Indian institute of technology madras
Presents

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Video Lecture on
Convective Heat Transfer
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Lecture 37
Similarity solution in natural convective for vertical
Isothermal plate-part2

Good morning all of you yesterday we were looking at one of the fundamental solutions to natural convective boundary layer flow past a vertical flat plate and this was initially attempted by paulhassan again and the theory behind this similarity solution is a continuation of the Belasis solution we assumed that the similarity variable is of the same function as in the Belasis solution only difference here will be that different dependence of boundary layer thickness.

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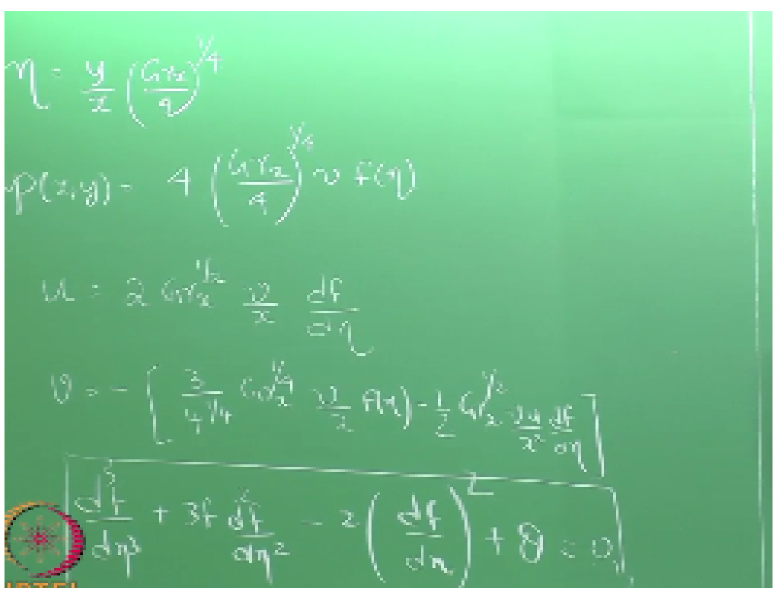

Instead of being on Reynolds number we suitably modified to be a function of gash of number okay accordingly pole house and choose the similarity variable something like this and then we proceed in the same lines as we derived the Belasis equation starting from the stream function definition and then substituting this to find the U velocity V velocity and also the gradients of all the velocity components and then finally when we substitute into the momentum equation we end up with the following similarity equation okay similarly if you do the similar transformation.

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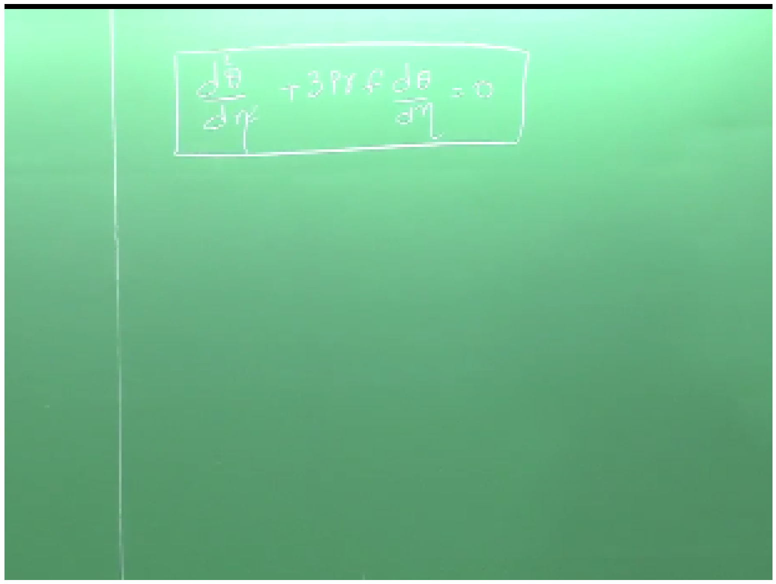

From the energy equation you get the following similarity equation for the temperature θ okay so here we have defined θ to be the non-dimensional temperature based on t - T ∞ by T w in a steel okay so these are some of the fundamental components to deriving a similarity equation and they are pretty much common so if you do this a couple of times you will become familiar for the third time the only cache is how do we find the right similarity variable.

To transform this solution so that is the most important point and once you find it either by trial and error or by intelligent guess okay so you will be able to most likely convert the PD into a similarity ordinary differential equation so now you should understand that these two equations are coupled in the sense that you have this term θ which is sitting in the momentum equation and therefore unlike the external force convection case you cannot solve this separated.

So you have to do this simultaneously when you use the shooting method and we will see that in a short time but before that I asked you to check whether if you get these similarity equations I hope some of you could try it so are you able to get the same equations anybody who tried so far no anyway so I think I request you to please check because these are all no very critical part of the similarity questions okay do not because a substitution if you do it a couple of times.

You will understand now how the derivatives are computed and things like that so now let us write down the boundary condition see unless we write the boundary conditions we cannot complete the you know we cannot say for sure that this is a similarity equation okay so the Boundary conditions also should be independent of x and y so there are cases in mixed convection where you can get a similarity equation but the boundary conditions does not show self similar set of boundary condition in that case what happens is you cannot solve those equations anyway even though they are converted okay so we have to first also ensure that your boundary conditions satisfy the requirement for similar equation.

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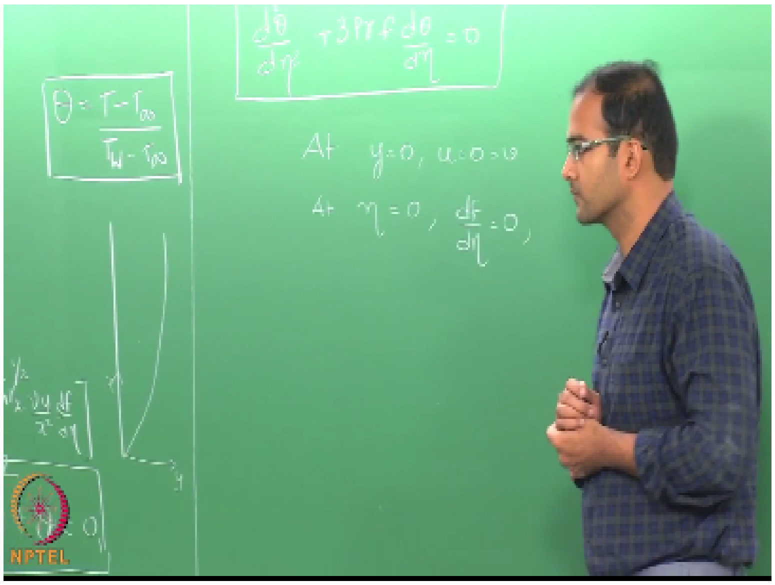

So what are the boundary here so if you take the case of the vertical plate pretty much similar conditions as in the case of external forced convection at we have the coordinate here x and y so therefore to solve this we need to therefore define three conditions for f and two conditions for θ right so and therefore at when we said do this transformation from size which is a function of XY to η so we apply also the transformation for the for the similarity variable.

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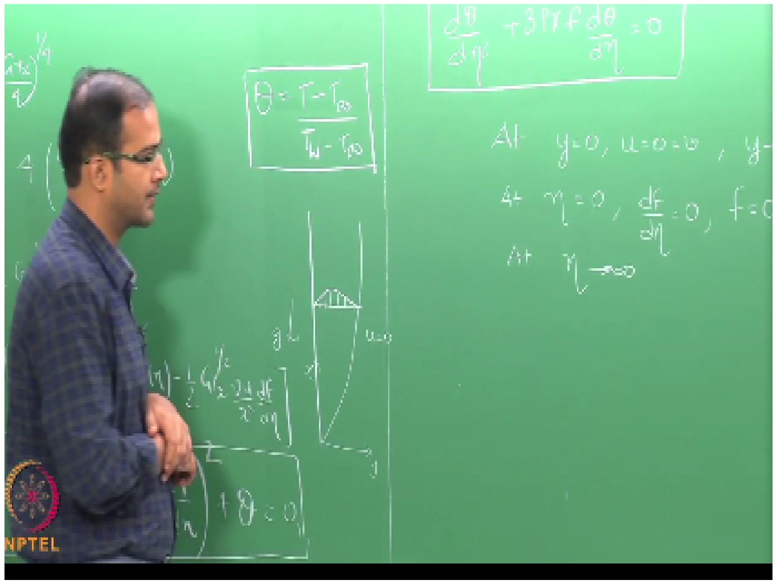

η so therefore at y = 0 since we have the no slip boundary condition okay so we can also say that at using our similarity variable η = 0 right y = 0 η = 0 what will be the condition for DF / D η 0 so we have based on the fact that u = 0 we can say DF / DT = 0 and what about can we say something else from the V velocity when w= 0.so DF / D η at η = 0 V = 0 so therefore f = 0 okay so we need one more condition that can we apply at going to so Y going to ∞.

Okay what will be the condition let us look at the condition with respect to Y going to ∞ what do you think will be the condition hmm y = 0 okay so can we also say Du / d y = 0 if you draw the velocity profile here the pure natural convection case the gravity acting downward okay so this is zero outside and also at the plate so we can say both u =0 and also the derivative of this =0 so therefore if you put the condition u =0 what happens DF / DT =0 again.

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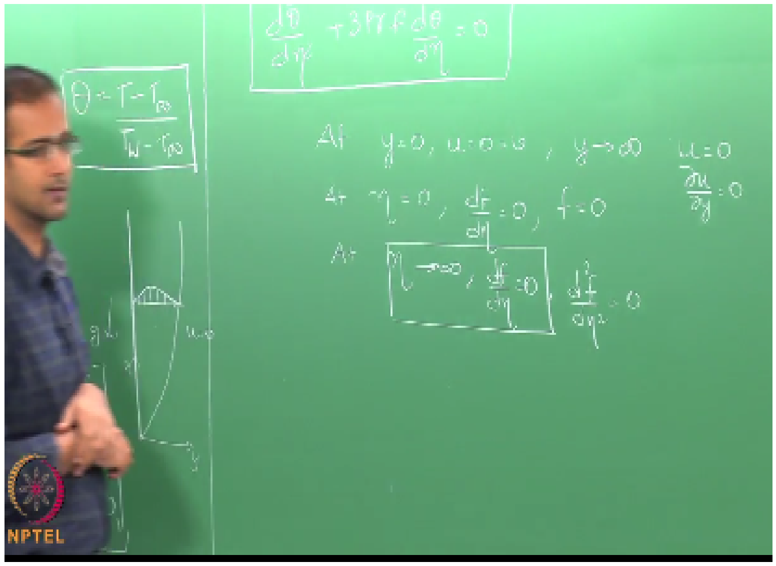

And if you put the condition D u / dy = d square F by DT square = zero okay so now we should also remember that the condition at X = is also part of Heat are going to Z ∞ correct so therefore at X going to 0 η are going to ∞ what is the condition u should be=0 okay so the X going to 0 condition is satisfied by this condition as well right so these are the conditions now similarly let is let us write it down for the thermal boundary layer so at for the energy equation at y =0 what is the temperature this is the wall temperature which is constant and therefore correspondingly at θ = 0 θ will be 1 and at Y going to ∞ okay.

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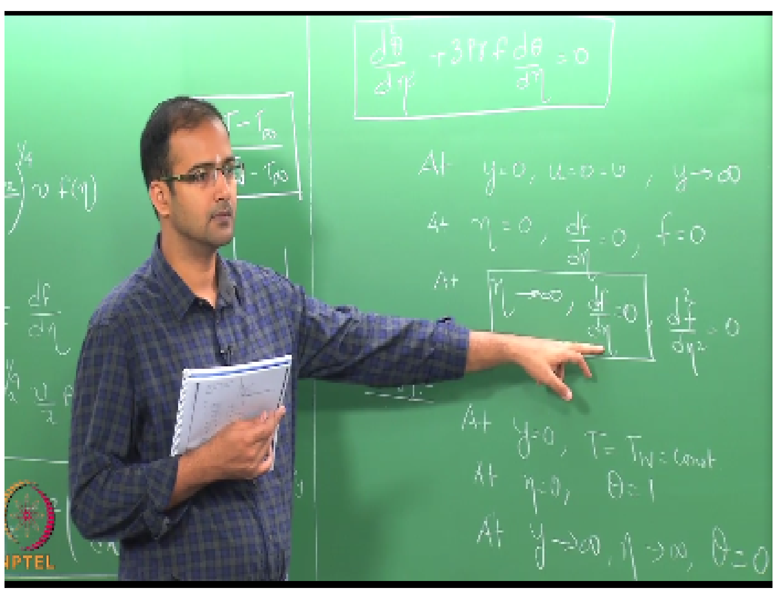

Corresponding to η going to ∞ θ will be going to 0 so this is pretty much the same as the original block below sense boundary conditions only that the case of momentum you have now

DF/ D η = 0 whereas what is the condition in the blazes equation going to 1 u / u ∞= DF / dη okay so now you should understand actually this entire component.

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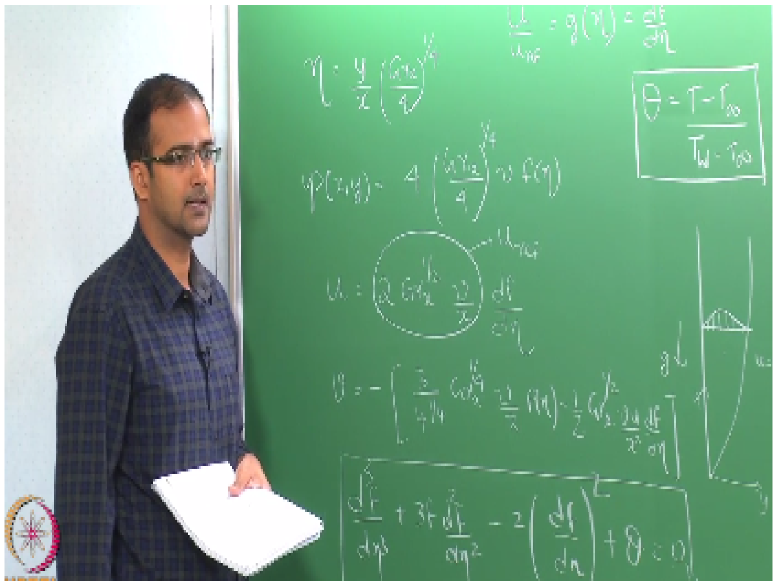

Ok yes you are what you reference correct so this is the starting point we assume u by u reference is actually a function of Meet up ok which is nothing but df/ DT so this 2 times crash of number to the power half nu by X should be = u reference you can please check that ok so we are our new reference definition is 4 times G beta T w - T ∞ into X ok so you can replace

This with rush of number so G beta into T 1- T ∞ is what how do you rewrite this in terms of gash of number gash of number into $\mu 2 / X^3$ okay so therefore you have two times so this is local gash of number you take this is μ and this is X^2 if you take this is new by X right so this is nothing but you our new reference here so in the blazes case this was your u ∞.

And the blazes case since here you do not have any u ∞ you introduce a reference velocity and this is the reference so in the original blazes case therefore he η going to ∞ DF / D η was 1 so u by u ∞ u = u ∞ so therefore DF/ D 8 hours 1 but here we cannot say u approaches u reference okay it is U =0 so therefore DF / D η is therefore zero here so this is the difference also in the boundary condition now you if you therefore look at the boundary.

Conditions all of them show that this is a these are similar equation with the similar boundary condition so you can say that all of them do not involve any X or Y components in that okay so therefore we have we can say for sure that we have found a self-similar solution to this problem and the next step is to go ahead and solve it so we will once again use the shooting method that you are so familiar by now you have been using it for so many different problems.

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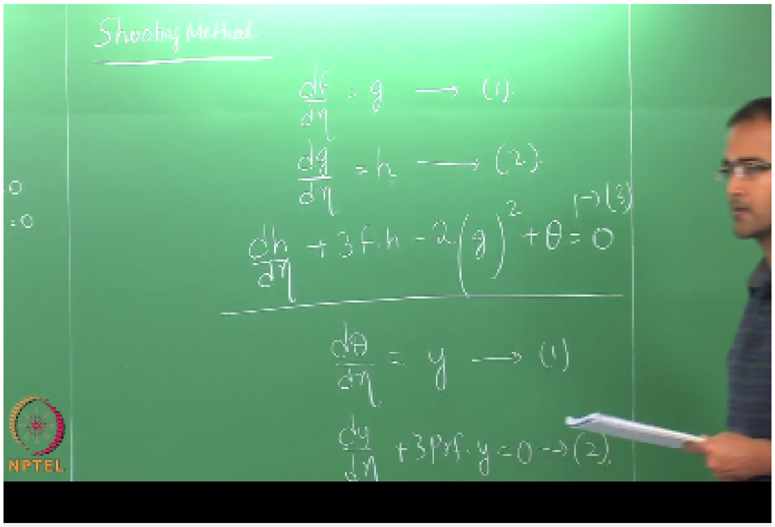

Both in external convection as well as in internal conviction okay so you know the equations can you please convert them into Ode's OD is our first order so we have a third order ode please try to reduce this into three first order roadies and similarly the energy equation so what do you start with DF/ is = G this is your first order OD and then therefore next o d Dg /dh H this is your second first order OD and then finally you put it into the master OD okay.

So what will be d square F / D η cube D H by D η okay plus three times F into H - two times D G into whole square plus θ equal to zero so this is your third first order ode similarly you are energy equations we can say D θ by D η so what do you use generally D θ by D η is equal to let us say something like Y okay this is your first order ode next you have dy/ D η plus three PRF into y is = okay this is how you have converted.

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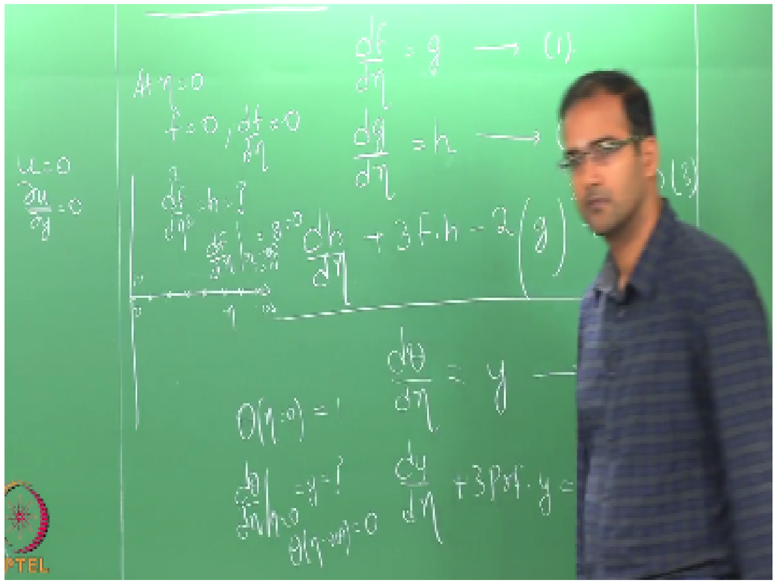

So now just you can take the vertical plate discretize this in the η coordinate system okay so break this into finite domain with discreet number of points going from η =0 all the way to ∞ will be some large value okay it could be 10, 15 whatever so you have points I = 1 all the way to I = n right so therefore you will be marching in space so these are all.

First order ode is you can use a simple Euler backward differencing and then you can just simply march in space forward so once you know the solution at the previous location you can use that to calculate solution at the next location and keep going forward till you reach I = n okay now therefore in order to march forward in space you need to know the boundary conditions for F G and H right at I =1 that is that η =0 then only you can stay much.

In a straightforward manner okay let us see how do we convert this into D current boundary conditions okay as far as F is concerned so at η =0 we have F = 0 is known okay so therefore the first equation can be solved straight away what about DF / D η DF / D e η is also known right so this second equation also can be solved now coming to the third equation we need a boundary condition for d square F / D η square at η =0 which we do not know okay.

So we do not know therefore the boundary condition for d square F / D η square which is nothing but H okay at η =0 is unknown but what we know is DF / D η at η going to ∞ that is we know the boundary condition for G which is =0 okay this is like your blazes solution again so there you have DF by D η = 1 okay same way here also if you look at for θ so you know that θ at η= 0 is equal to 1 okay but you do not know what is D θ by D η at η= 0.

So this is your y this is not known but what you know is θ at η R going to ∞ which is = 0 so therefore we have to now introduce the shooting method so you have to shoot a guess here in this case for H which is d square F / D η square okay and simultaneous case these two have to Be solved within the same loop okay both with respect to iteration and with within the same space loop okay so it has to be it has to be all both of them have to be solved together ok first you solve the three equations three first-order Rudy's as getting some value of θ for example okay and then you put that in to this and then solve for θ okay and now this has to be hydrated again and again till you reach a final solution okay so you have the iteration loop.

Outside and inside you have this space loop within which you have to solve these two simultaneously so when you solve this the problem is here you do not know again the boundary condition for H here and also for Y here so then therefore you have to shoot a guess for in this case H and also in this case Y and then use newton-raphson to make more accurate estimate of these boundary conditions provided you satisfy the condition that G R θ going to ∞.

Is equal to zero okay similarly here θ R θ going to ∞ equal to zero so once you satisfy these conditions that means you have reached a final converged solution okay so till that point you have to keep guessing the values of both H as well as θ and then solve these two equations simultaneously so that means at each point you should be putting the corresponding value of θ and then you get the solution and then that value of f is put in the equation.

For θ and solved okay so once you do this finally you will have the equations solved for different values of Prandtl number so now you will have therefore unique solution for also velocity as a function of prattle number because Prandtl number is the one which decides the value of know the temperature profile and that temperature profile will go into this and decide the velocity solution so now prattle number will govern both the temperature as well as the momentum profiles so I will just give you how the gradients look the gradient of velocity as well as temperature at the wall because these are the important quantities to calculate the skin friction coefficient and the nusselt number so and then we will go ahead and see how the profiles of velocity and temperature look so.

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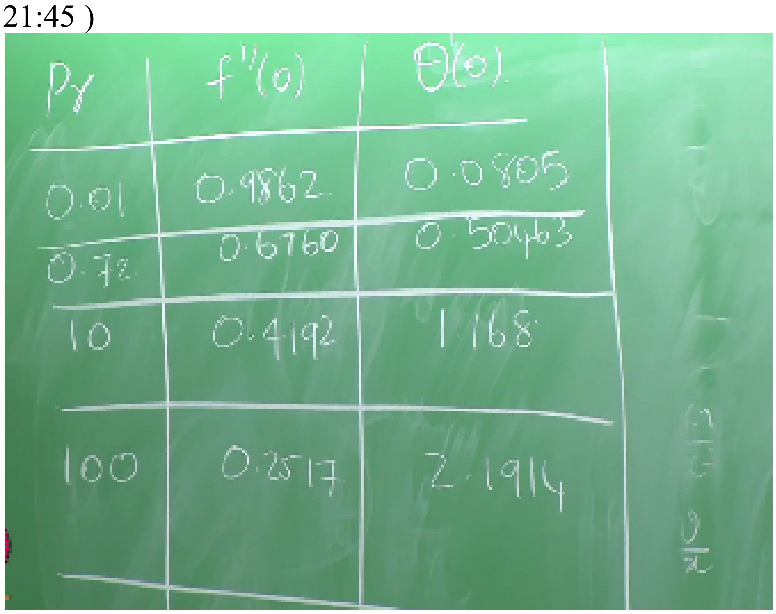

Prattle number we have f double prime of 0 and then θ of 0 so finally this is what you guess and after you reach a converged solution this will be your final values so if you vary the Prattle number for different Prattle numbers if you can find the solution by shooting method you will end up with 0.9862 and 0.6760 0.01 so now you therefore see that the gradient of velocity is also a strong function of prattle number unlike the external force convection.

So that value was simply 0.33 - okay so here you have a function of prattle number okay because it is coupled with the energy equation the corresponding values of temperature gradient right sorry this will be θ - right so you have the corresponding values here you can just compare for your understanding so approximately this is close to Prandtl number one so you have 0.6760 as the value of f double Prime what was the corresponding value.

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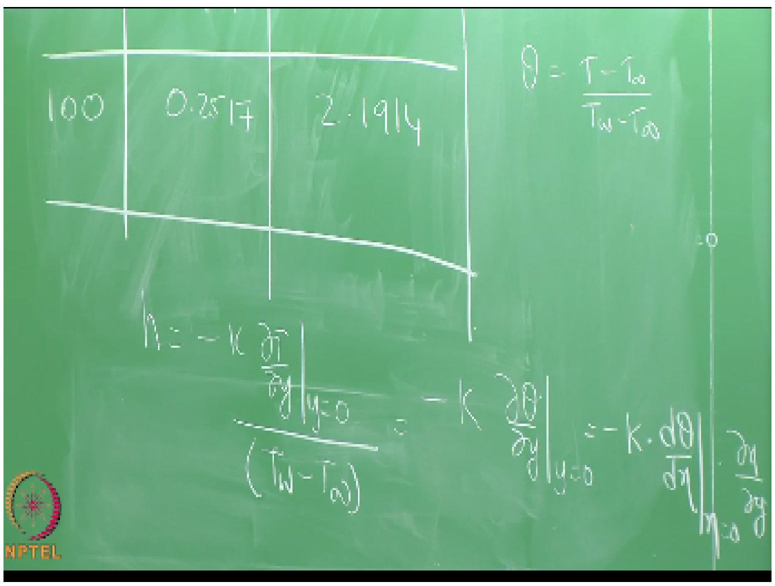

For the external post-convention flat plate oh point three two okay and what was the corresponding value of D θ by DT for Prattle number equal to one that is all the same 0.332 okay so these values are now slightly higher than that but does not mean now the final nusselt number is going to be larger than the force convention we will see that now we will convert that into an expression for nusselt number okay so can you all do that so once you know the temperature gradient at the wall now you should write down what is the expression for nusselt number yes so in this case it should be negative right so let us say - of θ - it is correct.

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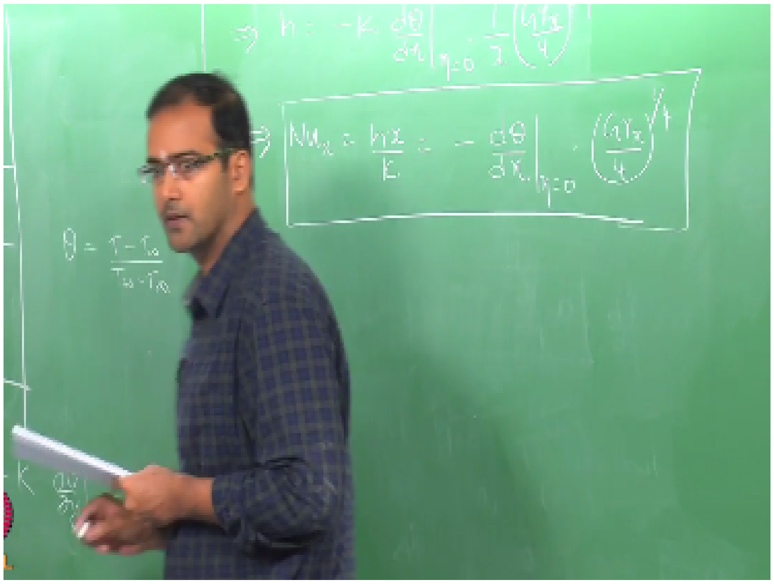

So what is d η/ dy 1 / X into rash off number by 4 place to the power 1 / 4 okay therefore if you define a local nusselt number as HX / K we simply have - D θ / DT =0 into a gash of number by 4 okay so this is your expression in the case of force convection what did you have D - D θ by D $\eta\zeta$ =0 into Reynolds number raised to the power 1 / 2 okay so you have this factor now by 4 because we have defined this in the similarity variable so accordingly this will get scaled okay so if you do not use the factor of 4 the value of the temperature gradient will also be different it will be lower it will be 4 times 4 to the power 1 by 4 times lower so then therefore you do not have this factor so now that you have put this constant so this will be 4 to the power 1 /4 times larger than the normal case okay so everything gets proportionally.

Scaled okay so the question is now we have a functional dependence of prantle number so we cannot directly say some value of D θ / DT that we have to fit a curve now as a function of prattle number so this was done actually by ooze-struck so initially when Pohlossan and solved he could not solve for so many Prattle numbers because he did not have access to numerical techniques so therefore he stopped with the equation and maybe found just one solution but it was later ooze-struck extended it to so many parental numbers prattle number much < 1 to much >1 and then he fit a curve to this D θ / Dη as a function of prattle number.

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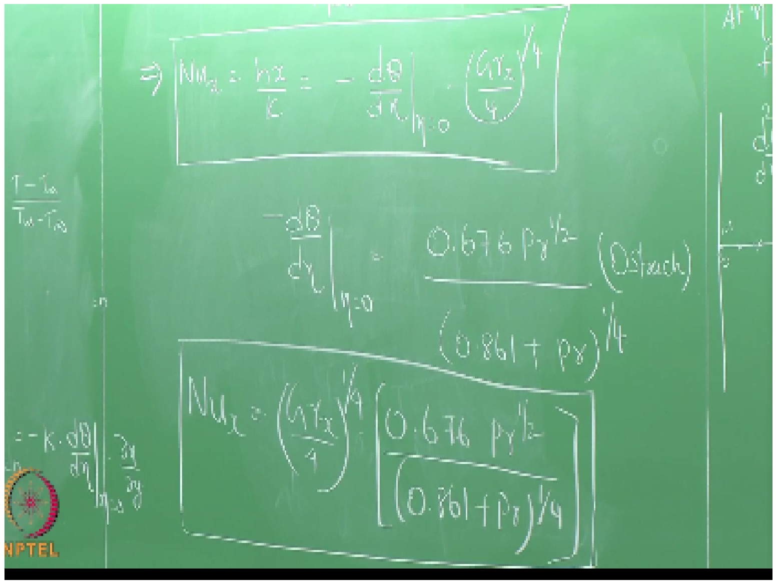

So according to that it comes out to be - D θ by D η η equal to zero according to Oasis struck is 0.676 okay so this was by Ostia you can do a quick check for prattle number equal to 0.7 whether you recover the value shown here if you have a calculator you can quickly check that so therefore with this curve fit we can write down the final expression for nusselt number right this is your exact solution okay so now that unlike the case of no classical polo's an equation where you have 0.33 two times re power half prattle number to the power one-third so now you have a more complex dependence on frontal number and instead of Reynolds number you have gash of number okay so this is how the nusselt number varies now what we will do is also plot the solution for θ and the velocities okay so once you solve these OD by shooting method whatever is described here you should be able to also plot DF / D η which is nothing but u / Eu and θ so first I will show the variation of θ which all of you are mostly familiar.

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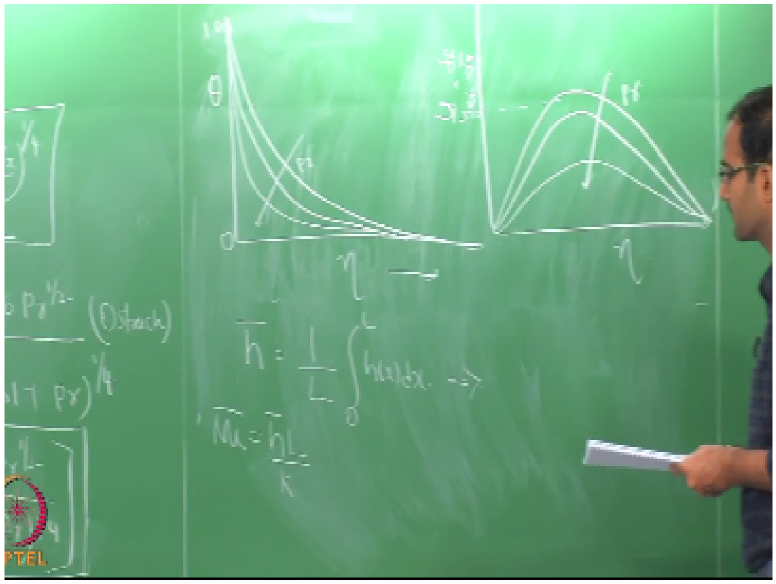

So this is 1 0 θ going from 0 to a large value so at η=0 T = T wall right so this will be = 1 and then but large values of θ it will decay to 0 and what will happen to the temperature gradient with increasing prattle number it increases okay so this will also shift downward right this indicates higher slope at the wall so for increasing prattle numbers the temperature.

Profile will be varying like this similarly if you plot you are you buy your eu/ef which is nothing but DF/ D η as a function of η ok so this also shows a functionality with prattle number so you find that something like this so it reaches a value of depending on what you take as your F okay so suppose somewhere here it is your eu it reaches one here okay and if the peak is not your Eu that will exceed one okay and similarly with prattle number what happens.

What happens to the slope of the velocity and the wall it decreases with increasing prattle number the slope decreases so therefore how should the curve shift with increasing prattle number downward right so this is your increasing frontal okay so for a given value of D η so your D U will be smaller for this compared to the other one okay so this is how your similarity solutions look now you should remember that the similarity solutions for natural convection.

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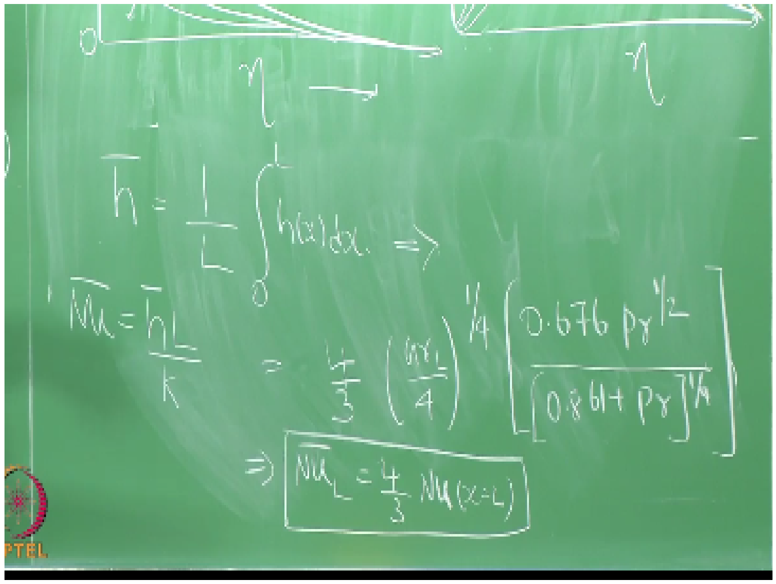

Both velocity and temperature are functions of prattle number okay so we have to be careful about it so next thing what I am interested is if you want to calculate an average heat transfer coefficient just like we did for the you know the classical pole of sand solution sometimes when you solve heat transfer problems you are not interested in the local heat transfer coefficient but average values okay so let us try to therefore start from this point here okay try to calculate.

An average heat transfer coefficient which we will call has H^{-} b which is nothing but 1 by L integral 0 to L H of X DX and then define a nusselt number based on the average heat transfer coefficient and based on the total length of the plate HL / K so can you do this and tell me what will be the average nusselt number so if you complete this exercise so you will find out that you will have 4 / 3 into grashof number now this grashof number will be defined based on the plate.

Length okay after you evaluate the integral you will convert all your X into L ok so the grashof number will be defined based on the plate length and therefore you will get the expression that the average nusselt number will be 4/ 3 times the nusselt number local nusselt number then your X is replaced with L ok so this is another useful relation that you can use when you are doing calculations for heat transfer rate and so on the way you define your heat transfer coefficient.

Based on the entire plate length so what was the corresponding relation for the external force convection twice of nu at X = L right so here we have 4/3okay they have any questions so far I hope the procedure is clear because we have now done enough similarity solutions so it is just you have to go through these mathematical procedure and substitution.

And you will get it there now what we will do next is move on quickly to the case of constant wall heat flux boundary condition okay so this is the extension to what pole house and did so he Started with the constant wall temperature case and later on austral completed the solution for different prattle number ranges now the extension to this is to find the solution for constant wall flux boundary condition so this was attempted by two people Sparrow and Greg okay so now similar to the flat plate case where we had constant heat flux boundary condition.

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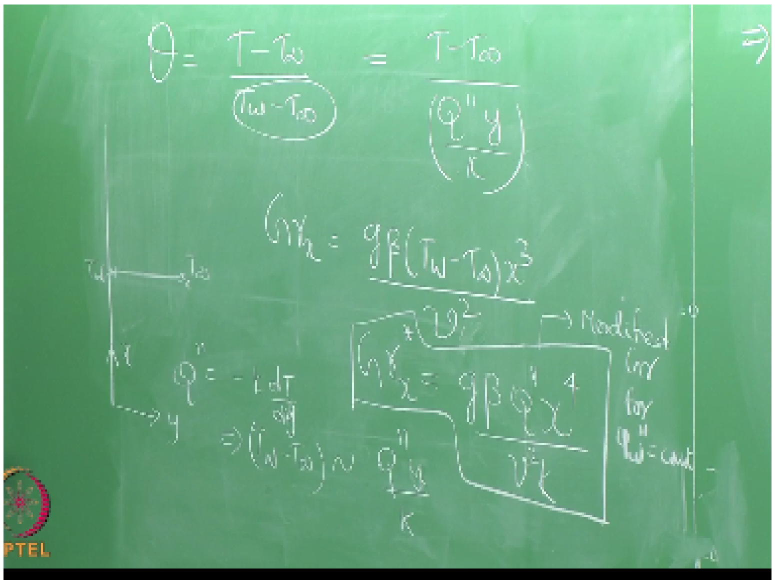

Now we have to find another similarity variable we use the same similarity variable more or less there but the way we have defined our non-dimensional temperature θ okay so the constant wall temperature case we use this definition and in the constant heat flux case since T 1 - T ∞ is not a constant we have to replace this in terms of heat flux which is known which is fixed so similar to the flat plate case so what we can do is assume this is your T wall this is your T ∞

And therefore we can relate some kind of scaling to convert this temperature difference into heat flux using the Fourier conduction equation so what we can say is that your Q double prime is equal to - K say DT / DX therefore this can be written as T w - T ∞ the order of magnitude of this will be Q double Prime into X / K we are just looking at the order of magnitude so therefore we will write θ here as t - T ∞ by instead of T w m- T ∞ we have Q Double Dash.

Now we are coordinate this is actually Y and this is your X so therefore we have to be careful we will be using Y here okay bye so this is how we try to transform temperature dimensional temperature T to a non-dimensional temperature θ okay just doing order of magnitude analysis now we have to also find the right definition of grashof number okay so your conventional definition of grashof number is also based on temperature differences now how do we convert again this temperature difference into what is known to us in terms of heat flux again we will use a similar scaling okay so we can simply write therefore T wall - T ∞ as something like Q Double prime X by K so x and y they are just the order of magnitude of x and y are comparable now okay so we will just interchange them so that we can define a grashof number we will use rush of number star now to differentiate this from the conventional grashof number now this will be based on your heat flux so therefore this will become Q double prime and then what would be X^4 / nu square K so this will be the modified grashof number this is called the modified.

Rush of number for constant wall heat flux boundary condition so you understand so we know what is the wall heat flux if you so we are replacing Y with X now we cannot use in the grashof number Y into X^3 you understand so we are just doing some scaling where we say the order of magnitude of x and y are kind of similar and therefore we are converting that also into X why do you want to complicate the definition of grashof number the grashof number.

Is only based on one local coordinates why should we use Y times X how do you then compute your ground is it a two dimensional variable okay so grashof number is just a local coordinate which is attached to the plate length okay so we cannot therefore define a two dimensional graph number here it is like defining a two dimensional Reynolds number okay so therefore this is their definition okay there is nothing wrong in defining a grashof number.

The way you want it provided it is dimensionally consistent okay so this might not be an exact transformation of your conventional grashof number but you say get a non dimensional group okay so that is all you have to be worried about we should not think about complicating it further and all you need to do is once you know your heat flux put it into this and therefore now you have a local grashof number varying as a function of X so this will be the starting point for defining the similarity variable right so and once again now when you look at the similarity variable.

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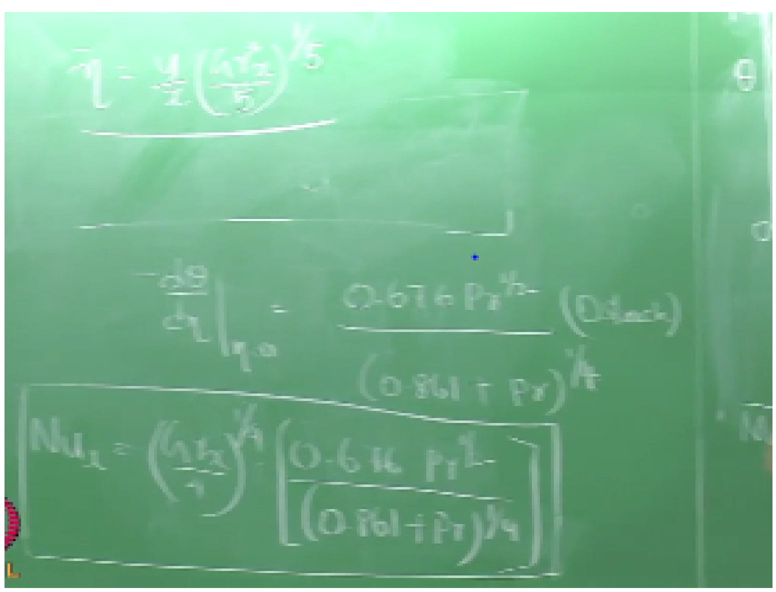

What was the similarity variable in the constant wall temperature case so we have η = Y by X your normal grashof number by 4 raised to the power 1 by 4 now if you simply replace this normal grashof number with grashof number star what happens to the similarity variable hmm it becomes y yes so that means you will not find the correct similarity variable because there will not be any function of X now your grashof number.

Star is X power 4 so X power 4 1 by 4 is xxx cancels so you will have a similarity variable which is only a function of Y which is not possible correct so therefore now accordingly you have to modify the definition of similarity variable for this case so how Sparrow and Greg did it was just changing the 4 to 5 okay so now the functional dependence of η on x and y are retained that clear if you use the default similarity variable you cannot find a similarity variable.

For this therefore you have to modify it a little bit okay they I mean if this is the right modification you should be able to find a similarity equation so that will check how we convert this into a similarity equation so but basically this is the starting point you be redefine the definition of similarity variable in the constant heat flux case because we have now defined a new modified grashof number okay so please keep this point in mind and then in the next class we will continue from here now the rest of the day rest are details so once you identify the similarity variable then finding velocities substitution they are the standard procedures and then maybe you can do this as an exercise and check what kind of similarity equation that you get okay so then we will meet in the next class.

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