

**Indian Institute of Technology Madras  
NPTEL**

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Video Lectures on  
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**Lecture 33  
Approximate method for laminar internal flows**

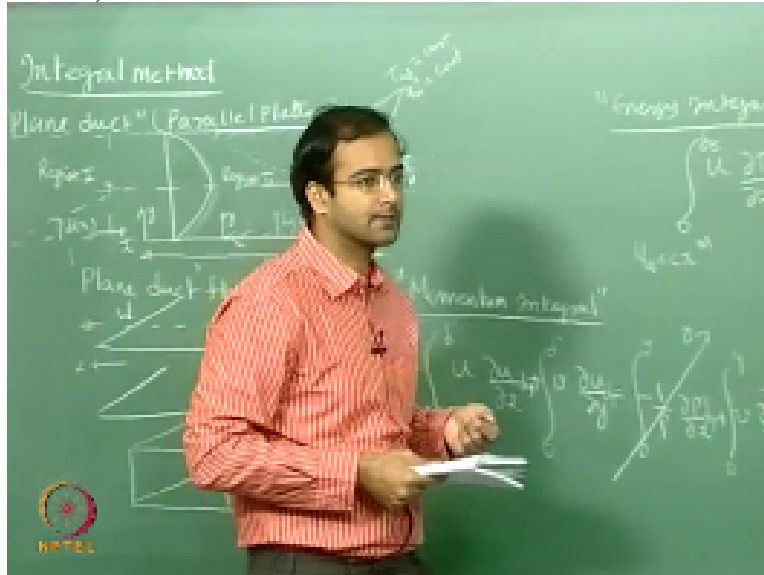
Okay so good morning all of you today the next two classes we will focus on the approximate techniques applied to the laminar internal forced convection we have seen the use of these approximate techniques in the case of external flows very powerful techniques we can apply them to flat plate flows without any pressure gradient as well as the case where we have a pressure gradient so even for the case of circular cylinder with the when we make use of the walls approximation .

So it is a very powerful technique we can very nicely to a great approximation we can get the profiles of boundary layer thickness the thermal boundary layer momentum boundary layer thickness the thermal boundary layer thickness and also the expression for Nusselt number which we obtain from using a higher order polynomial is very close to the actual analytical solution now sometimes as close as about 10-15 percent error.

So this integral techniques or the approximate methods are very useful techniques as far as the external boundary layer is concerned we will see whether such techniques can be applied to internal flows because in internal flows the main problem we do not have a very clear boundary layer kind of a flow which we can visualize in an external flow especially when the flow is in a state where it's about to become fully developed .

So they are definitely the boundary layer approximations cannot be used so we can see till what extent we can make use of these techniques in the case of internal flows and what is the regime where these approximations can be valid so this is the topic for the next two classes before we wrap up the laminar internal flows.

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So I am going to be talking about the integral method which is an approximate technique when applied to the boundary layer flows and we will see how this can be applied to our internal flow problems okay so if you consider say a duct flow a plane that flow that is typically the flow between two parallel plates okay so that is a plane duct so if you look at the cross-section of a plane that flow.

So if we look at the typical cross-section you may see that is something like this so you can assume that the width of this plate is some  $W$  and the length of these plates along the  $X$  Direction is  $L$  okay you can see that the width is towards the  $Z$  direction yeah okay and then you have your  $Y$  direction perpendicular to  $X$  and  $Z$  all right.

So this is a plane that the plane duct is nothing but you are not confining it in the  $Z$  direction it is just extended to a certain width and in fact you can also consider that these parallel plate case which were seeing in two dimension is a special case of the plane duct flow where the width can be assumed unity are sometimes you know going extending to  $\infty$ .

So where you do not have any confinement in the lateral directions whereas if you have a certain cross section to this suppose you replace this / a rectangular cross section okay so here you can see the boundary layers may start growing on this lateral walls also apart from the top and bottom so then clearly this is not a two-dimensional flow this becomes three dimension same way with the case of a circular cross section.

Okay the boundary layers start going simultaneously from all the walls but in all these cases we have really not looked into the three-dimensional nature of flow whatever we have done so far with the circular cross section was just look at the two dimensional approximation of it okay we neglected the radial and the tangential components.

Okay and the radial component was included in the conduction part of course but when we looked at the energy balance okay we neglected the dependence on the  $\theta$  direction the azimuthally direction so therefore strictly speaking there were 2d axis symmetric what we considered so far okay so this is if you rotate about the axis of symmetry that is going to give you the three-dimensional profile okay.

So that is why they are they were called as 2d axis symmetric flows now this is a 2d axis symmetric flow however this is not a 2d axis symmetric flow this is a clearly three-dimensional flow so there you cannot rotate it about any axis and get the profile variation okay so therefore you have to solve this in all the three coordinate directions and the limiting case the 2-dimensional case of this duct with a finite cross-section is your plain duct flow .

Okay so this is where we consider the two dimensional approximation of the plain duct flow where you look at the 2d flow in varying along the x and y direction only without looking at the Z direction okay .

So this is nothing but flow between two parallel plates or a plain duct so there are different names to it they all mean the same thing parallel plates so whatever we did under internal flows if you look at the Cartesian coordinate system they are all essentially plane duct flows or flow between two parallel plates so in such a case if you look at the boundary layer growth if you assume that first your velocity boundary layer has fully developed.

So you can see that the velocity profile at any axial location will be fully developed profile okay it will be perfectly parabolic and now if you start heating your plate from a certain location okay so either I can maintain a constant wall temperature or flux constant and you can see that the thermal boundary layer will start developing okay so from here you can take your coordinate system and you consider a thermally developing flows where X starts from the point where your heating starts okay.

So from here you find the thermal boundary layer growth happening so this is your  $\Delta T$  which is now a function of  $X$  now at the point when these two merge then this becomes thermally fully developed so this is basically your region III here fully developed most hydro dynamically and thermally this is your region II where your velocity profile is fully developed your thermal boundary layer is developing and if you go just before several distance axial locations before you will be in region I where the hydrodynamic boundary layer also is still developing okay. Now the question is where I can apply the momentum integral method okay.

In which region I can apply yeah so if you look at region II if you have the fully developed velocity profile then you can focus on region II where you have a boundary layer like structure okay so that is only up to a certain distance downstream if you go further down you can see somewhere here it loses the boundary layer like structure the boundary layers the 2 boundary layers growing from the top and the bottom walls they come close and then finally.

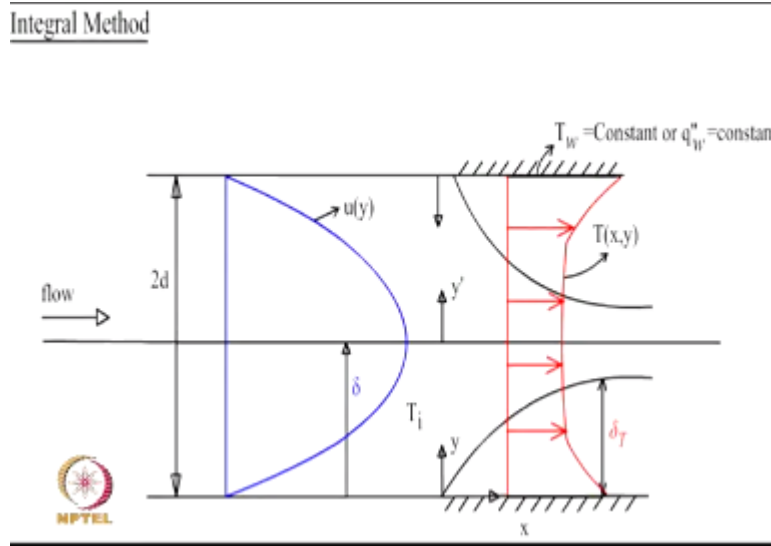
They are about to merge in such case your boundary layer approximation will not be accurate okay so you cannot that therefore apply your momentum integral or your energy integral approximations in this region and definitely your boundary layer equations are not valid once they fully merged because there is no boundary layers everywhere it is dominated / viscous effects.

So a boundary layer has as the definition goes it is a region which is if you look at the total length of the plate compared to that this is very small so your  $\Delta / L$  has to be very small and away from the boundary layer the flow has to be in viscous okay so once the viscous effects start coming then the definition of your actual boundary layer approximation will not be valid okay.

So therefore it is clearly you can see within region II it can be applied from the start of the boundary layer to a certain distance downstream okay so these momentum and energy integral approximations can be valid same way if you look at the region I if you go several distances ahead so you can find the momentum boundary layer also starts developing here and once again you can apply the momentum integral equations for the place where the momentum boundary layer is growing.

But once it is about to merge and once it has merged again the momentum integral equation loses meaning okay so therefore when it comes to using integral method for internal flows you have to be very careful in which region you are applying.

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Okay as far as the momentum integral is concerned okay so now you start from your momentum equation okay.

So you said  $u \frac{Du}{DX} + V \frac{Du}{dy} = - \frac{1}{\rho} \frac{DP}{DX} + \mu \frac{d^2 u}{dy^2}$  so this is your boundary layer momentum equation correct without the viscous dissipation term now we will integrate this from 0 to  $\Delta$  0 to  $\Delta$   $dy$  and you get your momentum integral equation okay so this can be applied for the momentum boundary layer when it is still in a developing state okay so somewhere here you can find that the boundary layer thickness and the boundary layer variation looks somewhat similar to a flat plate case okay.

So one plate here at the bottom one plate at the top but the additional problem that you have is this pressure gradient okay so this makes it a little bit more complicated so when you have a pressure gradient term you should know how the free stream velocity is varying so for example in the external flow case you took free stream velocity which are varying of this form this is the Falkner Skan kind of a profile so you should know what is the variation and then you can substitute for  $DP/DX$  as  $D u_\infty / DX$  but however in the internal flow case it is not very clear about this.

So therefore one level of approximation will would be to directly neglect this because the pressure gradient terms will not be so much when you look at the initial part of the boundary layer growth they will become more and more dominant when the two boundary layers start

merging and once it is fully merged then only the pressure gradient term will balance the viscous terms the inertial terms will not be there .

So if you look at the initial part of the boundary layer development you can make an approximation that this is 0 and this is like now a flat plate flow preposterous the same way that you did the momentum integral there the same thing can be applied but this will be a very big crude approximation and you cannot get an asymptotic solution for the fully developed case from this case because the momentum integral is not valid for the fully developed case.

Okay so that you should keep in mind so this is basically the structure of the hydrodynamic development okay it is not very different from a flat plate case however lot of approximations are involved coming to the energy integral .

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Handwritten mathematical derivation on a green chalkboard showing the energy integral for a thermally developing region. The derivation starts with the energy equation:  $u \frac{dT}{dx} = \frac{d}{dx} \left[ k \frac{dT}{dy} \right] - \rho c_p u T$ . It then integrates from  $y=0$  to  $y=\delta_t$  to get:  $\frac{d}{dx} \left[ \int_0^{\delta_t} \rho c_p u (T - T_i) dy \right] = k \frac{dT}{dy} \Big|_{y=0} - k \frac{dT}{dy} \Big|_{y=\delta_t}$ . The term at  $y=\delta_t$  is circled and labeled as  $-Q_w''$ . Below this, the heat flux  $Q_w''$  is defined as:  $Q_w'' = \frac{d}{dx} \left[ \int_0^{\delta_t} \rho c_p u (T - T_i) dy \right] \rightarrow (1)$ . The NPTEL logo is visible in the bottom left corner.

So in the 2d plane duct case you can of course write your energy equation which is  $U DT / DX = \alpha D^2 t / dy^2$  we are neglecting the axial conduction term in comparison to the radial conduction and also we are neglecting of course the radial velocities here okay so these are some approximations with which we are writing the energy equation okay we wrote this for the case where we have a thermally developing region okay .

So this is the energy equation for thermally developing region or thermal entry length region in the Cartesian coordinate system okay. So in the classes we were doing this for cylindrical coordinate system for the case of circular duct so right now what I am doing is we are looking at only Cartesian coordinate system that is for plane ducts okay.

So in that case you can integrate this and you can integrate that from 0 to  $\Delta T$  and once again this is very similar to your flat plate energy equation in fact even much simpler than that you do not have your  $V \frac{DT}{DX} = \frac{V}{dy} \frac{DT}{dy}$  so that becomes much easier to handle rather than the flat plate case okay.

So if you look at the energy integral as such if you know the fully developed velocity profile okay which you already know from your analytical solution that can be substituted here and then you can directly use the integral method for calculating the temperature profile ok so this is perfectly valid as far as the thermal entry length is concerned and once again you should be careful that you cannot reach the solution of region III that is the fully developed thermally region as an asymptotic solution to the integral method.

Because this is not valid in that particular region the moment the thermal the energy integral is not valid in that particular region so this is what we can say about the an integral method now once it comes to non Cartesian coordinates that is the circular coordinate that is the case of circular duct then you cannot even apply the energy at the momentum integral there why because you have a complicated.

So you have here  $u \frac{dt}{DX} = \alpha \frac{1}{r} \frac{d}{dr} (r \frac{dt}{dr})$  so now if you are integrating from 0 to  $\Delta t$   $dr$  so this becomes quite difficult to evaluate ok so you cannot just like that say this is  $DT/dy$  at  $y=0$  so this is a function of  $R$  and you have a function of  $R$  inside also so as such this becomes a little bit more cumbersome to apply this to a circular cross section okay.

So therefore most of the solutions with using the integral technique are restricted to Cartesian coordinate system so that is typically the plain duct so when you are looking at two-dimensional flow between a channel in a channel or between parallel plates you can look at either the mostly they are valid when you look at the thermal entry length problem where you already know the velocity profile but you do not want to go through the rigorous analytical solution for getting the temperature profile.

So there we can use the energy integral okay once again when it comes to the hydrodynamic development part there we have to make some approximations and the profile will be not as accurate okay so most of the textbooks that deal with the integral method with the internal flows they look mainly at only the thermal entry length and that too for a 2d problem okay as

far as I know I did not come across any literature where they have extended this to 3d circular cross-section okay.

So therefore its utility is very much restricted in internal flows it is not as wide as in external flows where you used it for almost all the problems where you had a similarity solution you replace that with an approximate solution and in fact even more than that for cases where you had an unheated starting length for example.

So there you cannot use the solution from similarity variables and again the cases where you talked about with the pressure gradient were much simpler and also the case where you had a non-uniform wall temperature or wall heat flux so you had a linear variation or some other variation any variation.

So there those were dealt with the approximate methods but here because of the geometry constraint so we are fixed to only two-dimensional flows and also the fact that it is valid only in the region close to the thermal entry length okay and you cannot reach an asymptotic solution to the fully developed case from this solution.

So that so this you have to be very clear about okay apart from that the solution method is very similar to what we did with the external flows only we will have a difference in the variables that we use okay so any questions on this so therefore what we will now do is assume that we have a fully developed velocity profile and start with our energy equation and look into the thermal entry length okay and get the temperature profiles and the expression for the nusselt number all right.

So we will have a coordinate system where we will start from here and at the center of the duct I will use the coordinate system  $Y'$  and  $X$  so the one which is starting from the bottom plate I will call the coordinate system as  $Y$  in the one which is starting from the center of the plate I will call this is  $y'$  and from the center of the plate so this is at a distance of  $+D$  and this is a distance of  $-D$  this is the locations so the total height of this separation between the two plates is  $2D$  okay.

So this is basically my coordinate system so what I am going to do is first I am going to solve for the case of constant wall flux boundary condition okay and then the constant wall temperature is very similar and you can extend it.



So I am going to look at uniform and now when we define my non-dimensional variables rather I use Reynolds number or nusselt number I in the case of plain duct flows it is common to use in terms of the hydraulic diameter or the equivalent diameter although it does not have any cross section in the Z direction you can replace your this is just the spacing between the plates so you have to write this in terms of the equivalent diameter .

Okay so we will use the definition of hydraulic diameter okay which is  $4A/P$  and if you look at the plain duct if you imagine the width is very long of course in the Z direction so that is nothing but you have  $4XL$  times  $W$  okay so in this case this will be  $2D \times W$  so  $2D$  is this particular and you have in the Z Direction  $W$  divided by the perimeter what is the perimeter  $2XW$  okay.

So therefore this becomes  $4d$  so your high equivalent diameter if you talk in terms of some cross section so you give some representative diameter to that so in similarly you talk in terms of an equivalent diameter so that is related to 4 times the or 2 times the separation between the plates okay so this is your characteristic dimension with which you define non-dimensional variables like Reynolds number and nusselt number .

Okay and we can look at the fully developed parabolic velocity profile so what we will do is now we will first start with this step and we will integrate it from 0 to the edge of the boundary layer so we will look at the coordinate system of Y when we do this integral not Y' okay so that is why I have written this as  $dy$  .

So I can express this as  $d/Dx \int_0^{\Delta T} I$  I can write a  $\alpha$  as  $K/\rho CP$  and I can take  $\rho CP$  to the left  $\rho CPu (T - Ti) dy$  should be  $= \int D^2 t / dy^2$  so that is basically  $K DT / dy$  between the limits 0 and  $\Delta T$  so this is a  $\Delta t - K DT dy$  at 0 . so I have just introduced  $T - Ti$  on both the sides because  $Ti$  is the inlet temperature you can assume that the profile which is coming temperature profile at  $X = zero$  is uniform and this temperature is basically  $Ti$  okay so there is no harm in just introducing  $Ti$  into the derivative okay .

So now so this is very similar and in fact it is simpler than the flat plate case so there I had a  $VD t / dy$  and I had to rewrite my  $V$  in terms of  $U$  and convert all my derivatives in terms of derivative with X direction so now I have directly only a derivative with respect to X direction here and this is of course related to the wall heat flux .

Okay which is known so what is the value of this  $K \frac{DT}{dy}$  at  $\Delta T$  zero okay that is the boundary layer condition and  $-K \frac{DT}{dy}$  this is nothing but your given heat flux which is a constant okay so therefore in this case your energy integral simplifies to  $0$  to  $\Delta T \rho C_p \int u (T - T_i) dy$  okay so now what do we need to do so we have the energy integral equation.

We will call this as number 1 so this is my energy integral equation so how do I proceed from here in the integral methods what is the where is the approximation so far there is no approximation made so this is called an approximate method why we have to assume a profile now that is where the approximation comes we do not know what is the nature of the velocity and the temperature profile .

So we have to guess some profile and I mean the best guess could be something like a cubic profile as well as the velocity profile is concerned we are not going to solve the momentum integral because it is now hydro dynamically fully developed so therefore we cannot guess any velocity profile but we have to take the exact profile as it is okay because if we had guessed a profile there and we applied the momentum integral that will be valid only in region I and that too in the developing section .

Once it is close to becoming fully developed then those profiles are not valid and we are looking at region II now so therefore we cannot guess anything for velocity we have to take the exact parabolic profile .

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$$\frac{d^2 + 2y d - k}{d^2}$$

$$T - T_i = \frac{q_w'' \delta_t}{3k} \left[ 2 - 3 \left( \frac{y}{\delta_t} \right) + \left( \frac{y}{\delta_t} \right)^3 \right]$$

$$\left. \frac{dT}{dy} \right|_w = -\frac{q_w''}{k}$$

Okay so that is basically your  $u / u_c = 1 - (y/d)^2$  I have not derived this ok I have derived the velocity profile only for the circular duct but if you do this for the Cartesian coordinate system you will get a very similar velocity profile which is like this so instead of  $r$  you will now have the half separation distance  $D$  coming in the denominator and this is your centerline velocity and in your channel case your  $u_c$  is basically  $3/2 u_m$  okay whereas in your circular duct your  $u_c$  was  $2 u_m$  okay.

So I have not derived it in class but I think already you have done this in fluid mechanics and you should be able to recollect so this is basically the velocity profile now this is written in terms of what the coordinate system for this is from the center okay so therefore this is  $y'$  okay so if you look at the profile at  $y'$  of  $D$  and  $-D$  then this will be equal to 0.

Okay that is only possible if the coordinate system is at the center so therefore we have to now convert a coordinate system from  $y'$  to  $Y$  because our energy integral is in  $y$  coordinate so that is because we have we are operating in a flat plate coordinate system which is attached to the wall whereas now here it is somewhere in the center of the duct okay so how do we do that what is the transformation.

So I want a system such a way that when your  $y' = 0$  then your  $Y$  should be  $= D$  and when your  $y'$  is  $-D$  your  $Y$  should be  $= 0$  okay that is what your  $y$  should be  $= +D$  so when your  $y' = 0$   $Y = D$  and when your  $y' = -D$   $Y = 0$  okay so this is basically your relation so you can substitute for this  $y'$  as  $Y - D$  into the profile correct.

So therefore this becomes  $u / 1 - (y' / D)^2 / d^2$  which if you expand you get  $d^2 - y'^2 + 2y'D - d^2 / d^2$  or if you so this is  $d^2$  and  $-d^2$  cancels this can be written as twice of  $y'D$  so here again this is  $d$  so twice of  $y' / d - (y' / D)^2$  so this is your velocity profile with the  $Y$   $XY$  coordinate system attached to the bottom plate alright so you can again check so for  $y = 0$  this will be 0 and  $y = 2D$  so again so this becomes 0 so this is  $2 \times 2 - 4 - 4 = 0$  okay so that satisfies the boundary conditions of the velocity.

So now once we know the velocity profile what else so we have we have taken the exact analytical solution for velocity profile so what do we do next still we have not done anything approximate here right so next we have to approximate the temperature here where the approximation comes. So we can assume a cubic temperature profile okay  $a + by + cy^2 + dy^3$  so this is where you approximate or assume now in order to get the coefficients of the polynomial.

We need to have how many boundary conditions 4 boundary conditions okay so what are the boundary conditions at  $y=0$   $\partial T / \partial Y$  at  $y=0$  should be  $= - Q_{\text{wall}} / K$  this is one boundary condition and what is the other condition at  $y = \Delta T$   $DT / dy$  should be 0.

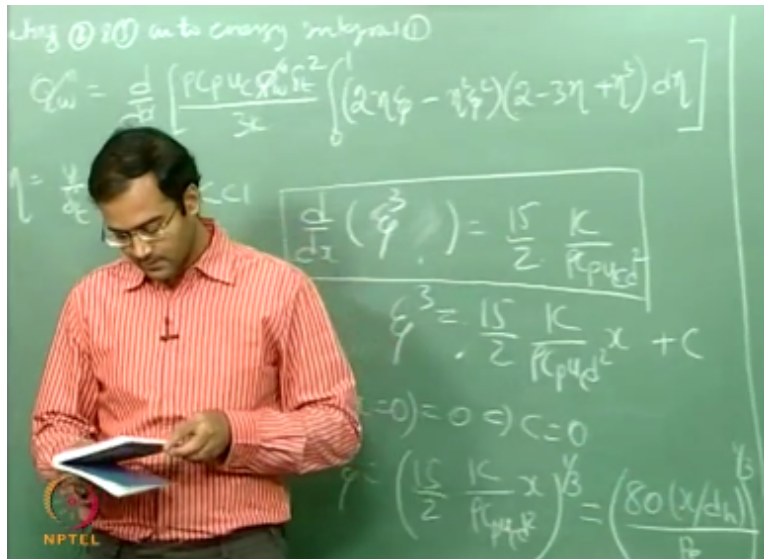
So two boundary conditions the third one okay so at  $y=0$  if you look at the energy integral your velocity is 0 therefore  $d^2 t / dy^2$  has to be 0 right so we need one more and that should be nice well you can see that it should be at  $y=\Delta T$  second order should be directly give a second order condition we will not given condition for the profile temperature itself. We should not go to higher order derivatives without giving boundary condition for the temperature.

The first preference is for the temperature if you do not have that then you go to the higher order boundary conditions what will be the value of temperature at  $y = \Delta t$  TI right because outside the thermal boundary layer whatever inlet temperature that will be the same it is like your flow past a flat plate you have your  $T \rightarrow \infty$  so outside the boundary layer so instead of that you have TI here so therefore you have 4 boundary conditions.

So if you substitute these four boundary conditions one after the other you will get all the four coefficients and your final profile will look like this  $T - T_i =$  okay so I will call this as I will call this velocity profile as number 2 and this is equation number 3 all right.

So if you substitute these profiles this is your cubic temperature profile that you will be getting so now you have your velocity profile and your approximate temperature profile which you can directly substitute into the energy integral.

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So substituting 2 and 3 into energy integral 1 okay so I can write this as  $Q_w = D / DX$  and as far as the velocity profile is concerned so you have 2 times  $u_c$  in two of course  $u_c \times 2 y / D - (y / D)^2$  I am going to introduce non-dimensional variables here one I call as  $\eta$  which is  $y / \Delta T$  similar very similar to the external flow okay you introduced in terms of  $\eta$  and you remember you introduced  $\xi$  there to denote  $\Delta T / \Delta$  right.

So here we do not have any  $\Delta$  because we do not talk about boundary layer momentum boundary layer thickness so rather than  $\Delta$  what we should use  $D$  okay you look at the profile it is  $y / D$  whereas in the external flow it is  $y / \Delta$  okay so then this will be replaced by  $\rho C_p u$  so these are the two non-dimensional variables and now you can write them in terms of non-dimensional.

So this is  $\rho C_p x U_c$  and  $t - t_i$  is nothing but  $Q_w \Delta T / 3k$  so I can write that as  $Q_w \Delta T / 3k$  which I can pull out of the integral and now I can integrate from 0 to 1 so instead of  $\Delta T$  set  $y = \Delta T$  becomes 1 and you have  $u \times T - T_i$  and you are you by you have already written as you see in this so this will be 2 times  $y / y / D$  can be written as  $H$  into  $Z$  okay so this is  $H Z$  minus  $H$  square  $H$  square this is your velocity profile into  $t - t_i$  already you have taken the constant out so that will be  $2-3 H + H Q d \eta$ .

Okay so now you have to simply integrate the resulting equation so if you what you can do is you can see that for the boundary layer approximation to be valid your  $\Delta T$  upon  $\Delta$  that is your  $Z$  should be small okay just like in your earlier case where your prandtl number greater than

one your  $\Delta T / \Delta$  was small in your case for the thermal boundary layer approximation to be valid your  $Z$  should be small.

Okay so in that you can neglect all the higher-order terms of  $Z$  of the order two under book so that means all these is multiplied by  $Z^2$  and this can be completely neglected and the resulting profile with the first order term can be integrated out and you should be getting an equation which is  $d / DX$ .

So when you write in terms of  $H$  here one more thing so your  $y$  was  $H$  into  $\Delta T$  so this becomes  $\Delta T^2$  okay so this becomes  $\Delta T^2$  into if you integrate it out you will get a constant times your  $Z$  because you are integrating with respect to  $H$  so you will still have this  $Z$  here that will come out as  $15 / 2$  and you can take  $K / \rho CP$  you see on the other side okay so  $Q$  wall  $Q$  wall cancels of course the constant.

So this is your equation for your thermal boundary layer thickness okay variation of thermal boundary layer thickness along  $X$  so with this you can now again substitute your  $\Delta T$  in terms of  $Z$  so this becomes  $Z Q x D$  okay so your  $\Delta T$  Square so that so when you had basically your  $\Delta T^2$  that will be  $Z$  square into  $d^2$  and  $d^2$  is a constant I can take it to the right hand side okay so I can now integrate it right away and the resulting profile will be  $15 / 2 K / \rho CP u C$  into  $D^2 x + a$  constant okay.

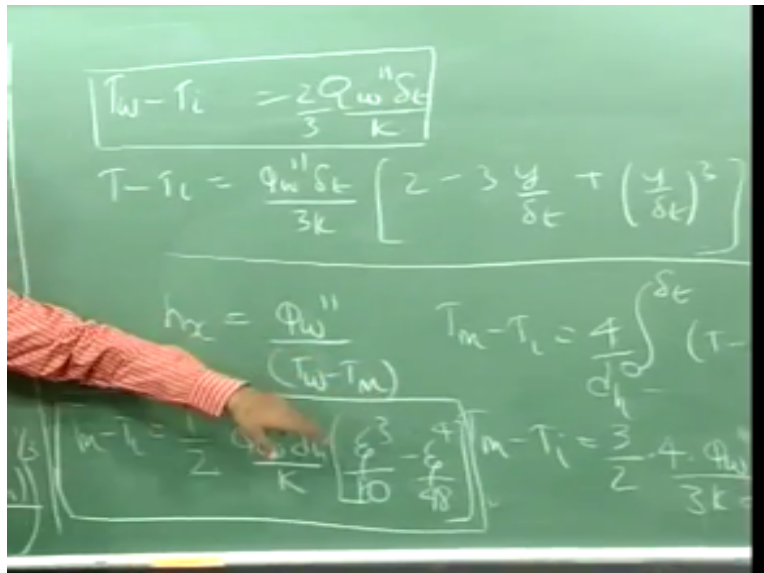
So I can evaluate this constant with the condition that at  $X = 0$   $Z$  should be what 0 okay so therefore this gives the constant is 0 so therefore my  $Z$  will be  $15 / 2 K / \rho CP u C y d^2 x X$  the whole power  $1 / 3$  okay so now what I am going to do is just recast it in the little bit so that I can construct a non dimensional variable out of this so my  $K / \rho CP$  is  $\alpha$  I have  $u c D / \alpha$  so I know my relationship  $u c$  is basically  $3 / 2 u m$  I can define my peclet number in this case as  $u m$  and what is the diameter I am going to use my equivalent hydraulic diameter  $D H / \alpha$ .

Okay where my  $u m = 2 / 3 u c$  and my  $D H = 4$  times  $D$  okay and so I have  $\rho CP u m$  so  $u m x D / \alpha$  that is my peclet number and I have another  $D$  which I can write as  $X / D$  okay so of course I have these multiplication factors which come up so finally this will come out as  $80$  the multiplication factors  $X / D h$  by peclet number the whole power one-third okay so peclet number by  $X / D$  is nothing but the grades number.

So you can you can now directly say that you are  $\delta$  which is nothing but your thermal boundary layer thickness this is directly a function of the grades number okay so once you get this the rest of the problem is a little bit straightforward that is you have to so once you got your thermal boundary layer thickness you can calculate your mean temperature and the wall temperature and from there you can get an expression for the inertial number .

So I can calculate my of course I think I stuck off my  $T - T_L$  I will write it again so this was my temperature profile cubic temperature profile now from here I can calculate  $T_{wall} - T_i$  that is at  $y = 0$   $\frac{2}{3} Q_{wall} \Delta T / K$  right the other two terms are 0 and.

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I can also calculate my mean temperature. Okay I can so now if you look at this particular expression right here this is still not complete we have to get the mean temperature to finally calculate the heat transfer coefficient because my local heat transfer coefficient is defined as  $Q_{wall} / (T_{wall} - T_{mean})$  so I need to get another expression for  $T_{mean} - T_i$  and then subtract both that will give me  $T_{wall} - T_{mean}$ .

So how do I get my  $T_m - T_i$  how do I get my mean temperature that is basically  $\int (T - T_i) u \, dy$  correct now this is the case of Cartesian duct so you are  $\rho u \, dy$  will be the flow rate so divided by  $\int_0^{\delta_c} dy$  and from the definition of mean velocity you know that  $u_m$  is equal to  $\int u \, dy / D$  so in this case it is  $2\delta_c$  okay so I have to integrate to get my mean

temperature from 0 to  $\Delta T$  and I can actually replace this as  $1/D$  and my  $D$  will be  $D/4$  so therefore my  $u_m$  will be  $4/DH \int_0^{\Delta T} u \, dy$ .

So that can be this can be written as  $u_m \, dx \, H/4$  okay so now you get an expression where you have  $4/DH$  and you have  $u$  by  $u_m$  here so  $u$  by  $u_m$  I already have an expression so I also have an expression for  $T - T_i$  which is this so I can substitute both of them into this expression and get an expression for  $T_m - T_i$  so this will be  $3/2 \times 4 \times Q \Delta T^2 / 3K \times D/4H$  and within the integral 0 to 1 the velocity profile was  $2\eta - \eta^2$  into the temperature profile was  $2 - 3\eta + \eta^3$   $\times d\eta$  okay.

So this was your if you integrate it once again you already did it but now we are not going to neglect any higher order terms will keep that as it is so your temperature profile  $T_m - T_i$  comes out to be  $1/2 \times Q \, 1'' \, DH/K \times \theta^3 / 10 \, \epsilon^4 / 48$  okay so this is the relationship for  $T_m - T_i$ .

So therefore from this we can we know  $T - T_{wall} - T_i$  and  $T_m - T_i$  we can take the difference between these two and that will give me  $T_{wall} - T_m$  okay so I will continue on Saturday and complete this so we can just substitute here and finally you will get an expression for your local heat transfer coefficient as a function of your  $Z$ . Okay and  $Z$  you already have the expression in terms of the glide number.

So finally everything you will get in terms of writes numbers okay so these are all just straight forward you know you have to just go step by step and you have to do the integration there is nothing much to explain here very similar to what we did in the case the only thing in the internal flows you do not use  $T_\infty$  so therefore additionally you need to calculate your mean temperature which is not required in your external flow case okay so we will stop here today and Saturday we will finish the last part of this and we look at the result for the constant wall temperature case also.

**Approximate method for laminar internal flows**

**End of Lecture 33**

**Next: Integral method for thermal entry length problem**

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