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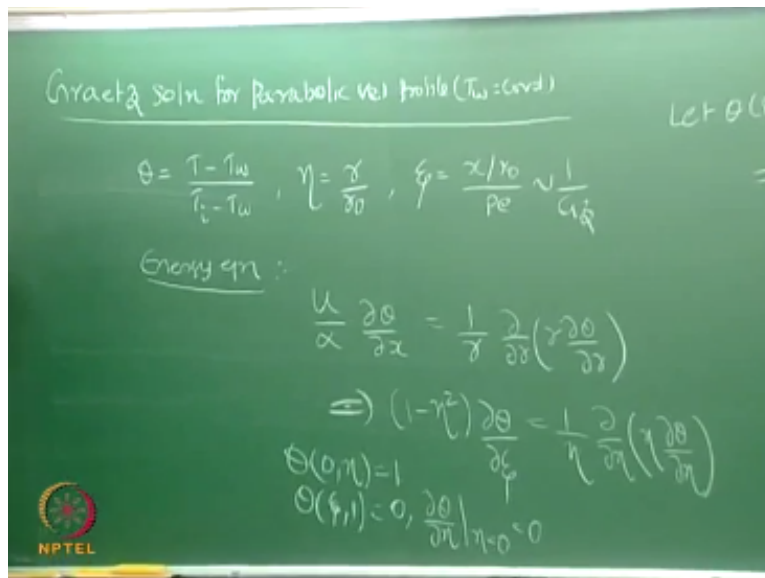
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Lecture 31

Extended Graetz problem

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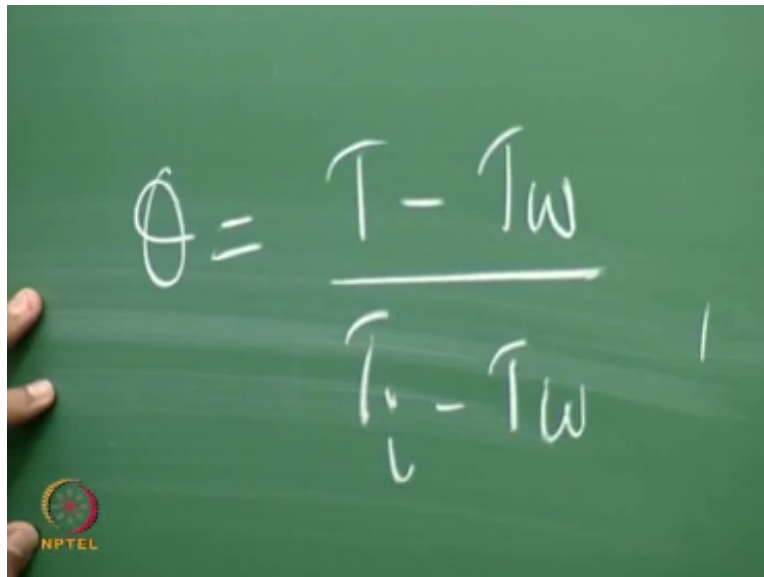


So good morning all of you so today we will look at in fact continuation of the last class which was extending right solution the great solution was originally proposed by grades 4 flow profile and it looks at the thermally developing region or the thermal entry region wherein your hydro dynamically fully developed and you are looking at only the entrance region where the thermal boundary layer is developing for such kind of a that is what we call as region2 so grades.

Assumed a sluk flow so where anyway the sluk flow does not vary and it is a uniform everywhere and here developed solution for constant wall temperature which we had seen earlier so they are the Eigen functions were what were the Eigen functions essentially in the original grades problem so the Eigen the eigenvalue problem there was actually a Bessel equation right so now the same problem can be extended to a case which is bone realistic that is for a parabolic.

Velocity profile okay it was this extension was done by a group of people sell or settle all sell or strippers and I have posted that in the moodle you can just have a look originally 1954 how they did the extension to the rights problem of course nowadays the solution is more by numerical Methods they try to do a approximate technique where they did something for are close to the wall  $r$  which is somewhere in the middle and  $r_0$  which is far away from the wall and they Patched up all the solutions together so what we are going nowadays is directly go for a numerical solution to the eigenvalue problem so the easier way to start is to introduce these non-dimensional variables for temperature as you shall be defined  $\theta$  as  $\theta = (T - T_w) / (T_i - T_w)$ .

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$$\theta = \frac{T - T_w}{T_i - T_w}$$

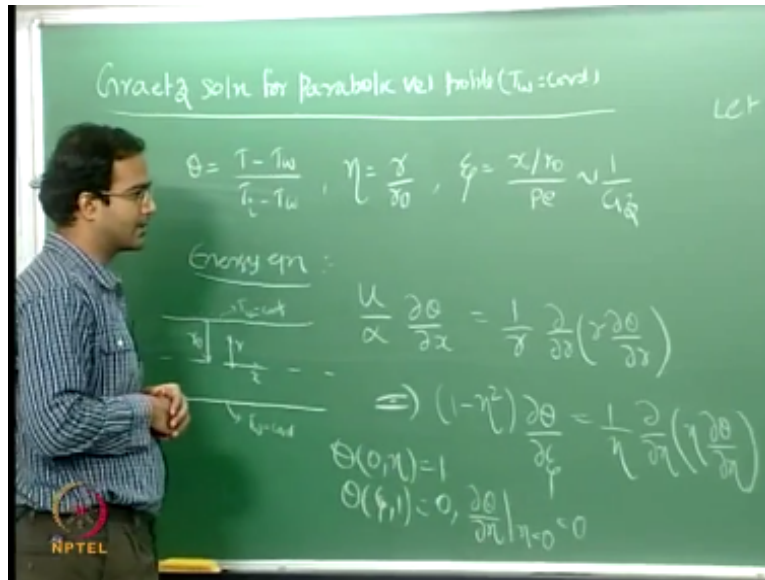
Since this is a constant wall temperature boundary condition and your inlet temperature is also assumed constant so this is an appropriate definition for  $\theta$  once you have a constant wall flux boundary condition then you cannot define  $\theta$  this way okay because your when you take the differential with respect to  $X$  your wall temperature will not be a constant and therefore they will not cancel out on both the sides and this is the non-dimensional radial coordinate and your axial non-dimensional axial coordinate can be non dimensionalized in this manner which is somewhat similar to an inverse of the grades number your grades.

Number one by grades number is actually  $X / D$  by pecelet number okay so this is somewhat similar exactly now we are not using  $X / D$  here but  $x/r_0$  so when we substitute into the energy equation okay the energy equation for thermally developing flow is this and if it had been fully developed your  $D \theta / DX$  would have been zero but in this case your flow is still developing and therefore if you substitute you get a partial differential.

equation in terms of  $\theta$  which is a function of both the axial coordinate  $\zeta$  and your non-dimensional radial coordinate  $\eta$  and the boundary conditions are at the entry region that is at  $\zeta = 0$  your  $T = T_i$  so therefore your  $\theta$  will be one and at the location  $R = R_0$  that is at the wall okay

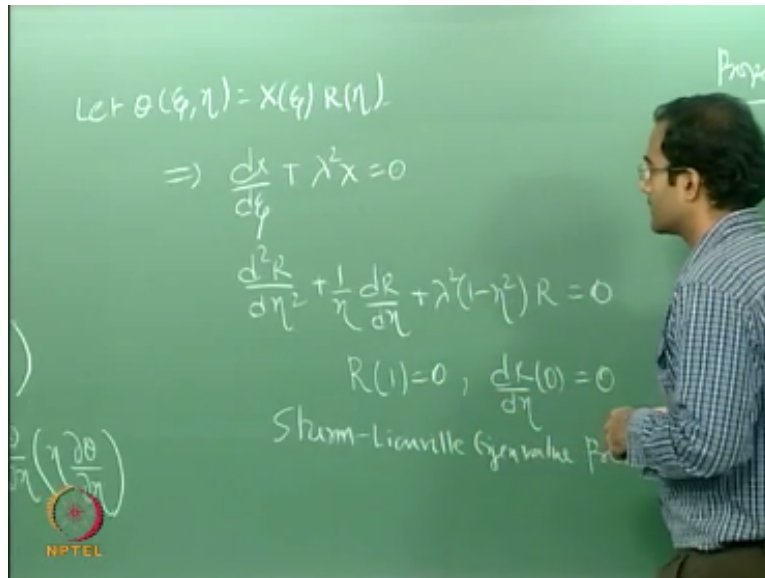
so your coordinate system that you are looking is something like this is your R and this is your X okay so this is your R0 so at R.

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=R0 that is your  $\eta = 0$  one that is where your  $\eta$  equal to zero where  $y = 0$  so this is where you apply a constant wall temperature and at  $\eta$  equal to 1 that is at  $R = \delta$  centerline there is a symmetry in the profile so therefore your gradient at the centerline should be 0 so now once you assume that you can use separation of variables to solve this we introduce  $\theta$  and we break up the solution as a product of two independent.

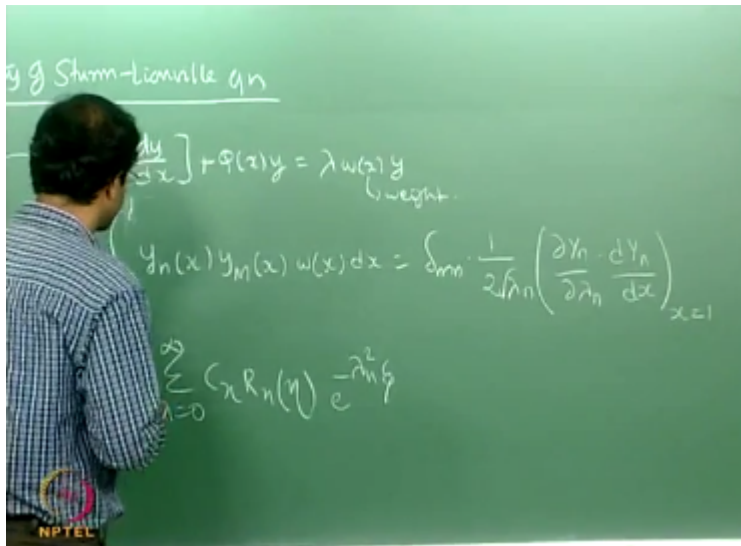
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Solutions one is a function of only  $\eta$  the other is a function of  $\zeta$  so we introduce  $X$  as a function of  $\zeta$  and  $R$  as a function of  $\eta$  and then substitute into the PD and then we get to we reduce the PD to two ordinary differential equations through the eigenvalue  $\Lambda$  square okay and the cell the eigenvalue problem here is basically the one with homogenous boundary conditions which is basically in then direction and if you compare this to the Bessel equation.

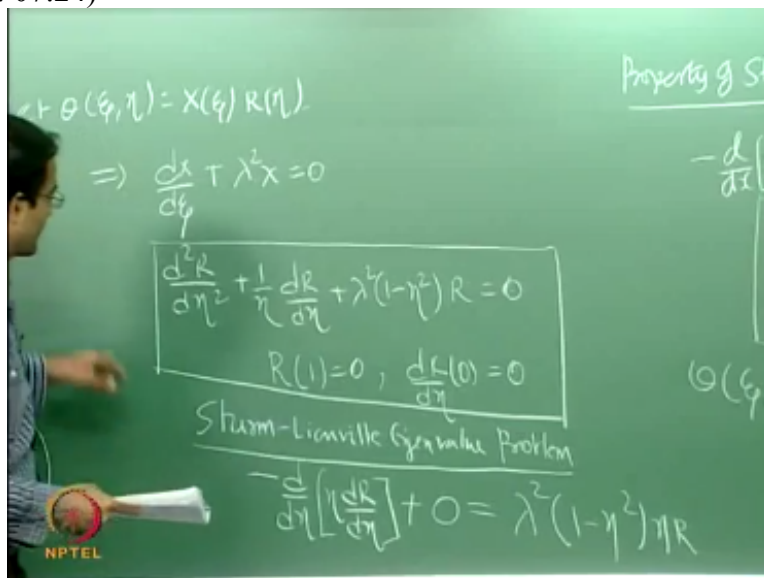
You can find that this last term here is not same so therefore this is not a Bessel equation if you look at these first two terms they appear similar but the third term is not the same as it is a function of you have  $\eta$  if you multiply by  $\eta$  square you will have  $\eta$  square into  $1 - \eta$  square that comes different from the Bessel equation and this kind of general eigenvalue problems are called strongly will Eigen value problems okay so any kind of an eigenvalue problem whether this is a Bessel equation or

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A basic bode can be represented as a sturm-liouville eigen value problem so the sturm liouville eigenvalue problem can be cast into a differential equation like this of course you can express this also in this particular form and this sturm Louisville problems have a particular property that when you integrate your Eigen function you multiply your eigen function and you multiply it with the weighing function the weighing function is this which appears on the right-

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Hand side so this is the particular property this is your Chronicle  $\delta$  so if  $m$  equal to and then this will be = 1 otherwise it will be 0 ok so now if you compare your eigenvalue problem to the sturm liouville problem ok so you can write your Eigen value problem into the sturm liouville form so that will come out as - d / d  $\eta$  into  $\eta$  dr / d $\eta$  + the term corresponding to Q of X is 0

here and this term can go to the right hand side and that can be expressed as  $\lambda^2$  into  $1 - \eta^2$

$\zeta$  square into  $\eta$  into  $R$  so I am multiplying throughout by  $\eta$  and the first two terms I can combine and write it in this particular format ok now the format is the same as the term Louisville format you can compare the coefficients your  $(p) \times$  you is nothing but  $\eta Q$  of  $X$  is zero and you are weighing function is  $1 - \eta^2$  into  $\eta$  so this is your weighing function as a function of either okay so this can be written as 0 to 1 so.

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The image shows a green chalkboard with handwritten mathematical formulas. At the top, there is a note "weight" with an arrow pointing to the variable  $\eta$  in the formula below. The formula is:

$$x = \sum_{mn} \frac{1}{2\lambda_n} \left( \frac{\partial Y_n}{\partial \lambda_n} \cdot \frac{dY_n}{dx} \right)_{x=1}$$

Below this, there is another formula enclosed in a box:

$$\int_0^1 R_n(\eta) R_m(\eta) \eta (1-\eta^2) d\eta = \delta_{mn} \frac{1}{2\lambda_n} \left( \frac{\partial R_n}{\partial \lambda_n} \frac{dR_n}{d\eta} \right)_{\eta=1}$$

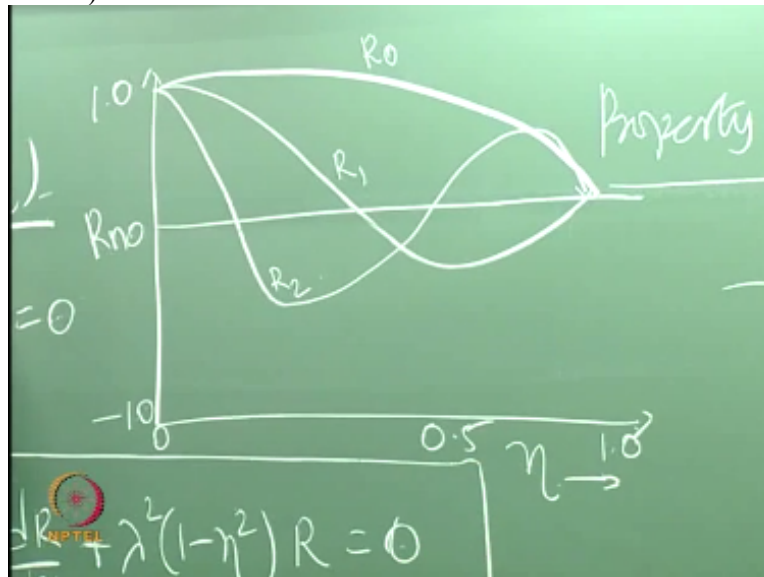
This is strictly speaking a to be okay in your case your  $\eta$  goes from 0 to 1 that is why I have just used to 0 to 1 there so this is 0 to 1 and this will be our end of  $\eta$  into our  $M$  of  $\eta$  into weighting function here which is  $\eta$  into  $1 - \eta^2$  Square  $D \eta$  is equal to  $\delta_{MN}$  times  $1$  by  $2 \lambda$  because here your  $\lambda^2$  is your  $\lambda$  so this is square root of  $\lambda$  square which is  $\lambda$  into  $D$  our  $n$  by  $B \lambda$  into  $D$  our  $n$  by  $d\eta$  at  $\eta$  equal to 1 okay.

So this is this is how the corresponding properties of Sturm Liouville comes out to be in this case and if you want to write the final solution for  $\eta$  which is basically  $X$  into  $R$  so you know the Eigen functions now in this particular case this is an OD and to get the Eigen function you have no other option but to solve it numerically ok you can once again go back to your shooting method you can reduce it to two first order Rudy is and then here guess the value of  $\lambda$ .

Because you do not know the value of  $\lambda$  unless you know the value of  $\lambda$  you cannot find out the Eigen function so guess the value of  $\lambda$  and once again you have to satisfy the other boundary condition and that is hydrate of  $\eta$  found out and that is the suitable value of so like this for the given value of  $\lambda$  you have a particular value of eigenfunction  $\lambda$  so for each value of  $\lambda$  you have to find the Eigen functions and finally the solution will be.

A superposition of all these eigenfunctions okay so that can be written as a constant times the eigenfunction into the other solution for X the other four solution for X as a function of  $\zeta$  that is a very straightforward ODE which can be directly integrated and that will be in terms of constant times  $E$  power -  $\Lambda$  square  $\zeta$  okay so this is your final solution for  $\theta$  having determined your Eigen functions and your Eigen values you can therefore plug in into.

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This expression and find the variation of the temperature I will just give you a representation of how the eigenfunctions look if you solve for them and if you plot the first three Eigen functions are  $n = 0$  it varies between negative -1 to 1 0.5 so this is your  $r_0$ ,  $r_1$  and  $r_2$  this is a this is a representation of so these  $r_0$ ,  $r_1$ ,  $r_2$  so here you have plotted with respect to  $\eta$  okay so this the so this is the Eigen function corresponding to  $n$  equal to 0 that is for  $\Lambda = 0$  okay and.

This is the Eigen function corresponding to  $\Lambda = 1$  so for different each Eigen value sorry for eigenvalue  $\Lambda = 0$ ,  $\Lambda = 1$ ,  $\Lambda = 2$  you substitute into the ODE and you can get the corresponding Eigen function variation with respect to  $H$  so these are the first three Eigen functions so these are the most important ones there are other higher-order I cannot functions but their contribution will be relatively smaller so when you sum them you take only the first three.

Or four important Eigen functions into account okay so now the thing is this is the solution but still we have to find the constant  $C_N$  okay so for this we have to apply the initial condition that is at  $\zeta = 0$  and we make use of the property of the store movie when you integrate with the weighing function so this is basically since this is an orthogonal All storm level problems a lot of orthogonal so this is a property which satisfies the orthogonal condition so this is an.

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weight.  $\rightarrow$   $k$

$$dx = \sum_{m=0}^{\infty} \frac{1}{2\sqrt{\lambda_m}} \left( \frac{\partial Y_n}{\partial \lambda_n} \frac{dY_n}{dx} \right)_{x=1}$$

Orthogonality condition

$$\int_0^1 R_n(\eta) R_m(\eta) \eta(1-\eta)^2 d\eta = \delta_{mn} \frac{1}{2\sqrt{\lambda_n}} \left( \frac{\partial R_n}{\partial \lambda_n} \frac{dR_n}{d\eta} \right)_{\eta=1}$$

Orthogonality condition of the storm level system of problems and we will make use of this in calculating the constant so let me call this as equation number one and I am going to multiply both sides by  $R_m$  and integrate so for first before doing that I will apply the condition at  $\zeta = 0$  which is equal to one therefore one equal to summation  $n$  equal to 0 to infinity  $C_n$  you have  $m$  or  $PH$  into  $(E) - \lambda n^2 0$  that is 1 so I can.

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Exact soln for parabolic vel profile ( $T_w = \text{const}$ )

Let  $\theta$

$$\theta(\eta=0, \tau) = 1.0$$

$$R_m(\eta) = \int_0^1 \left( \sum_{n=0}^{\infty} C_n R_n(\eta) \right) R_m(\eta) \eta(1-\eta)^2 d\eta$$

$$\Rightarrow C_n = \frac{\int_0^1 R_n(\eta) \eta(1-\eta)^2 d\eta}{\int_0^1 R_n^2(\eta) \eta(1-\eta)^2 d\eta}$$

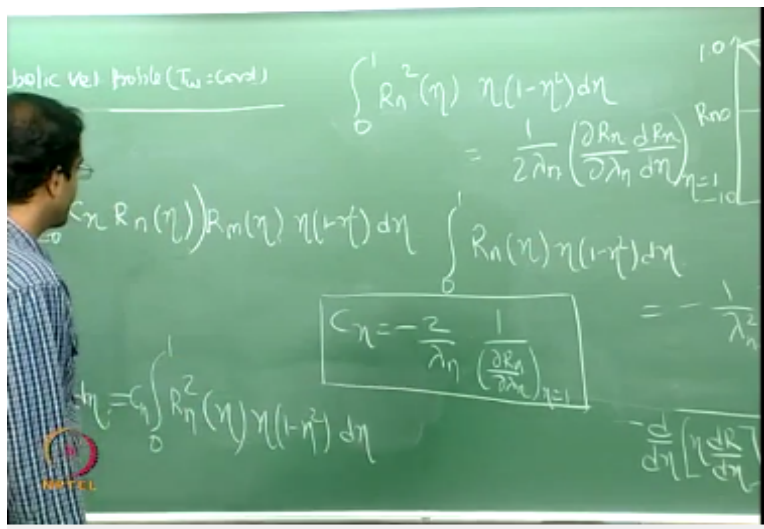
Now multiply both sides by  $R_m$  of Vita and the weighing function okay so the weighing function is here  $\eta$  into  $1 - \eta$  Square  $D \eta$  and integrate from 0 to 1 so this is also the same I have  $r$   $m \eta$  into  $\eta$  into  $1 - \eta$  Square  $D$  okay so multiplied by this  $r m \eta$  into the weighing function and integrate both sides so now I make use of my orthogonality property so therefore only for  $m$ . Equal to  $n$  this will be nonzero okay so this will turn out to be 0 to 1 our  $n$  of  $\eta$  into  $\eta$  into  $1 - \eta$



Square  $D_\eta$  on this side you can sum only if  $m$  equal to  $n$  so therefore this will be 0 to 1 and the constant can come out okay so this will be our  $n$  square of  $\eta$  because then if  $m$  equal to  $n$  then only this will be 1 so this will be our  $n$  square  $\eta$  into  $\eta$  into  $1 - \eta$  square okay so therefore my constant  $C_n$  will be zero to one  $R$  and  $\eta$  into  $1 - \eta$  Square  $D_\eta$  divided by 0 to 1 integral.

$R$  and square  $\eta$  into so now I have to evaluate these integrals how do I do that for example the integral in the denominator how do I calculate integral 0 to 1  $\eta$  square  $\eta$  into  $1 - \eta$  squared 8 yes so that will be  $1$  by  $2 \Lambda n$  into this particular thing right so I already know from the property of Sturm-Liouville system of equations that the denominator 0 to 1 are  $n$  square into  $\eta$  into  $\eta$  into  $1 - \eta$  Square  $D_\eta$  should be equal to  $1$  by  $2 \Lambda n$ .

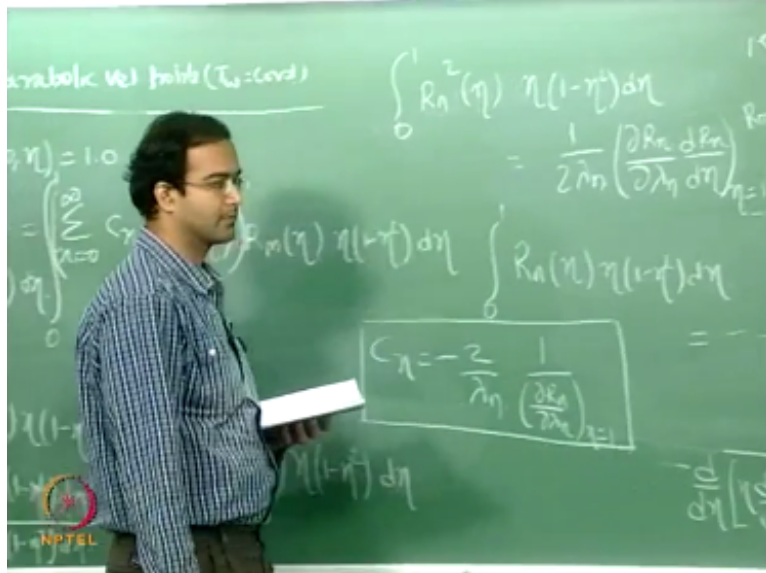
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Into  $D$  are  $n$  by  $D \Lambda n$  into  $D$  are  $n$  by  $D \eta$  at  $\eta$  equal to 1 ok so this is what I get from integrating the denominator and how about the numerator however the numerator you have a very nice clue if you have basically the eigenvalue problem here from this if you integrate both the sides okay so if you integrate this that is basically you have already  $R$  into  $\eta$  into  $1 - \eta$  Square  $D_\eta$  and this will be therefore on this side -  $1$  by  $\Lambda$  square into you have  $\eta$  into  $D R$  by  $D \eta$ .

And you are integrating between 0 & 1 so at 0  $\eta$  will be 0 so therefore this should be at  $\eta$  equal to 1 this will be 1 right so this comes from the eigenvalue problem itself so i can just integrate and i can find out the value of this okay so therefore now I can write my constant  $C_n$  so if I substitute for this integral in this integral what will be the constant  $-2 / \Lambda n$  and so this will be  $dr / dt$  at  $\eta$  equal.

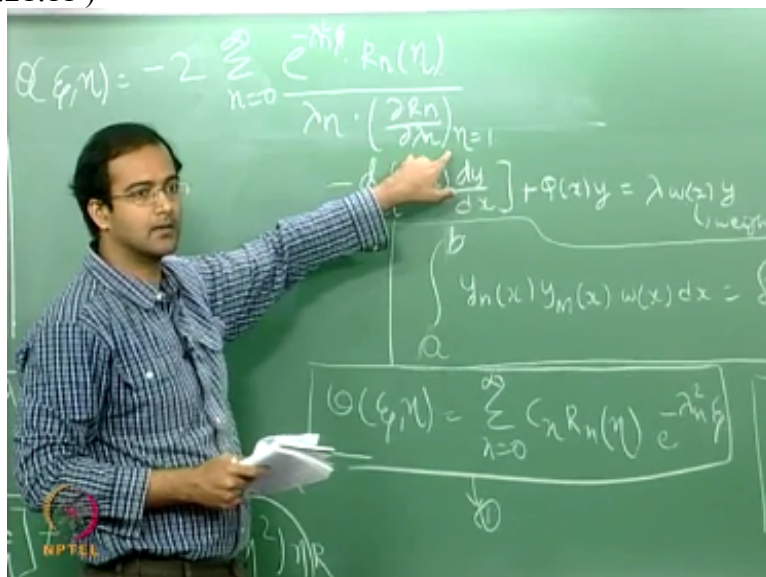
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To 1 and denominator dr by D H these two will cancel off so you have only this in the denominator so you have into 1 by dr n by D  $\Lambda$  n corresponding to H equal to 1 correct okay so this and this cancels of you have  $-2 / \Lambda$  n and this in the denominator so now for calculating this constant C as a function of n this is a function of n so for different values of eigenvalue  $\Lambda$  0  $\Lambda$  1  $\Lambda$  2 I need to know what is the derivative of the eigenfunction with respect to  $\Lambda$  okay corresponding to H equal to . /

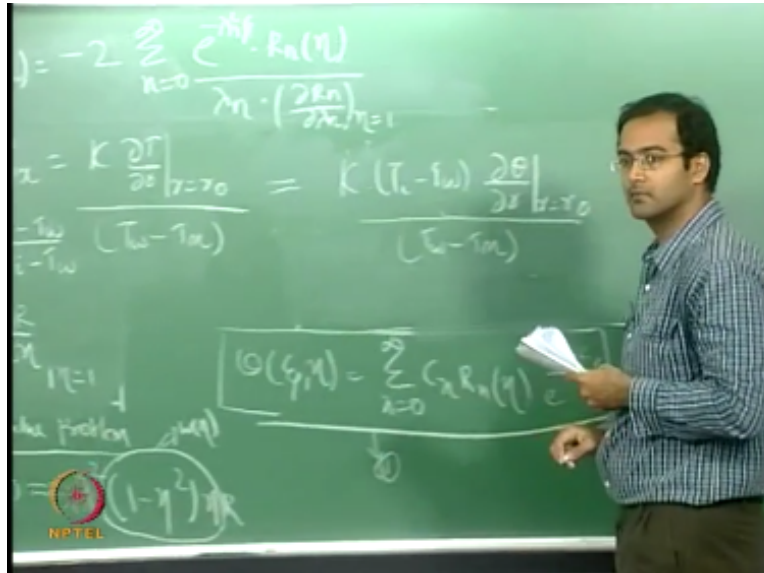
So therefore now for different values of  $\Lambda$  for H equal to 1 I should know what is the value of the eigenfunction and then i should fit some approximate curve and calculate the slope okay so that basically corresponding to H equal to 1 from there I can calculate my constant C okay so I can finally substitute for C into my equation number 1 for the solution turn therefore the.

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Solution for  $\theta_{\eta}$  can be written as - two this can be taken out and I have summation  $n$  equal to 0 to infinity and for  $C_n$  so the other these are all functions of  $n$  so I have to just keep it inside the summation I have  $E$  power -  $\Lambda n^2 Z$  into  $R_n$  of  $H$  divided by  $\Lambda n$  into  $D R$  and by  $D \Lambda n$  at  $H$  equal to 1 okay so I have simply substituted for  $C$  or  $C$  of  $n$  from what I have obtained here all right so finally one so once I know my eigen.

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Functions my Eigen values and this derivative I can now finally find the solution for  $\theta_{\eta}$  so this is my final solution so I will go further and I will calculate the expression for the nusselt number the local nusselt number so therefore now I will define my local heat transfer coefficient as  $K D T / D R$  at  $R = R_0$  divided by  $T_w - T_m$  this is my definition of local  $H$  and from substituting the temperatures in terms of  $\eta$  and everything in terms of non-

Dimensional radial coordinates okay so this will be if you substitute for  $T$  in terms of  $\eta$  you remember that because  $t - T_I$  by  $t - T_{wall}$  by  $T_I - T_{wall}$  okay so this will be  $T_I - T_{wall}$  into  $D \eta$  by  $d R$  at  $R$  equal to  $R_0$  divided by  $T_{wall} - T_M$  okay now I can define a mean temperature non-dimensional mean temperature which is  $\theta_{\eta M}$  as  $T_M - T_{wall}$  by  $E I - T_I$  so here  $T$  is a function of both  $R$  and  $X$  here  $T$  mean will.

Be a function of only  $X$  okay so I can define a non-dimensional mean temperature and you see that  $T_{wall} - T_M$  by  $P A - T_{wall}$  is nothing but  $\theta_{\eta M}$  okay so I can write this as  $K D \eta$  by  $D R$  at  $R$  equal to  $R_0$  by  $\theta_{\eta n}$  what I can also do is replace  $R$  in terms of  $H$  so therefore this will be  $H$  into  $R_0$  and this will be at  $H$  equal to 1 because you are  $\eta$  is higher by are not so I can replace directly with respect to  $H$  so all I need to know is my mean temperature  $\theta_{\eta M}$  and also

the non-dimensional gradient of temperature at the wall so once these two are calculated I can find the expression for H X so.

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$$= \frac{-k \left. \frac{\partial \theta}{\partial r} \right|_{r=r_0}}{\rho_0 c_p u_m}$$

From the equation for  $\theta_{m1}$  let me call this as equation number two now I can go on and calculate the expression for first  $\theta_{m1}$  so therefore I can write my  $\theta_{m1}$  as follows so how do I define my bulk mean temperature according to the basic.

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Solve for parabolic vel profile ( $T_w = \text{const}$ )

$$\theta_m = \frac{2\pi \int_0^{r_0} \theta u(r) r dr}{2\pi \int_0^{r_0} u(r) r dr} = \frac{2 \int_0^{r_0} \theta u(r) r dr}{\int_0^{r_0} u(r) r dr}$$

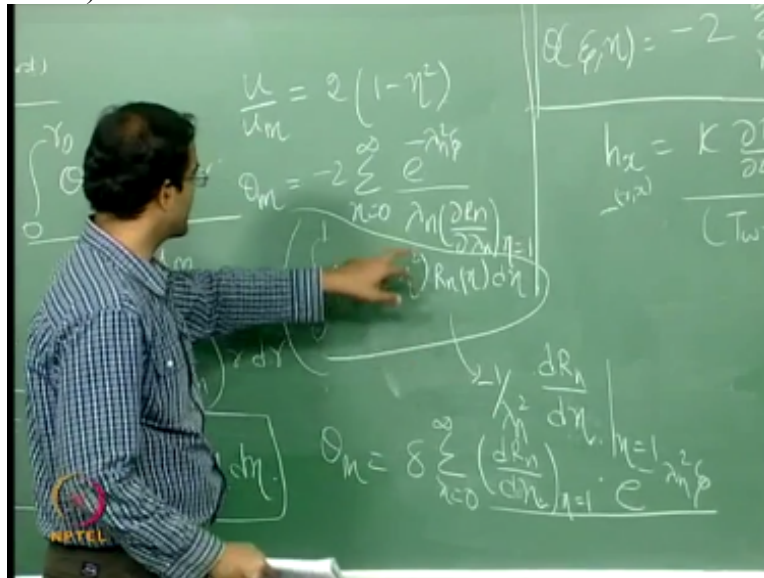
$$u_m = \frac{2}{r_0^2} \int_0^{r_0} u(r) r dr$$

$$\theta_m = \frac{2 \int_0^{r_0} \theta u(r) r dr}{\int_0^{r_0} u(r) r dr} = 2 \int_0^{r_0} \theta \left( \frac{u(r)}{u_m} \right) r dr$$

Definition I take my non-dimensional temperature multiply it by the velocity and integrate across the cross sectional area so that is  $2\pi \int_0^R \theta u(r) r dr$  okay so this is from zero to R so this I divided it again by the mass flow rate so that is  $2\pi \int_0^R u(r) r dr$

okay this is this cancels now I define my bulk mean velocity or mean velocity as once again 1 by so this is 0 to R 0 U of R into RDR so this will be 1 2 this will be 2 by R 0 square.

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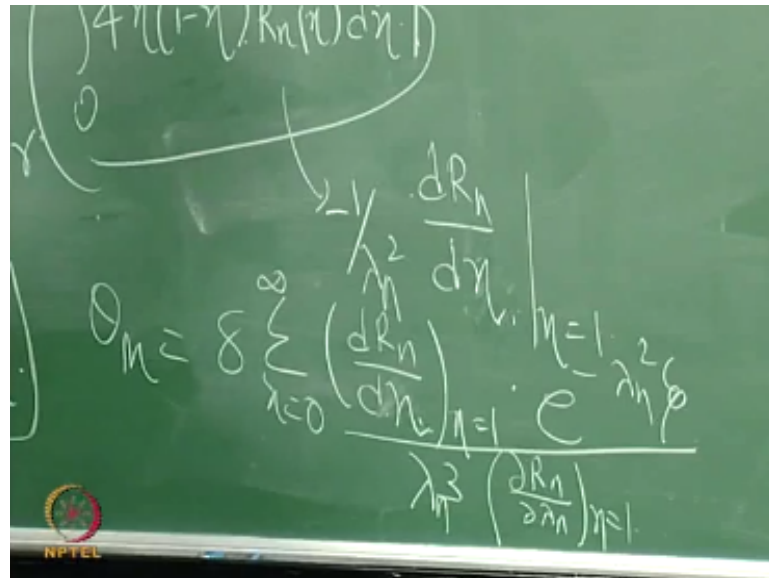


Correct okay so I can now substitute for this right here in terms of the mean velocity so this is basically integral 0 to R 0 η into U of R into Rd R divided by this will be hard not square into um by 2 okay so since um is only a function of X I can take this inside the integral and I can write this as u by u n into Rd R so I can now convert this completely in terms of non-dimensional coordinates okay so this will be 2 zero to 1 η into you by um into H into.

DITA because I have are by are not here dr by or not okay so this will be my expression for the M as a function of non-dimensional η + u bi um okay so I can now substitute I already have my expression for u bi um what is the expression for u by u a that is from the fully developed parabolic velocity profile right that is twice 1 - η square in terms of the non dimensional coordinate okay so I can substitute for η coming from Equation 2 and the

velocity profile from this into η M so this will become - 2 summation of n equal to 0 to infinity e power - λ n square H divided by λ n into D RN by D λ n responding to H equal to 1 into 0 to 1 then I have so 2 into 2 4 ok into H into 1 - H square into R which is also a function of H ok RN of H into D H this is okay so I am the substituting for η and you buy um into this and I am grouping all the terms which are

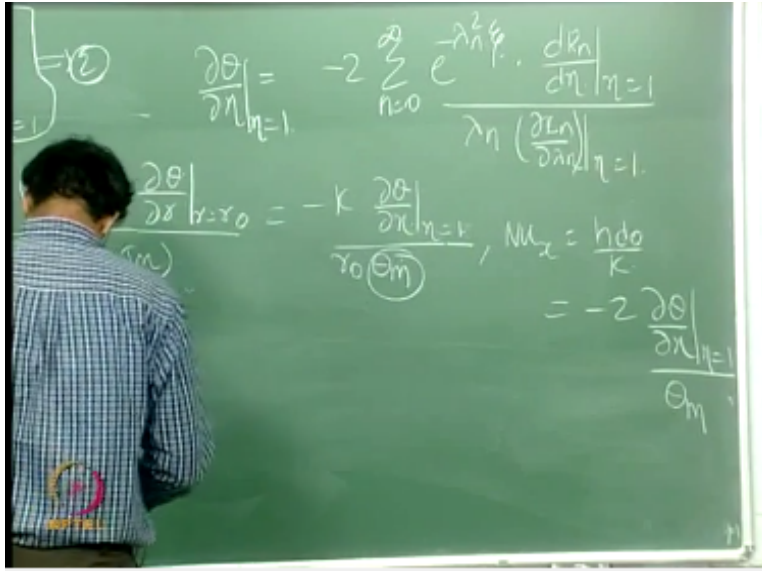
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Function of H and that I am integrating from zero to one so that is basically four times H into 1 - H square into R into D H right so this H into 1 - H square R and H D H what is this value so we have already seen that this is nothing but 1 by  $\Lambda n$  square into d r by D  $\Lambda$  that H equal to 1 okay so we will just substitute that and my  $\eta M$  now becomes so 4 into 2 becomes 8 here this is a - sign here okay this take note of.

It so - and - become + here so 8 into summation n equal to 0 to infinity into so that is a DR by D R by D  $\Lambda$  and at H equal to one no this should be with respect to H so therefore this comes out with respect to H please correct it okay so this is d dn by D H here okay so this is one of the terms multiplied by E power -  $\Lambda n$  square into Z and divided by you have  $\Lambda n$  square here and there is a  $\Lambda$  in here this becomes  $\Lambda NQ$  into.

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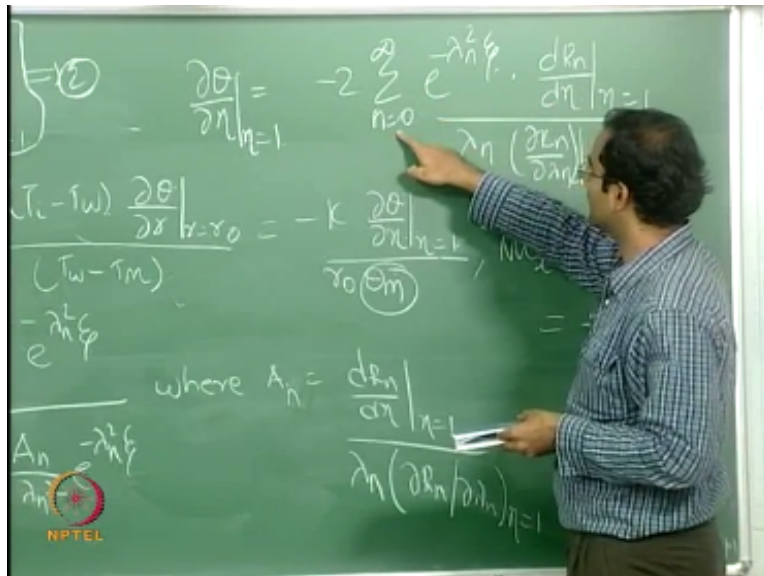


Already you have a term  $dR_n$  by  $D \Lambda n$  so this becomes  $D R$  and by  $D \Lambda n$  corresponding to  $H$  equal to 1 ok so this is your expression for  $\theta \eta M$  so now you have a closed form expression for  $\theta \eta M$  as a function of the Eigen value and the derivative of Eigen function both derivative as a with respect to  $H$  as well as with respect to  $\Lambda$  okay so now we need to still find  $D \theta \eta$  by  $D H$  with respect to  $H$  equal to 1 what is this value.

Directly you can differentiate that is  $-2 \sum_{n=0}^{\infty} e^{-\Lambda n^2} Z$  into this will be your  $dR$  and by  $dH$  at  $H$  equal to 1 divided by  $\Lambda n$  into  $dR_n$  by  $D \Lambda$  and a  $T$  type photo what okay so therefore we will substitute for  $\theta \eta m$  and  $D \theta \eta$  by  $DT$  a so therefore if I substitute I can so I can also write my so I have my expression for  $HX$  as  $-K$  by  $r-0$  I can directly get an expression for  $\nu$  which is.

Defined as  $H$  into  $R_0$  by  $K$  so it should be actually defined with respect to  $D$  therefore I will write this as  $H$  into  $D_0$  and so this will be  $D_0$  by 2 therefore this will be  $-2$  into  $D \theta \eta$  by  $D H$  at  $H$  equal to 1 divided by  $\theta \eta$  okay so I can substitute for  $D \theta \eta$  by  $D H$  and my  $\theta \eta M$  so that gives my  $\nu X$  as  $\sum_{n=0}^{\infty} e^{-\Lambda n^2} Z$  divided by  $2 \sum_{n=0}^{\infty} e^{-\Lambda n^2} \Lambda n$ .

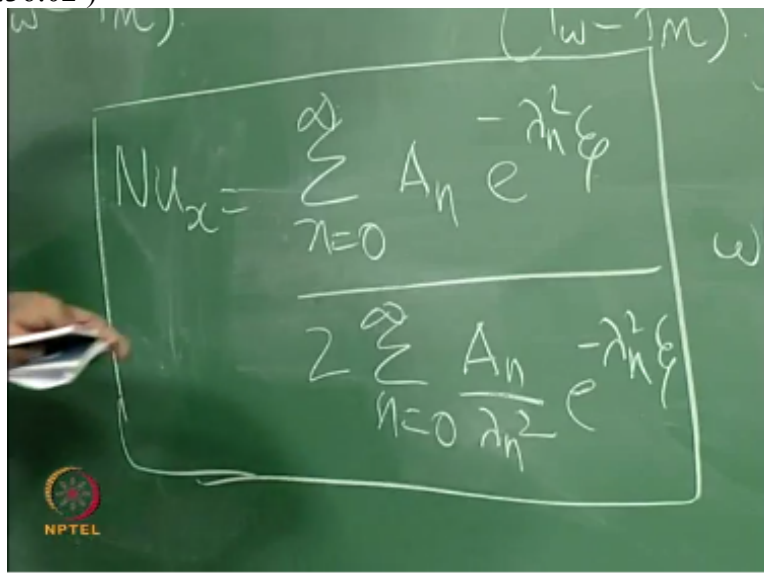
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Square e power -  $\Lambda$  and square Z where a n is nothing but d RN by D H at H equal to 1 divided by  $\Lambda$  n into D RN by okay so I will give you a couple of minutes you can substitute and check for yourself it is just straightforward I am just grouping this entire term d R by D H divided by  $\Lambda$  into D R by D  $\Lambda$  this has a constant which is a function of n okay so I call this as a n okay so this will be nothing but therefore a and into summation of.

N equal to 0 to infinity power -  $\Lambda$  n square that is the numerator divided by the  $\eta$  M also has the same term so in addition I have  $\Lambda$  n cube here so therefore this will be a n by  $\Lambda$  n square into this okay the numerator has basically 2 into 2 for denominator as 8 therefore there is 1/ 2 there okay I hope all of you are clear I am going the pretty slow here so therefore this is your final expression now to calculate my Nestlé's number here I need to.

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Basically know numerically the values of all these slopes of eigen function with respect to  $H$  as well as with respect to  $\Lambda$  and also the eigen values so from there I can calculate my nusselt number once I know the location axial location where I need to so - alt number is now only a function of my axial location non-dimensional axial location so what sellers did if you look at that particular paper which I posted so he did it numerically and he has.

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$n$	$\lambda_n$	$C_n$	$A_n$
0	2.7043	1.466	0.748
1	6.6790	-0.802	0.544
2	10.67	0.587	0.462
3	14.67	-0.474	0.415
4	18.66	+0.400	0.382

Predetermined the values of all these constants and tabulated them so I am just going to give only the final values tabulated by sellers okay so you can also see the therefore the value of corresponding to  $n$   $\Lambda_n$  the value of  $C_n$  and  $A_n$  okay so  $C_n$  is required where in your solution for  $\eta$  okay so that also everything has been computed numerically so for  $n$  equal to zero the most important Eigen value that is your 2 point 7 0 for 3 the value of  $C_n$ .

1.466, 0.748 and similarly for one two three will also give the fourth value this is a 6.6790, 10.67, 14.67 and 18.66 - zero point eight zero two zero point five eight seven - zero point four seven four + zero point four zero four zero point five or for 0.462 to 0.38 so the most important the first five eigenvalues a corresponding constant  $C_n$  and  $A_n$  and they were all numerically calculated and tabulated by sellers at a 1954 so therefore you can just directly use.

These values you do not have to sum them to so many terms you can just sum them to the first four or five terms okay you can directly substitute the corresponding value of  $\Lambda_n$  and in the solution for temperature the value of  $C$  then you can get an expression directly in terms of  $Z$  okay so which is basically a non-dimensional axial coordinate so for different values of  $Z$  you can actually plot and see how the nusselt number varies right from the location where.

Your thermal entry length starts okay now for the limiting case if you look at a very far away distance axially so that is for large values of  $\Lambda$  but  $\Lambda$  very large you can see that this is an exponentially decaying function okay so except the smallest value of  $\Lambda$  if you go for larger values of  $\Lambda$  that is corresponding to  $n = 1, 2, 3, 4$  so these are large values and it is already an exponentially decaying function so there will be very small okay. so only the first term corresponding to  $n = 0$  will be important when you look at large values of  $Z$  so for that.

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The image shows a green chalkboard with handwritten mathematical work. At the top right, there is a small sketch of a tube with a temperature profile  $T_w$ . Below it, the text  $\xi \rightarrow \text{large}$  is written. The main derivation is for the Nusselt number  $Nu_{\xi \rightarrow \text{large}}$ . It starts with the expression  $\frac{\lambda_0^2 (2.7)^2}{2} \frac{A_0 e^{-\lambda_0^2 \xi}}{\lambda_0^2 e^{-\lambda_0^2 \xi}}$ . The exponential terms cancel out, leaving  $\frac{\lambda_0^2 (2.7)^2}{2}$ . This is further simplified to  $3.6566 (FD)$ , where  $(FD)$  likely stands for 'Fully Developed'.

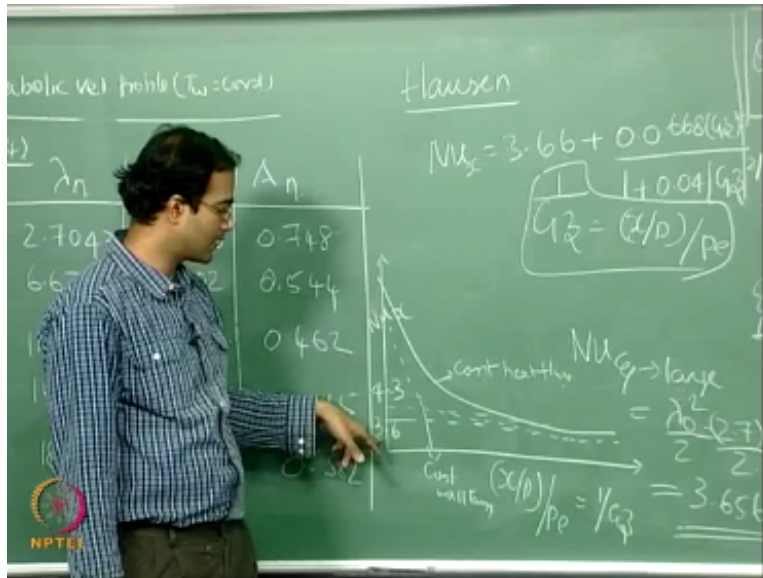
This will be reducing to your  $\Lambda_0$  into so basically you have  $E$  power -  $\Lambda_0^2$  x  $e$  power -  $\Lambda_0$  square  $Z$  right why I am retaining only the first term and neglecting all the higher terms so these terms how and this will become

So your nusselt number corresponding to large values of  $Z$  will be nothing but two  $\Lambda_0$  square divided by two so if you substitute the value of  $\Lambda_0$  corresponding to  $n$  equal to 0 what will be the value of  $Nu$  so that is basically 2.7 the whole square divided by 2 any of you anyone who is having a calculator can quickly check that 3.6566 anybody remembers the significance of this number this is for the case where we started fully.

Developed flows in the fully developed both thermally and hydrodynamically fully developed Region three and constant wall temperature boundary condition this was the nusselt number okay so now that we are getting as a asymptotic solution to the thermal thermally developing case for large values of  $Z$  the same value so this is your fully developed okay that comes out naturally as an asymptotic solution now to wrap up this particular case as you can see.

The Sturm-Liouville problems, any general Sturm-Liouville problems you cannot find direct closed form solution you have to do a numerical solution and therefore sometimes it is difficult to do so once the numerical values are calculated and tabulated and.

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There was a person called Heusen who actually fitted the empirical curves to this numerical solution for different values of  $n$  he took and then he substituted them there and then he fitted set of numerical empirical curves to The numerical solution and then he directly proposed an empirical correlation which is independent of all these constants and the eigenvalues okay so that is a very famous correlation so that goes as  $nu$  is equal to three point six six + zero point zero six six eight into grades number divided by one + zero point zero four Brides number to the power two-thirds okay where your grades number is defined as one over or we can say one bike rides number is

basically  $X$  by  $D$  by pecllet number okay so this is a very simpler correlation you just substitute the axial location non-dimensional form as Christ number into this expression and you directly get your local nusselt number okay you don't have to find out Eigen values the corresponding constant corresponding to  $n$  equal to 0 1 2 3 4 this okay this is an empirical correlation which fits very well with the exact solution okay so you can see for the limiting case where your  $x$  by  $d$

goes to very large values your one bike rides number is basically say here your grades number becomes very small for large values of  $x$  by  $d$  so then this particular term here disappears and it will lead to the limiting case of 3.66 which is the nusselt number for fully developed flow okay so if you want to just plot the axial variation of the local nusselt number with respect to one our grades numbers so you will find that this is for the constant heat flux

case anybody remember what is the asymptotic solution for point four point three four point four point three roughly this is for a constant heat flux and this dotted line here is for constant wall temperature and the asymptotic case leads to the value of 3.6 okay so if you plot the local variation you know once you have the first five dominant terms you plot it as a function of  $1$  over grades number and you will find that exponentially it is a decaying function

Okay, and also you can plot both the constant wall flux boundary condition case constant wall temperature boundary condition case the constant flux case has a slightly higher nusselt number and asymptotically that will reach to a value of 4.3 and constant wall temperature case reaches to three point six okay, so this is to give an idea about the thermally developing region now as I said how do we get the constant heat flux case okay, so that is a there is a slightly different problem okay, the problems that we have solved in the class corresponding to constant wall temperature boundary condition ok and parabolic velocity distribution.

This was an extension which was proposed by Sellars extension of the grades problem now the other extension is for a non uniform wall temperature that is for case where you can have an axial variation of wall temperature which is a linear variation or you can also have a constant heat flux boundary condition so these were proposed also by sellers and I have uploaded a document on the Moodle which gives the extended solution for the case of a plug flow the velocity profile is plug flow but the boundary condition is a constant wall flux boundary condition okay.

So there I will probably in the tomorrow's class give you hints how to approach that problem there the condition is non-homogeneous because you have a defined heat flux so the thing is how do we homogenize that boundary condition for getting an eigenvalue problem so that that is the key once you know how to do that after that the rest of the things are straightforward so to get an idea you can just look into the Moodle Lovera have posted for the case of parallel plates okay that's a much simpler case to deal with say Cartesian coordinate system and taken case of plug flow and constant water wall flux condition okay.

So I will give you I will just describe the procedure briefly and you can go over the document and you can do the same thing for the case of circular tube balls okay so with that the developing case will be over the thermally developing case of the thermal entry length cases will be done and finally we look at the case of simultaneously developing simultaneous entry length problems so those problems we cannot do analytical solutions because you cannot neglect any terms strictly speaking okay we have to go for a full numerical solution.

To the navier-stokes equations there have been some solutions where some approximations have been made but still they were involving a numerical solution so I will just give you the empirical correlations coming out of those solutions and I think tomorrow we should be able to complete this part and the last two classes on Thursday and on Saturday we look at the

approximate solution to the internal flow problems so how do we use the integral method here okay.

## **Extended Graetz problem**

### **End of Lecture 31**

**Next: Extended Graetz problem with  
Wall flux boundary condition  
Online Video Editing / Post Production**

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M.V. Ramachandran

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Soju Francis  
S.Subash  
Selvam  
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Kannan Krishnamurthy & Team

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