

Indian Institute of Technology Madras
NPTEL

National Programme on Technology Enhanced Learning
Video Lectures on
Convective Heat Transfer

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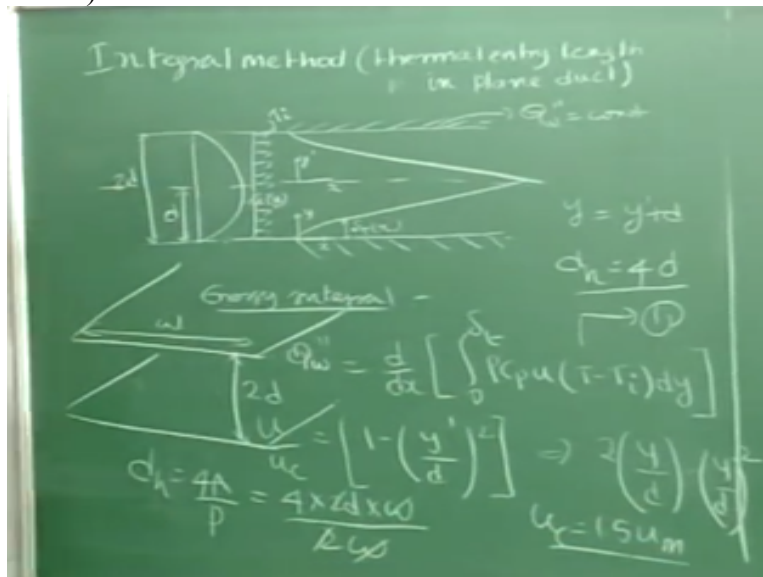
Department of Mechanical Engineering
Indian Institute of Technology Madras

Lecture 34

Integral method for thermal entry length problem

okay so let us quickly go through what we covered last time so we were looking at using integral methods for thermal entry length region and here we can find this integral method for internal flows to only Cartesian coordinate system so since it is difficult to integrate your energy equation in cylindrical coordinate system and apply the integral technique so most of the discussion related to integral method for internal flows is confined to plane ducts so that is basically flow between two parallel plates so we will take up one such example where we have a thermally developing region that is basically region two. Where we have the fully developed velocity profile the parabolic velocity profile.

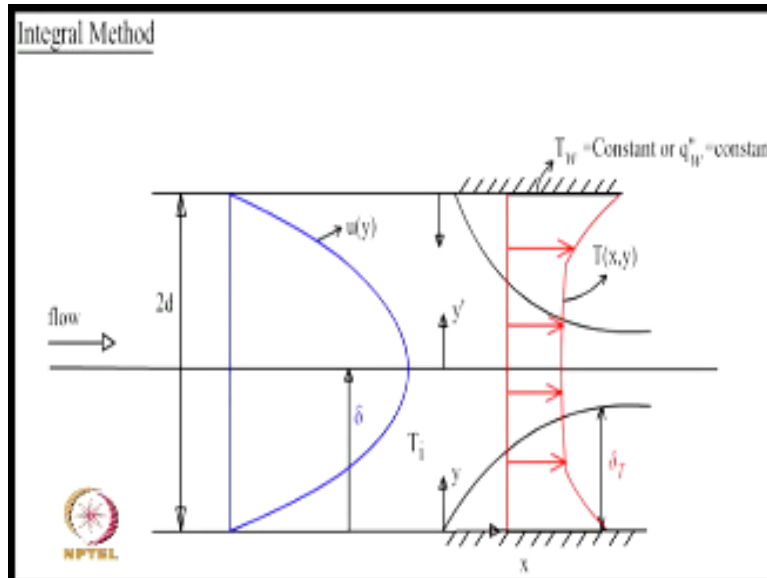
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And we have applied the constant w flux boundary condition so then when you write the velocity profile basically you use a coordinate system which is aligned to the center of the channel okay

so that we have to transform to a coordinate system which is starting from the plate because the integral method is basically integrated from the plate till the edge of the boundary layer.

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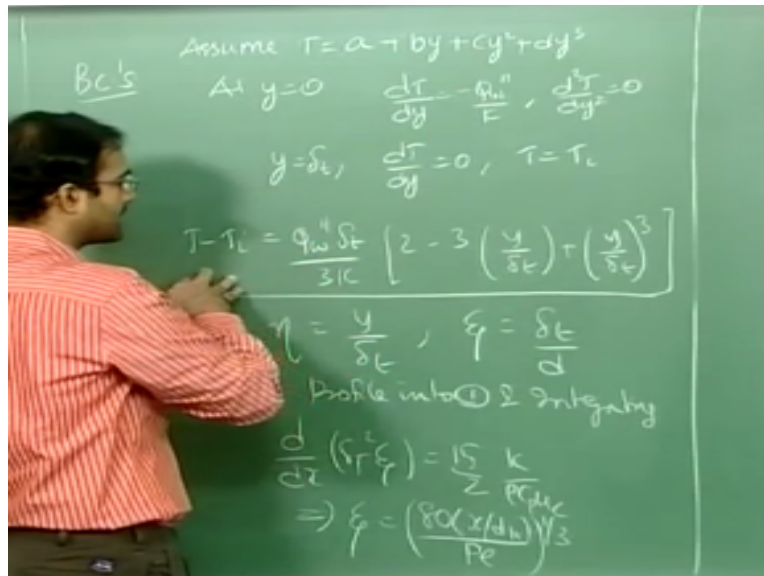


So that is where we transform the coordinates and then this is the velocity profile in the transformed coordinate okay and of course if you integrate the energy equation it's very straightforward integration you directly get the energy integral which is the equation number one.

So all you need to do is know the velocity profile and then guess some value for the temperature profile. The velocity profile here is coming from the exact solution because you cannot again use an integral method solution in the fully developed region okay, so once the boundary layers merge then the integral method cannot be applied because these are all boundary layer equations so therefore we use the fully developed velocity profile from the analytical solution but for the temperature profile in the developing region we just make a guess for a cubic polynomial as usual and these are the boundary conditions specified with flux at $y = 0$ and at $y = \Delta t$.

The gradient is 0 and the temperature at the edge of the boundary layer is equal to the inlet temperature okay so from here we get if you substitute all the boundary conditions you can end up calculating all the coefficients and this is the resulting.

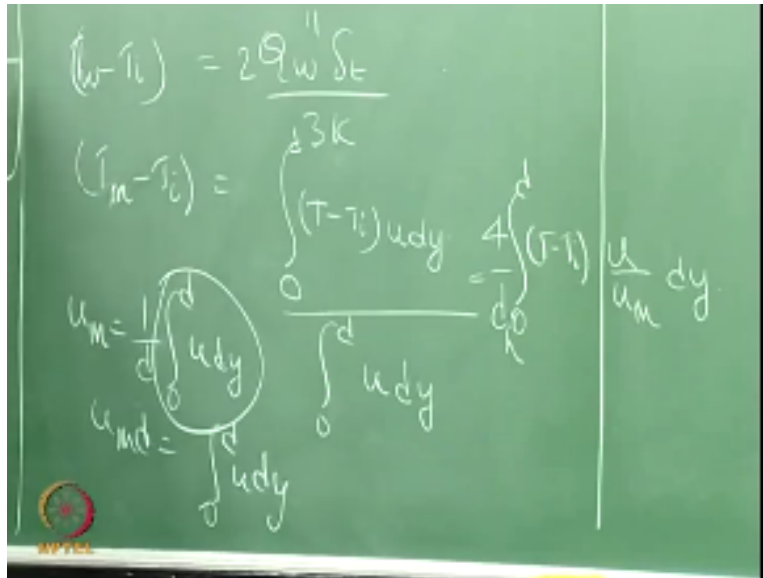
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Cubic profile for temperature that you get so the temperature profile is a function of your w flux your thermal boundary layer thickness and of course the non-dimensional y coordinate so we will introduce certain non-dimensional variables here your B is your non-dimensional y coordinate same way this is the same nomenclature I am using similar to your external boundary layer flows so $Y / \delta T$ and similar to your external flows where you use the non-dimensional ζ for δT by Δ here we do not have any boundary layer the momentum boundary layer thickness because they are considering already fully developed flow therefore instead of Δ we use D here which is the half separation distance okay.

And we substitute this profile into your momentum integral you integrate it out and we neglect all the higher-order terms of ζ since you are looking at the thermal entry length you are supposed to be looking at region where your δT is much smaller than Δ okay so therefore since your $\varepsilon < \Delta$ is your $\zeta < 1$ we neglect all the higher-order powers of ζ of the order of quadratic powers and higher so if you do that you get a direct expression for the ζ which is nothing but you are non-dimensional thermal boundary layer thickness okay so from here we have to go on and get the expression for the heat transfer coefficient.

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So we define our heat transfer coefficient h of X . Which is varying locally the thermally developing region as $Q_x / (T_w - T_m)$ so this is our definition in internal flows so now we need to get the denominator which is $T_w - T_m$ okay so we can get $T_w - T_i$ from the profile that we have this is our profile let me call this as equation number 2 so $T_i - T_i$ here is at $y = 0$ okay and we also need to know what is my T_w what is my T_m so once you know $T_w - T_m$ I take the difference I get $T_w - T_m$ okay and that will give me the final expression for heat transfer coefficient so now we have to determine what is the mean temperature so to do that how do we determine mean temperature we integrate our temperature profile this is a mass weighted average okay so that is $T_i - T_i$ and wait it with the velocity and here for the channel case we have to consider only the integration along.

This Y now the velocity profile that we have now we are looking at boundary layer growth from the top plate and bottom plate okay now we are focusing on only one plate at a time so for this the velocity profile we have to consider is from zero to D okay where you have your velocity profile that is the maximum velocity there all right so therefore we will do this integration from zero to D because this is a mass weighted average we are taking and divided by zero to D u dy okay so we know that I can calculate my mean velocity based on $1/D$ integral 0 to D u dy so I can define a mean velocity from 0 till D .

Because the boundary layer growth at the bottom plate is affected by the velocity profile from 0 to D and for the top profile it is exactly symmetric okay so I am looking at the mean velocity which is basically affecting the thermal boundary layer growth on the bottom plate okay so therefore I am concerned only with the region from 0 to D here okay not the complete 0 to $2D$ okay so if you integrate it out so you get your relation between your 0 to D u dy as u_m in two so therefore you can write this as 0 to D $(T_i - T_w) \times u / u_m$ dy $\times 1/D$ okay so now I can also express the relationship between D and the hydraulic diameter.

So the hydraulic diameter for the case of plain duct we have seen that this is basically four times D so therefore we can replace D by D H by four okay so this will be the H by four so how we got this I think all of you know if you consider a plain duct in the three dimensional sense and this is your separation between the plates which is too deep to consider width in the third direction as W so the equivalent hydraulic diameter here will be for a / P which will be four thanks to D x W bikes perimeter will be twice W so that for D H will be four times B okay so finally this is my relationship for calculating the mean temperature.

So I simply have to substitute my temperature profile into this I know my velocity profile you buy um okay which is so you are in the case of channel flows again your centerline profile is 1.5 times your mean velocity okay so this can be substituted and you can integrate and therefore determine the expression for T mean - T1.

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The image shows a green chalkboard with handwritten mathematical derivations. On the left, there is a boxed expression $\frac{u}{u_m} \frac{dy}{\delta}$. The main derivation starts with $T_m - T_1 = \frac{2q_w'' \delta_t^2}{k d_h} \left[\frac{2\zeta}{5} - \frac{\zeta^2}{12} \right]$. This is followed by $T_w - T_m = \frac{2}{4} \frac{q_w'' d_h}{k} \left[\frac{\zeta^3}{10} - \frac{\zeta^4}{48} \right]$. A note in parentheses says $(T_w - T_1) - (T_m - T_1)$. The final expression is $T_w - T_m = \frac{2q_w''}{k} d_h \left[\frac{\zeta}{12} - \frac{\zeta^3}{40} + \frac{\zeta^4}{192} \right]$.

So we will quickly do that so I am just substituting my velocity profile as well as the temperature profile so this becomes $3/2 \times 3 k d h$ and I am integrating over non-dimensional Y I am converting this into B which is y by δT okay in place of Y so therefore I am substituting all of this becomes zero to one essentially and this is $2 B \zeta - B^2 \zeta^2 \times 2 - 3 + B^3$ indeed okay so if you plug in for a temperature profile which you already have it here in terms of B so your $Y / \delta T$ here can be expressed in terms of B and in case of your velocity profile your Y / D can be expressed as $B \times \zeta$ right so you plug it in have plug it in terms of B and ζ and you integrate or the non-dimensional y coordinate that is basically your ζ between zero and one.

So this gives my expression for $t_m - t_1$ is $2 Q'' \delta T^2$ by $K D h$ and I do not now eliminate any higher order terms of ζ okay so unlike the case where I substitute into the momentum integral and there the moment the expression for $\zeta \zeta^2$ is under the derivative D / DX so there if I make

the approximation that the higher order terms for ζ can be eliminated so the differential equation becomes easy to solve whereas here I do not have any differential this is an algebraic equation so I or in all the higher order terms as much as possible okay so this is my expression for $t_m - t_I$ let me call this as equation number 4 and this is the expression for $t_w - t_I$ this is equation number 3.

So that for $T_w - T_M$ can be written as $T_w - T_I - T_N - t_a$ okay so you see there are common terms here your Q to Q " δT by K is common okay so we will just take that out so that I can write my expression for $T_I - T_m$ 2 times Q " $D H$ by K ix ζ by $12 - \beta^4 T + \beta^4 / 192$ so in fact the $T_M - T_I$ can also be substituted you can simplify this a little bit and write it in terms of β^3 $10 \beta^4 / 48$ so here I am using the relation that my δT is nothing but ζ times D so I am substituting for δT in terms of β times T so this becomes β^2 so this multiplies this becomes 8 here power 4 and this is my D so this is my expression for $T_M - T_I$ and this is my expression for $T_I - t_a$ when I subtract both this is what I get for $T_I - T_M$.

Let me call this as equation number 1 so therefore so once you have the denominator here so directly.

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$$h_x = \frac{q_w}{\Delta T}$$

$$\frac{2 q_w d_h}{4 K} \left[\frac{8}{3} - \frac{\beta^3}{10} + \frac{\beta^4}{48} \right]$$

$$Nu_x = \frac{h_x d_h}{K} = \frac{2}{\left[\frac{8}{3} - \frac{\beta^3}{10} + \frac{\beta^4}{48} \right]}$$

$$\beta = \left(\frac{80(x/d_h)}{Pe} \right)^{1/3}$$

$$Pe = \frac{U d_h}{\alpha}$$

I can get the expression for H_x so H_x will be nothing but $Q R$ "that is your constant heat flux applied to the plates divided by this particular expression here for $T_I - T_M$ so you all " cancels here now I can directly get the expression for nusselt number you can see that this can be written as $H D$ by okay so nusselt number in the terminally developing region which is based on the local heat transfer coefficient times the hydraulic diameter divided by K comes out to be of this form which is I can also take some terms common from here I can take write this as $2x4$ and I

can make it $1/2$ here okay so that this becomes 310 and 48 okay so therefore this becomes 1 by 2 and goes to the numerator this becomes 2 by ζ by $3 - \beta^3$ by $10 + \beta^4 / 40$ okay.

Where your ζ is nothing but we have already derived the expression for ζ by solving the energy integral that is $80 \text{ times } X / D H$ divided by reline umber volt power one-third so this non-dimensional ratio here quickly number by exploit is nothing but great cinema ok so you can also call this as grates number one this is inverse of grates number so this is pecelet number by X pe okay so once you basically know your non-dimensional ζ so this gives you the variation of your local nusselt number with respect to the non-dimensional axial location okay so this is the relation for the case of flat plate.

Now the question is whether if you use the asymptotic value whether the asymptotic solution leads to the case of thermally fully developed flows that is the question so therefore if you look at the asymptotic case where your ζ goes to very large value okay whether this particular solution becomes equivalent to your solution for fully developed thermally fully developed flows okay so in that case if I substitute since my Z ties in terms of $\delta T / B$ what happens if the flow is thermally fully double what should be the value of asymptotic value of z_1 okay so if for large values of ζ so that should asymptotically go to one so your nusselt number for ζ going to one from this expression comes out to be 7.86.

Now I think in the last class I have given you a table for flow past channel and what is the value of you have apply a constant heat flux in the fully developed region 8.23 so that is the actual analytical solution so compare that with what you are getting from the approximate method so it is not leading to the correct solution you know it is there is a quite a bit of an error between these two values this is because the integral method is not valid for the asymptotic limit okay why the boundary layer is noise so that part the boundary layer approximation is not valid anymore and the other thing we have in the energy integral we have neglected the higher order terms of ζ correct so in fact for we are looking at does not Artic case which is for the large values of ζ and that we have neglected here.

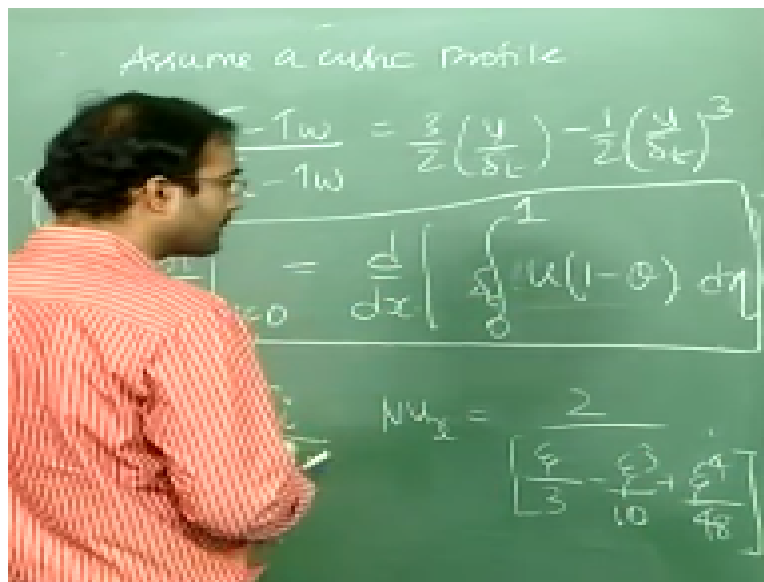
So even if the boundary layer approximations we are assumed to be valid since we have made an approximation here so together put together finally the asymptotic values are not that close to the exact values okay so therefore you should consider the integral method when applied to the internal flow case this is a very approximate method and it is valid only in the region where your boundary layer approximation is valid that is the initial part of the thermal entry the entry length if you go down downstream to the place where it merges and finally becomes fully developed then you cannot apply the approximate methods okay so this is to just give you an overview one example how we can use the approximate method.

In internal flows and that that is also restricted to a Cartesian system Cartesian coordinates and again you use the exact solution for the velocity profile and you work out the case for thermal entry length so you can do this for also a similar analysis can be done with the constant w

temperature so I am not going to do this in fact I have posted assignment number four on the Model you can just go and check that and I left that as an exercise in the assignment for where you are supposed to sit and work out for the constant w temperature case it's a very similar exercise very straightforward this is the same similar kind of an exercise which you did for the external flows okay.

So I will just give you the final solution for that before we stop the discussion on laminar internal flows so if you consider that instead of constant w flux you have constant w temperature boundary condition okay so for the case again.

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You can assume a cubic polynomial for temperature and you should of course use the appropriate boundary conditions in this case so rather than saying the gradient is equal to $-QL$ by Ku we say that at $y=0$ where T is $=t_1$ which is fixed boundary condition and the other boundary conditions remain the same you get profile which is something like this $t - T_w / T_w -$

$T_w 3/2$ okay so this is this is if you recollect this is this is the exact cubic profile that you got for external flows okay the same profile only the non dimensionalization is a little bit different you have $t - T_w / T_1$ instead of $T_\infty - T_1$ here your free stream temperature is replaced by your inlet temperature at the edge of the boundary layer.

Okay so you can assume this to be a non-dimensional temperature β for the thermal entry length problem and your energy integral now so this is your energy integral this can be rewritten so your QR can be written as $-KDT/dy$ at $y=0$ okay and that is $= D/DX$ you have 0 to δ_T now I am going to use the non dimensional coordinate for Y which I define as B which is $=Y/\delta_T$ so therefore I can integrate across the non dimensional coordinates so that limits of integration becomes 0 to 1 okay so $P CP x u xt - T_1 I$ can write in terms of β so how do I

express $t - T_i$ in terms of β so that becomes $1 - \beta$ will be $t - T_i$ by $t_1 - T_i$ -clear okay so therefore $t_1 - T_i$ is constant because my w temperature is constant inlet temperature is constant so I can introduce $t_1 - T_i$ and both sides.

It will get cancelled off so this can be written as $u = \frac{1}{\beta} \times D \frac{dY}{dx}$ so instead of dy I substitute in terms of B okay and if I convert $Y = 2B$ how multiply by δT okay so there is a δT also which is coming here so once again this K can be written as $\alpha P / CP$ right so P / CP cancels on both the sides and I can convert my Y in terms of β so I have to also multiply by δT in the denominator so this becomes $B=0$ okay so this is my modified energy integral slightly modified form for the case of constant w temperature so now I have the cubic profile and of course the velocity profile is again known the fully developed velocity profile okay parabolic profile so I substitute and then.

I proceed ahead the same way I did for the constant heat flux calculate my mean temperature and then finally I get an expression for the local variation of the nusselt number I will just give you the final expression okay.

So the final expression comes out to be $h_{local} X = \frac{2}{\zeta} \beta^3 \frac{10 + \zeta^4}{48}$ okay so you can compare the expressions that you got from the constant heat flux they are coming to be the same here I think you have to do it and check probably I made a mistake here okay you please check in the assignment when you do that in the I have given this as an assignment problem so you please check.

What is the expression so I am not going to give you this right now okay so I myself have not worked it out so you please do that and check it has to be of course different so this is to conclude our discussion on approximate methods in the last class. I have also given you a tabulation where I have compiled all the fully developed nusselt number values for different geometries, we start from plane duct with the parabolic velocity profile then circular duct for a slug flow profile plug flow profile a parabolic flow profile and also for other cross-sections like triangular cross section and the order of decreasing nusselt number starts from your channel flow the highest value comes for the plug flow case with the constant w flux.

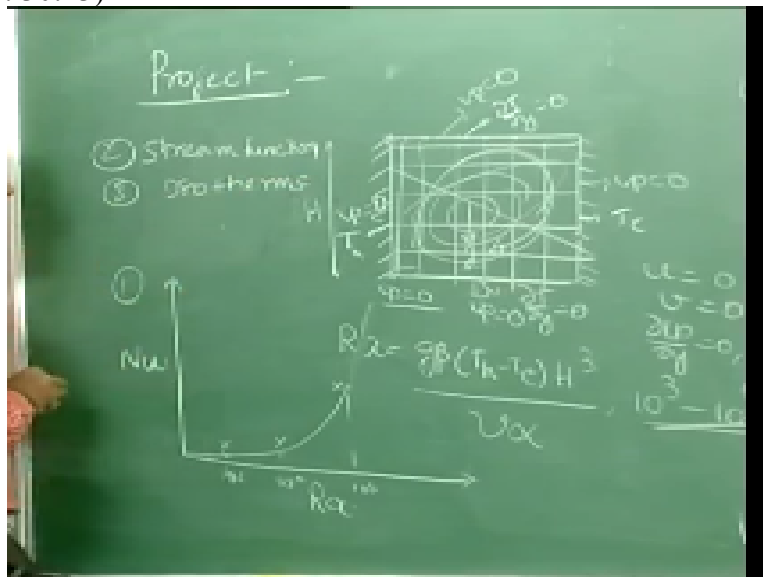
Then constant w temperature then your parabolic flow velocity flow so it goes in that order starts from to duct flow and then you are 3d circular ducts and then finally your triangular cross section so this is the hierarchy in which you are decreasing nusselt number follows that pattern and before. I conclude I just want to talk a little bit about the project that I was thinking in mind. I also posted another document on Model which you will see it is called project natural convection in a square cavity. I will just briefly describe the problem and also how you are going to solve it and I since you have now more than a month's time with you. I think you should probably take it up a little bit seriously and try to do the project you know.

So for whatever you learnt in the class they are all based on some theoretical discussion and deriving some analytical expressions okay and that is not probably sufficient for you to gain appreciation of for subject like convective heat transfer okay a few of you are coming and asking me in this subject only dealing with deriving expressions okay there are so many equations and the mathematics is very rigorous and is it limited to that okay, so in order to satisfy those kind of people it is the better that you also do a practical hands-on project take up a case where you can really feel that you can apply the concept that you learnt of course.

You know I am not asking you to do analytical solutions for everything so you can also try some numerical solutions okay which are nowadays gaining immense popularity and replacing the analytical solutions you should try that and see for yourself as a fundamental research problem how you are able to gain your number better your understanding of what you learnt in the class so far that I want to know give this small problem of course they take some coding now you have to write a small code you know based on finite difference method it is a very straightforward technique once you know that and the document which I posted as all the details including.

How to write the finite difference expression for the governing equations how to apply the boundary conditions and how are you going to solve iteratively and it also gives the solution so I want you to just simply read the document thoroughly and understand and try to implement it may take some time that is why I am announcing this one month before your final exams okay so I will just give you a brief overview any questions on what we have covered I hope everything is clear okay so anyway when you do the assignment problems I think things will get much more clearer alright okay so coming to your project of course.

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I am being a little bit hasty here before the topic of natural convection is thought to you I am giving you the problem but I think most of you have a basic heat transfer background you will be able to quickly appreciate what I am explaining here so you can consider a square cavity just for geometrical as well as computational simplicity nothing else it can have any aspect ratio and you can consider that the left-hand w of this cavity is given some temperature you know you can call this as the temperature which is higher than the right side temperature okay, so this is a heated cavity and the top and the bottom was are insulated okay, so in this case you don't have basically any flow to start with okay.

So there is no inflow outflow here it is just cavity which is enclosing a space and you it is filled with fluid and now you already know some basics of natural convection so because of the temperature difference there is a boundary layer growth which happens due to the density difference and this will start a convection process okay so this convection process is called the natural convection okay so it is also called as a free convection so now for small temperature differences so in natural convection we characterize the non-dimensional number which is called the grashof number also written sometimes as Rayleigh number this is your $G \beta \times \rho H^3 / \mu^2$ into if you look at the dimension.

If it is a square cavity dimension is H cube by kinematic viscosity into thermal diffusivity okay so this is a non dimensional number which characterizes the ratio of buoyancy force to the viscous force and this will give you the strength of the convection which has happen so now you can see that that is linked the buoyancy force is linked to a temperature difference the higher the temperature difference the greater is be the density difference and that creates the convection patterns to be stronger and stronger so initially for small ΔT 's you will find the convection pattern is very a mild and it will be mostly conduction.

So if you look at the isotherms you will find that the isotherms go like this almost and there is a linear profile from the left to the right that is indicating that it's conductive mode of heat transport and as the dial a number increases due to their temperature increasing temperature difference so the convection pattern becomes dominant and you will see non-linearity coming in the temperature profiles do not look so good and then you start seeing streamline patterns so initially your streamline patterns will show a circulation like this later on it may become more and more convection dominated okay.

So you can start from Rayleigh number of say 10^3 and go up to 10^5 okay so from something which is close to conduction to something which where you can see the dominant effects of convection but still it is laminar okay so this is what, I want you to do with the project you take a small cavity square cavity and you vary the Rayleigh number from 10^3 to 10^5 and you look at the modes of heat transfer when it when it starts from being from conduction then transistor transition to convection and then becomes pure natural convection and to solve this you do not have to solve the navier-stokes equations in the true sense you do not solve the momentum equations but you solve it.

In a stream function vorticity method which I have derived in the very beginning okay so we introduce a stream function and vorticity and solve the equation so just to give you an overview how the equations look for the case of natural convection ok so anyway if you introduce your stream function the advantage is that your continuity is automatically satisfied so you do not have to solve for the continuity equation and now you can eliminate the two momentum equations by taking for example to differentiate the X momentum with respect to Y and the Y momentum with respect to X and you subtract the two therefore you can eliminate the pressure gradients and finally you can get a single equation divide of pressure okay so you have one variable less to solve now okay and therefore the number of equations also come down so in that case your vorticity equation finally.

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2-D incompressible steady state

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right] - \beta \frac{\partial T}{\partial y}$$

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right]$$

velocity transport eqn

$$\frac{\partial p}{\partial y} - \frac{\partial \omega}{\partial x} - \frac{\partial p}{\partial x} \frac{\partial \omega}{\partial y} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right] - \beta \frac{\partial T}{\partial y} \quad (1)$$

$$\frac{\partial p}{\partial x} - \frac{\partial \omega}{\partial y} - \frac{\partial p}{\partial y} \frac{\partial \omega}{\partial x} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right] - \beta \frac{\partial T}{\partial x} \quad (2) \text{ (Energy)}$$

If you do that can be written as $u = \frac{\partial \psi}{\partial y}$ so I am writing this under 2d incompressible and steady state approximation so you do not have to look at the unsteady or transient patterns you just directly go to the steady-state and want to get the steady-state solution okay so therefore the steady-state equations will be $-\rho g \beta \frac{\partial T}{\partial y}$ okay so now you introduce stream function in terms of ψ to replace the velocity in terms of stream functions so I can say my $U = \frac{\partial \psi}{\partial y}$ if I assume a stream function and $V = -\frac{\partial \psi}{\partial x}$ so I can write this in terms of stream functions so $\frac{\partial p}{\partial y} - \frac{\partial \omega}{\partial x} - \frac{\partial p}{\partial x} \frac{\partial \omega}{\partial y} = \nu \left[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right] - \beta \frac{\partial T}{\partial y}$ is the rest of the things on the right hand side is the same okay.

So this term is a body force term which comes in natural convection and you take the derivative of the Y momentum equation with respect to Y so therefore you have a $\frac{\partial T}{\partial y}$ and the gravity is acting downward here so therefore you it acts only on the Y momentum equation okay if it was a inclined cavity then you have gravity acting in both the momentum equations so therefore

when you take the derivative of the Y momentum with respect to Y and subtract it so therefore you get this $-G \beta x \frac{DT}{dy}$ okay so now this is your equation for vorticity so what is a transport equation.

So this is the first equation that you have to solve and obviously you see to solve this you also need to solve the energy equation along with it because you need the information of temperature gradient so therefore we solve the standard 2d incompressible energy equation we do not use the viscous dissipation we neglect the viscous dissipation so and that will be $u \frac{DT}{DX} + V \frac{DT}{DY}$ by D where this is the convection term on the right hand side you have $\alpha \frac{D^2 T}{DX^2} + \frac{D^2 T}{DY^2}$ so once again for u and V you can write in terms of the stream function so this becomes this a by dy -D by D.

So this is the second equation this is your energy equation so you have your vorticity transport you have your energy and what else is required do you have enough equations to solve for all the variables so this is a governing equation for vorticity right and this is the governing equation for solving temperature but what is the equation for getting the stream function so there we use the definition of vorticity okay you define your vorticity how do you define participations so your DV by DX -D u by dy so there we substitute in terms of stream function for u and V here okay so your V is - DZ by DX okay so therefore this becomes $\frac{d^2 \psi}{DX^2} + \frac{d^2 \psi}{DY^2}$ for you it will be DC / dy this is $= -\Omega$.

So this is my third equation for stream function so I call this as equation number three so therefore you have three equations three unknowns okay so one for stream function one for vorticity and one for temperature three partial differential equations and three unknowns which you can solve by using finite difference method and that is very clearly explained I think all of you have some basic understanding of finite difference in the heat transfer course in the in the undergraduate course already I think some of you we have done the finite difference expressions okay for the 2d conduction problem.

And I think all the m-tech students also have taken numerical methods in thermal engineering this semester so should be fairly straightforward to express all the derivatives in terms of the finite difference expressions so you can have a look at the document I have posted and that clearly gives you how to write the finite difference expression for each of the derivatives the first order derivative the second order derivative and so on and you solve these equations together I think if you okay so solution for Ω requires of course the knowledge of stream form as well as a temperature solution.

For temperature requires a knowledge of stream function solution for stream function requires a knowledge of vorticity so all these three equations have to be solved simultaneously but it can be solved iteratively okay that means you do not have to invert a matrix together so first you start with the solution to stream function equation where you guess the value for vorticity field to begin with that is the initial guess value you solve this and get the field for the stream

function once you have the stream function filled you come to your vorticity equation make use of the guessed value for so that is your hydrated value for the stream function the field that comes out so that field.

You make use of you also guess the field for temperature and use this equation to solve for the vorticity okay once you have the vorticity field and your stream function field then directly your temperature field also can be solved so like this you keep solving take the newer values the latest values use that in the next iteration and so on till the difference between two consecutive iterations is very small you can use that as sum 1×10^{-5} or 1×10^{-6} mind the same way that you are using shooting methods okay.

You have a convergence criteria and then finally stop and then when you plot these contours in mat lab or wherever tech plot or mat lab and you will find this nice convective patterns in terms of a stream function if you plot you can directly find out the convective patterns and isotherms also can be plotted so and they are coming to the boundary conditions to solve this you need boundary conditions okay so the boundary conditions here as far as the temperature is concerned these two left and the right was are fixed temperature so you know the temperature at those points you do not have to solve them the top and bottom have adiabatic conditions.

So in the simplest case will be extrapolation from the inner point so you divide the domain into of course your grid which you all know and you have your locations where you are solving for the governing equations and then at the boundary at the top and bottom you simply extrapolate from the interior point okay and for as far as the boundary condition for stream function is concerned you have to apply the no slip condition at the was $u = 0$ and $V = 0$ what does it mean in terms of stream function it means my DC by $dy = 0$ and $DC / DX = 0$ so if I take the value of stream function say at this bottom left corner as sum 0 I can take any value.

So the thing is the boundary condition for stream function is in terms of gradients so it doesn't depend on the value as long as the gradient condition is satisfied so if you take for example the value here as 0 then you can look at this isothermal w you can apply DC by $dy = 0$ so therefore the stream function has to be 0 everywhere ok so if it is 0 here again for this w you can apply DC by $DX = 0$ right so therefore it has to be 0 here even here so therefore in all the ws you can directly put stream function as 0 ok and for vorticity you can use the remaining condition for example for the left w you have used $DC / dy = 0$ but you have not used $DC / DX = 0$.

So this can be used in the vorticity condition so you have you have the expression for vorticity here so for the left boundary your DC by $dy = 0$ therefore your $d^2 \psi / dy^2 = 0$ so this will be 0 for the left bond ok so therefore your what is it is $d^2 \psi / DX^2$ you can write as simple Taylor series expression for the boundary in terms of the interior point and you can express this in terms of the vorticity at the ball the stream function at the boundary so therefore you can directly calculate your vorticity at the boundaries so all these is very clearly explained the document okay so please read through the document.

If you have any questions you can ask me okay but it is very straight forward and just you have to construct the finite difference expressions and you will be therefore converting the partial differential equation into an a set of algebraic equations so that is the method how you have to numerically solve differential equations finally you get a resulting set of algebraic equations which you solve by any method whether you have to consider this as a matrix and use inversion methods or do iteratively okay so here you can do a very iterative solution which is going to take only a few lines of code do loops.

Where you just iteratively keep doing it till the convergence criteria satisfied and then you can plot it plot the results for streamlines already in the document for ten power three I think has given the streamlines and also for 10^4 and you should also plot the nusselt number that you get okay as a function of the Rally number I think in the document he has also given that you have four you start with say 10^3 and then say 10^4 10^5 and then so you can stop the 10^5 you know this just a representative and you can you can see the computed values which is given there and your computations you compare both of them on the same plot.

For nusselt number apart from the nusselt number you also have your stream function which you have to plot and compare with this stream function and you also have your isotherms which you can plot ok so this gives you a very good idea practically how you solve the navier-stokes equation in two-dimensional case so there you have the gift of basically reducing that in terms of stream function vorticity and you can get a direct solution through differences and I think that will hopefully give you a better appreciation of subject when you go to the research level okay in the search level we need not do everything by similarity solution so that gives you also an exposure to using some numerical methods okay.

So that the deadline for the submission of the project will be on the last day of the class or maybe if required if required you feel some difficulty then we will have it on the day of the final exam okay and my suggestion is that if you feel that individually you have problem working with that you can also discuss with your colleagues you know there is no harm in discussion but whatever code that you write should be individual it should not be copied and you have to submit your code along with your report.

Integral method for thermal entry length problem
End of Lecture 34

Next: Introduction to turbulent heat transfer

Online Video Editing / Post Production

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