**Indian Institute of Technology Madras**

**NPTEL**
**National Programme on Technology Enhanced Learning**

**Video Lecture on**

# Convective Heat Transfer

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**Lecture 35**

## Introduction to Natural Convention Heat Transfer

So good and today we will start a new topic this will be on natural convective heat transfer whatever we did so far you know the external force convection and internal force convection so most of you already have done the hydrodynamics part of it before in fluid mechanics you start with the Belasis boundary layer theory for the external force convection are the fully developed internal flows for a channel and for a duct and so on but in this particular topic.

Natural convective heat transfer definitely would not have done this in a separate fluid mechanics course because this is one problem where hydrodynamics and heat transfer are coupled together so unlike the other external flows and internal force convection so you cannot separate the hydrodynamics from the heat transfer part in the case of natural convection because it is basically the energy equation which drives the momentum in this case so today from today the next the focus for the next five or six hours will be on natural convective heat transfer so what is the fundamental physics behind the natural convective heat transfer.

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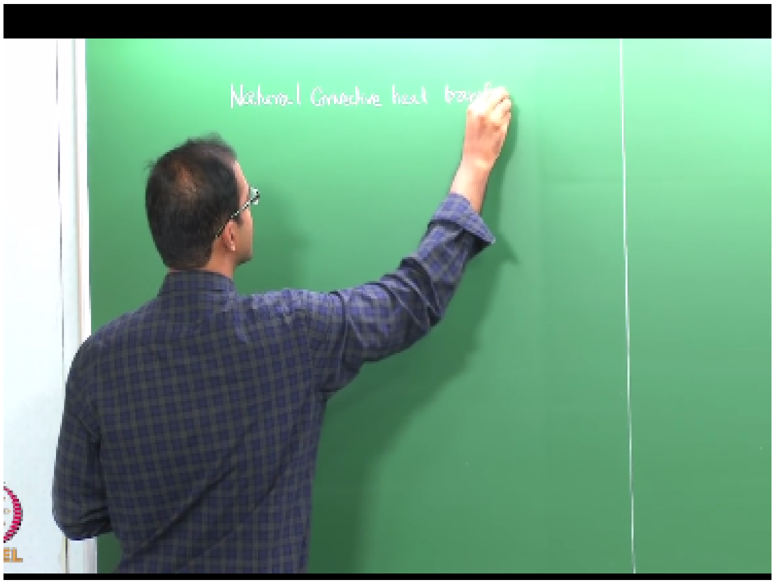


Are the motion the driving force behind natural convection we will look at the example of a simple flat plate which is aligned vertical okay so in the case of external forced convection it does not matter whether you have a flat plate placed vertically or horizontally since you push the air by external means you create a boundary layer and we do not take into account the effect of gravity in those cases but when we study natural convection the effect of gravity becomes important in fact the buoyancy is the driving force here so it matters what is the orientation of the particular configuration so let us look at a flat plate which is oriented vertically.

And we have gravity acting downwards so initially the ambient air is quite and that means there is no forced convection of anything so you just place the flat plate in a quiescent atmosphere and then you heat this plate you can either maintain a constant wall temperature or constant heat flux boundary condition and you can heat this vertical plate now naturally what happens is that the fluid layer which is in contact with this plate will be at a higher temperature compared to the ambient since you are heating the plate so let us assume temperature of the ambient to be T $\infty$ therefore the fluid layer will be at some temperature.

T which will be > Infant so now therefore you know that the density is a function of temperature especially very strongly for gases so when you look at say kwasind atmosphere now the heated air here will therefore have a density which is dif from the density somewhere outside where you have T $\infty$ so the density here P of T therefore will be < through what $\infty$ so that means you have a lower density air close to the hot surface now the tendency of the lower density air is to naturally rise up right so therefore you will have over a period of time you will see a visually a boundary layer which is actually forming and growing.

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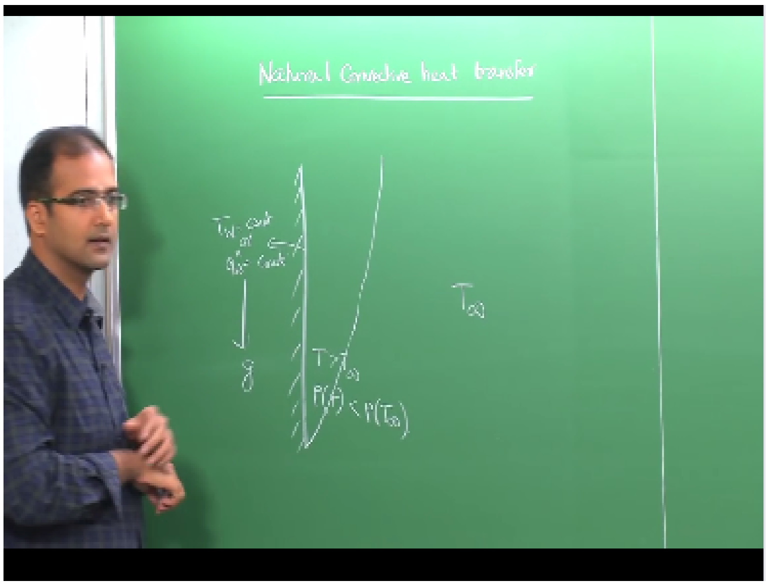


From the leading edge the leading edge here is actually the bottom of the plate here all the way up okay so we will have a coordinate system x and y in such a way that X is in the direction of the along the plate and Y is perpendicular to the plate so our origin will be starting from the bottom this is your X and the perpendicular coordinate is your Y okay so in the case of therefore the natural convective boundary layer the boundary layer formation happens essentially due to a temperature difference so this is the starting point of the convection to happen and because of this temperature difference this essentially maintains a density difference between the hot air close to the plate and the ambient quiescent air outside.

So this density difference will cause the lighter air which is in contact with the plate to rise up and therefore a boundary layer is formed okay so now you can very clearly see that the cause for the boundary layer is essentially due to the temperature difference so the temperature difference is the driving potential in the case of natural convection unlike the pressure gradient in the case of internal force convection or the external flow of air or water in the case of the external boundary layers so in this case you have temperature difference as the driving potential so this is a very important aspect of natural convection and therefore or intuitively you should Understand that the momentum and the energy equations have to be coupled in some way so on unless the energy the information from the energy equation goes into the momentum you cannot actually solve for the boundary layer growth now you can also do the same way by reversing the temperature direction that means you can have a coal plate and you can have heated air okay suppose you have a plate where your Tw is < T ∞ okay so your T ∞ is somewhere here and tea ball is < to ∞ so what would be the direction of the boundary layer growth in this case from top to bottom because here this density will be a function of temperature which will be lower so this temperature is lower than T ∞ therefore this density will be > P at T ∞ so essentially the heavier fluid has a tendency to go down and therefore you have a boundary layer in this case.

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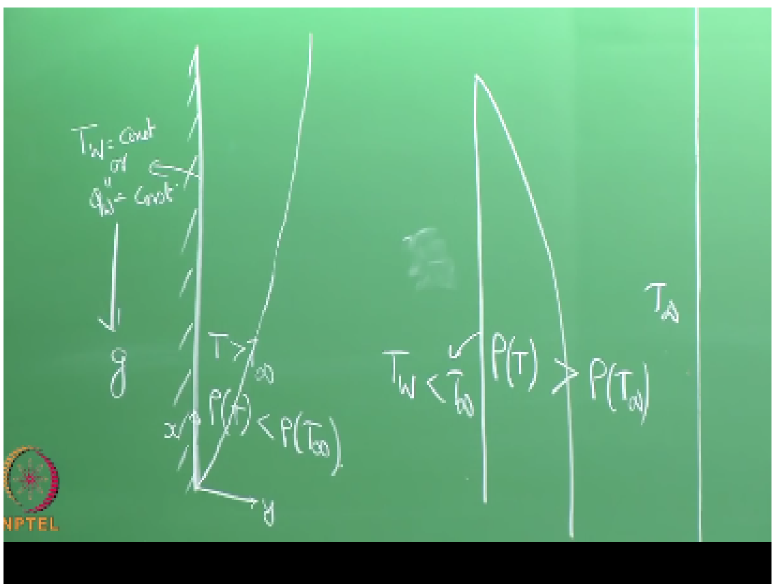


Which essentially goes from top to bottom okay so now if you want to represent how the velocity and the temperature profile varies at a particular location X location for this case so unlike the case of external boundary layer where outside you have a bulk motion u ∞ in this case this is completely stationary air so when you want to draw the velocity profile at some location so therefore the velocity has to be 0 at the wall and also 0 at the edge of the boundary layer right so you have 2 points where the velocity.

Becomes 0 in this case so therefore the velocity can attain a maximum somewhere within the boundary layer okay so obviously this is quite different from the external force convection so where the velocity Julie increases from the plate and attains a Maxima with the boundary layer so here you have 0 velocities both at the solid wall and at the boundary layer the boundary layer and therefore it has to reach a maximum somewhere within the boundary layer we do not know which what is this location we'll find it out the new course and also we do not know what is the value of this maximum velocity right so let us say this is your U max in the case of external forced convection you know that u max is = u ∞ but in this case we do not know that we have to get it from the solution right and how will the temperature profile look in this case.

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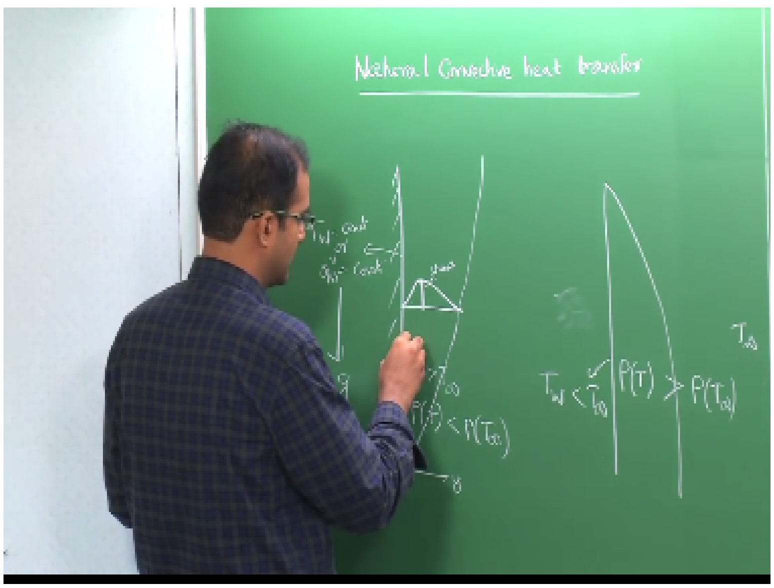


So you have the maximum temperature here and minimum temperature so this will be similar to your external convection forced convection for a flat plate right so the same way if you draw the velocity profile for this case so it will be 0 at the 2edges and then it will peaks somewhere we do not know where it Peaks and similarly if you draw the temperature profile so this is your plate so you have higher temperature outside and then lower temperature close to the plate okay so this is how the velocity varies as a function of Y and temperature as a function of Y.

Correct so we have demonstrated I mean the fundamentals of the motion of convective boundary layer when you have buoyancy in the case where we have a heated plate and a cold plate now let us try to derive the governing equations for this case okay we will keep the boundary layer as the example so you have a vertical flat plate boundary layer natural convection and let us try to derive the governing equation so let us assume that the length of this plate to be capital n okay this is one of the characteristic dimensions that we will use in non dimensionalzing these three boundary layer equations but before we go to the boundary layer equation let us first write down what will be the navies-stokes equations 2 dimensional steady state now we have Stokes equations for the natural convective boundary layer faster vertical flat plate.

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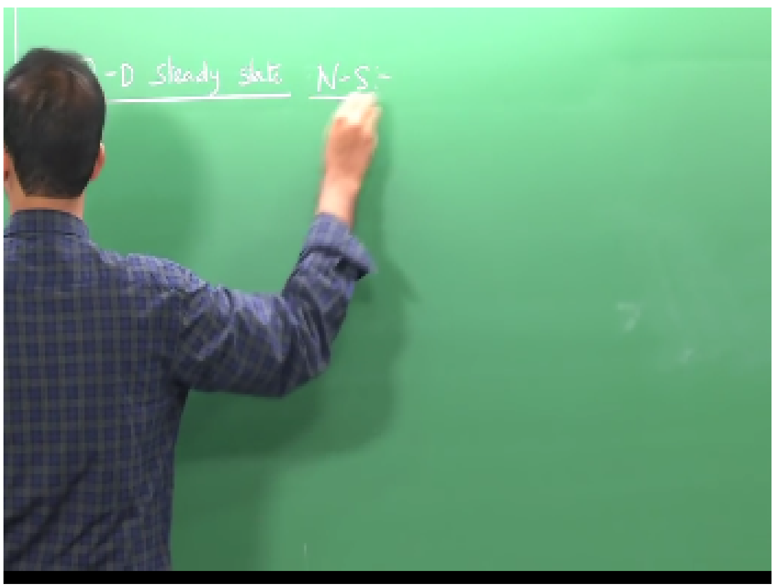


So how will the continuity equation look what will be the convective continuity equation in this case d by DX of P u so unlike the external forced convection we cannot claim that density is a constant this is an incompressible fluid but density is now a function of temperature and therefore since temperature is a function of position so density becomes a function of space therefore we cannot take P outside the derivative okay so locally it will look like it is a compressible fluid because the density keeps varying the different position so you cannot therefore there Li at a first cut say that this is a incompressible approximation straightaway.

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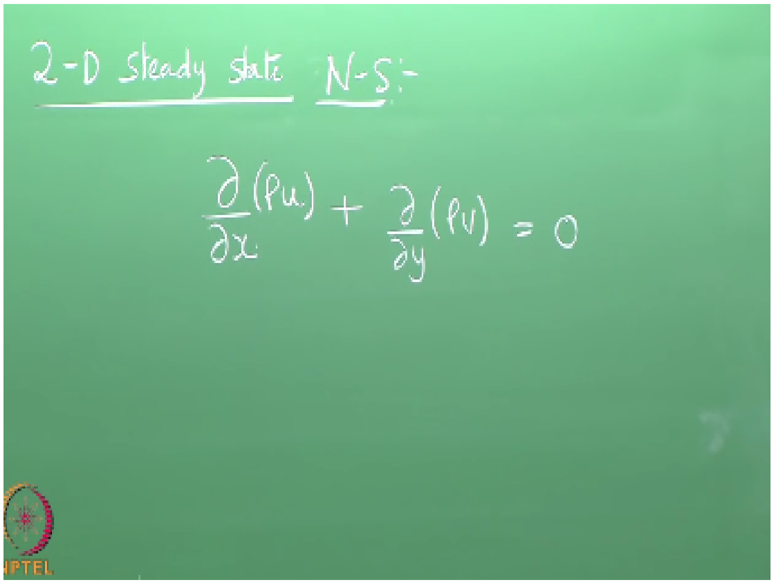


We cannot directly pull density outside the derivative and say it is a constant right however it was shown later by bossiness okay so bossiness has made the approximation that it is fairly reasonable to treat density as a constant in the continuity equation and also for the most part of the momentum equation except in the body force term of the momentum equations so the body force where this is where the driving potential the density difference emerges as a function of temperature difference so except for the body force term which is the driving potential it is reasonably good enough to approximate the density to be a constant.

Everywhere else in the other equations are other parts of the equations provided the temperature differences are small enough if the temperature differences are very large even the bossiness approximation will not be held valid so what we will do is just to start off we will make a approximation as done by bossiness and therefore try to pull density outside as a constant from the continuity equation okay and also the convective part of the momentum equation so therefore if you write down the X momentum and Y momentum and invoking the bossiness approximation.

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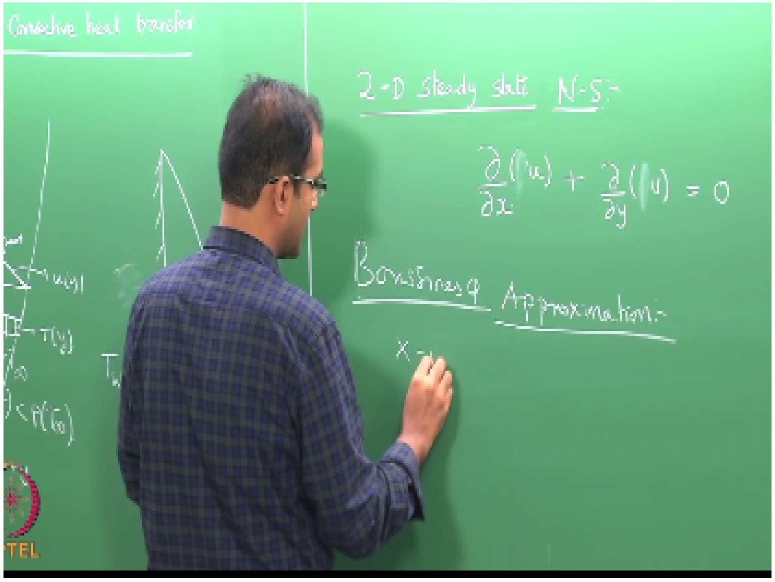


So let us write down the X momentum equation how does the X momentum equation look so you have let us keep density outside the derivative but let us not divide it by density right away so P of you D u by DX + V G u by D y this is your convective term according to the bossiness approximation he says both in the continuity equation and in the convective part of the convective acceleration part of the momentum equation you can take density to be a constant.

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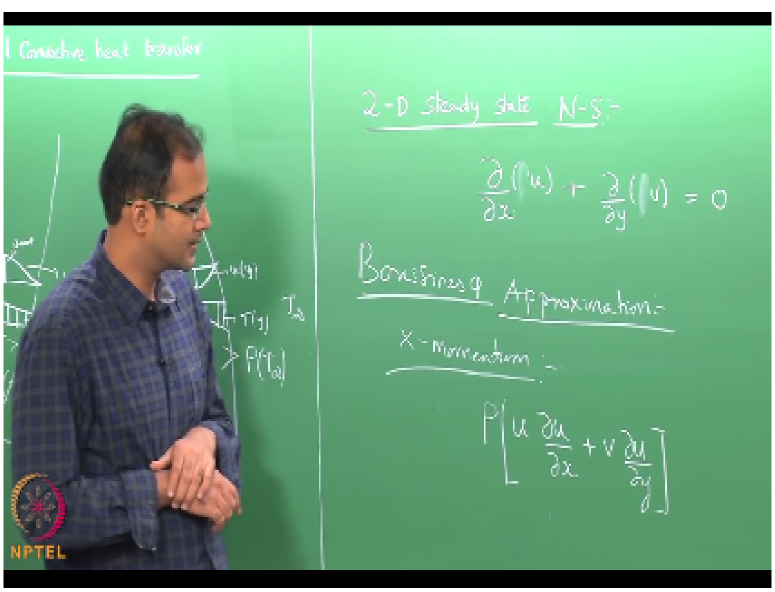

So this is =what are all the terms on the right-hand side - of DP by DX so now this is the momentum in the direction of the plate that is in the vertical direction so you have DP by DX + μ into d square u by DX Square t square u by D y square and then what else you have the body force so here definitely the gravitational acceleration cannot be neglected so the body force is nothing but the gravitational acceleration G which is acting downwards.

So therefore we put a _ G right so we have P G correct  right so now once again similar to the external force convective boundary layer  if you do a scaling analysis we can order of magnitude analysis we can  show that the diffusion in the vertical direction and that the diffusion in the  Y direction here the Y direction is the horizontal direction now Cartesian X Cartesian Y and is this is much > your diffusion along the length of the plate okay so this is the same  conclusion that we also got from external force convection.

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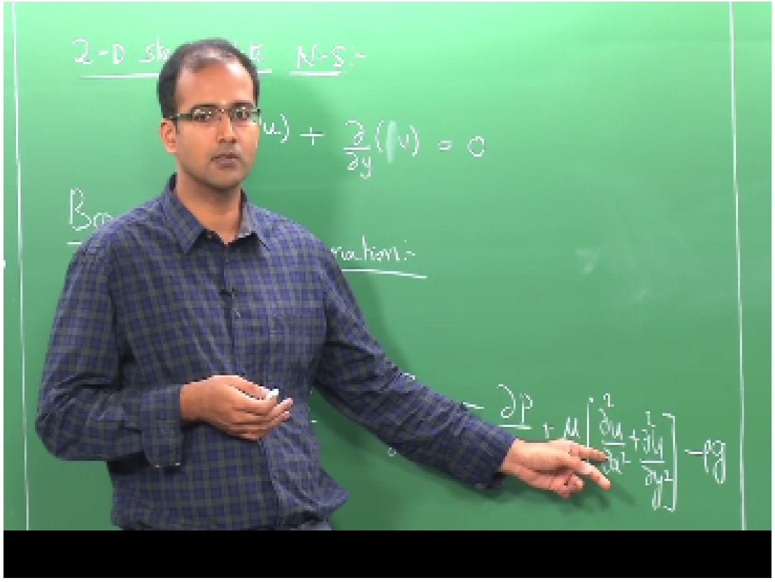

And therefore for the sake of simplicity we will only write the diffusion perpendicular to this place not in the direction across the boundary layer okay so this is the most dominant direction of the viscous diffusion right so therefore we have the X momentum can you write down the Y momentum similarly.

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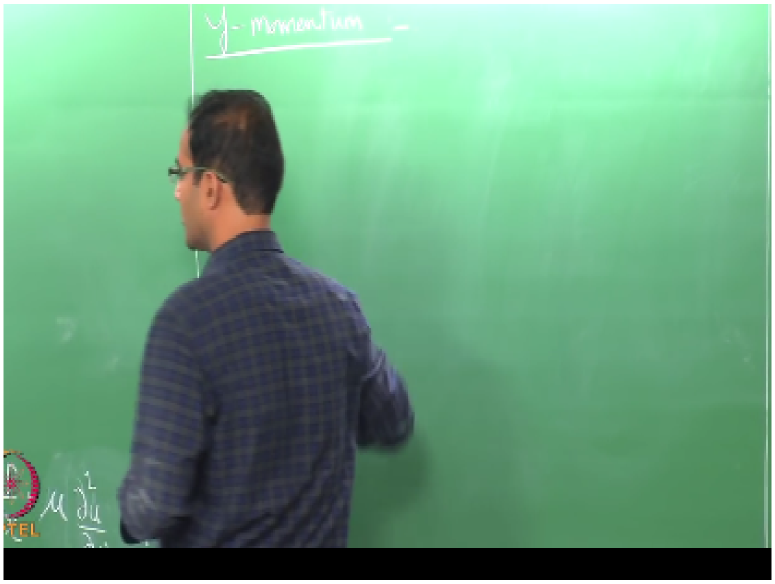


You can we write down the so what do we have for the convective terms in the Y momentum you DV by DX + V DV by D y but we do not have any V velocity in this case we have only u velocity which is varying as a function of Y we do not have any V velocity that is velocity perpendicular to the plate length okay so therefore the adjective term the convective term of the Y momentum equation will be 0 on the right hand side you will have pressure gradient term DP by D y there is no diffusion of the V momentum also and there is no body force in the Y direction okay so essentially DP by D y is =approximately 0 very smooth the others are very small so we can approximate it to 0 so that means P is not a function of Y that means P is a constant along way so this is the same conclusion.

That we also got for the external forced convection okay that means the pressure that you calculate outside the boundary layer the same variation also happens inside the boundary layer so if you draw therefore a line here the pressure at this point outside the boundary layer will be the same as the pressure within the boundary layer ok so that is why you are DP by D y is approximately 0 so this also says that the pressure variation that you find outside the boundary layer that is DP by DX here will be the same as what you have is DP by DX inside the boundary layer so therefore now since we have this continuity and the X momentum equation we do not have an additional equation for pressure so how do we therefore approximate.

DP by DX in this case so now therefore we have to calculate DP by DX by writing down the momentum equation outside the boundary layer since we say that DP by DX can be obtained from applying it outside the boundary layer so outside the boundary layer these become the Euler equations okay in this case there is no advection at all so essentially the convective term is completely 0 so if you write the momentum equation.

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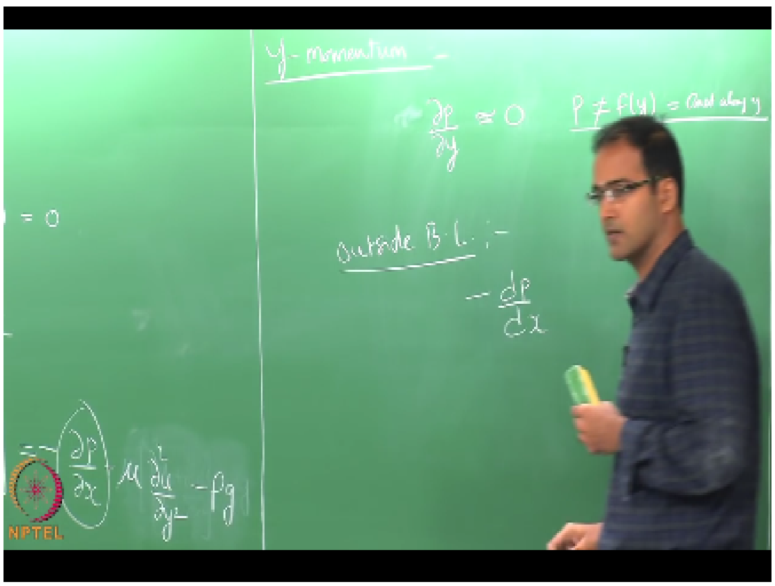

Outside the boundary layer so you end up with _ DP by DX and then what else so you do not have advection term you have you do not have any diffusion term outside there is no viscosity and you have but body force okay but we will distinguish the density within the boundary layer from outside so we will express the density here as P ∞ okay so outside the boundary layer we will use P ∞ here therefore we will write this as _ P ∞ G is =0 so this gives that my DP by _ DP by DX is =P ∞ times G so therefore I can find my pressure gradient along the plate by applying the equation outside the boundary layer.

And I for determined that this is nothing but the gravitational acceleration outside yeah so in the case that inside we don't know how the variation is otherwise we have to solve for this and we have to build another equation to solve for it or we have to use the equation of state correct so in order to therefore simplify it we take it outside the boundary layer and we see that already from momentum equation we get the clue that there is no variation of pressure along Y so that for DP by DX does not matter whether you calculate it inside or outside and outside it simply is =the gravitational acceleration so if you directly substitute it now you are eliminating DP by DX from the momentum equation therefore now if you substitute for DP by DX you have μ d square u by D y square here so _ DP by DX is P ∞ so therefore you have P ∞ _ P times G so this will be the body force okay so the effect of P ∞ G is coming from DP by DX and the default body force is P G so this difference P ∞ _ P what is this force this is your buoyancy force so this is the net buoyancy force which is now driving the momentum in the natural convection.

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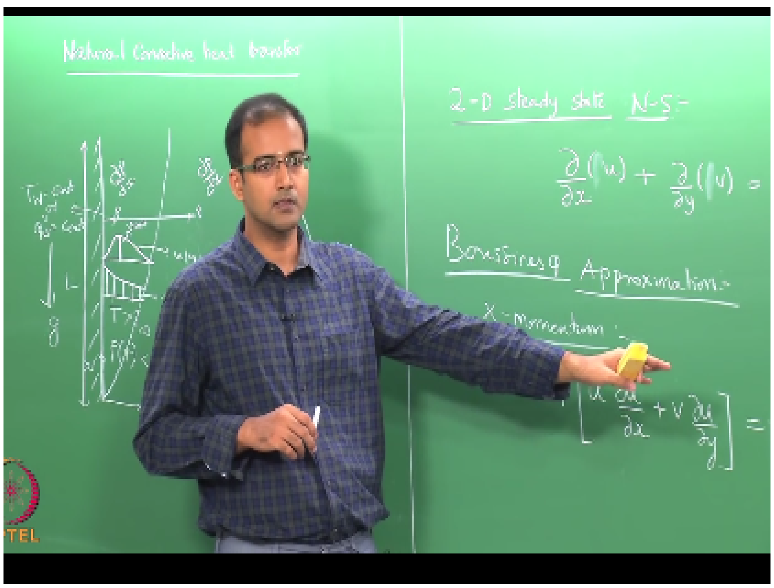


Right so if the buoyancy force is 0 then you do not have any boundary layer growth okay the boundary layer growth happens in this case only because of this density difference and what is causing this density difference temperature difference so now this is where bossiness approximation is used in the sense we are ignoring the variation of density as a function of temperature elsewhere except in the body force term okay so now to invoke the bossiness approximation so we will define the coefficient of thermal expansion β okay so β is the coefficient of thermal expansion so this is written as _ 1 / P D P / T so what it simply measures is the variation of density of a particular fluid with respect to temperature.

Okay so if this coefficient is high that means you have potential that this fluid can expand or contract very quickly very strongly as a function of temperature okay so the higher the value of β indicates that the potential for this density difference can be higher okay and these are usually measured as a part of the thermo physical properties just like thermal conductivity specific heat capacity and so on and they are tabulated for different gases okay and for ideal gas what will be the value of β how do you calculate β if you make the ideal gas equation of state if you put in the ideal gas equation of state into this so it will come out simply as 1 / T okay so why we are putting a negative sign.

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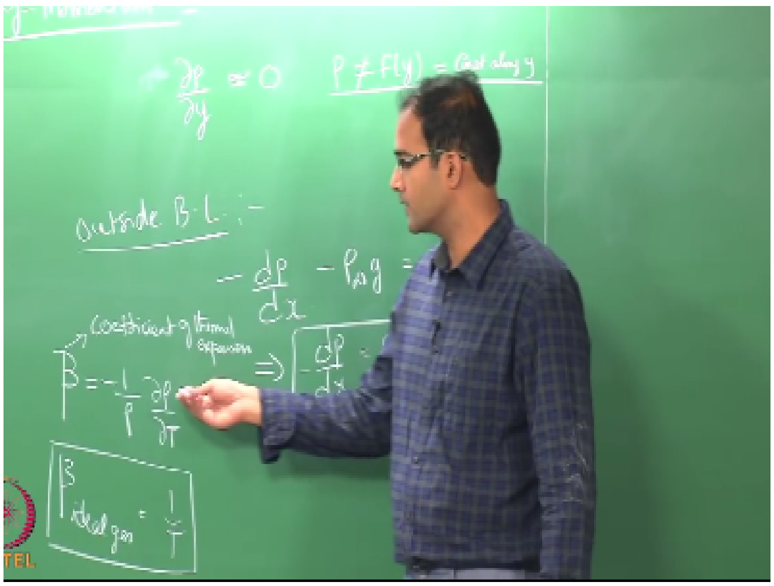


Here because usually the density decreases as your temperature increases okay so in order to make sure that this coefficient is positive okay thermal expansion coefficient is positive we put a negative sign here alright so therefore now if you apply the calculate use a simple finite difference assuming a linear variation of density with temperature okay if you want to calculate the variation from some reference temperature T ∞ to actual temperature T okay so how will this look you have _ 1 / P _ P ∞ ÷ / t _ T ∞ okay if you assume that for small changes in temperature we can assume a linear variation in density.

Okay and therefore we can just approximate the derivative d ϱ / dt as P _ P ∞ by t _ T ∞ now you can therefore substitute for this buoyancy force P ∞ _ P from this particular coefficient so therefore what do you get ϱ _ ϱ ∞ is =_ g β into t _ T ∞ or P ∞ _ ϱ is = g β t _ t ∞ so this is basically the relation between the buoyancy force to the driving potential which is the temperature difference so you can therefore substitute for P ∞ _ P from there so I just take G should not be here I am sorry so once you substitute into this you have G so you have therefore G β into T_ T also that is the P here right yeah okay P β into t _ T right okay so therefore now your buoyancy force is now written as a function of temperature difference so this is now what we call as the momentum equation in working the bossiness approximation so bossiness approximation says that the density can be treated as a constant in the advection part whereas you invoke that as a function of temperature.

In the body force term so now if you divide it throughout by P so now this looks similar to your external force convection boundary layer equation except the last term which is G β x t _ T ∞ right so this is your buoyancy term or body force term so if you our temperature difference is 0 there is no natural convection boundary layer and therefore the boundary layer grows because of this temperature difference so now you can write down the energy equation also so how does the energy equation look so it will be no different from your external force convective boundary layer equation right Q DT by DX + me DT by D y is =α into T Square t by D y square you can neglect again heat diffusion in the X direction.

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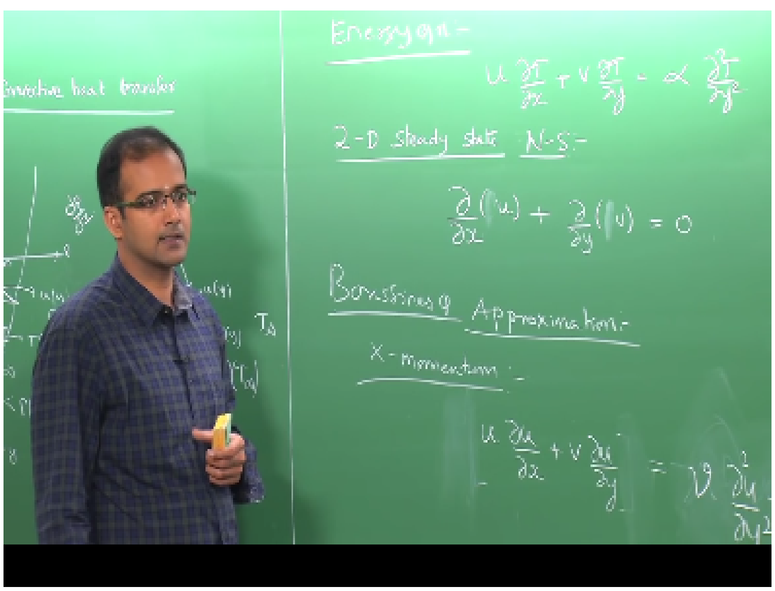


With respect to Y and if you also neglect the viscous dissipation okay we are talking about small values of Eckert numbers so in that case this will be your same as your external laminar forced convection past a flat plate okay so this will not change now what is the major differences the inclusion of the buoyancy term into the momentum equation so now you can see that unlike the other case where you solve the momentum equation first get the velocity profile so this is how Blazes did first blush is solve the hydrodynamic part you got the velocity profile then Paul Hassan.

Used that then he solved the energy equation but here the velocity profile itself is coming from the temperature okay so you cannot therefore do it in a serial sequential fashion so all of these has to be simultaneously coupled and solved now this is where the complication comes okay so that means you cannot find a simple sequential segregated solution unlike the case of external force convection so you have to couple all these equations and solve them okay now we will see in the due course of another 1 or 2 lectures we will see how to solve these equations one after the other for different boundary conditions.

But before doing that so now that we derived the governing equations let us try to non-dimensionalize them taking some reference parameters and see what are the non dimensional numbers that come out okay so I request all of you to scale the all the variables here that means you take your position X and scale it with the length of the plate this will be a non-dimensional X similarly your non-dimensional wide and how do you scale velocity here.

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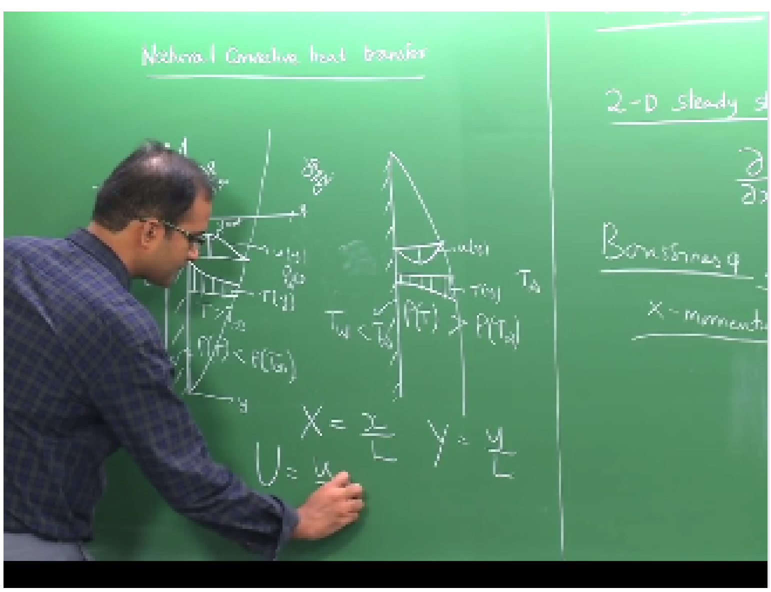


U max because we do not have you ∞ but the complication here is we do not know what is u max a priori right this is happening within the boundary layer which comes out of the solution but for the time being you do not worry about it you just assume that u max is your reference okay so we will call this as a reference velocity u subscript R some reference velocity it need not be u max also it can be any other velocity okay so some reference velocity which we don't similarly your V OHS okay now we don't have pressure term explicitly.

So you don't have to worry about non dimensionalizing the pressure and what about temperature now so again we introduce non-dimensional temperature Θ okay when we do the non dimensionalization let us do it assuming a constant wall temperature so that we can write this as T _ $T_\infty$ by $T_w$ _ $T_\infty$ all right so I will give you about five to ten minutes time you please substitute this into the governing equation and find out what are the non-dimensional groups so I will write the final expression on the board but you please work it out and check so all of you let us check whether you get the same non-dimensional groups so is that okay so you have therefore 1 over Reynolds number here okay and you have 1 over Reynolds number times prantle number okay so if you define your Reynolds number.

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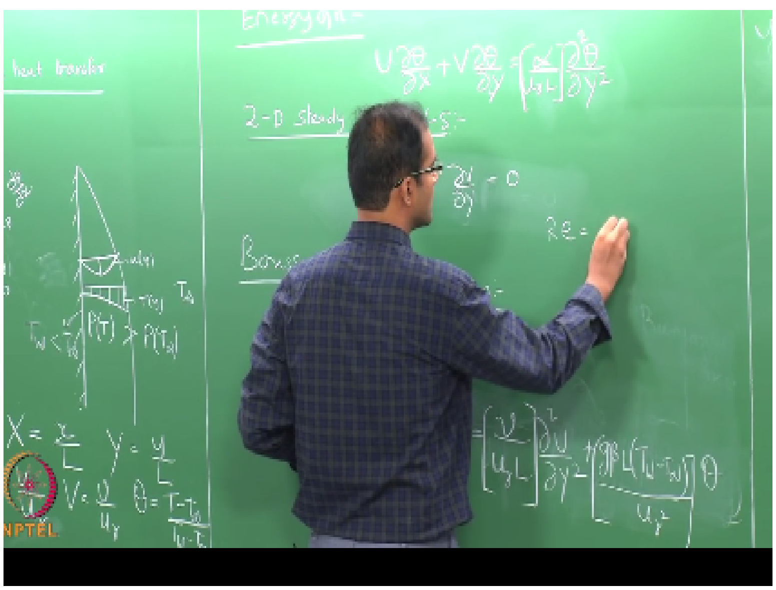


Now as u are x L by nu some reference velocity times the total plate lengths so you can therefore write this as 1 over Reynolds number and this is 1 over Reynolds number times Prandtle number what about this now you have an additional non dimensional group if you see this is no units what is the unit of β Kelvin inverse okay so this entire thing will be again a non dimensional group okay now this represents the ratio of two forces the numerator is nothing but the buoyancy force okay where G $\beta$ into T w _ T $\infty$ is nothing but the density difference P _ P $\infty$ and the denominator is your inertial force okay so we will now define a non-dimensional number in natural convection this is called the Grashalf number.

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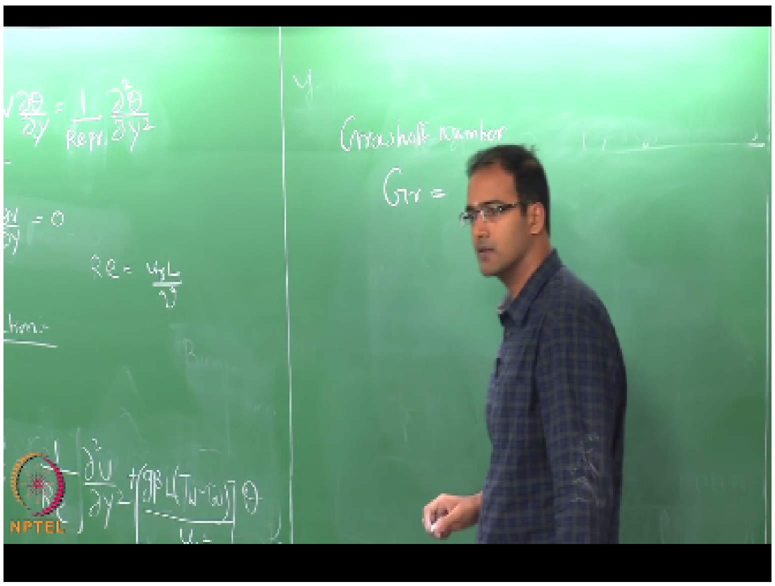
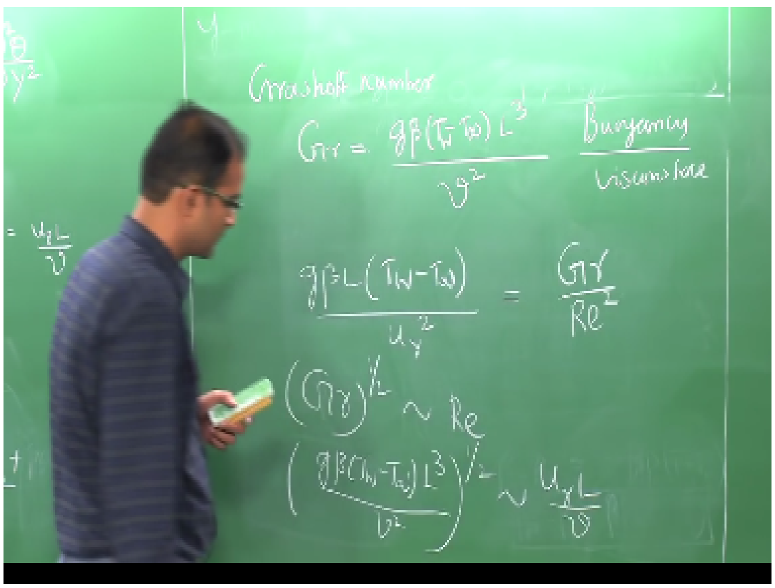



So usually denoted as Gr okay this is the ratio of buoyancy force G $\beta$ into T w _ T $\infty$ x L cube ÷ by the viscous force u square okay so this is basically the ratio of buoyancy and viscous force so you can imagine that this Grash of number is somewhat analogous to the Reynolds number in force convection so there you have inertial force here the inertial force is replaced by the buoyancy force or in fact the inertia here is driven by buoyancy okay and therefore you can write this ratio of buoyancy to viscous from using the grashalf number and also number because grashalf number is a function of buoyancy and viscous forces now somewhere function of is nothing but inertia and viscous force.

So therefore you can write G $\beta$ L x T w_ T $\infty$ / $\mu$ R square how do you express this in terms of grashalf and Reynolds number turns out to be that this is nothing but trash of by nonce number square okay so therefore this entire non dimensional group is nothing but the ratio of rush off to Reynolds number square okay so in the process we have therefore defined a new non-dimensional number which is very much relevant to natural convection which is now called as a grashalf number ratio of buoyancy to viscous force analogous to the Reynolds number now from this can we kind of estimate what is the order of the reference velocity okay so what is the equivalence of grashalf under north number so you can say that grashalf number to the power half is of the same order as the Reynolds number correct.

Because we have Gr by re square that means the order of Re square should be of the same order as Gr now therefore you can substitute the expression for grashalf number Reynolds number then calculate what is the order of the expression for calculating the order of view reference okay so this is nothing but G β T w _ T ∞ 1 q ÷ / nu square to the power half which is =u r l /.




So from this what do you get first you are hmm so you have L power 3 by 2 _ 1 which is L power half okay so you have nu which cancels therefore you are should be square root of G β x T w _ T ∞ x L okay so to get your reference velocity at least the order of it you can therefore use this particular expression.

Okay otherwise from your you know when you non dimensionalize e do you do not know what is your reference velocity but you can now using the order of magnitude between grashalf and re square you can therefore come to some reasonably good estimate of reference velocity okay so you can see that this reference velocity is nothing but what is going into your annals number definition and here it is driven by your temperature difference if there is no temperature difference therefore there is no inertia okay the inertia is essentially arising from the buoyancy term which is actually a function of your temperature difference therefore now you can classify different regimes now what we have seen is a pure natural convection case but you can also have a case where you can combine your force convection.

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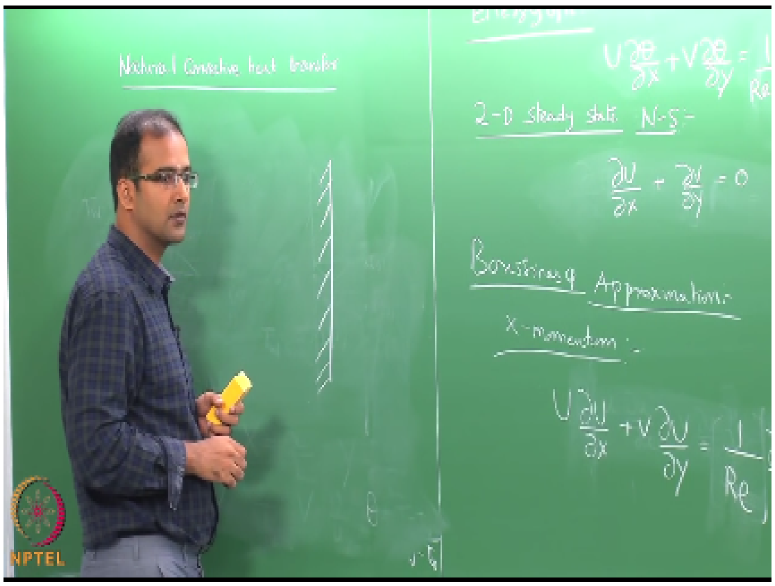


With buoyancy that means you can have a regular bulk velocity through let us say this is your u ∞ and the temperature difference is also substantial so that you can have a boundary layer growth now this is a combined effect of both your forced convection and natural convection okay so in such a case the same equations are valid right so but in that case you are you reference that you can actually use as u ∞ because even if you do not have any temperature difference the force convective boundary layer will still exist okay so when you define the Reynolds number in that case you can define Reynolds number using u ∞.

Okay which indicates the external bulk convective motion and the grashalf number is still decided by the buoyancy ratio of buoyancy to viscous forces okay so in that case what is important what are the different regimes? And what are they? How are the different regimes classified? is by the ratio of inertia of bytes re square okay so if you are talking about these values much lesser than 1 that means your bulk velocity is now overpowering your buoyancy force so in that case you can ignore natural convection and therefore this is only pure force convection so remember in this case we define a nonce number as u ∞ L by μ correct so in this case.

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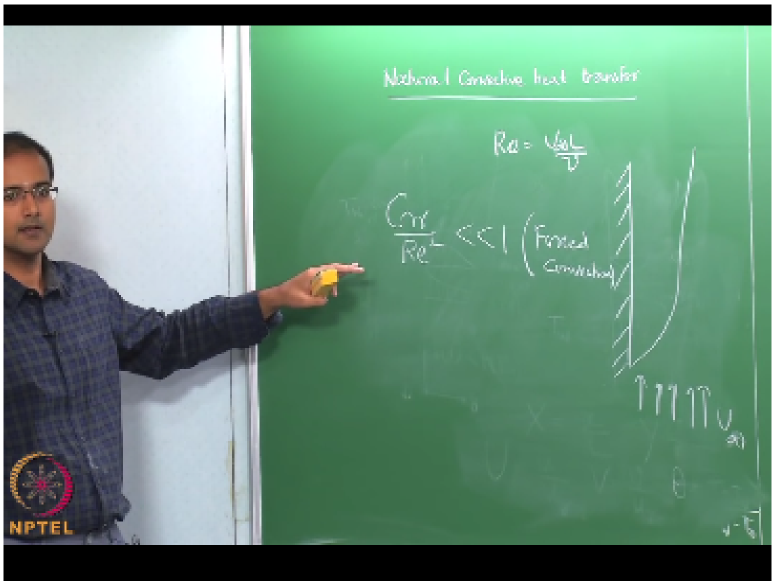


When grashalf by re square is very small you ignore your natural convective effects buoyancy fits on the other hand if you are grashalf by re square is very large that means your buoyancy force is dominating your bulk motion so here this will be your natural convection so by the way the other name for natural convection is called free convection since you do not spend you know you are not putting any effort in driving happening in making this convection happen.

It happens naturally therefore it is called free convection so the regions where this is significant that is the order of one okay so here this is called mixed convection so in the case of mixed convection both the effects of forced convection and natural convection will be equally significant you cannot completely ignore therefore either of them so we have already seen cases of forced convection derive the correlations for nusselt number and so on similarly for natural convection we will do it now what happens in mixed convection..

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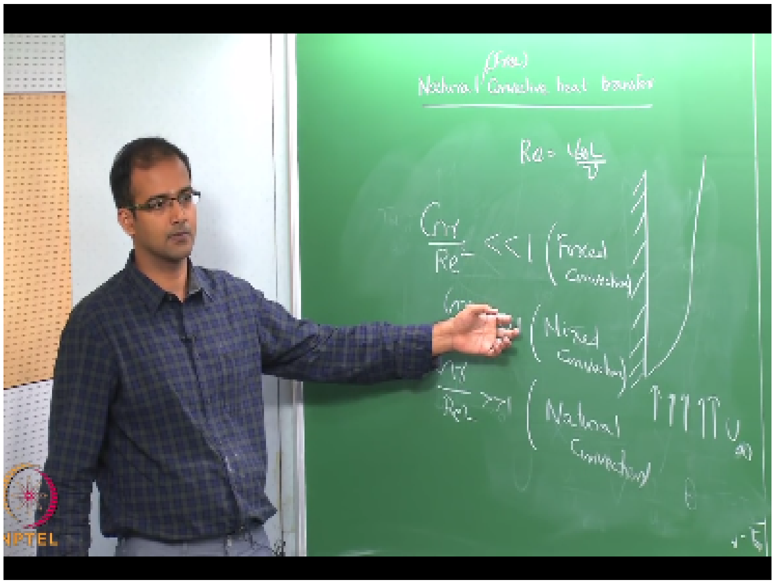


Okay so in weeks convection the most simplest way of approaching this is the Nu in mixed convection is simply calculated from independent correlations for post convection and free convection and we just use some power law to blend these two okay so this is one of the simplest approaches so here the value of M could be either 0.3 or 0.4 okay so we will stop here so tomorrows class we will look at the different ways of solving the governing equations okay so for first starting with the constant wall temperature case then the constant heat flux case and so on okay thank you.

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