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Video Lecture on
Convective Heat Transfer

Dr. Arvind Pattamatta
Department of mechanical engineering
Indian institute of technology madras

Lecture 36

Similarity Solution in Natural Convection Vertical isothermal Plate –Part 1

So good morning yesterday we started looking at fundamentals of how natural convective heat transfer occurs and also we derived the governing equations and the non-dimensional form of the governing equations we find that the more important term coming in the buoyancy force is the non-dimensional group.

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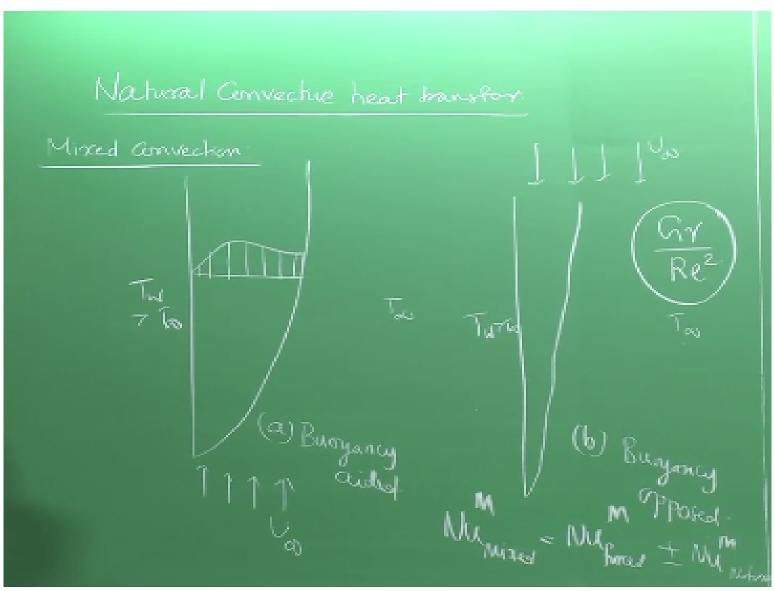

Which is grashof / re ^{2} so this is important non-dimensional parameter which is governing basically the strength of natural convection with respect to the forced convection so the Reynolds number that we defined you should remember when we non-dimensionalize that governing equations were such that.

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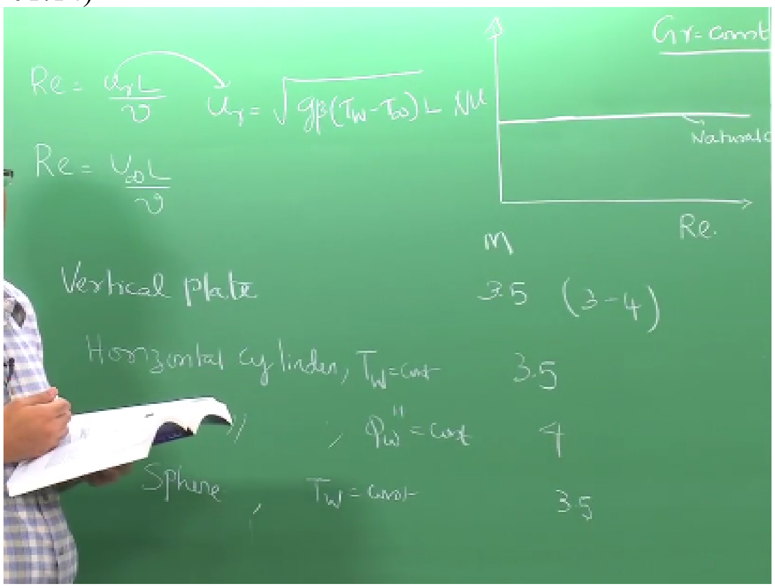

We used re = ur x L / u so in this case the reference velocity u r from the order of magnitude analysis we have also expressed this s do you remember what it is G β Tw - T ∞ x L correct so essentially the velocity here what we are referring to is nothing but a velocity scale arising out of the buoyancy force okay, now there could also be a case where you can have a bulk motion well along with the natural convection you can also have a bulk motion in that case you have to modify the definition of Reynolds number to the conventional definition where we define it based on the free stream velocity.

Okay so however still the same non-dimensional combination will be there only that the Reynolds number in that case will be defined based on the bulk velocity so this is the important non-dimensional number therefore governing the strength of the natural convection to the forced convection so this is the ratio of your buoyancy force to the inertial force no so you can have considering therefore two scenarios where you can have the force convection acting vertically upward or going downward okay, so we call this case where the buoyancy and the direction of the bulk motion are in acting together.

So this is called buoyancy aided natural convection or buoyancy aided you can say forced convection okay so these are two different scenarios in mixed convection okay so one is a buoyancy aided convection the other is a buoyancy or post conviction where the buoyancy is in this case is trying to act upward because the wall temperature is greater than the free stream temperature however the bulk motion is acting downward.

So in fact depending on the strength of the forced convection the boundary layer growth could be either way so if your forced convection dominant then the boundary layer will actually start growing from top to bottom in this case okay so this is decided by the ratio of grashof number to re ^{2} so this strength is around 1 then both of them will be equally dominant if this is very small then the nusselt's number will be the dominant one and therefore you might actually end up having a boundary layer growing in this way so depending on this if you draw the velocity profile.

So unlike the pure natural convection case you will have a finite value of velocity profile at the edge of the boundary layer and they should approach your u ∞ right so you don't have actually minima at this point it will be a finite value depending on what the u ∞ is and this will vary so if your u ∞ is constable large you can actually end up with actually going for a gradual profile and then which will approach your conventional boundary layer velocity profile okay without going through the maxima and then up new ∞ depends on the strength of the forced convection in this case nevertheless.

Now you can use the same set of governing equations that we derived yesterday and we can find the solution for the mixed convection case also however one of the more simpler approaches the empirical manner will be to find the nusselt number independently for forced convection and the natural convection and then simply blend it blend these two values together depending on whether it is a buoyancy assisted case or a buoyancy a post case so for the case of buoyancy 8 at one you see that the boundary layer growth is much faster and it is thicker than the case of buoyancy opposed okay.

So therefore the when you look at the design it can be either plus or - depending on you have a buoyancy assisted or a buoyancy post case so now the value of M here depends on the kind of configuration so I will just list out the value of that index M for a few cases for a few configurations for example if you have a vertical plate and you have an so this index M will be something like 3.5 okay so this is usually between 3 & 4 okay so this value is generally taken to be 3.5 and if you have a for example a horizontal cylinder also this value if depending on the condition boundary condition.

If it is a constant wall temperature this is again taken to be three point five the same horizontal cylinder with constant wall flux is taken to be four and similarly for a sphere also with a uniform surface temperature this value is taken to be 3.5 okay so depending on the configurations you can use values of m appropriately and blend the values from the independent results so usually it is between three and four okay so now what we will do is that so I just also will give a representation of how the nusselt numbers might probably look in the buoyancy assisted case so when we plot muscle number as a function of re for a fixed value of grashof number.

So that is I maintain say the grashof number to be something like 10 power 5 and very only the Reynolds number okay ,so I can go anywhere from there for natural convection regime all the way to post convection regime depending on the value of Reynolds number that I vary okay so for very low values of Reynolds number so then what happens this will be in the natural convection regime so typically you end up with value for natural convection now this will be your natural convection value so you see what happens at low Reynolds numbers since it is dominated by buoyancy irrespective of what the Reynolds number is the grashof number is also a constant so the nusselt number will remain a constant it will be governed by the value of grashof number.

That you use now when you again go to very high value of Reynolds numbers so then the buoyancy will be insignificant compared to the inertial force and therefore nusselt number will be purely dictated by the force convection in forced convection case nusselt number is directly proportional to Reynolds number and therefore it progressively increases in the case of forced convection so if you plot forced convection case you will have something like this so this is your forced convection case so therefore the combined convection will have to transition from the natural to the force like this so this will be combined or mixed convection so this is the assisted case so initially when you talk about low Reynolds number.

So you have only natural convection which is governing the value of nusselt number very high Reynolds number it is only the forced convection which is governing it and in between where the ratio of GR by re square is of the order of 1 so you have both of them both the force convection as well as the natural convection so your result number values will be transitioning smoothly from the natural to the post convection is that okay so this is the kind of more realistic case in many applications you can actually have both the forced and natural convection to be equally significant and we cannot therefore neglect the effect of mixed convection in those cases so now what we will do is first we will take up the pure natural convection case.

Where we do not have any bulk motion and try to look at some solutions to the fluid flow and heat transfer problem so what are the different ways of solving it just like you have your external force convective boundary layer for which we how the flashes and pole house in solution we can approach by using similarity methods.

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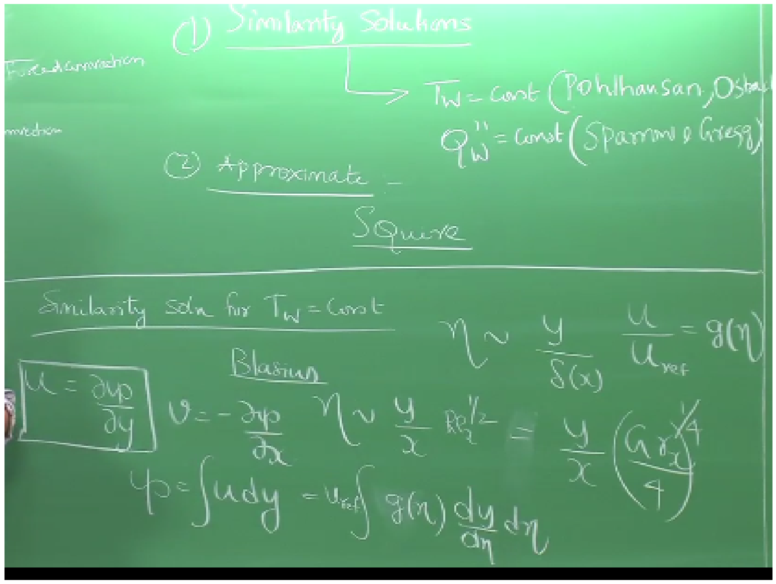

So similarity solutions are the exact solutions that we are going to do and once again this can be done for either a constant wall temperature boundary condition or a constant heat flux boundary contain so the constant wall temperature boundary condition case was originally attempted by Paul Hassan along with the external host convection boundary layer.

He also started looking at the case of natural convection and he derived the similarity equation for this case also but this was later on solved for wide range of cantle numbers by another person called awestruck okay, originally during polo since time there were numerical methods were very few so he could not find a very general solution for the nusselt number for different prantle number so only for a fixed prantle number he was able to get the solution but nevertheless that similarity equation was solved using numerical methods later on by was struck and he generalized to a different range of prantle number x' and the other solution is for the constant heat flux boundary condition and this was done by two people Sparrow and great so this is extension to the basic pole Haasan's solution to a constant heat flux boundary condition and apart from this you also have the approximate methods.

So one is the use of similarity solutions the other is the approximate technique which is not as rigorous but nevertheless gives you a very useful correlation which is close to the exact solution so the upper approximate method has been derived by a person called Squire okay so using the momentum and energy integral technique especially for the constant wall temperature boundary condition okay now later on people extended the solution to also constant heat flux boundary condition so like this similar to the external force convection you have similarity and approximate solutions also for the natural convection problem.

So we will start with the similarity solution with constant wall temperature boundary condition okay and later on we will move to the modification required in the similarity variable for the constant heat flux and then we will go to the approximate methods okay so the similarity solution was originally developed by pole housing similarity equation as such so we will look at the solution first before we go to any other extensions okay now just the same with blushes equations you have to first start with defining a similarity variable similar transformation from X y coordinate to coordinate.

Which is called the similarity variable coordinate so we do not know what is it but let us say that H will be a function of Y by some Δ right so we will go on the same lines as blushes did okay naturally because we are talking about boundary layer growth and therefore the similarity variable should be able to map the boundary layer thickness at different f x locations and when you plot Y by Δ this should be a similarity variable which should collapse all the velocity profiles okay the same starting point as the blushes solution but in this case we don't know what the order of Δ should be what was the similarity variable.

In the case of Blasius equation in the case of blushes solution the similarity variable was assumed to be y by x re x to the power half because δ the order of ninety dove δ was found out to be x by square root of Reynolds number local Reynolds number okay so now we have to find a similar transformation in the case of natural convection that we have to replace Reynolds number with our grashof number okay so yesterday we have seen that the equivalence between Reynolds and grashof number so according to this non-dimensional number we have grashof number to the power half is of the order of Reynolds number correct so therefore if you want to use Reynolds number to the power half this will be rash of number to the power 1/4 so what pollution did was to simply for sandwich right so it is re-raise to 1 / 2 this is going a square so in that case we may yeah I mean this is a generic representation yeah but you can you can plot it exactly you know I have just given an idea here the shape of the curve could be different ok so what you are suggesting is you should go like this right.

Okay you know at very low values of Reynolds number here so your nusselt number with the force convection will be very less so this will be insignificant okay so you are saying that this value should be shifted up yes no but that's what I'm saying for very low values of Reynolds number if you plot the value of nusselt number from the natural convection so this will be much larger than the forced convection value right so this becomes dominant only after the Reynolds number becomes sufficiently large this is a generic representation okay I am not plotting any values here so you can actually calculate it for fixing rash of number say 10 power 5 you will find that this value will be different for 10^{6} this will again change okay.

So but this is a generic representation saying that very low value of Reynolds numbers okay your force convection contribution is small okay if you that sort put it into that empirical correlation so it will be mostly governed by the natural convection value and once you cross a threshold Reynolds number then the force convection becomes dominant so then that will be decided only by the force convection value so in between these two is where you use this empirical correlation and blend both the values of forced and natural convection so that blending will give you a smooth transition all right.

So let us come back to the solution so when we replace their fourth Pole house and looked at the same similarity variable and here therefore he replaced Reynolds number with grashof number so in this case therefore what happens you have rash of number to the power 1 by 4 okay so this is of the order so what he did you can also use this find a similarity equation solve it so it will not change the solution but nevertheless to be precise he used rash of number to the by for the whole raised to the power 1 /4 this is anyway a constant this should not affect the final solution for say skin friction coefficient nestle number.

Because everything will get adjusted in the end okay so even if you do not do this you might get different constants in the similarity equation but then finally when you calculate the CF one nestled number they will get scaled proportionally okay by this constant right so he used this as the similarity variable many of the textbooks in fact show that this is your exact similarity variable there are a few textbooks which so meet a constant and they go ahead and solve they get slightly different constants in the similarity equation but finally the non dimensional numbers all will be the same okay and in fact the Blasius equation we did not use any of these constants so now the next step is to find the stream function okay.

So when we solve the governing equations we have to solve in terms of the stream function because then the continuity equation becomes redundant okay so therefore what is the appropriate transformation for transforming the stream function which is a function of X Y to another function which is only a function of the similarity variable H okay, so that is the next step that you have to find so how do we do that so once again we write down the equations for relating stream function and velocity field so U = DC by dy and V = - DC by DX so let us take this particular relation and we can integrate it.

So therefore Y will be nothing but integral u dy and let us also assume that if you find the right transformation variable and you buy say you reference will be a function of only this variable H right that is the purpose of finding the similarity variable so that finally when you plot u by U reference so it will not vary as x and y but it will all collapse as a function of H right so there for we can substitute for you into this so you have you reference which is taken out into G of H so now we have to transform the variable Y in terms of H so we will write this as dy / D H into DT correct and dy /D H can be expressed from the similarity variable here okay.

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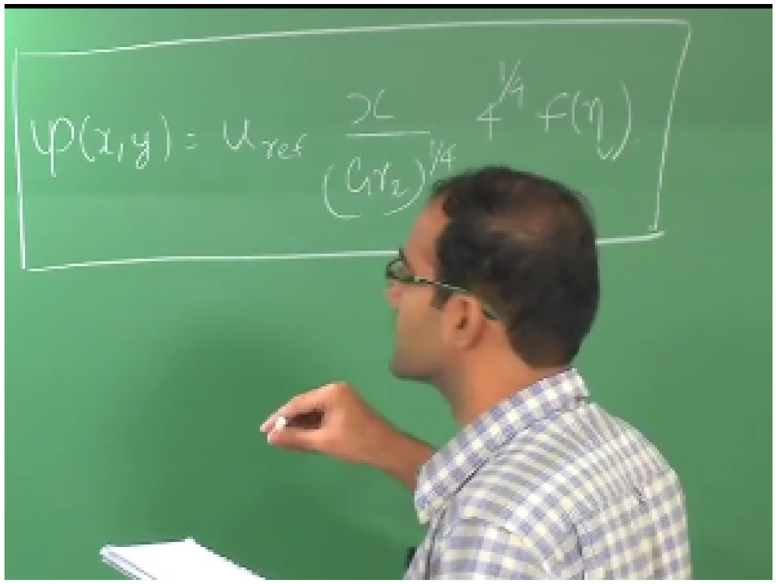

Now if you do that I will just give you what will be the expression for Sine please check that it comes out to be you reference times X /rash of number to the power 1 / 4 into 4 raised to the power 1 / 4 into this integral G of H D H will be nothing but another function of H which will say F of H and this dy / D η is nothing but X / X into 4 raised to the power 1 / 4 / crash of number to the power 1 by 4 okay and integral G of H D η is nothing but another function f of H so therefore this is your transformation from of the flow field from XY coordinate to η coordinate this is your transformation from X y coordinate to η our coordinate through the similarity variable so this is your similarity variable.

And this is the corresponding transformation of the flow field so now that you have you reference we also know that your reference can be related to G β δ T okay so in fact Paul Rosen gave he also introduced the constant four into this it does not matter if you do not introduce any of the constants we are here also finally it will not matter all these are scaled up or down accordingly so the final non dimensional numbers will not change but let us do it exactly the same way he did it so if you substitute for your reference here and therefore combine.

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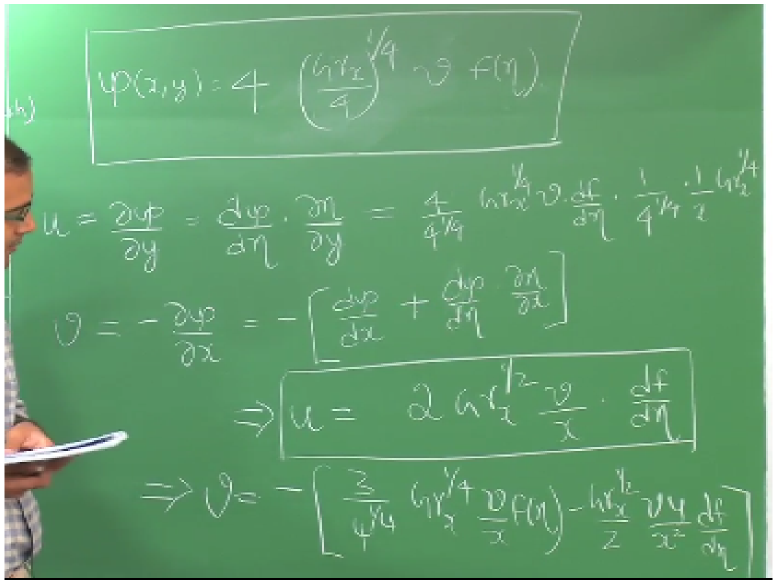

The terms you get four into grashof by four over 1/4 x nu times f of η okay so what we are doing is we are writing G β x T wall - T ∞ in terms of grashof number okay since we know the definition of thrush of number as G β into T w - T ∞ so this is a local grashof number we will define it with the local coordinate so I am just substituting for G β x T w - T ∞ as grashof number into nu square by X cube into this and already. I have a grashof number so I just combine the terms and finally I will get this as my relation between psy and F okay so therefore I find the transformation.

Now we can go ahead proceed start calculating u then V D u / DX D u by dy all the derivatives and then we can finally plug them into the governing equation so if this were the right similarity variable so we should be able to transform this PDE into similarity equation which is only a function of H so that shows that we have whatever we have assumed for H this correct so if you have some terms in X or Y that means we have still not found the right similarity variable so let us do that the next step is therefore to find U which is DC by dy and how do you find this now sy is a function of Y and you have to convert this in terms of H that means we can write this as d side by D H x D H by beveling.

Okay the side is a function of Y through H so now we will write D si by D H into similarity variable is a function of H so this is a partial derivative D H by D Y so using the chain rule of differentiation okay so I same way we can also calculate the V velocity how do we calculate V velocity now V velocity is nothing but - DC by DX now if you look at this psy is directly a function of X correct through grashof number and also function of X through H so therefore in this case we can write this as DZ / DX this is your normal derivative with respect to X plus it is a function of X through H so therefore this is DC by D H into partial derivative of H with respect to X so this is exactly the same way we did the blushes solution.

So can you calculate and tell me what the velocity is U and V are so when you write DC / D H in this case the other terms are all constants so only F is a function of H okay so essentially you have four by into D H by D Y which is nothing but 1 by 4 4 x 1 / X x grashof number raised to the power 4 okay so this gives my you to be 2 into rash of number raised to power 1 by 2 into new by X into DF by okay so we have 4 raised to the power 1 / 4 into 4 raised to the power 1 by 4 okay so this becomes therefore 2 4 / 2 right so this is 2 and we have grashof number raised to the power 1 /4 into so 1 by 4 +1 by 4.

So we have 1 by 2 okay and then we have new by X x DF by DT similarly you can find V also I will what I will do is I will write down the final expression here you can it will take some time about 5 to 10 minutes to simplify and do this you can check this a little bit later so it comes out to be - 3 / 4 raised to the power 1 / 4 / 4 u / H - okay so all you have to do is find the sine by DX treating other things as constant and differentiating with respect to X then the second term you have DF by D H x D H / DX okay.

So put together you have function of grashof number x nu + y okay so now using knowing therefore U and V we can calculate the derivatives okay so we will see how the derivatives are calculated therefore.

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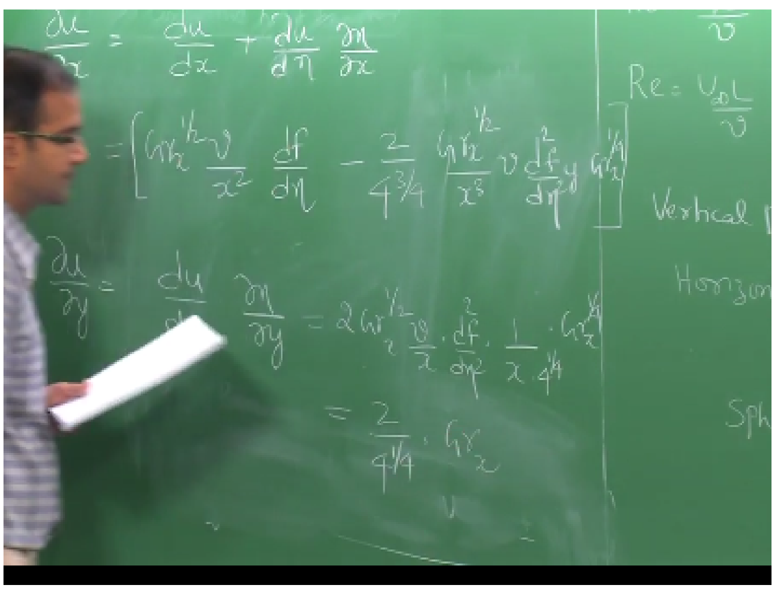

The first step is d u /DX okay now how do you calculate d u by D X now you should tell me so again you is a function of X directly and also through H okay so we can write this as therefore D u /DX + D u / D u /D H x D H / DX right okay so D u / DX so you have to differentiate by keeping the other terms constant only as a function of X only with respect to X you should differentiate D u / DX the other is d u /D H so in that case the other terms are all constant so you have only D / D η of DF /D H so you will have a d square F /D H ^{2}term into D H by DX right so if you again substitute for D H / DX and so on so this should come out to be so when you write d u by DX that DF / d H will be a constant term.

So that will just come as it is and from the second term you will have a d square F by D H square okay so you have to come one from D / DX and the other from D / D H xD H / DX that is this term where so here when you differentiate you only treat the other terms as constant huh right yeah that is what so you have grashof number to the power half / X so this is actually X^{3} / 2 - 1 okay so that is what 1 / 2 so we have that for 1 / 2 X power - 1 by 2 right so this is what you are writing here again you are splitting that as therefore Russia of number to the power half by X^{2}so here you have X^{3} / 2 - 2 that is what - 1 / 2 right okay.

So I we are finally putting wherever possible in terms of grashof number again you don't want to carry that G β δ T together so you are rewriting them in terms of grashof number again right so the next step is to calculate the gradient with respect to Y so how do you calculate D u by dy so U is a function of y only through H you look at the expression for u right we don't have any other term of Y sticking there so therefore we can write this as d u by D H into D H by D Y so now you should be able to tell me in terms of rush of number what this is this is much simpler term so you have grashof number raised to power 1 / 4 x grashof number H^{1} /2 1 / 2 1 / 4 so what will be you should have 3 / 4 3 / 4 and you have nu by X^{2}x u d^{2} F by meter square. So we can also find out the second derivative of velocity with respect to Y.

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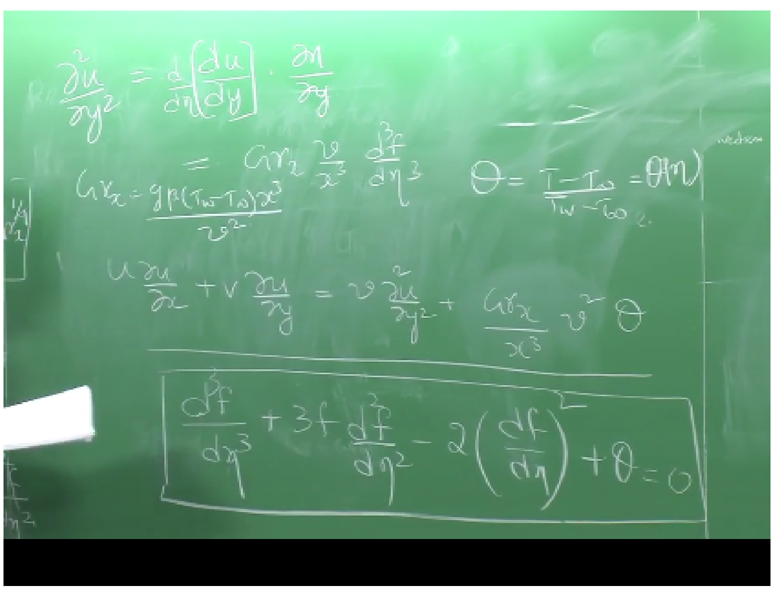

So d s^{2}u by dy ^{2}so this is again the same way you can write this as what you have D u / dy you already have d u / dy so this will be a function of Y only through H so you can write this as d by D H x D u / dy into D H / dy okay so that comes out to be rash of number into nu /y X cube into DQ F by meter cube please check once again so we have all this is constant you have to differentiate with respect to H D cube F / D H cube into D H / D Y so D H / D Y will again give gr X power 1 / 4 PI X x 4 raised ^{1}/4 okay.

So now that we have all the terms what I asked you to do is please substitute into the X momentum equation okay so our momentum equation for this case okay so we can write the last term in terms of η we can now define for the constant wall temperature case t - T ∞ / T one - T ∞ as η and therefore we can write this as G β into T one - T ∞ into η okay so you can go ahead and plug for you do u / DX VD u by dy d ^{2} u / dy ^{2} and G β x T w - T ∞ you can write this in terms of grashof number again right so what this will be in terms of grashof number so your grashof's number once again G β T w- T ∞ x X ^{3} /u ^{2}.
So this will be simply grashof number X / X ^{3} x ^{2} η so please plug in the other terms you will get some terms cancel some terms will cancel away and tell me what will be the final equation will you be able to transform the PD into a OD function of H okay as you keep doing .

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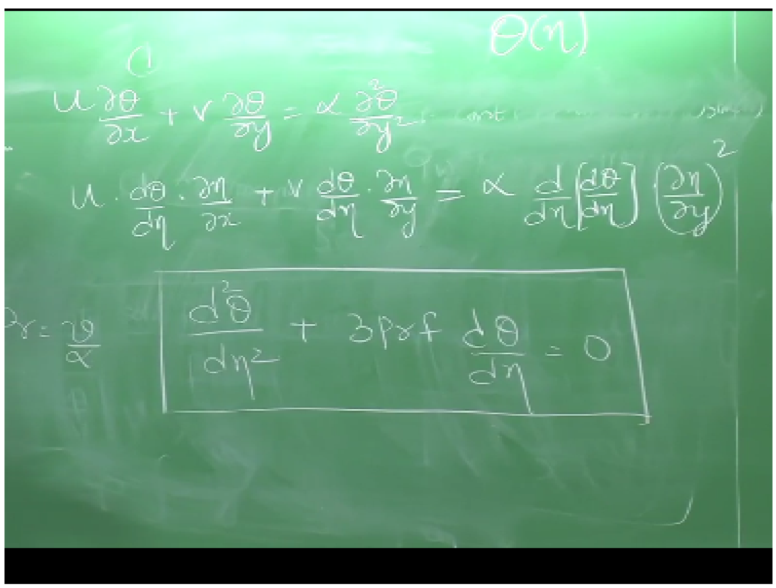

I will also write down the energy equation so the energy equation can also the we can use the differentiation by parts here so you can write this as you can assume now η to be only a function of H now let us use the variable η of η here okay so when you plot this non-dimensional temperature also as a function of the similarity variable they should all collapse same way like you are you by you reference okay.

In that case then η will be only a function of H so we can split this as D η /DT x D H /DX and so on so I will give you the final similarity equation here this comes out to be d cube F by D H cube plus 3 times F into d square F / D H ^{2} - twice DF by D H $^{(2)}$ +η = 0 so finally we have successfully transformed PDE into OD as a function of only eat up and now you see we also have the η term here so please note that finally when we are solving this equation by shooting method we have to make sure we solve both the momentum and the energy equations simultaneously okay.

So this is where the coupling comes from so you can compare this to the Blasius equation so there we have d ^{3}F / D H ^{3} +we had F by 2 into d square F by D H Square the other terms were not there so compared to that we have now additional terms okay, so to complete it now we can also transform the energy equation as a function of only H by substituting for you V D H by DX data / dy and so on okay, so I will again give you the final similarity equation for energy D square η by D H square plus so you get d square η /DT ^{2} +3 times prantle number where prantle number is the ratio of momentum to in thermal diffusion.

So in the case of pole how since solution for flat plate external force convection there was PR into F / 2 so now we have modified this okay, so tomorrow we will stop here you can also go home and verify whether you are getting the final equations so tomorrow we will apply the boundary conditions and then look at the application of shooting method for solving this equation then how the velocity and temperature profiles look after solution okay, based on that we will develop the expression for nusselt number alright thank you.

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