case in the flat plate case external force convection you know that this is your u ∞ but in the natural convection case we do not know the U f okay we do not know precisely u refs.

Therefore what are all the unknowns here we do not know you rap and then what is the outcome of solving the momentum integral equation what do we get an expression for hydrodynamic boundary layer and then by solving the energy integral Δ T so in your integral equation applied to external force convection you have only 2 unknowns Δ and Δ T your reference velocity is your free stream velocity there but now in natural convection we have 3 unknowns okay unfortunately we still have only 2 equations two integral equations so therefore what do we do is in the case of Squire he made a very simple.

Assumption that since we are talking about natural convection for mostly gases we do not talk about very high Pr very low Pr but for Pr approximately around 1 so it is reasonably safe to make an approximation that Δ is = to Δ T so finally we therefore reduce the number of unknowns to the number of equations that we have okay so this is a an approximation that Squire makes but we will see whether this will impact the accuracy of the solution much still it will be close to the exact solution since we have to solve 2 equations we need to only reduce this to two number of unknowns okay and therefore when we integrate this irrespective of whether we are integrating the momentum or the energy equation will still go from 0 to Δ on okay so all of you can now try converting this into an integral equation also we need to write down the continuity equation.

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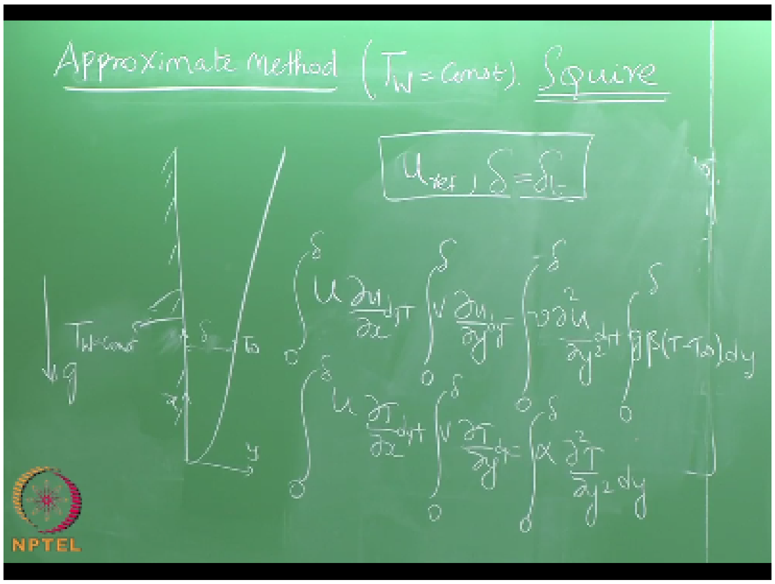

Please do not forget it we need continuity equations in order to relate the V velocity okay so this also will have to integrate so I will split the second integral here VD u by d y d y as the by d y s of u v _ u x D v / d y okay flit the second integral into these 2 parts so therefore what do you get when you integrate d by d y of UV from between the limits 0 and Δ huh what happens to this particular integral 0 okay there is no u velocity at y = to 0 and y = to Δ right that knocks away I can use continuity to write D v / Dx s D v / d y S d u /D x so now I can therefore combine these two terms right so I have 2 times0 2 this should be a + here.

So I have missed a sign somewhere okay so this is - here right so this is + okay therefore two times 0 to Δ u D u by D X D Y this is = to new when I say D u by d y between the limit 0 and Δ what is the value at y = to Δ 0 okay so this will be therefore _ T u /d y at y = to 0 and the last term will be as it is you cannot unless you know the temperature profile okay you cannot integrate it so we just have to remain this as 0 to Δ G be x new okay so this is therefore the momentum integral equations so let us call this as your momentum integral equation right so similarly if you integrate the energy equation.

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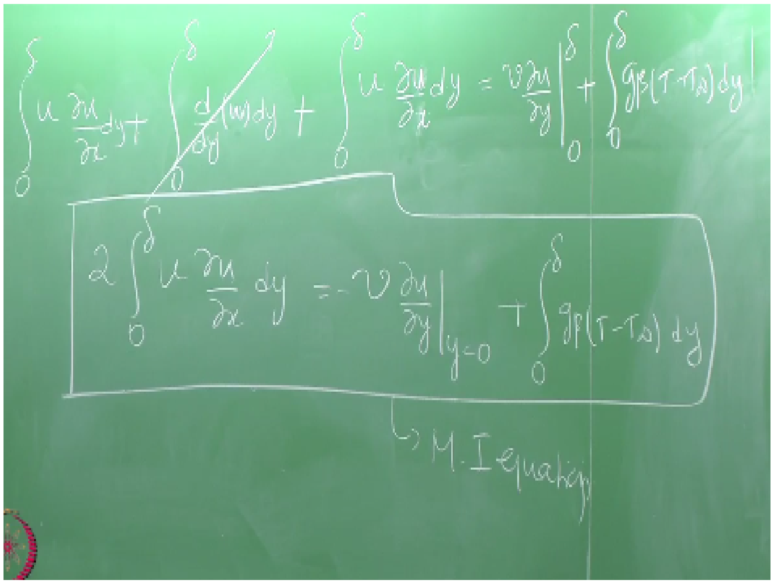

So you have 0 to Δ Q DT by DX d y and the second term can also be split you can write this as 0 to Δ D by d y of VT d y _ 0 to Δ T into DV by d y x d y which we can use the continuity equation and write it like this is = to α times once again DT by d y _ α DT by d y at y = to 0 so if you integrate this between the limits 0 to Δ what do you get at y = to 0 be = to 0 but at y = to Δ V is not = to 0 because we have an in Trainmen happening at the edge of the boundary layer for the boundary layer to grow okay so that we a Δ is obtained from the continuity equation so you can therefore substitute for the continuity here so you have 0 to Δ u dt by DX d y Plus becomes we at Δ times T ∞ okay so I am substituting for V of Δ as _ integral 0 to Δ into D u by DX d y into T ∞ slash 0 to Δ into TD u by DX d y okay this is = to _ VT by 0 so they say this can again be reduced to the form which will be D by DX of 0 to Δ u of t _ T ∞ d y.

Which is = to _ α DT / d y at y = to 0 okay so you can again write this as D/ DX of UT _ T x D u by DX and that and this will cancel and therefore finally you will have two terms which you can combine it so this is the same energy integral like what you got for the external force convention okay only the momentum integral is now much simpler the energy integral remains the same and even the momentum integral you can just write it you can take the D by DX term out you can write it as simply d by DX of U square d y which will be the same as 2 u x D u by DX okay so D by DX of U Square D Y so that now once you substitute the approximate profile for velocity now this becomes an ordinary differential equation with respect to X okay similarly once you substitute the approximate profile for temperature and integrate it you get a ordinary differential equation so you get 2 equations and two unknowns one is your reference velocity the other is your Δ okay so you should understand that in this case we are integrating both with between the limits 0 to Δ so we do not distinguish Δ and Δ T right so therefore once we derived the integral equation we have to make a guess for the approximate profiles of velocity and temperature now what will be the proper des for velocity can we make a linear approximation because I told you that the peak can occur somewhere in between and again drops to 0 so we in order to approximate this kind of a curve what kind of a polynomial should we take at least we should have a cubic polynomial.

Whether we take linear or quadratic cannot predict this Maxima here okay so we have to start off with a cubic velocity profile and that is what Square did so he assumed the cubic velocity profile however for temperature he used a quadratic profile which is not a bad approximation and so this is where he started off by making an assumption that you by your reference is = to a plus B x y by Δ is C into y by Δ the whole square plus D into y by Δ the whole cube you can make H is = to Y by Δ so this was the cubic assumption similarly for Θ okay you can say a1 plus b1 into y by Δ plus c1 into y by Δ Square.

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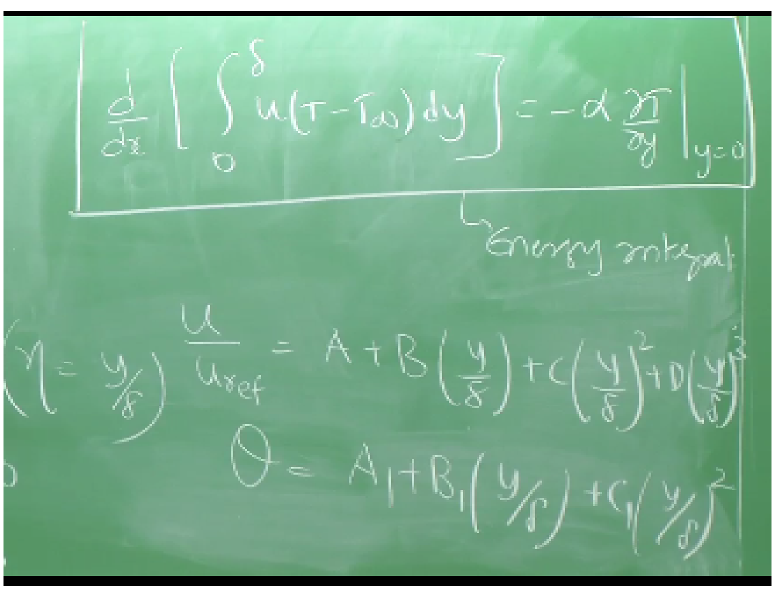

Okay so we will stop here tomorrow we will solve the 2 profiles by making the writing the boundary conditions down substitute them into the integral equations and precede further okay.

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