

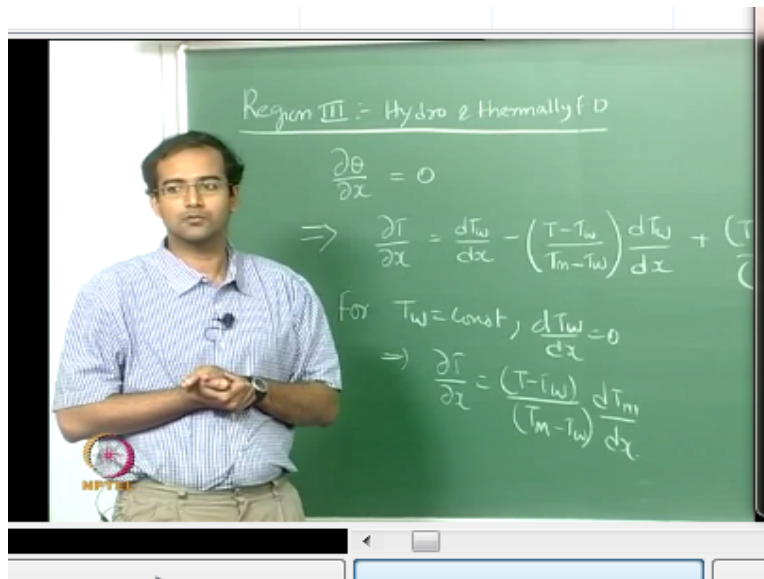
Indian institute of technology madras
Presents

NPTEL
National Programme on technology enhanced learning

Video Lecture on
Convective Heat Transfer
Dr. Arvind Pattamatta
Department of mechanical engineering
Indian institute of technology madras
Lecture 28

Shooting method for fully developed heat transfer
and thermal entry length problem

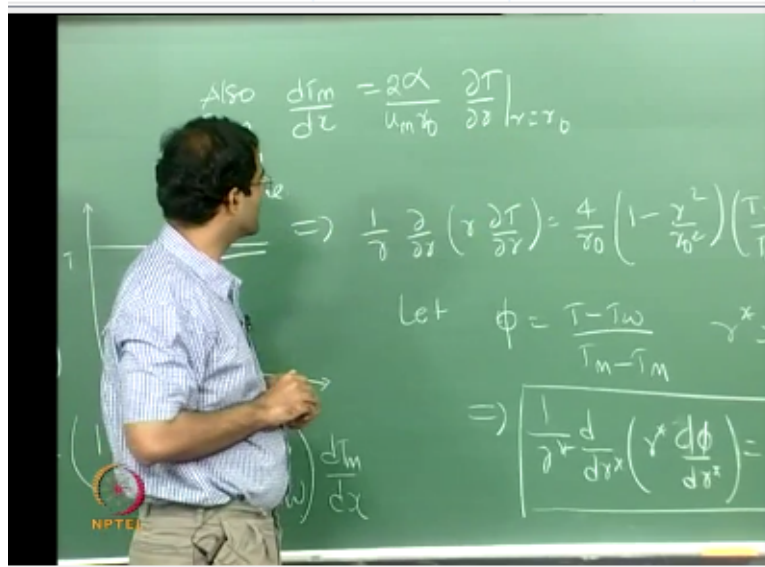
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So this is a recap of what we did in the last class on Thursday for those of you who could not make it in the morning class we were looking at the case of constant wall temperature boundary condition and when you apply earlier when we did the case of constant wall flux okay so there we came to the conclusion that your $\frac{DT_m}{DX}$ is $= \frac{DT_w}{DX}$ and these two terms got cancelled and your $\frac{DT}{DX}$ is $= \frac{DT_w}{DX}$ so all the three can be equal only if they are $=$ a constant and therefore the variation of the temperature at any radial location as well as your wall temperature and mean temperature they have to be parallel to each other.

And they have to be a linear line okay when it comes to the case of constant w temperature so this comes to the fact that DT_w / DX is a constant and therefore these two terms will get nullified so you have $DT / DX = T - T_w / T_m - T_w$ into DT_m / DX so therefore your DT / DX now related to your DT_m / DX now coming to the energy equation so we will substitute this whatever conclusion that we made.

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So we will instead of DT / DX we write that in terms of $T - T_w / T_m - T_w$ into DT_m / DX okay also since we have already calculated the velocity profile which is a parabolic profile we have substituted the parabolic profile into this expression now what is the problem with this unlike the case of constant w flux boundary condition there your DT_m / DX was a constant so therefore so this side this was not there you had only DT_m / DX this side it was T as a function of r so you could directly integrate this was this was the integral straightaway and you can find the temperature profile with respect to R and apply the boundary condition okay.

So that was a very straightforward way now if you look at it this DT_m / DX is not a constant okay so this is now changing and we do not know how exactly it is changing strictly speaking I I we last time derived how the profile should vary if you plot your profile as a function of X the w temperature is a constant however your mean temperature will vary such that the difference between the mean temperature and w temperature is exponentially decaying function so our X going to infinity the difference goes to 0 correct.

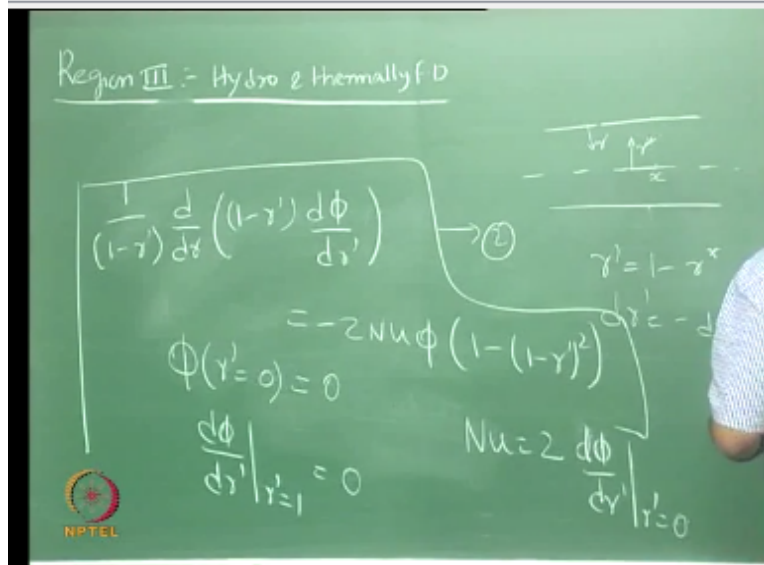
So therefore this is the way that the mean temperature varies okay so having known this you have again on the right hand side T which is a function of both X and R so you look at this equation now it is a partial differential equation and there is a chance that you can convert this into an OD provided you can express your DTM / DX as a function of DT / DR okay so then you can also substitute for $T - T_w / TM - T_w$ in terms of θ and now since $D\theta / DX$ is 0 so θ is only a function of R so then you can convert this PDE into an ISO D so this is how we are doing it so from energy balance my dTM / DR can be related in terms of the w heat flux as DT / DR at $R = 0$.

And when I substitute that into the earlier energy equation so now I have an equation where I can use θ to denote $T - T_w / TM - T_w$ okay so therefore I can non dimensionalized my radial coordinate also I can introduce a non dimensional radial coordinate which is R / R_0 which is R_0 is the radius of the duct so final resulting expression if I substitute that will come out in the form that is presented here this is nothing but an OD which can be solved / shooting method.

Once again I try table so we look at the boundary conditions So θ at our star $= 1$ that is corresponding to $R = R_0$ so that should be 0 because there your T will be T_w and at $R_{star} = 0$ you should make use of the fact that the profile is symmetric and therefore the slope has to be 0 so these are the two boundary conditions this is a second order ordinary differential equation so we can solve the OD with the two boundary conditions so how do we do this once again we can make use of the shooting technique to get the to solve this equation and also to get the value of nusselt number okay.

I will just briefly explain how you will be doing this it is very similar to the earlier shooting technique problems only thing .

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see now the coordinate system what you have been working with starts with the centre so what I would like to do is when you solve this with shooting technique you can start working from the coordinate system which starts from the w okay so why we can do that is that at the w you do not know the gradient okay also you do not the gradient is related directly to your nusselt number .

So in this equation you do not know what is the nusselt number also you do not know what is the gradient at $R_{star} = 1$ correct so both are not known so in order to simplify this problem if we start working directly with a coordinate system starting from the w and marching to the center you can directly guess the value of Nu and therefore that will give you a guess for directly the slope at the w right so you understood the problem so right now if you are marching from the center okay.

You need a boundary condition for fee at $R_{star} = 0$ so you need to guess a value of V at $R_{star} = 0$ and again you need to guess the value of nusselt number both are unknowns that will involve two guesses and that will become little tedious whereas if you start from a coordinate system from the w and proceed to the center you can you should need a guess for the slope at the w and that is directly related to your nusselt number so both in one shot you get it okay.

So that what I am going to ship the coordinate system from the center to the w okay so I will have a coordinate system right now I have an R_{star} non dimensional coordinate system ranging from zero to one I will shift that to R' how do I shift it $1 - R_{star}$ so that when R_{star} is $= 0$ R'

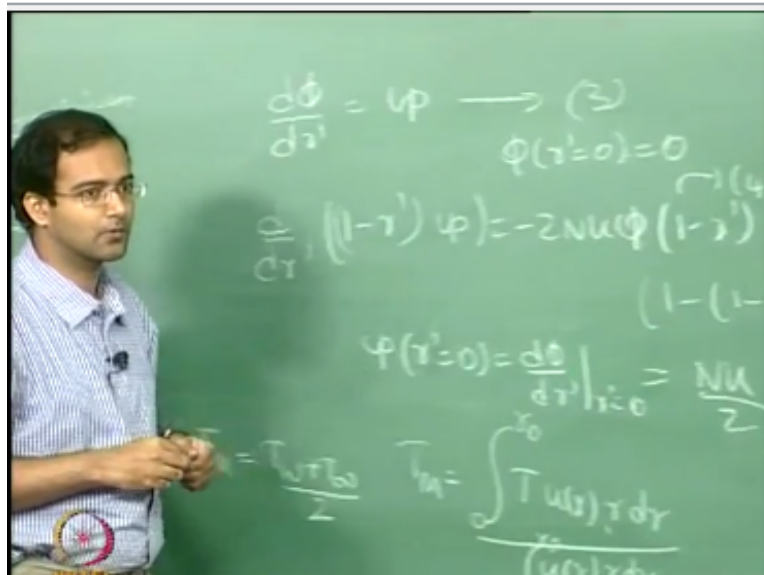
= 1 when $R^* = 1$ R' will be 0 so then I can substitute that into let me call this as equation number 1 substitute into equation number 1 and I can rewrite in terms of R' okay.

That will come as one - $R' D / DR$ $1 - R' D \pi /$ mere ' of course your DR' will be - $D r^*$ right so that should be = - twice nusselt number into π into $1 - 1, 1 - 1 - R'$ the 2 instead of R^* 2 I have $1 - R'$ the 2 so this is how my OD will be and the boundary conditions will be fee at $R' = 0$ will be what so now I should also make the boundary condition transform to R' instead of R^* this should be 0.

And $D \pi / DR'$ at $r' = 1$ should be okay so now i can hear your nusselt number is defined as twice $D \pi / DR'$ at $R' = 0$ this is how your result number is defined correct so earlier it was defined as in this case your result number was defined as - twice $D \pi / D R^*$ at $R^* = 1$ now when you substitute for D for $D R^*$ as - $D R'$ so that will become result number will be twice $D \pi / DR'$ at $R' = 0$ so that is starting from the top from the w and towards the center.

So now this equation is easy to be solved by the shooting method so we will call this as 2 so by shooting method first I should reduce before for applying shooting technique I should reduce the second order Odes into two first order Odes so / introducing the fact.

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That my $D \pi / DR'$ is = side this is one of the this is one of the Odes the therefore if you substitute into equation number two so you should be getting D / DR this is all your DR' into 1

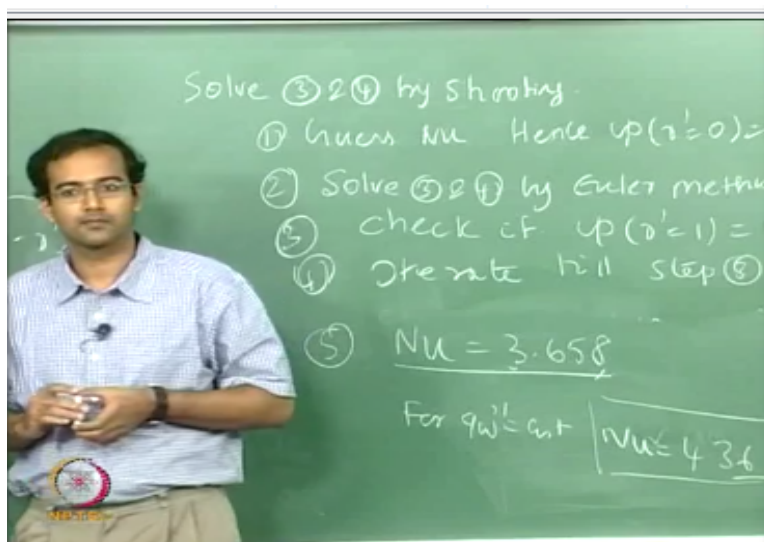
- R' into side should be $= -Nu \Pi$ into $1 - R'$ into $1 - 1 - R'$ the 2 okay so this is the equation number 3 this is the equation number 4 so now you have reduced equation 2 to 2 first order Odes 1 you have to solve for θ the other you have to solve for side.

So the boundary condition for this So θ at $R' = 0$ so that basically is 0 so that is sufficient for solving this now for solving this you need θ which is nothing but θ at $R' = 0$ which is $D \Pi / D R'$ at $R' = 0$ this is not known this is nothing but what nested / two therefore now you see you do not know the nusselt number anyway so you can guess a value of nested number and that is the guess for θ at $R' = 0$.

But what do you know is basically θ at $R' = 1$ okay so now this becomes a nitrated process again so you guess a value of nested number therefore you guess the value for θ at $R' = 0$ you keep marching / the shooting technique you shoot a new March and you should make sure that your θ at $R' = 1$ is $= 0.7$ so that satisfies the other boundary condition so you have to do this iteratively you keep guessing the value of nusselt number you change the value of nusselt number until you satisfy this condition.

So finally you end up directly getting the correct result number that is it okay so once again I will just write down the procedure.

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So solve 3 and 4 by shooting the step 1 is to guess Nu and therefore hence your θ at $r = 0$ is calculated as in $U / 2$ that is the I step number 2 you solve three and four by say oiler

method okay so you can take a large value of so you go you start from $R' = 0$ up to $R' = 1$ okay so unlike your external flows where your ETA was going to say 10 or 15 now your range is defined so you are our $R' = 0$ to 1 and then check if whatever from your solution if you are sigh at $R' = 1$ is $= 0$.

If that is true then whatever you guessed is correct that directly gives you or any if not you have to titrate till step 3 is satisfied now again for I iteration we make use of an intelligent guess using the Newton rap son method so here the Newton rap son algorithm will be sigh you need to basically guess the value of sigh at our $R' = 0$ so therefore sigh at our $R' = 0$ at $K + 2$ so you first guess once you go guess twice and then for the third guess you use the Newton rap son method.

Because again it needs at least two guesses to calculate the difference so this will be $=$ sigh at $R' = 0$ so $K + 1$ s + so this will be f of X divided / F' of X now f of X is a condition that sigh at $R' = 0$ should be $= 0$ so anyway that so this - 0 should be 0 or this is $= 0$ right Oh R' of sorry our $R' = 1$ so in this case this comes out as 0 therefore this term should be I think this entire term then should come out as zero here because your f of X should be anyway zero right.

Here f of X is anyway 0 so sigh at $R' = 1$ should be $= 0$ so therefore so this numerator completely so I think using the Newton's method here may not be that beneficial because anyway so this does not this term is absent here right this is it right because you are you have your sigh at $R' = 1$ this is your function which it has to be satisfied this is $= 0$ your refer of X is basically 0 okay so therefore you have to guess your Nusselt number so you can guess it iteratively okay.

Trial and error yeah okay but this is actually the function that it has to satisfy right so the function it has to satisfy is f of X is $= 0$ so this is basically on the right this is $+ \text{ of } X / F'$ of X so this we are writing as a difference but your f of X itself is 0 here okay so if you if you had a condition that this is $= 1$ then you can use it this - 1 $= 0$ is your F of X they are similar to the external boundary layer correct so here we have directly your F of X is 0 so there is no point in using this particular method will not give you lead you anywhere.

But this is what you have to satisfy so you have to say that you are F of when you say that this is the equation it has to satisfy this is a stating a we have to force it as 0 here right you can just try

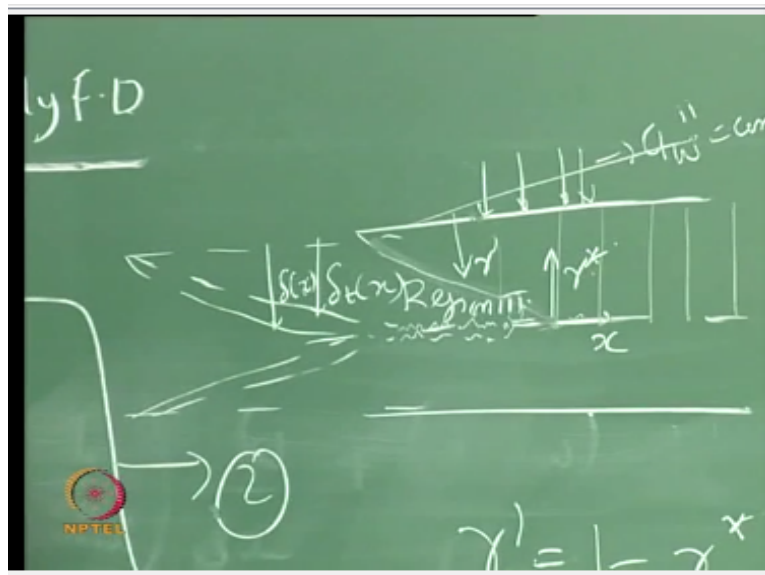
and check but I am myself from thinking whether if this will work out for this kind of a problem in this boundary condition because see when you had the condition clearly that say for examples $I = 1$ so you make sure that it has to satisfy $\text{sinh}^{-1} = 0$ so that is a function there it which is to which it has to satisfy whereas here sinh is $= 0$ directly okay.

So anyway you can try your whether you can apply your Newton's method and see but in my opinion this may not lead to the correct solution using this so you can just do a wild guess anyway I will give you the solution you can start with some guess close to the solution and you will find that your nusselt number for this case comes out to be 3.658 eight so it is a constant value once again as I said if you have a fully developed thermally and hydrodynamically fully developed case that is in Region.

Three you have a constant value whether it is isothermal or ISO flux case for ISO flux case or do you remember the value for point 4.36 so for the isothermal case if you do a trial and error you will finally see that at result number 3.658 this exactly satisfies this boundary condition sinh at R star add $R' = 1$ will be 0 okay so therefore if you compare this to the ISO flux case you see the ISO flux case has a higher nusselt number okay so there has been no concrete reason why people observe this experimentally also that this is higher.

But some kind of an explanation can be given from the thermal boundary layer.

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Now when you talk about the fully developed thermally so you talk about the region where the thermal boundary layers and the velocity boundary layers have met so this is your region 3 okay so what could possibly happen if you apply a constant w flux is that the w temperature keeps rising gradually and due to that if you look at the property calculations.

In the case of external flows all the properties that we calculated were based on T_{mean} which was $T_w + T_{\infty} / 2$ it is a linear average between the w temperature and the free stream temperature in the case of internal flows we defined a T_{mean} based on your mass weighted average for temperature U of R $\int_0^R U \, dR$ so you have divided / integral U of R $\int_0^R U \, dR$ so this is 0 to R 0 to R so you have to calculate the properties based on this mean temperature and this temperature varies from the w till the centerline okay

And if you have a case where your w temperature keeps varying then therefore and this will consequently result if you look at the property calculation the properties will be keep on changing due to the w temperature variation whereas if you have a w temperature which is constant if you take this kind of an average it will not vary the way that it varies for the case of $Q_{\text{all}} = \text{constant}$ so this will result possibly due to due to this dependence of the thermal boundary layer on the Prandtl number when it when this too much there will be a possible fluctuations in the thermal boundary layer okay.

So it will not be exactly located and it will not be exactly merging at the centre but there will be some small fluctuations due to the property variation okay because these boundary layer thickness are functions of prandtl number and locally the prandtl number keeps changing due to difference different temperature values axial so they attribute that these kind of disturbances in the thermal boundary layer will result in possibly a higher diffusion heat diffusion in the case of constant heat flux resulting in a higher nusselt number okay.

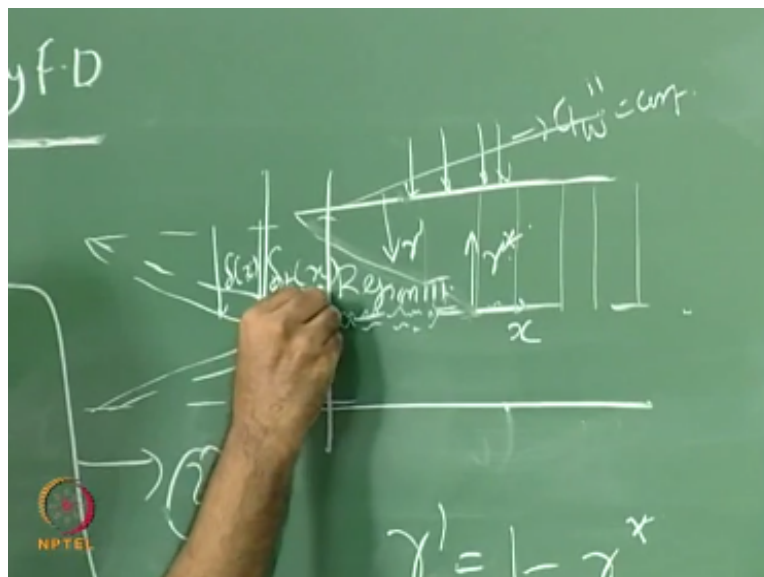
So this is about what 16 percent higher than the constant w temperature case so one possible reason this is a possible reason although you cannot attribute it to a particular reason

this is one possible reason what people say is that the variation in the properties due to varying much you know the variation property is much more in the constant wall flux boundary condition than the wall temperature case and this will result in instabilities in the thermal boundary layer which will possibly drive the heat transfer to be much higher than in the case of

constant wall temperature okay so this is one possible explanation although in the textbooks it is not even mentioned if they just say generally.

That this is higher and they stop there is no concrete reason why this should be higher but you should all remember that a constant heat flux case will generally result in a higher heat transfer coefficient than so this is true even for external boundary layers you observe it in external boundary layers the two boundary layers never merged they keep the boundary layer keeps on growing so there the fluctuations will be quite dominantly seen whereas here the two boundary layers merged and therefore you can only attribute some fluctuations towards the center centerline okay.

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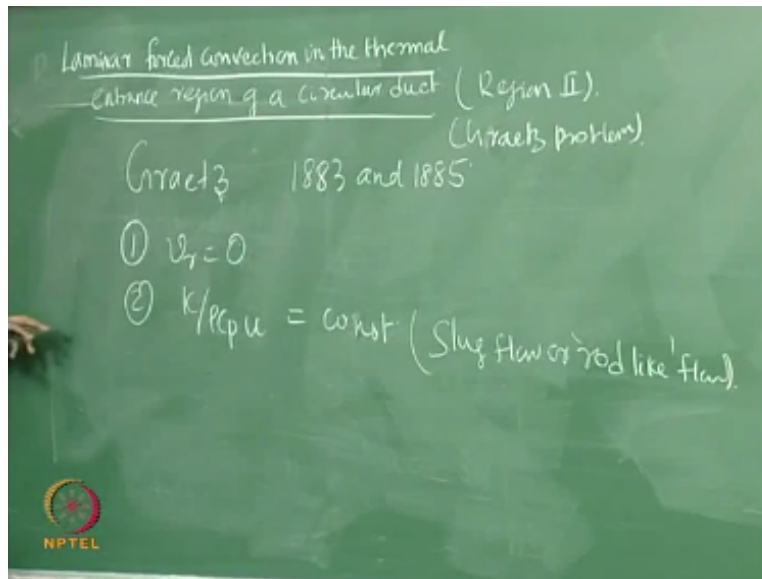


So this is something that that you have to pay attention now the next thing what we will do is we will move from region three so in Region three we looked at constant wall temperature and constant heat flux separately and we will now focus our attention to region two that is this region right here so where you are hydro dynamically fully developed and the boundary layer still have not merged in terms of you know the thermal boundary layers have not merged or you cannot say that your $D\theta / DX$ is $\neq 0$ okay.

So you can only say $D U / DX$ is 0 at the last profiles are fully developed but you cannot say the temperature profiles are fully developed so we for this particular region is a more interesting region we will focus the rest of the classes another three to three or four classes towards the

region to so any questions on this so far so I think I will give an assignment where you will be doing the shooting method you can you can try out with a knight relative guess you can start with one two three four and you will find that progressively this will be satisfying so once you reach you know close to the solution you can take the guess values much closer you know 4.1 , 4.2 then you should see that it should converge okay.

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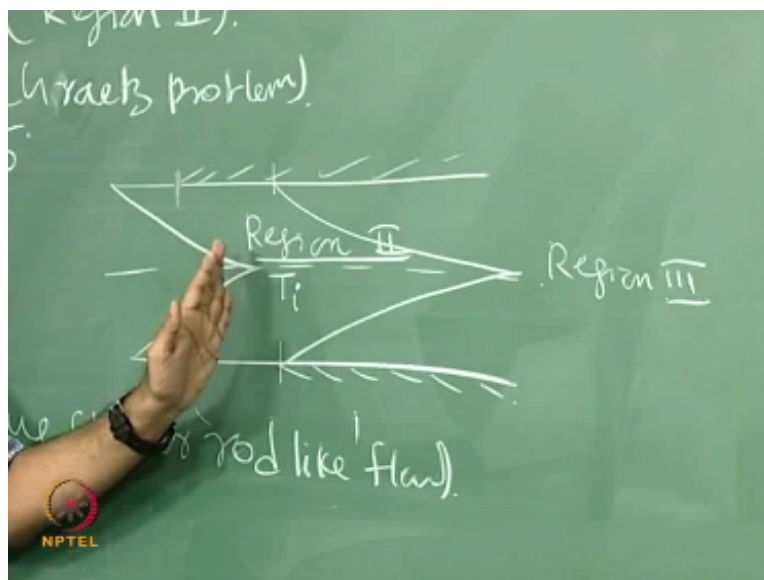
So we move on to the thermal entry length region or the region to which I denote it where you are hydrodynamically fully developed but thermally it is not developed yet so for this case the first solutions were given by Graetz so he did the first solutions for the region two as early as 1835 not 1835 it is 1883 and 1885 okay so he produced the first solutions in fact some of his papers there are still available but you have to go to a library and get it if you have the corresponding journal but whatever Graetz.

Did he did it for only one case for a very simplistic assumption that your velocity is completely uniform that is a slug flow or plug flow case okay so later on people extended the Graetz flow solution to other general cases where you can have a parabolic velocity distribution and you can have other boundary conditions such as constant wall flux and so on in fact there is a person called Sellers I have also uploaded a document on module yesterday so you can just have a look at it where the great solution.

Was extended by this person this group of people sellers is one of them is a first author so there is a document where they have proposed two solutions to parabolic velocity profiles and also to cases where you have linear variation of wall temperature wall flux is equal to constant so there are further extensions of this first original grid solution so this is popularly called as The Graetz problem also and I will just list down what are the assumptions that he made the first assumption.

That he made is that you are a radial velocity is zero everywhere the second assumption is the ratio of $K/\rho C_p$ into u is a constant now this is a big assumption okay when you say $K/\rho C_p$ this is your α so this you can understand that as a property could be constant but when you say also U is constant that means is as assuming everywhere plug flow okay so this is for a case where you have only plug flow and therefore you can assume that this is your slug flow or a rod-like flow he calls it okay.

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He Calls it as a rod-like flow so he assumes as it is like a solid rod which is passing through a circular tube okay so where do you have a velocity which is a constant and properties are also constant and at the starting point where you where you are looking at the thermal boundary layer okay so there the inlet temperature is a constant so the region that you are looking at now is where your velocity boundary layer smirched and then you start the thermal problem okay.

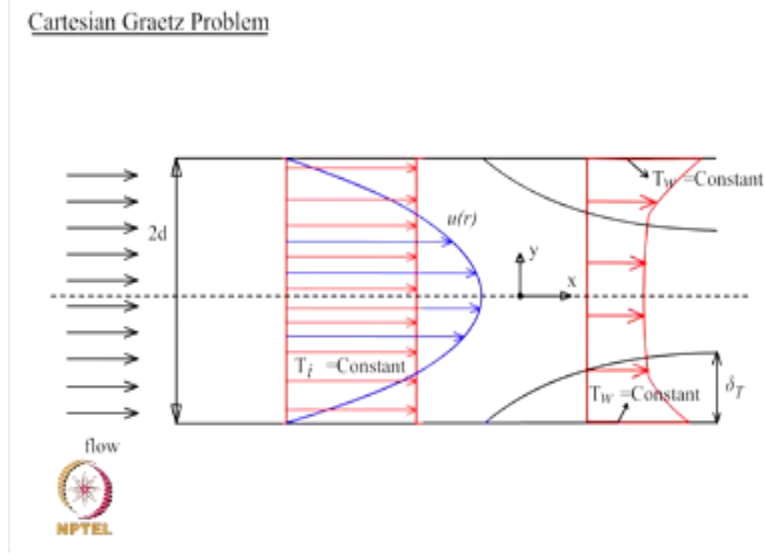
So suppose you start your thermal problem somewhere here okay so strictly speaking you have to start it somewhere after the velocity boundary layers have merged okay so from here you look

at the thermal problem where the two thermal boundary layers grow and then merge okay so this is completely your region two so now this is region three so you can now see that if you solve region two where you start from some initial temperature.

Here correct from where the two boundary layers start growing thermally already the velocity boundary layers have grown and they have merged then you start the growth of the thermal boundary layer at the inception of the growth of the thermal boundary layer your temperature is equal to the inlet temperature so from there they grow so you are developing solution for this region thermally developing region and asymptotically once the two boundary layers.

Merge you should asymptotically reach the solution that you had already developed for region three correct so that is an asymptotic solution of the solution to the thermally developing region so we will see that when we develop the final expression for asymptotic case you will directly find that we will be reaching these two limiting values okay for Region three okay therefore we could not be strictly speaking we need not look at region three separately we could have directly looked at region two and said that for the limiting case of X / D going to infinity you directly reach your fully developed solution okay.

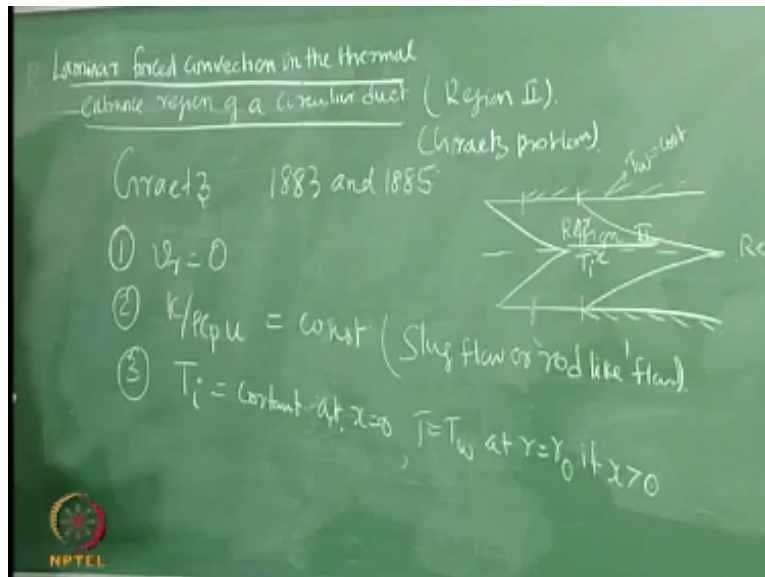
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You so this is your assumption that at the inlet everywhere Inlet where you start the growth of the thermal boundary layer your Inlet temperature is uniform it is constant and so this is for the case if at $X = 0$ so now your coordinate starts from a point where you start the thermal boundary

layer growth okay so this is your coordinate system so it does not start from the inlet of the pipe it starts from the point where your thermal boundary layer starts growing and where your fully developed velocity profiles are present okay.

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And $T = T_w$ at $R = R_0$ if you are $X < 0$ so he has looked at a constant wall temperature boundary condition okay so at $X > 0$ the wall temperature is applied less than that you do not have any thermal condition okay and the other assumption he makes is that your thermal conductivity in the axial direction is 0 or you can say you neglect your axial conduction in comparison to the radial conduction okay so then we can write down the energy equation for this case so now if you say the energy equation.

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$$u \frac{\partial T}{\partial x} = \alpha \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right)$$


$$\downarrow$$

$$u \frac{\partial T}{\partial x} = \alpha \left[\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right]$$


Is you DT / DX is equal to I am expanding original you have 1/ r d / dr of r dt / d r i can expand it i can say α is common so i can differentiate with respect to r keeping dt / d are constant okay so that will be 1 / r dt / dr + i can differentiate dt / d are keeping our constant so that will be $d^2 P / dr^2$ so now this is your energy equation so you are neglecting your axial conduction term with respect to radial conduction you have neglected your radial velocities the radial velocity is 0 so only your axial velocity is there so these are all the assumptions based on the great is problem so now you can assume.

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$$u \frac{\partial T}{\partial x} = \alpha \left[\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2} \right]$$

$$\Theta = \frac{T - T_w}{T_i - T_w}$$


A non-dimensional temperature θ like the way that you assumed before as $t - T_w / \sigma$ now rather than using T mean here I would like to use T_i okay so I am going to use here is that in earlier case you somehow manipulated such that your dt by DX was a constant okay so therefore you do not have to apply any boundary condition corresponding to $X = 0$ in this case this is a partial differential equation so for this you need a boundary condition for T at $X = X = 0$ so therefore at $X = 0$ you are $t = T_i$ so to do that you can non dimensionalized with respect to T_i such that at $x = 0$ so σ at $X = 0$ will be 1 okay.

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$$= \alpha \left[\frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right]$$

$$\theta = \frac{T - T_w}{T_i - T_w}$$

At $\theta(x=0) = 1$
 $\theta(r=1) = 0$
 $\theta(r=0) < \text{finite}$

And σ at $R = 1$ will be 0 that will be t_1 which is a constant so you need two boundary conditions with respect to radial direction so what is the other boundary condition should be finite or $D\sigma$ by dr should be 0 okay so substituting this we can rewrite this as I can say my $u = u_m$ which is a constant based.

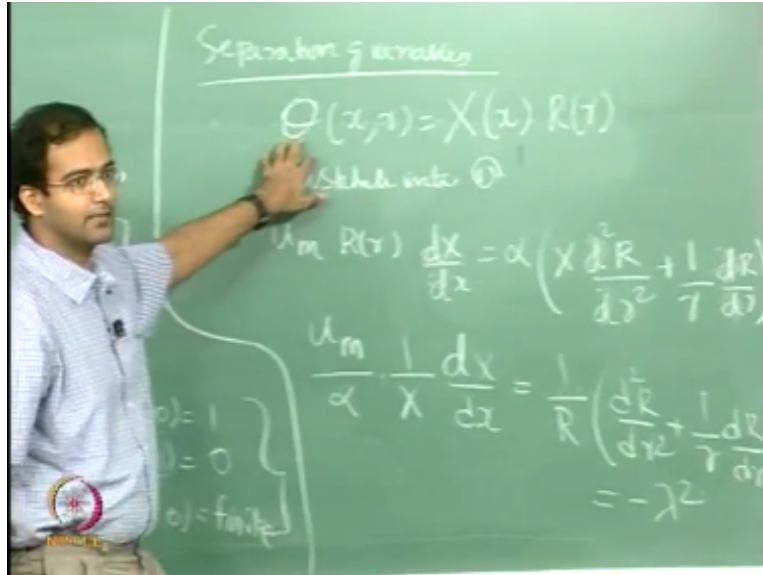
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$$u = u_m = \text{const} \quad \Theta = \frac{T - T_w}{T_i - T_w}$$

$$\Rightarrow u_m \frac{\partial \Theta}{\partial x} = \alpha \left(\frac{\partial^2 \Theta}{\partial r^2} + \frac{1}{r} \frac{\partial \Theta}{\partial r} \right)$$

On the plug flow assumption so this can be written as $u_m D \sigma / DX = K / \rho CP$ or I can just I can just leave it as α here $d^2 \sigma / dr^2 + 1 / r d\sigma / dr$ so this is my non-dimensional temperature and these are the boundary conditions so now in the case of thermally fully developed case depending on whether it is a uniform wall flux or a uniform temperature I can solve this directly I can convert this into an ordinary differential equation and I could have solved it but now in this case my σ is a function of both x and r and I cannot put the condition that $d\sigma / DX = 0$ so therefore this is a partial differential equation and how to solve the PD as it is so how do I solve the PD I think most of you have done the solution of PD is before what is the simplest technique to solve PD is separation of variables.

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Oh so how many of you have done separation of variables how many of you have done PD solution by separation of variable so how about the other M Tech students I think at least in the heat transfer advanced heat and mass transfer should be thought to you I think so I am not going to spend too much time explaining the separation of variables but you should understand that you assume now this is a linear equation once you have your velocity so any linear equation now we can assume the solution.

For σ in this case which is a function of X and R you can split it into two solutions you can assume that this is a product of two solutions one which is only a function of X and the other which is only a function of R this is the starting point of any problem for separation of variables so then this is the assumed solution you put it into the PD let me call this PD as number 1 so substitute the assumed solution into 1 what do you get so this will be when you differentiate with respect to X R will be held constant.

So this will be $u_m R$ into $\frac{dx}{dx}$ on the right hand side you have when you differentiate with respect to R your X will become constant here okay so you say $X \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr}$ right here so now I divide both sides by X into our capital X into R so this will be u_m by α I bring it here $\frac{1}{X}$ you have B now this since X is a function of only X all this partial differentials will get converted into ordinary differentials my X is a function of only X and capital R is a function.

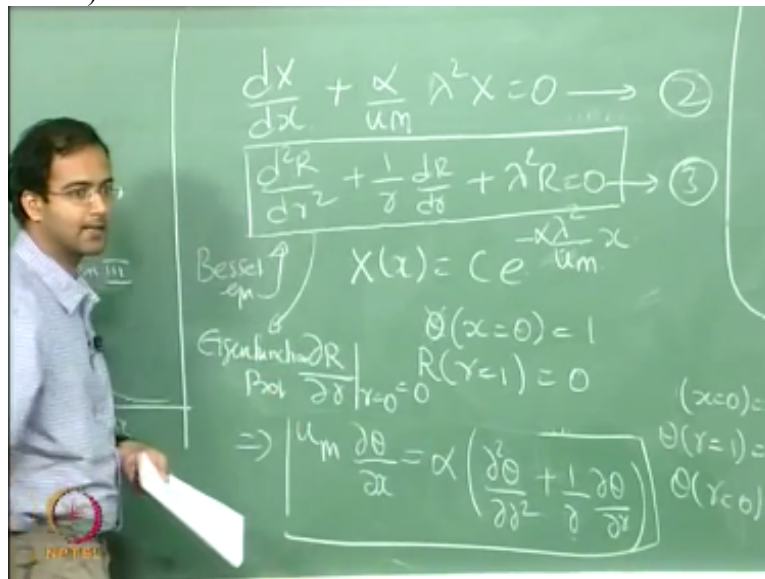
Of only R so all these partial differentials here should be written in terms of the normal differential so this will be $\frac{dx}{dx}$ on this side it will be $\frac{1}{r} \frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr}$ here

so on this side I have everything as a function of X on this side I have everything as a function of R so on these two have to be equal so this can be equal only if they are equal to a constant so I assume that this constant is negative minus λ^2 so why I put this is that if you look at the solution for X.

Then it will be an exponentially decaying function if I put a positive constant there it will be an exponentially increasing function and what is this X standing for here that is basically variation of θ with respect to X okay so if you look at the variation of σ u k you can see that as you start from this point your T will be = T1 okay and then that will be 1 so your θ will be 1 so if you plot your variation of θ with respect to X so it starts from some value 1 and from there it has to increase or decrease it has to decrease till it reaches the value of T wall.

Where it becomes 0 so that has to happen exponentially right so this can be possible only if your constant is negative here so this has to be an exponentially decaying function only then your θ will behave in that way ok so this λ square what I am using these are called as Eigen values okay this is the principle solution to the problem these are called as eigenvalues so therefore now I have two ordinary differential equations.

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So one is $\frac{dX}{dx} + \frac{\alpha}{u_m} \lambda^2 X = 0$ this I call as equation number two and $\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \lambda^2 R = 0$ so I can equate this separately to the constant this separately to the constant I have two equations to ODE so ultimately what I have done converted

the partial differential equation into two ODEs now ODE is I can always solve them you know I can at least use a simple numerical technique to solve the boundaries now what is the solution.

To the ODE number two it is an exponential solution okay so it will be X of X should be some constant into e power $-\lambda^2$ by $\alpha \lambda^2 / u$ so this is my solution with respect to X okay the other solution into X sorry okay the other solution is a solution to this ODE so how do I solve this so that is the question anybody can recognize what kind of an equation is this I think in heat transfer have taught so this is called as a Bessel equation.

We so if for people who do not know what Bessel equations are I will probably give a brief overview tomorrow about Bessel equation on the solution general solution to the Bessel equation okay generally for problems in cylindrical coordinate systems okay when you try to do a separation of variables you end up converting that into a Bessel equation and what are the boundary conditions to solve this so how many boundary conditions you would need for this.

How many boundary conditions for this for x_1 this is a first order now this is a second order ODE II you need two boundary conditions for R so what is the boundary condition for X at $X = 0$ should be do we have any concrete boundary condition for X now we do not have we have only boundary condition for σ so we know that θ at $X = 0$ should be 1 so you can write this as X into our X at $X = 0$ and τ should be 1 but from there we cannot deduce anything for X so we will hold on and apply this boundary condition in the end okay.

So now if you look at this problem we can apply this boundary conditions directly because they are homogeneous boundary conditions okay so already we know that your θ equal to 1 at $R = 1$ $\sigma = 0$ and it should be finite at $R = 0$ so correspondingly if you write σ as R into X you can say that R at $R = 1$ should be 0 and your dr / dr at $r = 0$ should be 0 or your solution for our should be finite okay so this is a ODE with two homogeneous boundary conditions and this becomes.

What is called the Eigen function problem so how do you identify which direction is the Eigen function problem you identify the direction where you have two homogeneous boundary conditions okay and that gives you the direction where you find the eigenfunction problem why do you need the eigenfunction problem because you apply the boundary conditions you find the roots of the eigenvalues so that that is why this is this equation is the eigenfunction.

Equation so once you know the Eigen function I can values then the our solution to our is the Eigen function okay so and then you can express your solution as a product of your Eigen function into X okay so this we will I will explain give you a brief introduction to Bessel functions and then I will give you the solution and then I will show you how to combine these two solutions okay so any questions on this so far so for people who have not had any course on partial differential I suggest you can you try to learn up at least the separation of variables this is a very fundamental thing the rest of the classes will be working only with the separation of variables you.

**Shooting method for fully developed heat transfer
And thermal entry length problem**

End of lecture 28

**Next: Thermal entry problem with plug velocity profile:
Geertz problem**

Online Video Editing / Post Production

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