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National programme on technology enhanced learning
Video lectures on convictive heat transfer

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Lecture 27
Fully developed laminar internal flow and heat transfer

So good morning all of you so let us today to continue on the discussion with respect to the fully developed both hydro dynamically and thermally.

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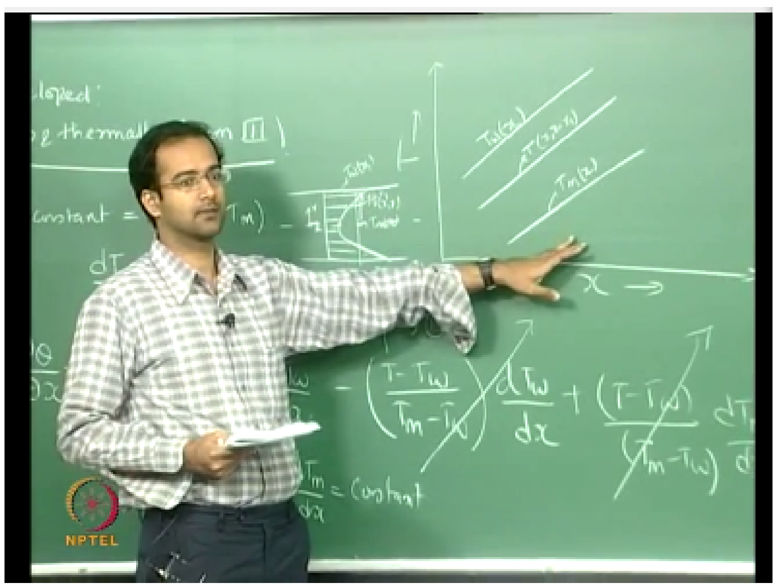

Fully developed region so I denote this as Region three and we solved for the velocity profile in the hydro dynamically fully developed region we got a parabolic velocity profile and we also started the solution to the heat transfer problem the first case that we to okay was a constant w flux boundary condition so Q w double prime is constant and for this particular case we have also shown that if your Q w is constant that means your DT w / DX should be = DT m / DX this comes because of the fact that this is equal to H into T w - TM.

Okay also we have expanded the fact for thermally fully developed flows you have a non dimensional temperature gradient variation axially is 0, so from which we had obtained an expression for DT / DX in terms of DT w / DX and DTM / DX that was DT w / DX - P - T w / DM - T w into DT w / DX + T - T w / TM - T w D TM DT mean / P X correct so you can note the partial and the standard derivative that I am using here so this is a function of one DX this is a function of only X whereas temperature is a function of both R and X therefore I use a partial derivative for DT / DX okay.

So now if you substitute this result okay if you call this as number 2 and this is number 1 substituting the consequence 2 into 1 you cancel of this terms because they are identical and therefore you come to the conclusion that my DT / DX should be = DT all / DX which in turn is = VT m / DX and we have seen that this can be possible only if these are = a constant because this is a function of both R and X but these are only a function of X so this can be holding true the slopes can be the same if this is = a constant.

And therefore if you plot the variation of temperature axially so you can visualize for example a temperature profile so where you have tea w X here you can calculate T mean X and of course this is your T X , R so if you plot your temperature somewhere here local temperature along the axial direction so you can say that this follows a profile linear profile like this maybe somewhere at this radial location okay so this is the center and the mean profile will be slightly lower than the temperature of the at this particular radial location.

So you can say that this is your so this is at some particular radial location we can say R = R 1 for example okay that is this particular location where I am plotting this R = and at the w of course that will be the highest temperature amongst all the three and that will also vary linearly like this okay so this is the characteristic of the temperature variation as far as the constant heat flux boundary condition is concerned so all the three variations are identical they are just parallel to each other.

But they keep vary with the axial position so that is the characteristic nature of the fully thermally fully developed of course they have to be hydro dynamically fully developed also now having seen this we wanted to calculate actually the temperature profile itself so if you substitute the fact that your DT / DX is actually a constant that is = some DT w or 3 divided / DX or ETM / DX into the energy equation okay so therefore it becomes much, much easier to integrate the energy equation directly so you have.

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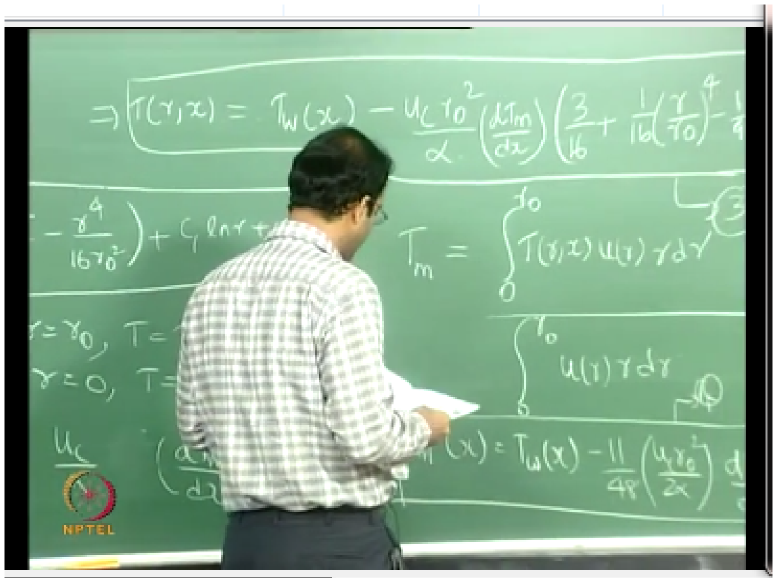

U / α now the DT / DX can be replaced as some DTM / D-X now which is a constant = T^2 / DX^2+ okay so we have neglected the axial conduction in comparison to the magnitude of the radial conduction so therefore we will retain only the radial conduction terms R DT / D R okay so now since the left hand side is a constant we can just integrate it out the same way we integrated the velocity profile where you are DP / DX is a constant so this is a much simpler way of looking at the solution.

So if you integrate it twice I am not going to spend time integrating I will give you the final expression 1 / α DT / DX which is actually a constant so I can either use DT / DX or DT m / DX they just mean the same and they are they are just constants so you have you see R^2 / 4 - R power 4 / 16 R 0 to the power 2 + c1 learn of R + c2 so this is what I get somewhat similar to the velocity profile where you had a pressure gradient in term in the place of this DT / DX you had DP / DX there and you had a similar variation okay.

So now we have to find the constants C 1 and C 2 and in order to do that we have to apply boundary conditions once again two boundary conditions are required okay so what are the two boundary conditions okay so at R = R 0 let us say the temperature is = some tea w of course which is a function of X okay so if you want to apply at a particular axial location so that that is a particular w temperature which is a boundary condition and at R equal to 0 the temperature has to be finite.

Now if you look at this directly similar to the velocity profile if for a finite temperature at the center your C 1 should directly be 0 because this term otherwise goes to infinity okay so therefore we directly eliminated C 1 so we can use the other boundary condition R = R 0 T is = T w to calculate the second constant C 2 so if you substitute you can determine the constant it will come out as so at R = 0 or = R 0 T will become T w this will be T w of X - this entire thing goes to the left hand side this is you see / α into so now this is evaluated at R 0 so this is R 0^2 / 4 - R 0^2 / 16 okay.

So if you simplify it comes out as so you can take 1 R 0 square into DT M / DX okay I can maintain this is DT M / DX into 3 R 0^2 / 16 I think and you just check if this is correct this is T w - you see / α R 0^2 this R 0^2 cannot come here I think this are not square should not come here right so it is just three are not square / 16 okay so this is your C 2 and therefore you can substitute for C 1 and C 2 and write your final expression if C 1 is anyway 0 so therefore T of R comma X so you can substitute for C 2 and you see, you see / α DT m / DX so this is a common term here and here.

So that can be just taken out as a common term so of course you have your T w of X which can be written as this - you see in - you can also pull out R 0^2 as common by α so you see you can take out 1 R 0^2 so that everything can be expressed as non-dimensional form and R 0^2 also is here into dt m / DX that is also common so this will be left with the 3 / 16 which is this + you have 1 / 16 so you have already R 0^2 taken out so this will be 1 / 16 R / R 0 the whole power 4 okay and this remaining term will be R / R 0 1 / 4 this will be - 1 / 4 R / R 0 the whole power 2 okay.

So this will be a final expression for temperature variation at this let us call this as equation number three so therefore you can see at this state it appeared like T is a function of only arc because your DT M / DX was constant but once you calculated this constant C 2 now it is a function of X through t1 therefore the final expression for T is a function of both R and X all right okay.

So once you got the expression for temperature we will go ahead and calculate the expression for the mean temperature which we define or the bulk temperature or the mixing Cup temperature so how is the mean temperature defined so we now know the variation of P with respect to both X and R now we should get an expression for the mean temperature varying with X okay and of course the w temperature R T - TM - T w okay the difference between them.

So how do we define the mean temperature yeah so it is basically a mass weighted average of temperature and how do you mass wait 0 to R 0 T into so what is the mass P into a into u okay in the numerator and denominator the P can cancel off so you have you and of course U is a function of only R okay and what is the differential area 2 π RDR 2 π cancels in the numerator and denominator you have RD R in the denominator it has to be divided / the mass flow rate okay so 0 to R 0 you have U of R into our dear okay so therefore you have the velocity profile I will just write down the velocity profile for you again.

So you are U of R is 1 / nu okay - 1 / 4 nu in through DP / DX R 0 square into 1 - r / r 0 whole square so this is your velocity profile and this is your temperature profile right here so you have to plug both these inside and you know so this DTM / DX is a constant DP / DX is a constant okay so therefore you will have to integrate this with respect to R you have to multiply all the terms with respect to R and then you have to integrate it is a little bit lengthy integration .

Which I am not doing you can also if you find it too difficult you can also use Mathematic and try to do the integral but finally if you do that the expression for PM comes out as T w of X - 11 / 48 into you see R 0^2 / 2 α into DT m / DX okay so this let us call this as number 4 this is the variation of mean temperature with respect to X okay so therefore you can see your t TM - T w is going to be a constant because d TM / DX is a constant okay.

So the difference between the two TM - T bar has to be a constant all right so once you got your TM now we can take calculate the non-dimensional temperature so your non dimensional temperature is defines as.

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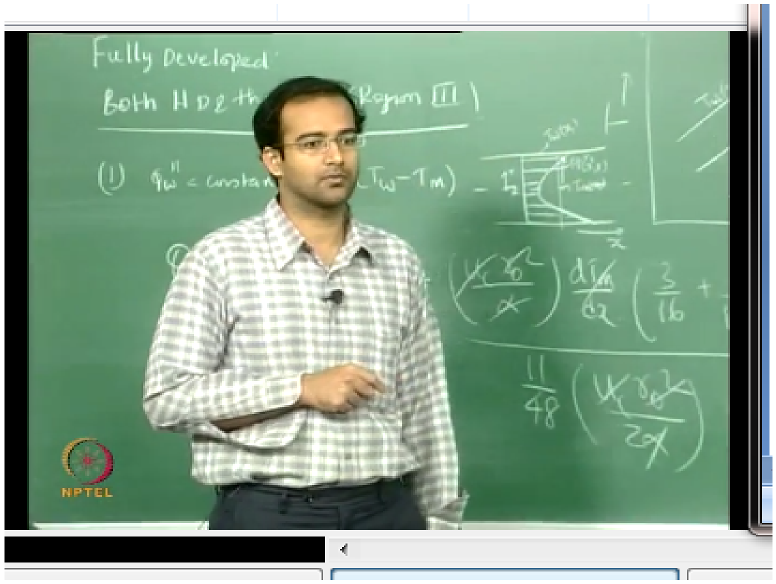

T - T w of X / T m of X - T w of X okay so now you have of course T - T w an expression for T - T w here from the temperature profile and also an expression for T M - T w here okay so if you just substitute those expressions so T - T w will be so you have your you see R 0 square / α into DT m / DX into 3 / 16 + 1 / 16 R / R 0 to the power 4 - 1 / 4 R / R 0^{2} that is basically t - TW there is a - sign but in the numerator and denominator the - sign cancels of similarly T M - T w will be - of that I cancel the - sign this will be 11 / 48 into you see R 0 square / 2 α into DT m / DX o dt m / DX cancels your you see R 0^{2} cancels α cancels out okay.

So this will give you an expression as 96 / 11 because you have 48 into 2 so that goes up 96 / 11 into this entire factor here 3 / 16 + 1 / 16 R / R 0 to the power 4 - 1 / 4 R / R 0^{2} so this is your final expression for θ now once you have calculated your non-dimensional profile now you can see the non-dimensional profile is not a function of X now okay the way we defined your θ in the thermally fully developed region θ is supposed to be independent of X so and that is what comes out correct okay.

So this proves the fact that the assumption what we did of course we use that assumption also but that correlates with what we find finally for θ so this is only a function of R now you can use this definition of θ and calculate your heat transfer coefficient we had shown that in the case of thermally and hydroynamically fully developed flows the heat transfer coefficient has to be a constant now we have to determine.

What is that constant value so for the constant heat flux case we have arrived at the particular profile for non-dimensional temperature so I will probably give you about 5 minutes you can from this step calculate what is the heat transfer coefficient.

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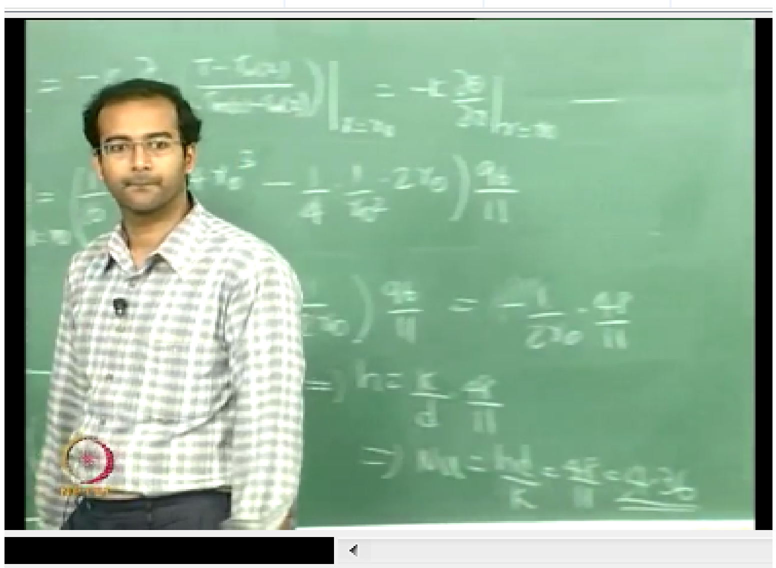

I suggest all of you to do it yourself and check okay so therefore finally you get your heat transfer coefficient as a constant value for a given diameter K / D into 48 / 11 therefore you can define an nusselt number okay for internal flows based on the diameter of the duct so it is not a local nusselt number anymore and if you define like that you get a constant value for point three six so for internal laminar flows both hydroynamically and fully developed and for a constant heat flux boundary condition this is your constant value of nusselt number 4.36 six okay.

So therefore you see that internal flows are quite different there is no local variation once you reach a completely fully developed condition and after that the heat transfer coefficient is becoming a constant value so even in your external flows one of the quiz problems had given you the poorest flat plate where your suction is continuously happening on the surface and you are asked to prove at large values of X your boundary layer thickness is becoming a constant.

And therefore as a consequence of this the heat transfer coefficient also becomes a constant so even in external flows if you maintain a suction boundary condition you can show that your heat transfer coefficient can become a constant somewhere down the length in the case of internal flows that is true if you have a fully developed flow region okay and now so this is a straightforward case as far as relatively straightforward when you look at in terms of the simplification that we have done for the constant heat flux now let us look at the other boundary condition.

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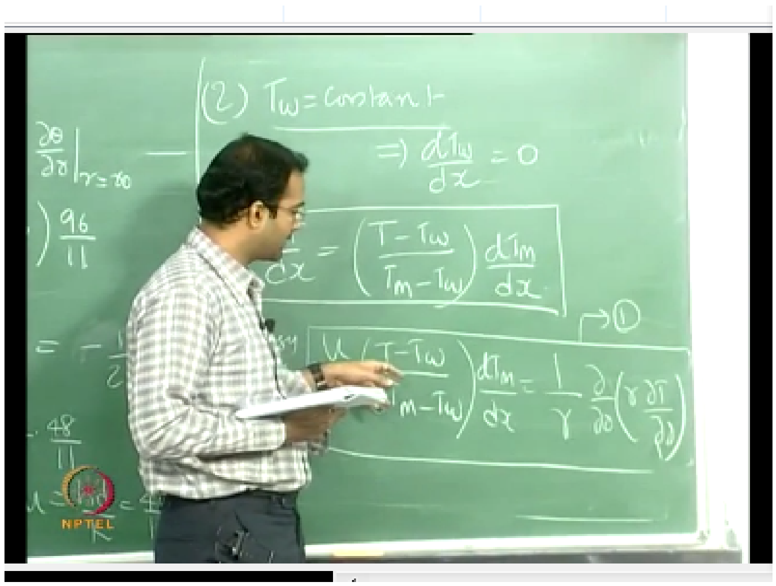

which is your what constant w temperature so only these two boundary conditions are the primary boundary condition that we will see one is additional boundary condition the other is in hyper boundary condition okay now when you look at constant w temperature what this means you are DT w / DX is 0 correct so now let us go back to our equation for DT / DX and substitute this expression what do you get so if you substitute DT w / DX is 0 in that expression yeah correct this will be partial derivative you are right.

So I want all of you to also participate you how to tell me if you apply this condition DT / DT w / DX is 0 what will be the resulting expression for DT / DX yeah t - T w / TM - T w into DT m / DX okay so the other first two terms get cancelled off you have only this last term so now this is your approximation for DT / DX right so of course your mean temperature is only a function of X but this T here this is the problem this is a function of both X and R .

So now if you substitute this into the energy equation now what happens so you in your energy equation on the left hand side you have your DT / DX so if you substitute in place of that you have now you / α into DT / DX I am going to substitute this particular expression here t - T w /

T - T w into DT m / DX so this is = of course your 1 / RD / DR into R DT here so now this is what you need to solve okay let me let me call this as another equation number one okay.

So this is the energy equation which you need to solve to find the temperature profile now the problem with this now earlier when you had constant heat flux the left-hand side was completely a constant okay you had only DT m / DX which again was a constant therefore you could directly integrate it out now you have t - T w / TM - T w now T is actually not a constant now so this entire expression now has cannot be solved analytically the way that we did last case.

So therefore we have to do it numerically and also iteratively so one way of doing it is you can guess some value of temperature profile preferably from the earlier case the constant heat flux case and substitute as a first guess and then you can integrate it along the radial direction okay and again get a new temperature profile and again keep putting it on the left hand side and keep doing it until you reach a convert solution okay so the other smarter way of doing it is again go back to a shooting technique.

So we should try to reduce this equation to another simpler form where we can apply shooting technique and solve that equation so that is what we are going to do now okay what I am going to do now first let me before going into the shooting technique we have to prepare we have to get appropriate expressions for DTM / DX so to do that we will do a small energy balance.

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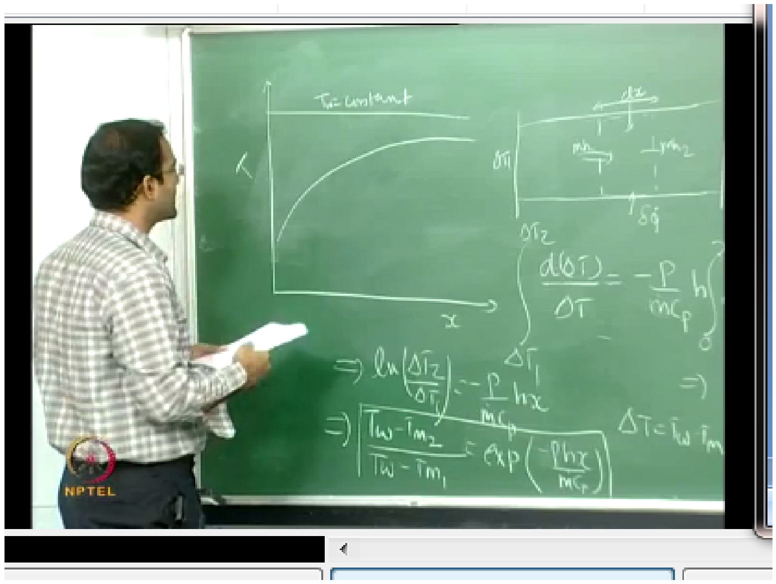

So let us look at the profiles of temperature T as a function of X of course your w temperature is now a constant okay now you want to know how the mean temperature is varying earlier w temperature mean temperature and temperature at any location were just straight lines with equal slopes so now it is not the case right so there from the Newton's law of cooling you directly showed that the slopes have to be the same but here it is not true.

So therefore in this case the w temperature is of course a constant but how the local temperature how the mean temperature vary is not clear so to determine the variation what we will do is just take a simple energy balance so you take a duct so you take a control volume so this is your heat transfer δ Q differential amount of heat transfer for this differential control volume and you have some enthalpy coming in and enthalpy which is leaving of course there is a mass flow associated with this enthalpy.

So therefore you can apply energy balance and say my ∇ Q dot is = m dot CP into DT M okay so that is ∇ G ∇ Q is = m dot into δ H D H and D H corresponds to CP into DT M because this enthalpy is the mean enthalpy or bulk mean or mean enthalpy that I am talking about so for the given heat transfer this is the corresponding change in the enthalpy of the fluid and I can relate my differential amount of heat input to the change in the mean temperature corresponding to the change in the enthalpy.

So this is the starting point of the energy balance so I can just divide everywhere / DX so this will be now d TM / DX will be = now this is your heat transfer rate okay so this can be expressed in terms of heat flux you can write this as Q double prime which is in what per meter square into if you assume that the control volume has a differential length DX into DX into parameter right so this will be your surface area where you are adding Heat alright so this will be divided / m . C P.

And of course you are dividing / DX so DX cancels and now your Q double prime you can apply Newton's law of cooling and write this as H into T w - T me okay therefore you have dt m / DX is = P / m dot CP p is your perimeter into H into T 1 - T means so this is this is like expression coming out of the simple energy balance that you are doing so we can what we can now do since your t w is a constant we can write this as d / DX of T w - TM correct T w is a constant so you can just introduce d / DX of T w - TF okay.

Now there will be a - sign so we can put a - sign on this side also P / m dot CP into H into T w - TM is already here okay so let us call T w - TM as some δ T okay so with this you can integrate this expression so this will be d δ T / δ T with this you can integrate this from δ T one to δ T two so what it means if you have a duct long duct at the inlet that will be your δ T one that that is the difference between your w temperature and the mean temperature and at the exit somewhere that is your δ T two okay.

So once I have an equation for this temperature difference T w - T mean I can just simply integrate it from the inlet to the outlet so that this should be = - P / m dot CP into h DX so this I integrate from 0 to your entire length or wherever whichever location there you want to find the appropriate temperature difference okay so this will give me Lon of δ T 2 / δ T 1 = - P / m dot CP H into X okay.

So our mighty w - T mean at some location 2 / T w - t mean 1 will be = exponential of - P H X / m dot CP so this is my expression which tells me how the mean temperature varies because my w temperature is constant if you know the mean temperature at some inlet from that you can calculate what is the mean temperature at some location X using this expression so if you plot this expression okay.

If you start with some mean temperature at the inlet you will find that the mean temperature keeps varying in a logarithmic fashion like this and it asymptotically go and meet this T 1 when your X goes to infinity so this becomes 0 and therefore your T mean will become T 1 where your X goes to infinity okay so this is the variation of your TM okay the mean temperature of the fluid okay.

Now having known this and we have this particular expression here let me let us call this as the expression number 4 okay I started with 1 okay let me call this a stool so we will use this expression now simplify for DT m / DX because why we are doing this is we have a DT m / DX term in this equation so we have to simplify this a little bit so for that we are using the energy balance now we will simplify it also we should know how the mean temperature profile varies so for that we can use that equation and integrate it out.

So now from 2 so you can write your.

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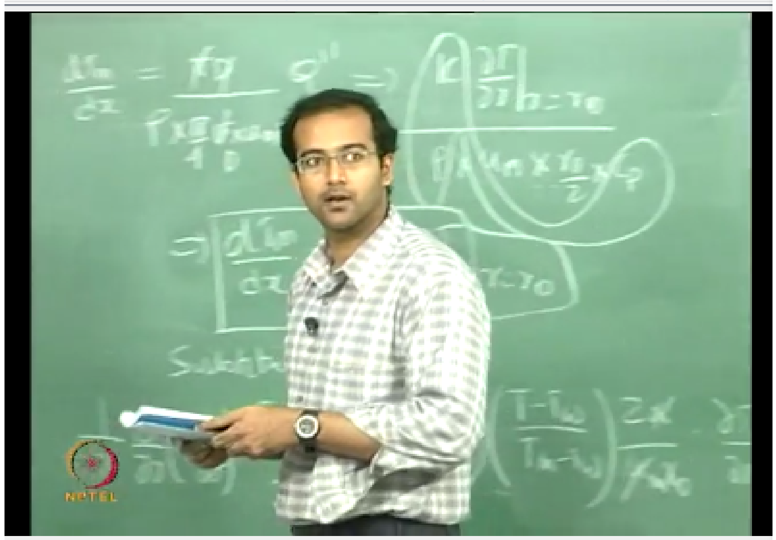

DTM / DX as okay for the case of circular duct your perimeter is 2 Π R so Π D divided / your mass flow rate will be P into Π / 4 D^2 P AV so the velocity is mean velocity into CP into H TM T w - T mean I can replace this / your heat flux Q double prime okay so this cancels Π cancels here V cancels so this will give me your Q double prime will be K DT / D R at R = R 0 this will be divided / your P into u M into D will be R 0 / 2 and I can write this as R 0 / 2 into CP so this will be 2 R 0 yes so that is correct.

So now I can Club this as K / P CP K / P CP is α so this can be written as DT M / DX is actually α times DT / D R at R = R 0 divided / this will be 2 times so UM R 0 so this is my expression for DT m / DX in terms of DT / D R at R = R 0 this a little bit of mathematical manipulation nothing more so as far as circular duct is concerned you are just writing in terms of your diameter and of course your mass flow rate little bit simplification and you get a relationship between DT m / DX and DT / D R at R = this is a gradient of temperature at the w.

So this can be substituted into equation 1 for DTM / DX so substituting into one so of course on the right hand side you have the same on the left hand side you have so I am going to write my I am going to substitute my velocity profile fully developed velocity profile for you so that will be twice UM into 1 - R / R 0^2 / α and DTM / DX is substituted from here so which will be now this is t - T w / T M - T w DT m / DX will be 2 α / UM R 0 into DT / D R at R = R 0 so this is equal to the right hand side which is 1 / RD / DR of R ET / here okay.

So here UM you have cancels α cancels here okay so therefore your final expression which you have to solve will be 1 / RD / DR of R VT / DR that should be = 4 / R 0 2 into 2 4 divided / R 0 into 1 - R / R 0^2 1 / R / R 0^2 into T - T w / T mean – T w into DT / DR at R = R0 okay so let us call this as my equation number 3 so this is the equation finally which comes to the form which I want to express as a OD in order to apply my shooting technique okay.

So you please stop and ask me if you have any questions any doubts anywhere so these are all just mathematical manipulations I am going a little bit fast here so now.

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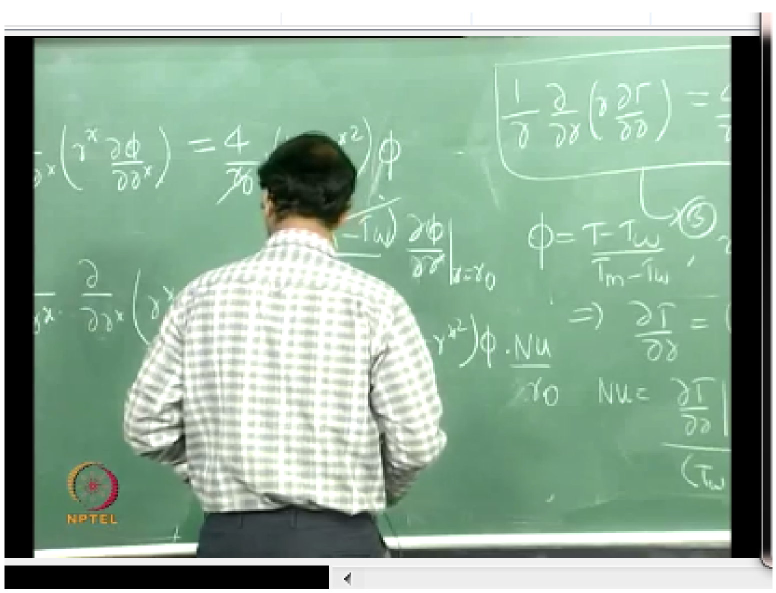

I am going to introduce a non-dimensional temperature θ or fee I am using the fee here to represent the non-dimensional temperature I can write it as t - T w / T mean - T w and a non-dimensional radial coordinate R star which is nothing but R / R 0 so this I am going to substitute into my equation number 3 and rewrite the entire equation in terms of non-dimensional variables V and R star okay .

So if I do that so from this expression I get that my DT / DR is = PM - T w into D5 / DR so i can substitute for D / DR as TM - T w into D Π / DR TM - T w cancels here okay and I can also define – nusselt number as DT / D R at R = R 0 divided / T w - PM into diameter the diameter is nothing but twice R 0 so if I use these expressions and substitute into this governing equation three I request all of you to do that and tell me what will be the final non-dimensional equation for fee.

In terms of V and R star so with that we will stop for today so what do I get so I have now I can write this one so 1 / R star D / D R star R star now for DT / DR i can substitute as team TM - T w TM - T w is a function of only X so that can be taken outside the derivative right so have TM - PR into D Π / D R star this should be = 4 / r 0 into 1 - this is R / R 0 is our star so this I can write as R star^2 T - T w / TM - T w this is fee so this will be x fee and DT / D R is nothing but again TM - T w into D Π / D R .

So this I can write as TM - TR T w into D Π / D R at R = R 0 so now so I have RR this cancels here so I have R 0 into R 0 R 0^2 so I have divided / R 0^2 here and here I can have again R0 and I can write this as R star so R 0^2 R 0^2 will cancel TM - TR TM - TR cancels so therefore I am left with the final non-dimensional form V / V R star into R star D Π / D R star which is equal to now.

I have got the expression for nusselt number as - 2 R 0 into D Π / so I can replace d Π / DR / r not as nusselt number / 2 r 0 so this will be a - 4 into 1 - our star square into fee into this can be written as nusselt number / two are not so there is a factor of our knot which is somewhere I have to cancel so this is this is are not are not here cancels this is are not square and on this side okay I think yeah so this is anyway I can write this as the D5 / DR star you are right so this is already 5 / DR star .

So therefore this will be in terms of nusselt number okay so this is my final expression you see this has reduced to an OD no I will call this as number 4 so this entire expression is now function of fee now remember fee is a function of only R so earlier I had a partial differential equation now I had reduced that somehow / any means of manipulation and non dimensionalization to an OD which is a function of R and this is of course a higher order OD I can break it up into two first order OD and you shooting method so we will see that in the next class okay I will just give a summary of the shooting method which you are already used to solve this equation okay.

Fully developed laminar internal flow and heat transfer
End of lecture 27
Next: Shooting method for fully developed heat transfer and thermal entry length problem

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