Indian Institute of Technology Madras
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Video Lectures on
Convective Heat Transfer

Dr. Arvind Pattamatta

Department of Mechanical Engineering
Indian Institute of Technology Madras

Lecture 25

Laminar internal forced convection – fundamentals

So good morning all of you so now that quiz 1 is over I am sure that many of you are feeling a little bit familiar to whatever we have covered until now so the person on external boundary layer flows external laminar boundary layers so from today onwards we will start looking at internal flows ok internal laminar flows internal flows are very important as far as no engineering heat transfer is concerned because most of the heat exchanges that you are looking at the tube in tube heat exchanger Sheldon tube heat exchangers so they are all having internal flows and how do you design these heat exchanges once you know the corresponding relationship between the nusselt number and the flow condition such as given by the Reynolds number as well as the properties given by a prantle number.

So you have to develop suitable correlations between linking the heat transfer coefficient and the flow and the properties in some manner so we will look at developing such kind of correlations for internal flows so far we had developed these correlations for external flows the technique for internal flows are a little bit different not exactly like the way we are going to do external flows where we had a clear boundary layer kind of a flow pattern so we were giving boundary conditions like at y = 0 you have whatever temperature and away from the wall you are giving free stream condition that is y going to ∞ far away you are giving the free stream condition but in the internal flows there is no classical boundary layer.

Kind of a pattern that you can follow especially when the flow is developing fully developed so there in those regions you cannot look at regions which are far away from the boundary layer because in the case of fully download flow everything is boundary layer itself ok the boundary layers from the body bottom wall the top wall of a channel can merge and they can everything will be a viscous region and the same way in a duct you can have boundary layer growing simultaneously from all the walls of the duct and you cannot really go and identify a point where you can look at potential application of potential flow.

And things like that so therefore the solution procedure for the laminar internal flows are slightly different from the external flows so therefore we will start looking at some basic aspects of internal flows before going x the mathematical theory of how to solve the laminar internal flow heat transfer I think most of you are familiar with the basics of internal flows from your fluid mechanics classes and your earlier heat transfer class so I am going to give an overview very brief overview and we will look at basic fully developed case first okay where we can get the solution for velocity profiles very easily okay and then we will slowly gradually go to the case where we have heat transfer so we will look at different regions again so first we will start with the simplest case that is both hydro dynamically and thermally fully developed case okay so before doing that let us let me give you some kind of an overview to laminar internal force convection.

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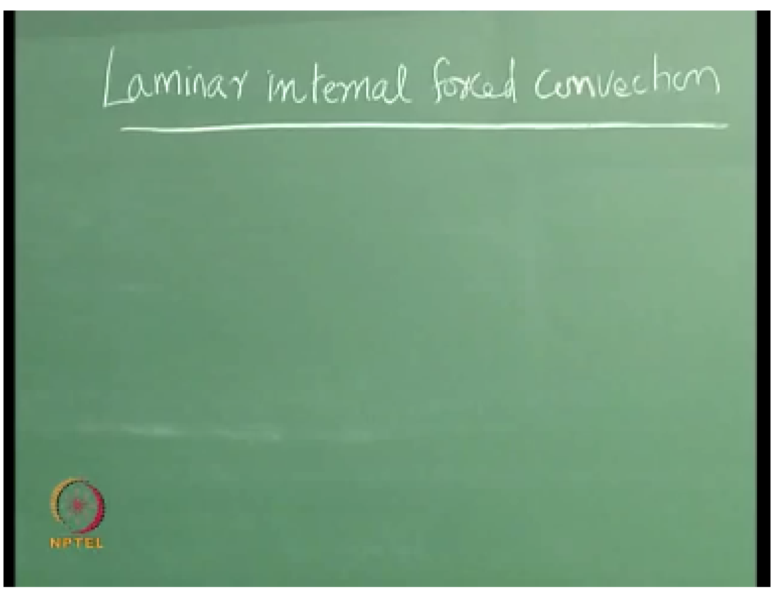

Okay so classically I would like to illustrate internal flow by means of flow between two parallel plates or flow you can take a sectional view of a duct and then look at how the flow pattern is so here when the flow is entering the inlet of the duct so you have a uniform velocity across the cross section and then you see a boundary layer growth from say the two walls of the duct and this growth keeps going on till a particular location.

Which is at the center of the that the duct diameter okay so exactly at the center the two boundary layers will merge and after that they look the region whatever you are looking here it is completely dominated by specifics so whereas at the inlet you have a very small region close to the plates where your viscous effects are important outside still the potential in viscid flow theory is valid whereas once the two boundary layers merge in this particular region everything is viscous dominated okay now if you look at how the profiles are drawn somewhere at the inlet you can have a profile something like this now as you keep going downstream the gradient becomes better and better.

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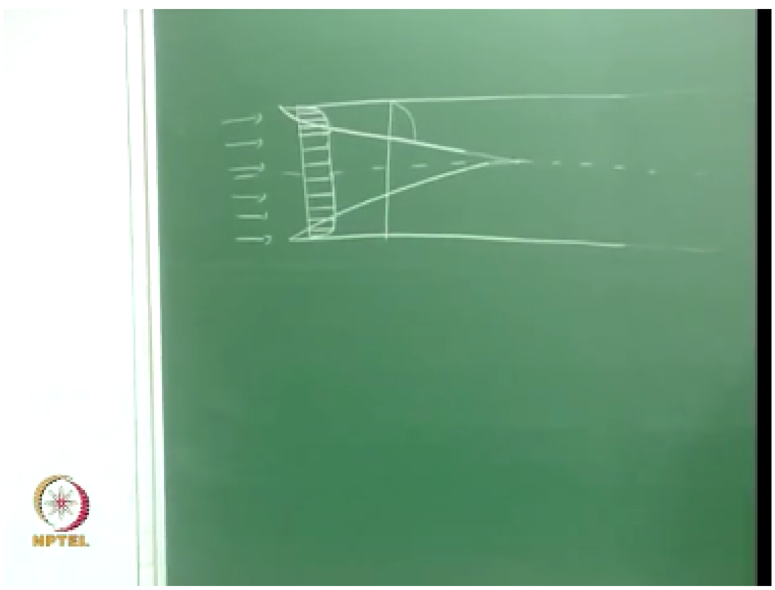

So as you keep going downstream the gradient becomes lesser and lesser here you have very high velocity gradients at the wall and slowly it get is smoother and smoother now once you reach this particular zone here both the boundary layers merge and the velocity profile becomes what is called a velocity so therefore looking at the different regions here you can classify the flow based on whether you are looking from the entrance till the region where both the plates meet okay from the entrance of the duct so this locus this region this length is called the hydrodynamic entrance length Ln.

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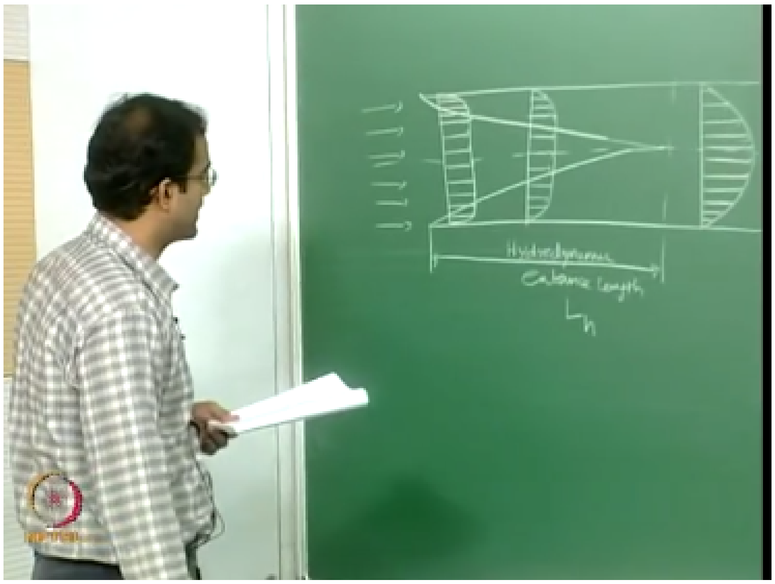

Okay so you can use the notation L subscript H to indicate that this is a hydrodynamic condensed length we'll also have a similar Renton length for temperature profiles so after the two boundary layers meet now the velocity profile looks similar wherever you go downstream so it all becomes parabolic velocity profile and this region is called fully developed region so therefore you can classify the different regions of laminar internal flow so now you can look at this region as entrance develop region or entry develop region or hydro dynamically develoπng region okay so therefore here you can see the boundary layers thickness is a function of position now once it merges then everywhere it is viscous dominated and the velocity profile is invariant of the axial location alright and this becomes completely fully developed.

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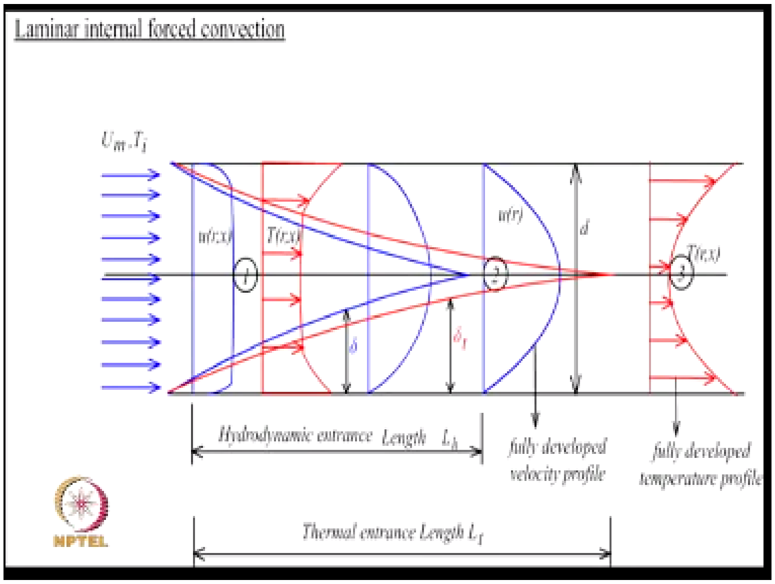

You so this is the characteristic of internal flows and in order to really understand how to calculate the length the hydrodynamic entrance length there are some several hand waving correlations there is no exact rigorous theory to determine the entrance length okay strictly speaking you have to solve the equation and understand where the velocity profile becomes parabolic and becomes invariant okay so just to give an empirical control for laminar flows the non-dimensional entry length so this is non dimensionalzed by the diameter if you have a duct or if you have a channel so this is the separation between the two plates of the channel okay so LH by D is roughly 0.05 times re based on the diameter of the duct okay so here the Reynolds number for internal flows is defined based on so how do we what kind of characteristic velocity that we have to use that is the question?

So, the kind of an average because here you see even if you look at a fully developed case you have a profile which is a function of Y okay so you do not have a fixed velocity value like in a case of external flow like a free stream velocity therefore so you can use what is called a mean velocity which we will define shortly okay so this is your mean velocity so if you replace this parabolic velocity profile with a constant profile everywhere across the cross section so this is referred to as the mean velocity so naturally how you have to define the mean velocity volume flow rate or the mass flow rate has to be constant okay so therefore the mass flow rate defined by this let us say P x UM x the area for a circular duct area is Π x R 0 square okay where are not is the dam with radius of the duct so this should be = the flow rate obtained with this particular profile parabolic profile so that is P x u x now if you if you say area is equal to Π R 0 square my DA will be = 2 Π R 0 we are not okay so I can average this integrate this over the entire cross sectional area okay so this will be 2 Π R 0 D or not correct so I have to integrate from so if I take the coordinate from the center my radius right from the center this is 0 and this is odd not okay so I integrate my profile across the entire cross sectional area so that will be P u x R 0 x dr not so this will give me the mass flow rate and this mass flow rate given by this profile should be identical to the mass flow rate if i replace this parabolic profile with a uniform velocity okay so from this I can define my mean velocity or the bulk velocity okay my Π cancels so this is if you look at incompressible flows my density is also constant so that is therefore this will be 2 by r 0 square so here my odd not so this is basically my R this is Π R square okay so this should be 2 by R 0 square integral 0 to R 0 u x so this will be the definition of my mean velocity or what I call as bulk velocity in internal flows.

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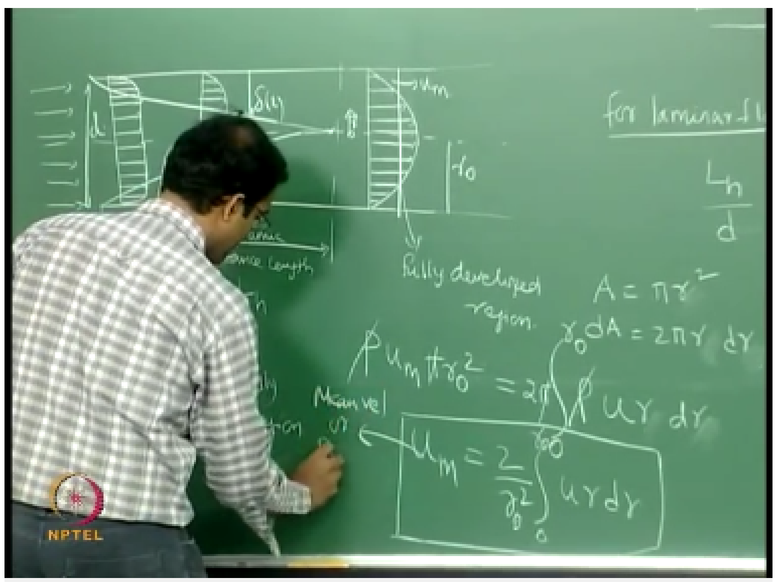

Okay so this will be used as a characteristic velocity to define my Reynolds number and it is also based on the diameter of the duct ÷ the kinematic viscosity okay is it clear okay so this is the area of at any location R okay so if you look at any location R so this will be the cross-sectional area Π R square okay so this is what I am using it so if I look at any differential area that will be 2 π r dr so i have to integrate it over the entire cross sectional area so that is why that is why I am doing this integral right.

Here okay now based on the Reynolds number you can again classify now when I say that for laminar flows this is the relation to calculate my entry length so how do I first return mine whether the flow is laminar or turbulent so once again I use the definition of Reynolds number to check if it satisfies a certain cutoff for a critical Reynolds number okay so far typically that flows our flow between two plates the critical Reynolds number for transition to turbulence will be approximately 2300 okay this is a once again an approximate value you know it need not exactly become turbulent at 2300 in fact if you maintain the walls of this channel to be extremely smooth it can remain laminar even as high as 7000 or 8000 okay so if the turbulent intensity at the inlet is so small that it cannot trigger transition to turbulence and if the walls of the channels are very smooth it can go as high as the critical value can be as high as 8,000 or 9,000 okay so if you check your RD is < Re critical so this is an approximate thumb rule for classifying the flow is laminar okay and otherwise you can look at either in a transition regime or not or Blount is it.

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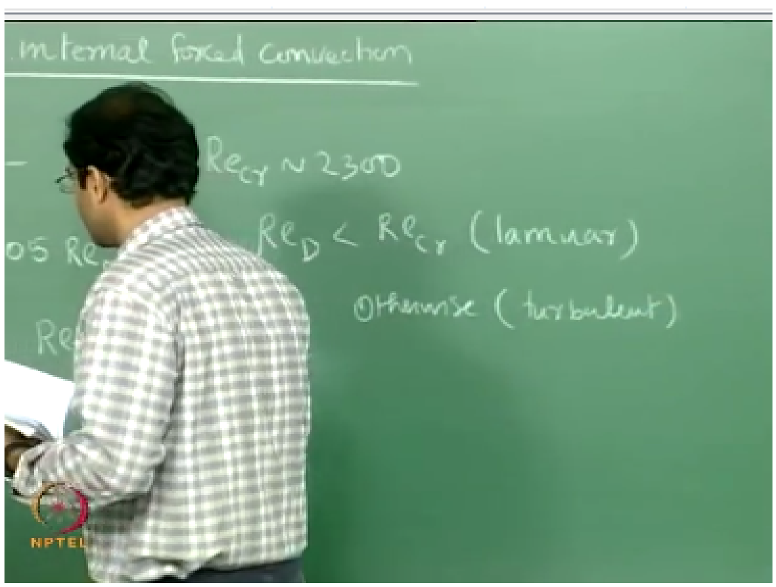

Okay as far as laminar flows are concerned this is the thumb rule for calculating the entry length all right and for turbulent flows interestingly what happens the entry length ceases to be a function of Reynolds number okay so the entry length non-dimensionalize comes out to be approximately thumb 10 okay so imagine that you have a Reynolds number of say 3000 okay if you classify that as a laminar flow thinking that the flow is still streamlined enough to be classified as laminar so there what should be the value of helix Brady 15 okay now in the case of turbulent flows if suppose the 3,000 was now turbulent.

∇ s by D would be 10 so now you can infer that the entry length entrance length for turbulent flows is actually smaller than the entrance length of laminar flows okay so the reason is the turbulence promotes lot of intense mixing or diffusion of flow so the mixing takes place due to the gradients along the Y direction so there will be intense mixing and therefore the profile can reach a fully developed state much earlier in the turbulent case than in the laminar case okay so therefore the entire lengths are much smaller in the case of turbulent flow than the laminar flow so now the condition that you have to apply to determine that the flow is fully developed okay now you can see from the shape of the velocity profile that if you plot this velocity profile somewhere downstream they are going to all look very similar so we have to introduce a mathematical criteria okay so what should be the mathematical criteria how do you hollow I say that I am in a fully developed region R exam yeah so you look at the velocity profile with the gradient with respect to the axial direction this is my axial direction X this is my radial direction and this is 0.

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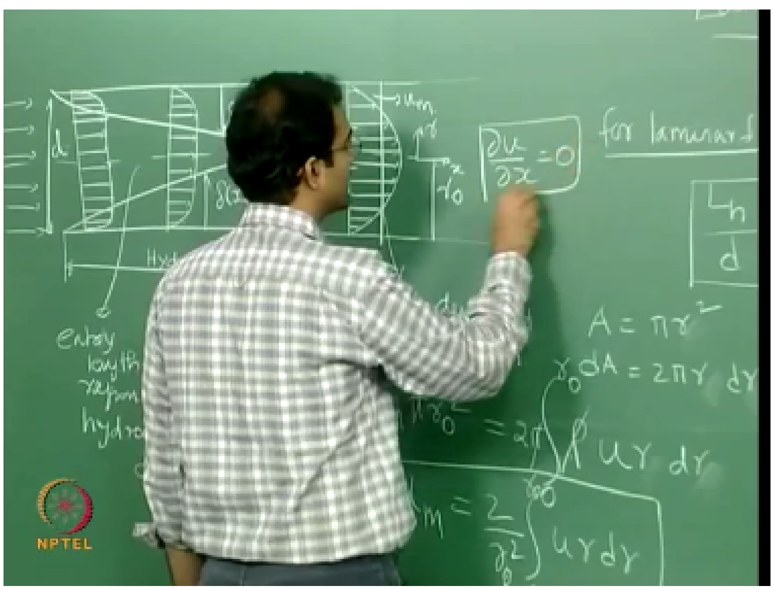

So this clearly tells me that I am in a fully developed region okay so this is the criteria as well as the flow is concerned okay now associated to the flow we can also have heat transfer now in the case of heat transfer suppose I take a case where I have prantle number > one okay so I have a growth suppose I apply either a uniform heat flux or a uniform temperature to the duct walls say there I say Q was in either of this case now associated to the boundary layer.

Growth of the velocity profile you can also have a thermal boundary layer growth now if my ∇ta prantle number is > 1 my ∇ T will be $< \nabla$ and therefore if you see the growth of the thermal boundary layer it will be smaller than the velocity boundary layer growth and therefore the point where they merged also will be slightly downstream the than the hydrodynamic and drilling okay so somewhere they will be merging here so this will be my ∇ TX now so similar to the hydrodynamic entrance length I can also define what is called as a thermal entry length.

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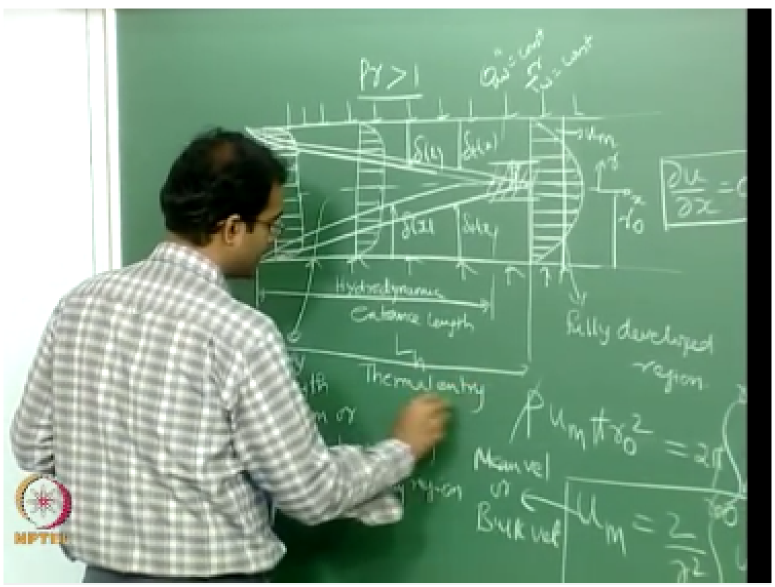

The point where the two thermal boundary layers must okay I can use the notation L subscript T to enter so say that the length corresponds to the thermal boundary layer and if I draw the profiles okay so typically if I draw a profile somewhere here so now the wall temperature is higher than your bulk temperature or the temperature of the fluid inside so it should it should look something like this right so the magnitude of your wall temperature will be higher than the magnitude of the temperature of the fluid at the center correct so this will be your temperature profile T as a function of Y.

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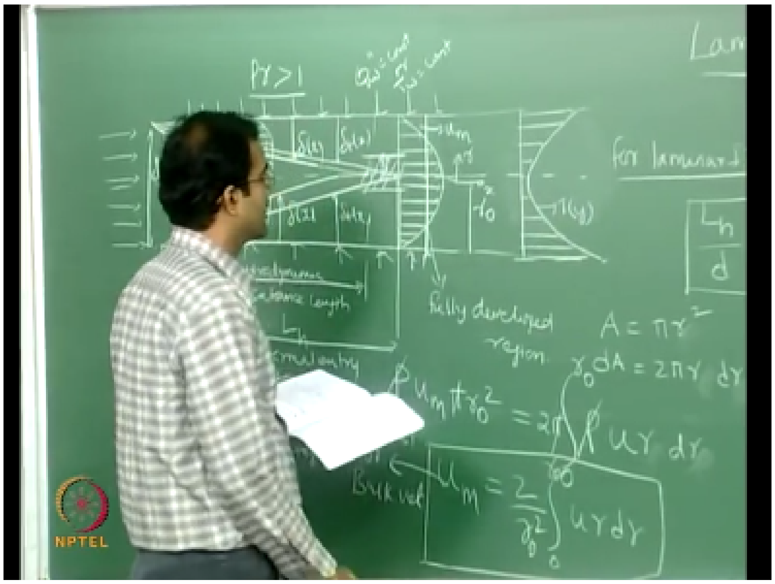

So now when you say that you have a hydro dynamically fully developed region and you have a hydrodynamic entry length similar to that you have a thermal entry length okay and you should also have a thermally fully developed region okay just analog is - your velocity profiles where the velocity profiles are invariant of the axial location however you can very clearly see when you apply heat transfer okay the for example if you apply a uniform heat flux what happens the wall temperature keeps on changing with respect to X the wall temperature will not be the same for example at the entrance and somewhere downstream because you are continuously adding Heat so therefore due to conduction.

So the wall temperature has to keep increasing downstream and also consequently the fluid also keeps getting heat continuously so the fluid temperature also has to vary along the X so therefore unlike the velocity profile which shows a constant behavior the temperature will never be constant value correct so you will find the temperature of the fluid anywhere that you plot will keep on changing with respect to X at any Y location or even at the wall that also keeps changing so how do we now look at a region and tell that that region corresponds to thermally fully developed okay so the criteria is we have to know develop a definition to call that thermally fully developed region okay so how do we now use that criteria okay so thermally fully developed.

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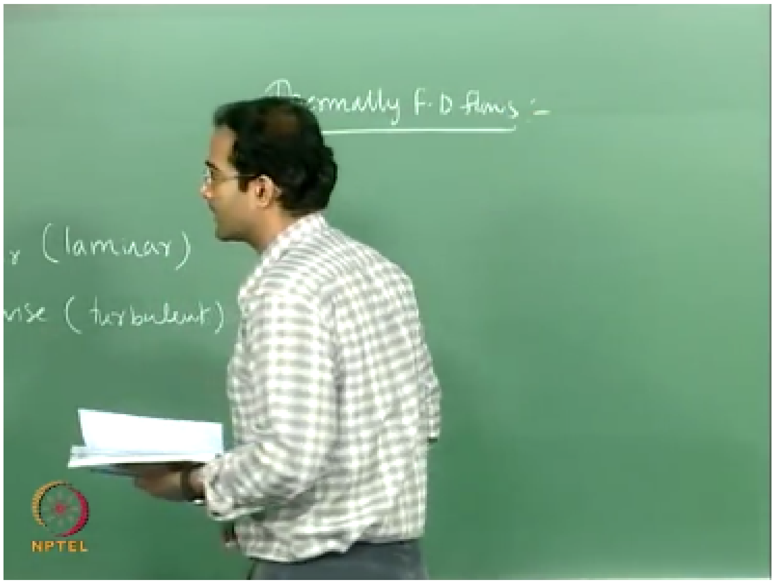

So I know that due to heat transfer my wall temperature is a function of X keeps changing my fluid temperature anywhere is also a function of both x and r okay now I have to same way similar to the way that I have defined what is called a mean velocity now I have to define another velocity which is something similar to a free stream velocity and external flow for internal flows okay so I will define what is called as a mean temperature so I need all of this to define a non-dimensional temperature.

Correct so I need to define another mean temperature now this mean temperature should be function of only X so therefore it has to be constant again something like this right so this should be my mean temperature the same way I define my mean velocity based on continuity principle I have to define the mean temperature based on what energy conservation so what I should say is that the enthalpy of this uniform temperature profile should be the same as the enthalpy of the actual profile that that you are getting okay so how do you now define enthalpy based on the temperature Cp Tm now this is the specific heat capacity so the total enthalpy will be you have to multiply by the mass flow rate now this is the this is the average enthalpy if you replace the temperature profile with the uniform profile.

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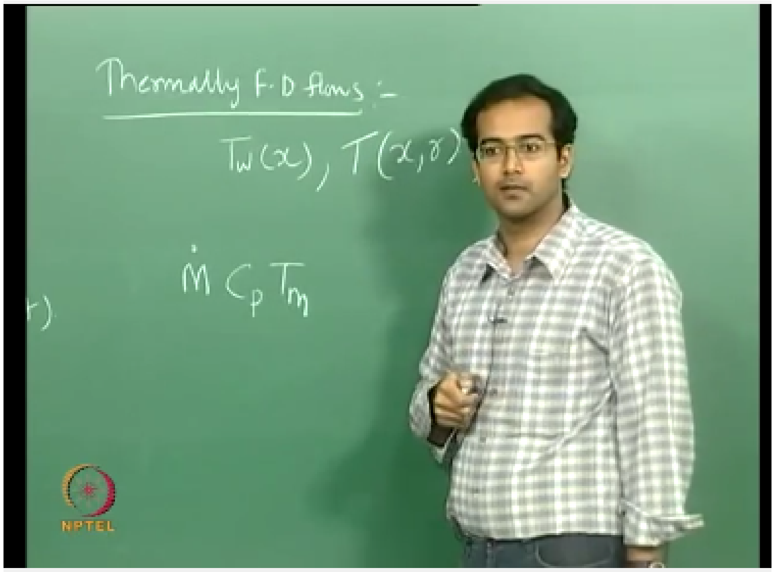

Correct now that should be equal to the enthalpy of this varying profile how do you how do you calculate the enthalpy of that integral 0 to R 0 so CP my MDOT now will be now there is a velocity profile also here please remember corresponding to this temperature profile there is a parabolic velocity profile so my MDOT there will be Du x you have already D a which is your 2 / x our Dr okay and of course you have your temperature x ∇ta enthalpy so this will this will be e x Rd R so this is the balance of enthalpy so this is satisfying enthalpy conservation.

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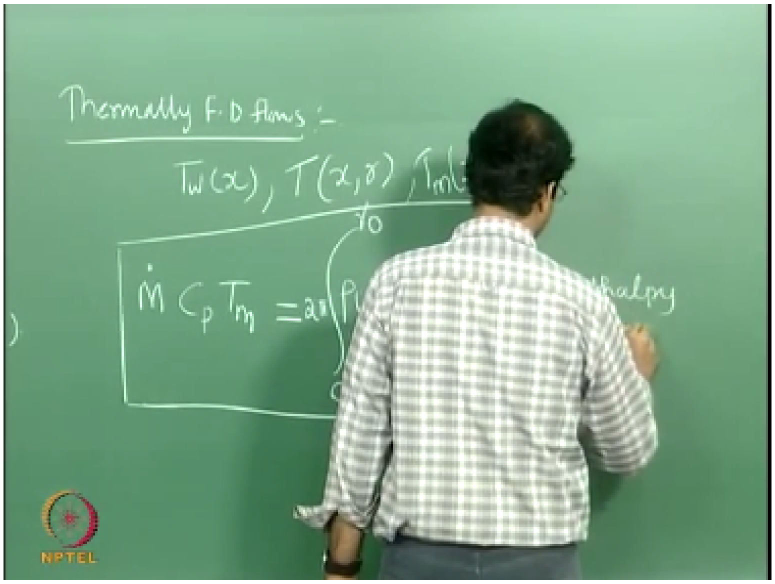

Okay so therefore for an incompressible flow with constant property you can so this is nothing but you are okay so let me just write it right now so I can knock up this and my TM will be 2π integral 0 to R 0 I can take my P out so T x u x Rd R ÷ so my flow rate there again will be a parabolic velocity profile so I can write this as integral 0 to R 0 x u x Rd R right so I will have a 2π x row again so this will get cancelled off.

So therefore this will be 0 2 or not you have P x u x Rd R ÷ integral 0 to R 0 u x R B so this is my definition of my mean temperature at any axial location mean temperature or sometimes people refer to this as bulk temperature sometimes they also refer to this as mixing up temperature so there are different names to the same mean temperature so this mean temperature is defined based on the enthalpy conservation the enthalpy if you replace you are varying temperature profile with a uniform temperature profile okay so therefore now we can see that we have defined all our necessary temperatures here.

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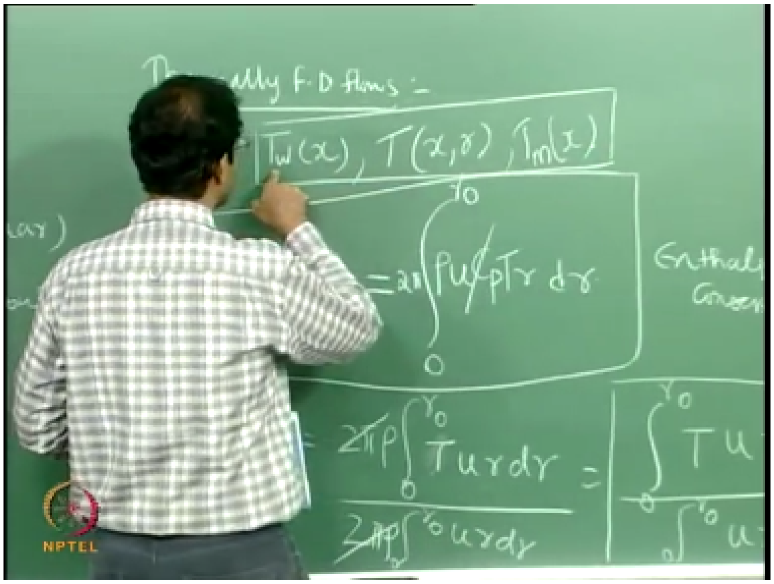

So you have T wall which is a function of X Pm. which is a function of X and you have e and therefore we can define a non-dimensional temperature Θ this similar to the external flows where we defined as t _ Tw/ T ∞ of a _ T w we can define T _ T wall so T is a function of both X and R ÷ Tw _ Tw x.

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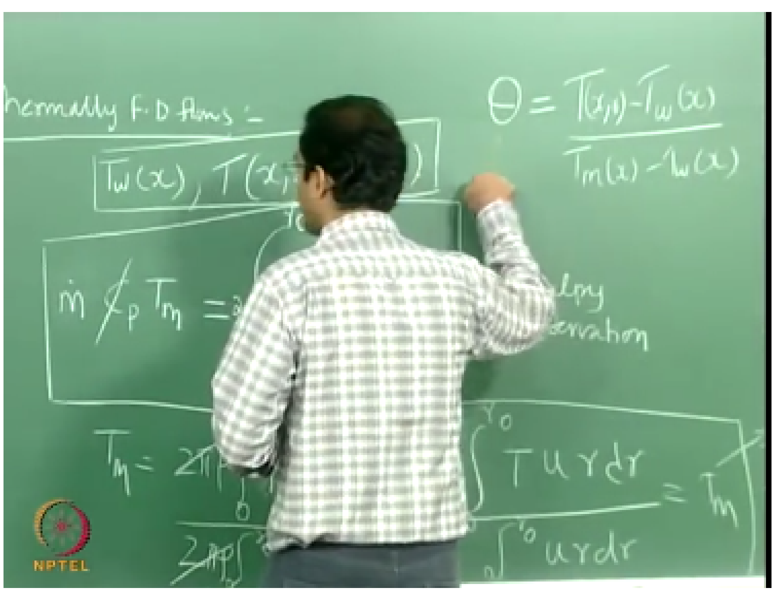

One okay so once you define a non-dimensional temperature profile like this now we can look at a region where this non-dimensional proper temperature profile is invariant of X because in that region your Tw _ Tw will be of particular function of X TM _ Tw will be a particular function of X such that the numerator and denominator are very in the same way so therefore there will not be any variation of Θ with respect to X so the condition for defining thermally fully-developed close will be d Θ by DX should be = 0 okay so once I identify that region where my D by DX is = 0 so then I can call that this is a thermally fully developed region.

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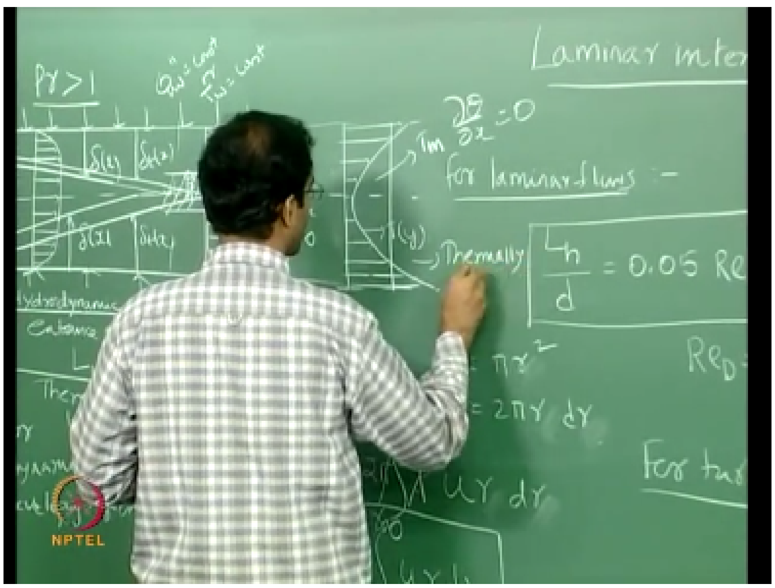

So I have to monitor my temperature profile Θ non-dimensional so if I plot the non-dimensional profile so although the dimensional profile will be something like this ok the non-dimensional profile will be something similar to the velocity profile okay so at wall it will become 0 right at the center it will be T me okay so therefore it becomes so it so it will be something like this ok so you have this is your dimensional profile and this is your non-dimensional this is your Θ and this is your now once you plot Θ okay so this Θ should be only a function of R.

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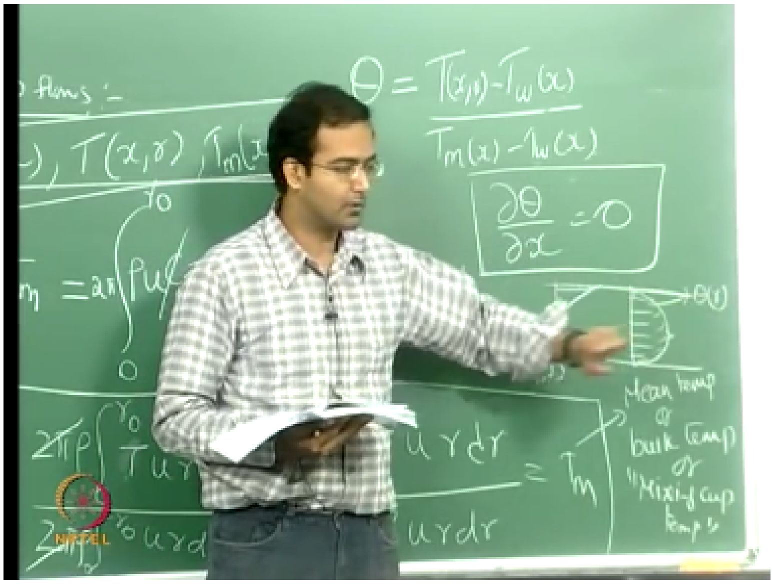

Now in this thermally fully developed region because it does not the non-dimensional temperature profile now should not vary with respect to X that is the condition which satisfies the thermally fully developed region ok so this should be a function of only our now based on this we can now define a criteria for the heat transfer coefficient in the fully developed region ok so therefore in the thermally fully developed region my Θ is a function of R only.

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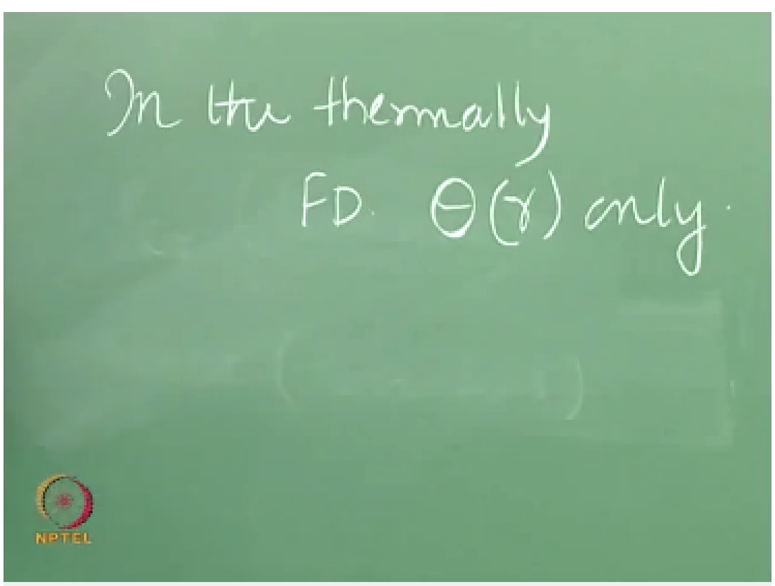

Whereas in the developing thermally developing region here thermal entrance length region my Θ is a function of both x enough okay but once it becomes thermally fully developed it does not become a function of R X it is a function of only our provider your velocity hydro dynamically is fully developed okay so this is a condition provided your hydro dynamically also fully develop okay so it should be hydro dynamically fully developed first and that is where we have taken a variation like this you have a parabolic velocity profile and on top of it if you if you put a condition that D Θ by DX is = 0 so then you are looking at a location where your Θ is a function of only R okay so for this particular region you can calculate your heat transfer coefficient as _ K DT by so now it is not Y anymore it is R and this will be at R = Y.

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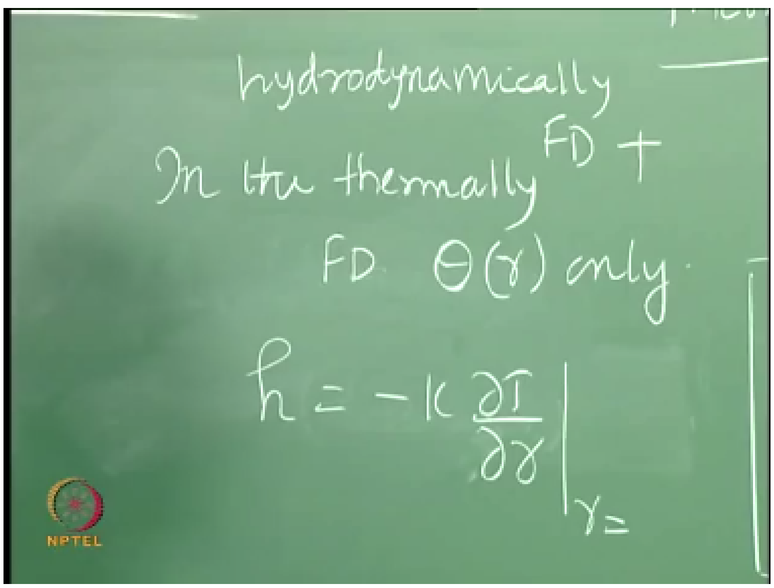

The heat flux should wall heat flux should be at R = R 0 okay now you have to be careful with the sign because the coordinate is going from the center it is not coming from the wall okay so therefore you see the temperature profile the temperature profile increases the temperature increases with increasing R so you do not have to put this negative sign here the gradient will be anyway positive okay so whereas if you have a coordinate from like this then the gradient will be negative so you have to put a negative sign okay so now your coordinate system is from the center of the duct so therefore you can just say this is K DT / D R at R = R 0

÷ (Tw- Tm).

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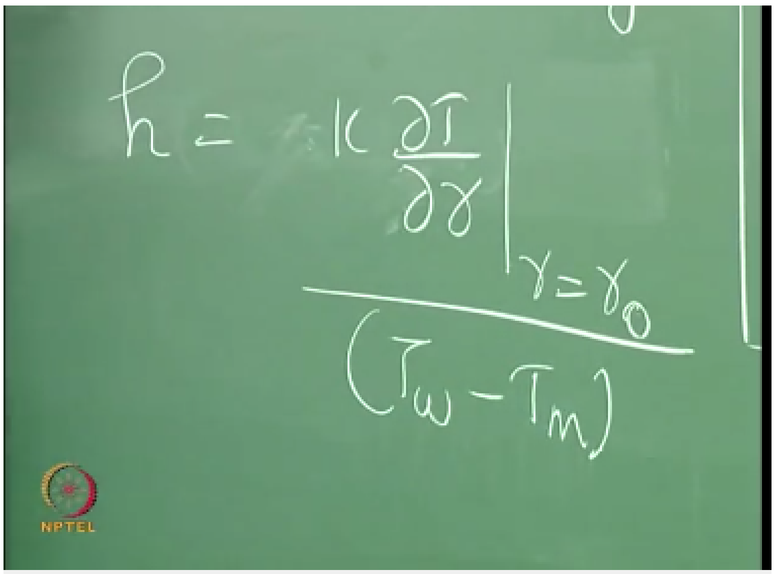

Me this is how you define your heat transfer coefficient for internal flows okay so now we can write this in terms of the non-dimensional temperature so I can write this as d by dr since this is a derivative with respect to R and Tw and T mean they are functions of only X okay so they can be directly taken inside this and you can also introduce T- Tw so T _ T w by Pm - 0 and you can put a negative sign I am just flipping this is t _ t1 okay so since T wall TM they are only functions of X I can just introduce.

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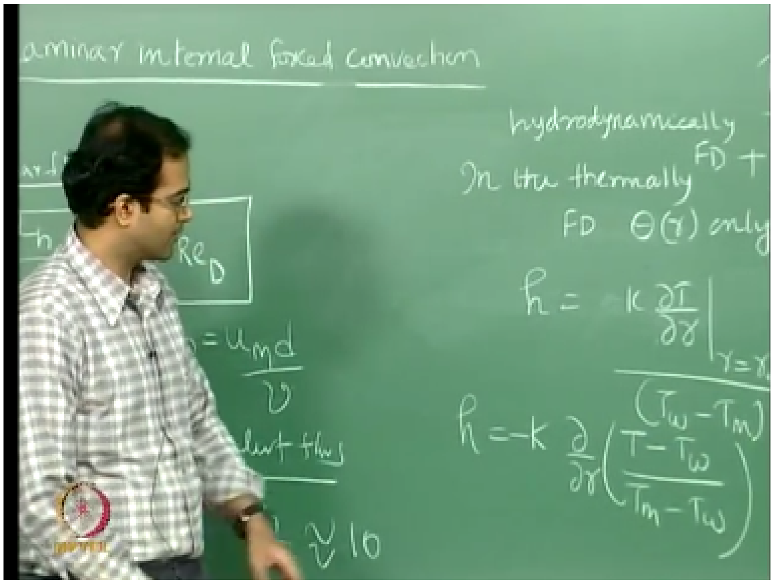

this as T_ Tw / TM _ Tw which is the same as this correct so this is nothing but Θ so this will be _ K D Θ /Dr now this is are R = R.

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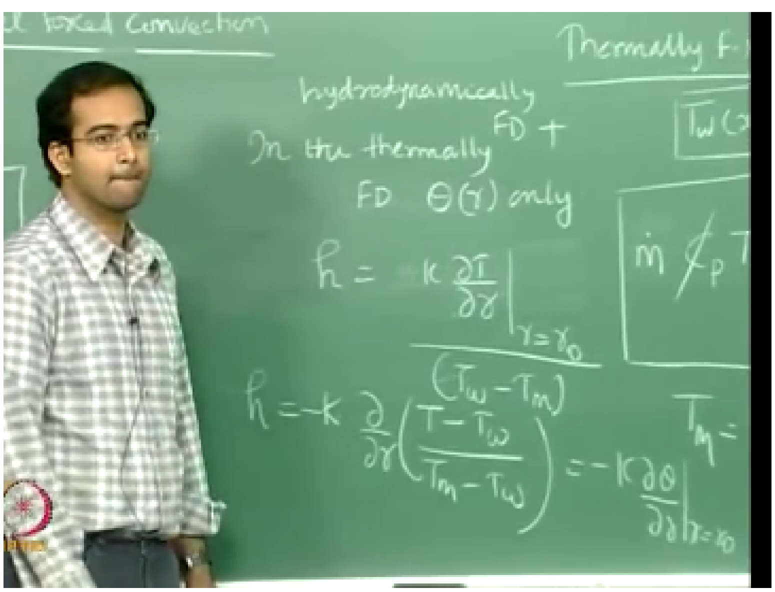

Now if you look carefully my Θ is a function of only R okay so therefore the slope of the profile at R = Rn and that is a slope at the w so that also has to be a function of only that is a fixed value the slope at any point will be a function of R at the at the w that will be a fixed value it now since Θ is not a function of X the slope also cannot be a function of X okay so whatever slope I calculate with respect to the profile whether it is here or if I use the non dimensional Θ so the slope I calculate will be the same because the profile is going to be the same the slope here.

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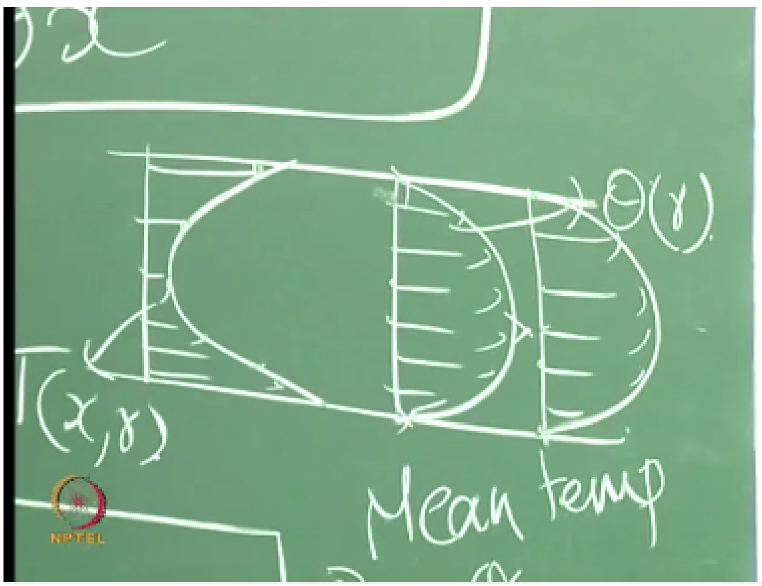

And here they ought to be identical so therefore so this D Θ by D R at R = R 0 has to be a constant value. So this tells me that my h is a constant so therefore it is not a function of X so this is a very important conclusion.

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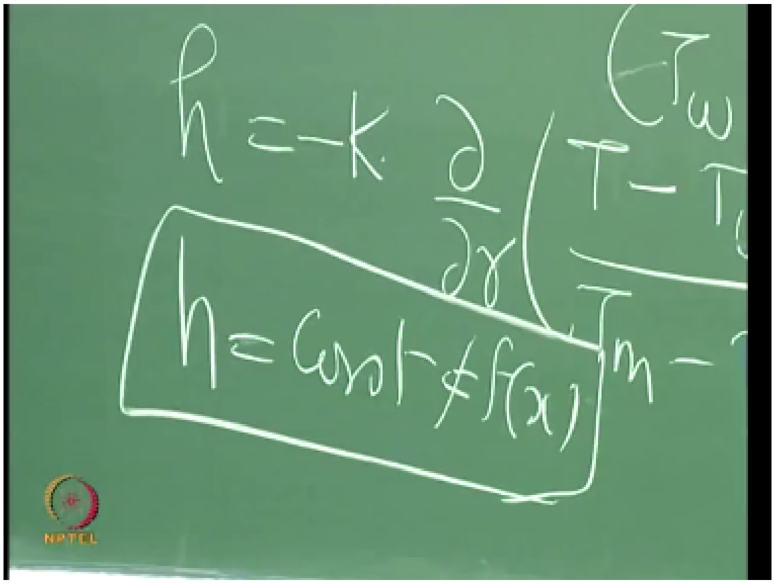

So as far as laminar internal flows both hydro dynamically and thermally fully developed so your heat transfer coefficient will become a constant it is not a function of X so that is going to simplify your correlations very drastically okay so this is a very important principle and this doesn't depend on the boundary condition that you employ whether it is a constant wall temperature are constant heat flux irrespective of that we have not used any boundary conditions here irrespective of the boundary condition this is the fact that your heat transfer coefficient is a constant for laminar internally fully developed flows.

Okay so with that this kind of gives you a brief overview what we are planning to do and in this particular the next 9 hours or so we will focus on three regions so first we will look at what is called region III. okay so let me identify this is region I write here where your velocity hydro dynamically is growing and also your thermal boundary layer is growing this is your region II where your velocity hydro dynamically has developed but your thermal development is still underway okay now this is your region III where both the boundary layers have merged and both the hydrodynamic and thermal boundary layers are fully developed okay so therefore the first part of the course will be on region III which is fully developed both hydro dynamically and thermal.

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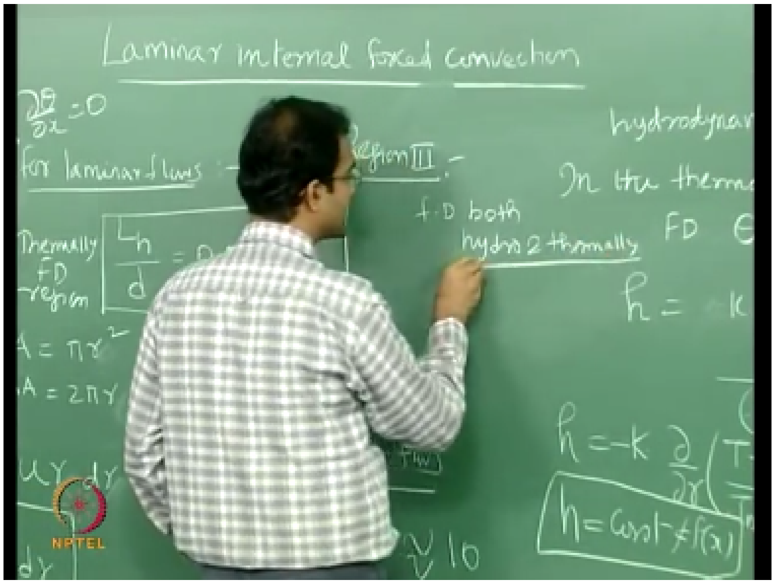

And as you can see that is the simplest to start with because you have constant heat transfer coefficient so you can calculate the probe elastic profile the temperature file and for a given boundary condition whether it is a heat flux or a wall temperature you can identify what is that constant value of hits okay so then slightly more complicated as region II where you have fully developed hydro dynamically and thermal entry length.

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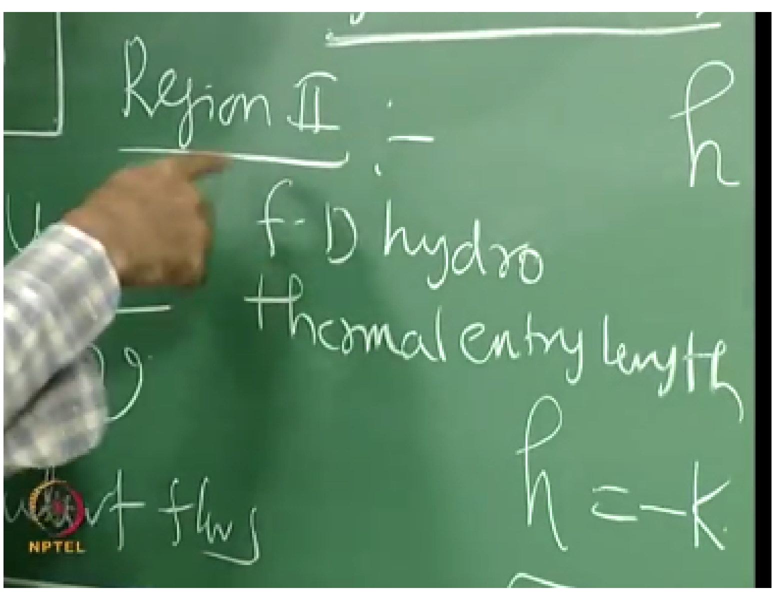

So here your velocity profile is fully develop profile but the equation that you are solving does not have that condition D Θ by DX 0 okay and finally the most complicated regions region number 1 where you have both hydrodynamic and thermal and filling so this region you cannot get good close form analytical solution you have to solve the complete nervier-stokes equation because you cannot neglect any terms the gradient of velocity is not 0 the gradient of non-dimensional temperature is not 0 so therefore all the terms in the nervier-stokes equation has to be present you cannot neglect any of the terms and therefore you have to do a complete numerical solution for the region I okay so in this particular course we will be focusing on region III and then lot of problems related to region II.

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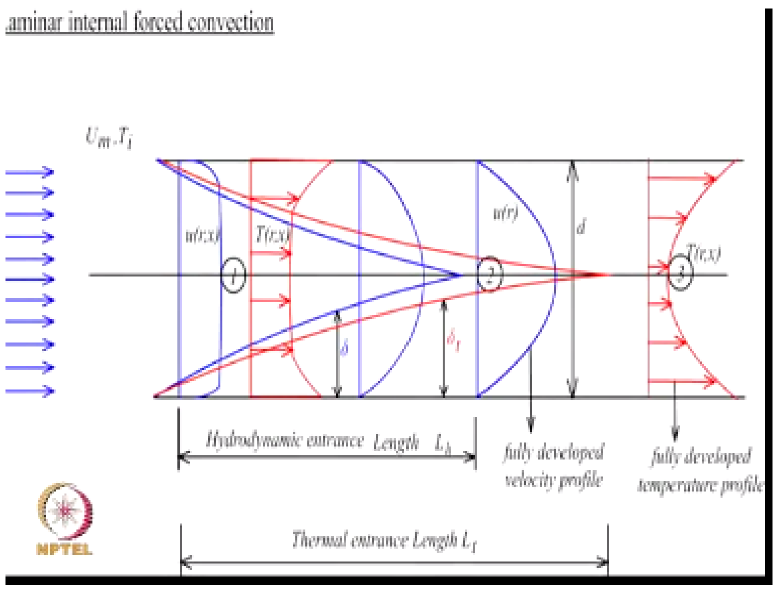

Okay so we will stop here and tomorrow we will look at we will start of course with region III we will look at the solution to first the hydrodynamic fully developed condition get the velocity profiles and then the depending on the thermal boundary condition we'll also get the fully developed temperature profiles.

Laminar internal forced convection – Fundamentals

End of Lecture 25

Next: Hydro dynamically and thermally fully

Developed internal laminar flows

Online Video Editing / Post Production

M. Karthikeyan
M.V. Ramachandran

P.Baskar

Camera
G.Ramesh
K. Athaullah

K.R. Mahendrababu
K. Vidhya
S. Pradeepa
Soju Francis
S.Subash
Selvam
Sridharan

Studio Assistants
Linuselvan
Krishnakumar
A.Saravanan

Additional Post –Production

Kannan Krishnamurty & Team

Animations
Dvijavanthi

NPTEL Web & Faculty Assistance Team

Allen Jacob Dinesh
Ashok Kumar
Banu. P
Deepa Venkatraman
Dinesh Babu. K .M
Karthikeyan .A

Lavanya . K
Manikandan. A

Manikandasivam. G
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Sivakumar. S
Soundhar Raja Pandain.R
Suman Dominic.J
Udayakumar. C
Vijaya. K.R
Vijayalakshmi
Vinolin Antony Joans
Adiministrative Assistant
K.S Janakiraman
Prinicipal Project Officer
Usha Nagarajan

Video Producers
K.R.Ravindranath
Kannan Krishnamurty

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